A

Q.13 The electric fields of two light sources with nearby frequencies ω_1 and ω_2, and wave vectors k_1 and k_2, are expressed as $\vec{E}_1 = E_{10}\hat{i}\, e^{-i(k_1 z-\omega_1 t)}$ and $\vec{E}_2 = E_{20}\hat{i}\, e^{-i(k_2 z-\omega_2 t)}$, respectively. The interference pattern on the screen is photographed at $t = t_0$; denote $(k_1 - k_2)z - (\omega_1 - \omega_2)t_0$ by θ. For this pattern

(A) a bright fringe will be obtained for $\cos\theta = -1$

(B) a bright fringe intensity is given by $(E_{10})^2 + (E_{20})^2$

(C) a dark fringe will be obtained for $\cos\theta = 1$

(D) a dark fringe intensity is given by $(E_{10} - E_{20})^2$

Q.14 A solid metallic cube of heat capacity S is at temperature 300 K. It is brought in contact with a reservoir at 600 K. If the heat transfer takes place only between the reservoir and the cube, the entropy change of the universe after reaching the thermal equilibrium is

(A) 0.69 S (B) 0.54 S (C) 0.27 S (D) 0.19 S

Q.15 If the surface integral of the field $\vec{A}(x, y, z) = 2\alpha x\hat{i} + \beta y\hat{j} - 3\gamma z\hat{k}$ over the closed surface of an arbitrary unit sphere is to be zero, then the relationship between α, β and γ is

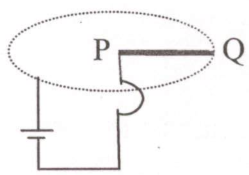
(A) $\alpha + \beta/6 - \gamma = 0$ (B) $\alpha/3 + \beta/6 - \gamma/2 = 0$

(C) $\alpha/2 + \beta - \gamma/3 = 0$ (D) $2/\alpha + 1/\beta - 3/\gamma = 0$

Q.16 The moment of inertia of a disc about one of its diameters is I_M. The mass per unit area of the disc is proportional to the distance from its centre. If the radius of the disc is R and its mass is M, the value of I_M is

(A) $\frac{1}{2}MR^2$ (B) $\frac{2}{5}MR^2$ (C) $\frac{3}{10}MR^2$ (D) $\frac{3}{5}MR^2$

Q.17 A rigid uniform horizontal wire PQ of mass M, pivoted at P, carries a constant current I. It rotates with a constant angular speed in a uniform vertical magnetic field B. If the current were switched off, the angular acceleration of the wire, in terms of B, M and I would be



(A) 0 (B) $\frac{2BI}{3M}$ (C) $\frac{3BI}{2M}$ (D) $\frac{BI}{M}$

Q.18 Two points N and S are located in the northern and southern hemisphere, respectively, on the same longitude. Projectiles P and Q are fired from N and S, respectively, towards each other. Which of the following options is correct for the projectiles as they approach the equator?

(A) Both P and Q will move towards the east

(B) Both P and Q will move towards the west

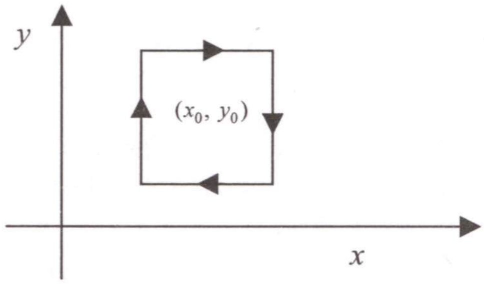
(C) P will move towards the east and Q towards the west

(D) P will move towards the west and Q towards the east

Q.19 Two particles A and B of mass m and one particle C of mass M are kept on the x axis in the order ABC. Particle A is given a velocity $v\hat{i}$. Consequently there are two collisions, both of which are completely inelastic. If the net energy loss because of these collisions is $\frac{7}{8}$ of the initial energy, the value of M is (ignore frictional losses)

(A) $8\,m$ (B) $6\,m$ (C) $4\,m$ (D) $2\,m$

Q.20 The line integral $\oint \vec{A}\cdot d\vec{l}$ of a vector field $\vec{A}(x, y) = \frac{1}{r^2}(-y\hat{i} + x\hat{j})$, where $r^2 = x^2 + y^2$, is taken around a square (see figure) of side unit length and centered at (x_0, y_0) with $|x_0| > \frac{1}{2}$ and $|y_0| > \frac{1}{2}$. If the value of the integral is L, then



(A) L depends on (x_0, y_0)

(B) L is independent of (x_0, y_0) and its value is -1

(C) L is independent of (x_0, y_0) and its value is 0

(D) L is independent of (x_0, y_0) and its value is 2

Q.21 Diamond lattice can be considered as a combination of two fcc lattices displaced along the body diagonal by one quarter of its length. There are eight atoms per unit cell. The packing fraction of the diamond structure is

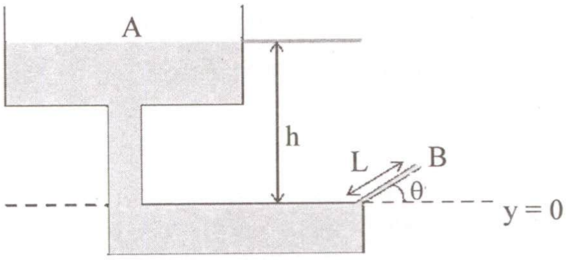
(A) 0.48 (B) 0.74 (C) 0.34 (D) 0.68

A

Q.22 Thermal neutrons (energy = 300 k_B = 0.025 eV) are sometimes used for structural determination of materials. The typical lattice spacing of a material for which these can be used is

(A) 0.01 nm (B) 0.05 nm (C) 0.1 nm (D) 0.15 nm

Q.23 What is the maximum height above the dashed line attained by the water stream coming out at B from a thin tube of the water tank assembly shown in the figure? Assume h = 10 m, L = 2 m, and $\theta = 30^\circ$.



(A) 10 m (B) 2 m (C) 1.2 m (D) 3.2 m

Q.24 A steady current in a straight conducting wire produces a surface charge on it. Let E_{out} and E_{in} be the magnitudes of the electric fields just outside and just inside the wire, respectively. Which of the following statements is true for these fields?

(A) E_{out} is always greater than E_{in}

(B) E_{out} is always smaller than E_{in}

(C) E_{out} could be greater or smaller than E_{in}

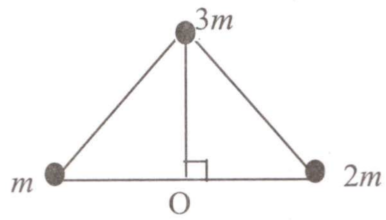
(D) E_{out} is equal to E_{in}

Q.25 A small charged spherical shell of radius 0.01 m is at a potential of 30 V. The electrostatic energy of the shell is

(A) 10^{-10} J (B) 5×10^{-10} J (C) 5×10^{-9} J (D) 10^{-9} J

A

Q.26 At an instant shown, three point masses m, $2m$ and $3m$ rest on a horizontal surface, and are at the vertices of an equilateral triangle of unit side length. Assuming that G is the gravitational constant, the magnitude and direction of the torque on the mass $3m$, about the point O, at that instant is



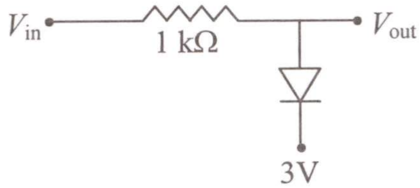
(A) Zero

(B) $\frac{3}{2}G\sqrt{3}m^2$, going into the paper

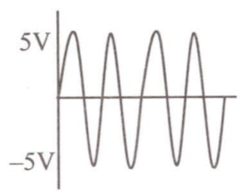
(C) $3G\sqrt{3}m^2$, coming out of the paper

(D) $\frac{3}{4}G\sqrt{3}m^2$, going into the paper

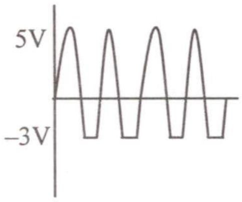
Q.27 A sine wave of 5 V amplitude is applied at the input of the circuit shown in the figure. Which of the following waveforms represents the output most closely?



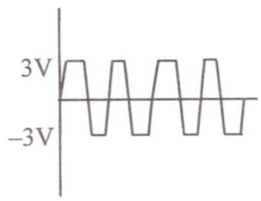
(A)



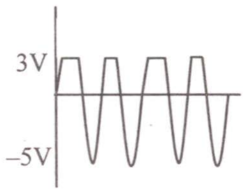
(B)



(C)

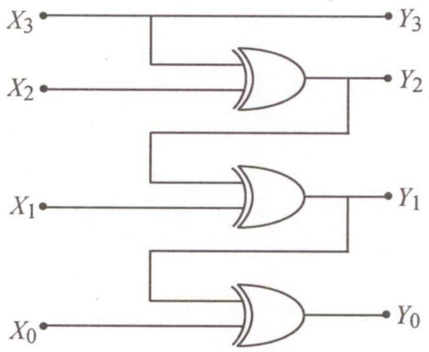


(D)



A

Q.28 1011 binary input have been applied at $X_3\,X_2\,X_1\,X_0$ input in the shown logic circuit made of *XOR* gates. The binary output $Y_3\,Y_2\,Y_1\,Y_0$ of the circuit will be



(A) 1101 (B) 1010 (C) 1111 (D) 0101

Q.29 A ring of radius R carries a linear charge density λ. It is rotating with angular speed ω. The magnetic field at its center is

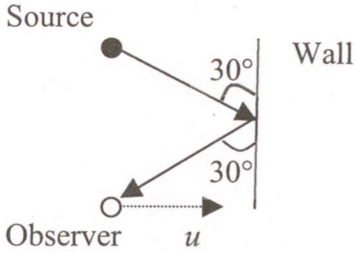
(A) $\frac{3\mu_0\lambda\omega}{2}$

(B) $\frac{\mu_0\lambda\omega}{2}$

(C) $\frac{\mu_0\lambda\omega}{\pi}$

(D) $\mu_0\lambda\omega$

Q.30 A stationary source (see figure below) emits sound waves of frequency f towards a wall. If an observer moving with speed u in a direction perpendicular to the wall, measures a frequency $f' = \frac{9}{8}f$ at the instant shown, then u is related to the speed of sound V_s as



(A) V_s (B) $V_s/2$ (C) $V_s/4$ (D) $V_s/8$

Q.31 A real gas has specific volume v at temperature T. Its coefficient of volume expansion and isothermal compressibility are α and k_T, respectively. Its molar specific heat at constant pressure C_p and molar specific heat at constant volume C_v are related as

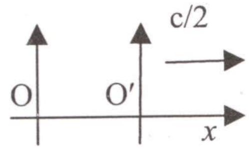
(A) $C_p = C_v + R$

(B) $C_p = C_v + \frac{Tv\alpha}{k_T}$

(C) $C_p = C_v + \frac{Tv\alpha^2}{k_T}$

(D) $C_p = C_v$

Q.32 Two frames, O and O′, are in relative motion as shown. O′ is moving with speed c/2, where c is the speed of light. In frame O, two separate events occur at (x_1, t_1) and (x_2, t_2). In frame O′, these events occur simultaneously. The value of $(x_2 - x_1)/(t_2 - t_1)$ is



(A) c/4 (B) c/2 (C) 2c (D) c

Q.33 White light is incident on a grating G_1 with groove density 600 lines/mm and width 50 mm. A small portion of the diffracted light is incident on another grating G_2 with groove density 1800 lines/mm and width 15 mm. The resolving power of the combined system is

(A) 3×10^3 (B) 57×10^3 (C) 81×10^7 (D) 108×10^5

Q.34 Four particles of mass m each are inside a two dimensional square box of side L. If each state obtained from the solution of the Schrodinger equation is occupied by only one particle, the minimum energy of the system in units of $\frac{h^2}{mL^2}$ is

(A) 2 (B) $\frac{5}{2}$ (C) $\frac{11}{2}$ (D) $\frac{25}{4}$

Q.35 At atmospheric pressure ($=10^5$ Pa), aluminium melts at 550 K. As it melts, its density decreases from 3×10^3 kg/m^3 to 2.9×10^3 kg/m^3. Latent heat of fusion of aluminium is 24×10^3 J/kg. The melting point of aluminium at a pressure of 10^7 Pa is closest to

(A) 551.3 K (B) 552.6 K (C) 558.7 K (D) 547.4 K

A

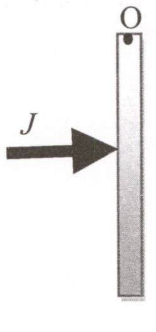
PART - II: Descriptive Questions

Q. 36 – Q. 43 carry five marks each.

Q.36 Find the solution of the differential equation $\frac{d^2y}{dx^2}+5\frac{dy}{dx}=0$ with the boundary condition $y(0)=2$ and $\left.\frac{dy}{dx}\right|_{x=0}=2$, giving all steps clearly. Find the value of x where $y=0$.

Q.37 The electric field in an electromagnetic (EM) wave is $\vec{E} = \sqrt{6\pi}\,\hat{i}\sin[2\pi(10^6 z - 3\times 10^{14} t)]$. What is the intensity of the EM wave and the number of photons per second falling on the unit area of a perfectly reflecting screen kept perpendicular to the direction of propagation? When a photon in this beam is reflected from the screen, what is the impulse it imparts to the screen? Use this to find the pressure exerted by the EM wave on the screen.

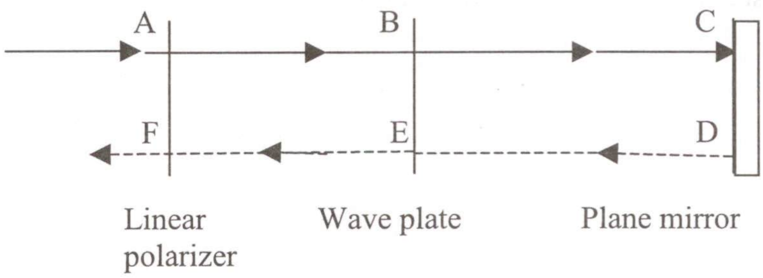
Q.38 A uniform rod of mass m and length l is hinged at one of its ends O and is hanging vertically. It is hit at its midpoint with a very short duration impulse J so that it starts rotating about O. Find the magnitude and direction of the horizontal impulse that O applies on the rod when it is hit.



A

Q.39 **An easy derivation of PV^γ = constant for an ideal gas undergoing an adiabatic process :** Consider P and V as the basic variables of an ideal gas and write the heat exchanged dQ in terms of dV and dP. Next, using the definition of C_P and C_V in the expression for dQ, obtain a differential equation relating P and V for an adiabatic process and solve it to get the desired relationship. Derivation SHOULD NOT use the first law of thermodynamics. [For a function $f(x, y)$ the differential $df = \left(\frac{\partial f}{\partial x}\right)_y dx + \left(\frac{\partial f}{\partial y}\right)_x dy$].

Q.40 As shown in the figure below, an unpolarised beam of light of wavelength 500 nm is incident on a linear polariser at AF with vertical polarisation. The light beam then passes through a wave plate BE (half wave or quarter wave plate) of thickness 1.00125 mm and gets reflected from a mirror CD. The reflected light is indicated by the dashed line (DEF) in the diagram. The ordinary and extraordinary refractive indices for the material of the wave plate are 1.658 and 1.558, respectively. Light is incident normally on all surfaces.

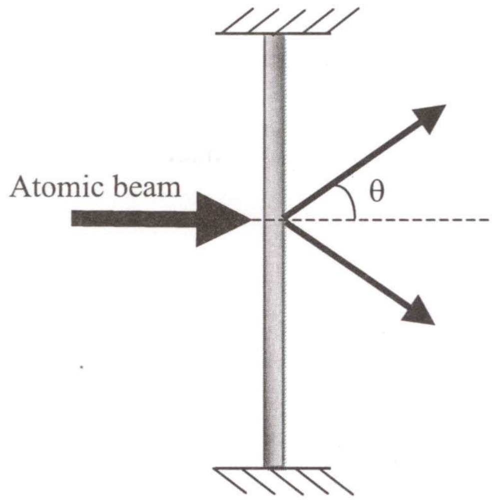


(a) What is the polarisation of the beam at C?

(b) What is the polarisation of the beam at E and F?

A

Q.41 A standing wave of light is formed between two mirrors and a beam of atoms is incident on it normally (see figure below) from the left. On the right side, atoms are detected in the direction of the beam and also at an angle θ as shown in the figure. This is due to material waves of atoms diffracted by the standing wave that acts like a grating; the slit width of this grating is given by the distance between two maxima of the light intensity. If the atomic beam is made of atoms of mass m moving with speed v and the light wave has wavelength λ_L, find the smallest angle θ by using the diffraction condition.



Q.42 According to Wien's theory of black body radiation, the spectral energy density in a blackbody cavity at temperature T is given as

$$u_T(\lambda)\,d\lambda = \frac{\alpha}{c^3\lambda^5} e^{-\beta/\lambda T}\,d\lambda$$

where α and β are constants and c is the speed of light. Further, the intensity of radiation coming out of the cavity is $\frac{u_T c}{4}$, where $u_T = \int_0^\infty u_T(\lambda)\,d\lambda$ is the total energy density of radiation. Given that Stefan-Boltzmann constant $\sigma = 5.67\times10^{-8}$ Wm^{-2}K^{-4} and $\lambda_{\max}T = 2.90\times10^{-3}$ m.K, find the values of α and β. The value of integral $\int_0^\infty x^3 e^{-x}\,dx = 6$.

Q.43 A horizontal rod of proper length L moves with uniform speed $V > 0$ along the x-axis of a coordinate frame. A ground observer measures the position coordinates of its two ends at two different times, with time difference $\Delta t > 0$. The observer finds that the difference between the two coordinates is L. Calculate Δt in terms of L, V and the speed of light c. If measured correctly, what would have been the length of the rod in the ground frame?

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36		
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Signature of Examiner(s)	:	
Signature of Head Examiner(s)	:	
Signature of Scrutinizer	:	
Signature of Chief Scrutinizer	:	
Signature of Coordinating Head Examiner	:	

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