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PAPER 1

CODE : 7

Time : 3 Hours **Maximum Marks : 210**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. This booklet is your Question Paper. Do not break the seals of this booklet before being instructed to do so by the invigilators.
2. The **question paper CODE** is printed on the right hand top corner of this page and on the back page (page No. 28) of this booklet.
3. Blank spaces and blank pages are provided in this booklet for your rough work. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets are NOT allowed inside the examination hall.
5. Answers to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. You should not separate these parts. The invigilator will separate them at the end of examination. The upper sheet is a machine-gradable Objective Response Sheet (ORS) which will be taken back by the invigilator. You will be allowed to take away the bottom sheet at the end of the examination.
6. **Using a black ball point pen, darken the bubbles on the upper original sheet.** Apply sufficient pressure so that the impression is created on the bottom sheet.
7. DO NOT TAMPER WITH/MUTILATE THE ORS OR THE BOOKLET.
8. On breaking the seals of the booklet check that it contains **28** pages and all the **60** questions and corresponding answer choices are legible. Read carefully the instructions printed at the beginning of each section.

B. Filling the Right Part of the ORS :

9. The ORS has **CODES** printed on its Left and Right parts.
10. Check that the same CODE is printed on the ORS and this booklet. **IF IT IS NOT THEM ASK FOR A CHANGE OF THE BOOKLET**. Sign at the place provided on the ORS affirming that you have verified that all the codes are same.
11. Write your name, Registration number and the name of the Examination centre and sign with pen in the boxes provided on the right part of the ORS. **Do not write any of this information anywhere else.** Darken the appropriate bubbles **UNDER** each digit of your Registration No. in such a way that the impression is created on the bottom sheet. Also darken the paper code given on the right side of ORS (R4).

C. Question Paper Format

The question paper consists of **3 parts** (Physics, Chemistry and Mathematics). Each part consists of **three sections.**

12. **Section I** contains **10 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.
13. **Section II** contains **5 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONE OR MORE** are correct.
14. **Section III** contains **5 questions.** The answer to each question is a **single digit integer**, ranging from **0 to 9** (both inclusive).

D. Marking scheme

15. For each question in **section I** you will be awarded **3 marks** if you darken the bubble corresponding to the correct answer **ONLY** and **zero marks** if no bubbles are darkened. In all other cases, **minus one (-1)** mark will be awarded in this section.
16. For each question in **Section II** you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY**. In all other cases **zero (0) marks** will be awarded. **No negative marks** will be awarded for incorrect answer in this section.
17. For each question in **Section III** you will be awarded **4 marks** if you darken the bubble corresponding to the correct answer **ONLY**. In all other cases **zero (0) marks** will be awarded. **No negative marks** will be awarded for incorrect answer in this section.

DO NOT BREAK THE SEALS WITHOUT BEING INSTRUCTED TO DO SO BY THE INVIGILATOR

Write your Name, registration number and sign in the space provided on the back page of this booklet

PART I : PHYSICS

SECTION I : Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A) , (B), (C) and (D) out of which **ONLY ONE** is correct.

1. Young's double slit experimental is carried out by using green, red and blue light, one color at a time. The fringe widths recorded are β_G, β_R and β_B, respectively. Then,

(A) $\beta_G > \beta_B > \beta_R$ (B) $\beta_B > \beta_G > \beta_R$ (C) $\beta_R > \beta_B > \beta_G$ (D) $\beta_R > \beta_G > \beta_B$

1. (D)

Fringe width is given by

$$B = \lambda\frac{D}{d}$$

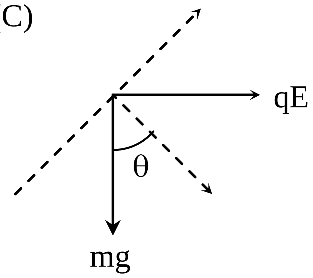
as $\lambda_R > \lambda_G > \lambda_B$

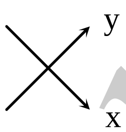
$\therefore$ $\beta_R > \beta_G > \beta_B$

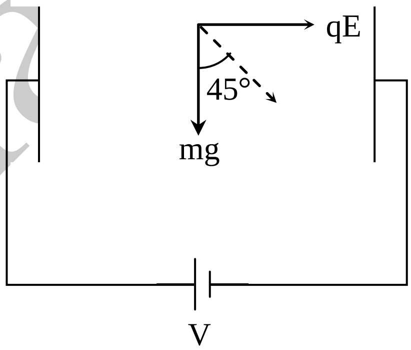
2. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45º to the vertical JUST after release. Then X is nearly

(A) 1×10^{-5} V (B) 1×10^{-7} V (C) 1×10^{-9} V (D) 1×10^{-10} V

2. (C)







Since the angle θ = 45°

$\therefore$ $mg = qE$

$$q\frac{V}{d} = mg$$

$$V = \frac{mg\,d}{q} = \frac{1.6\times10^{-27}\times10\times10^{-2}}{1.6\times10^{-19}} = 1 \times 10^{-9}\text{ Volts}$$

3. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the r m s speeds $\left(\frac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})}\right)$ is

(A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

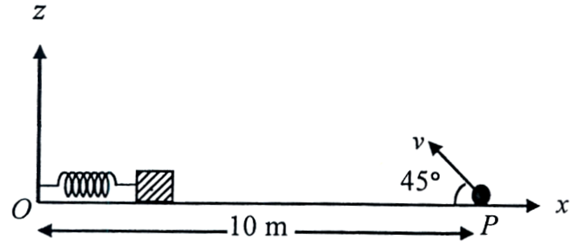
3. (D)

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\frac{(v_{He})_{rms}}{(v_{Ar})_{rms}} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = \sqrt{10}$$

$$\frac{(v_{He})_{rms}}{(v_{Ar})_{rms}} = 3.16$$

4. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at t = 0, a small pebble is projected with speed v from point P at an angle of 45º as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at t = 1 s, the value of v is (take g = 10 m/s^2)



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s (C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

4. (A)
Motion of block is given by

$$(x - 4.8) = (0.2)\cos\left(\frac{\pi t}{3}\right)$$

at t = 1 second

$$(x - 4.9) = (0.2) \times \cos\frac{\pi}{3}$$
$$= 0.1$$
$$\Rightarrow \quad x = 5.0$$

$\therefore$ Range of projectile must be

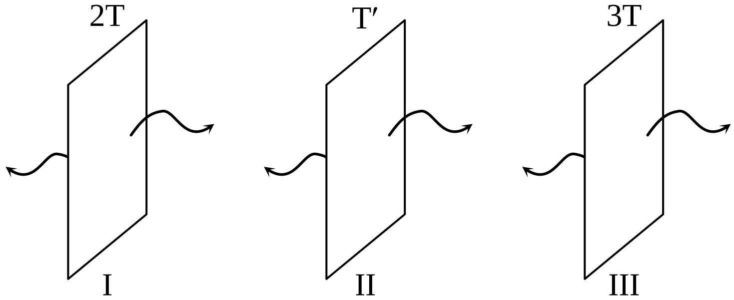
$$= 10\text{m} - 5\text{m} = 5\text{m} = \frac{u^2}{g}$$

$$\therefore \quad u = \sqrt{5g} = \sqrt{50}\ \text{ms}^{-1}$$

5. Three very large plates of same are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures 2T and 3T respectively. The temperature of the middle (i.e. second) plate under steady state condition is

(A) $\left(\frac{65}{2}\right)^{\frac{1}{4}}T$ (B) $\left(\frac{97}{4}\right)^{\frac{1}{4}}T$ (C) $\left(\frac{97}{2}\right)^{\frac{1}{4}}T$ (D) $97^{\frac{1}{4}}T$

5. (C)

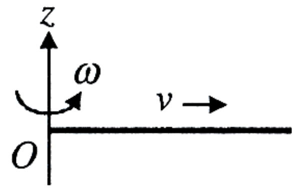


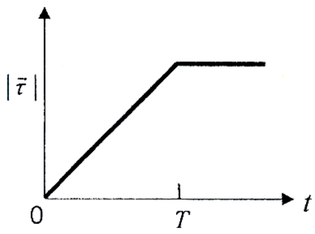
Consider unit area of each plate
II radiates at a rate $(2\sigma T'^4)$
Amount of radiation falling on plate II = $\sigma\,(2T)^4 + \sigma\,(3T)^4$
By steady state condition :

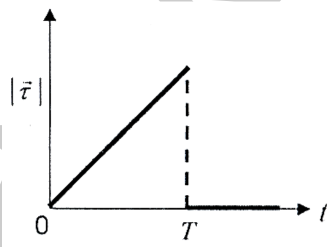
$$\sigma T'^4 = 8\sigma T^4 + \frac{81}{2}\sigma T^4$$

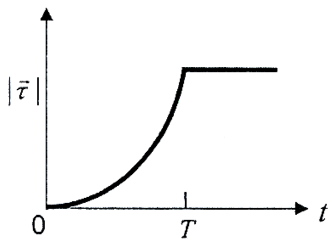
$$\Rightarrow \quad T'^4 = \frac{97}{2}T^4 \quad \Rightarrow \quad T'^4 = \left(\frac{97}{2}\right)^{1/4} \cdot T$$

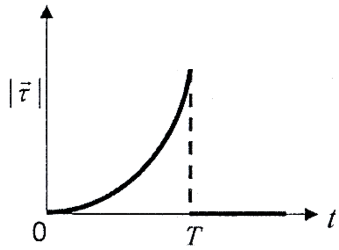
6. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time t = 0, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at t = T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $|\vec{\tau}|$ on the system about O, as a function of time is best represented by which plot?



(A) 

(B) 

(C) 

(D) 

6. (B)

$$\tau = \frac{dL}{dt} \quad \text{and} \quad L = I\omega_0$$

$$\therefore \quad \tau = \left(\frac{dI}{dt}\right)\omega_0 \qquad [\omega_0 = \text{constant}]$$

at time t, radial distance of insect from O = vt

$$\therefore \quad I = \frac{1}{3}m_r l^2 + m_i (vt)^2$$

$$\therefore \quad \frac{dI}{dt} \propto t$$

$$\Rightarrow \tau \propto t$$

When insect stop;

$$\frac{dI}{dt} = 0$$

7. In the determination of Young's modules $\left(Y = \dfrac{4MLg}{\pi \ell d^2}\right)$ by using Searle's method, a wire of length L = 2 m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension $\ell = 0.25$ mm in the length of the wire is observed. Quantities d and ℓ are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

(A) due to the errors in the measurements of d and ℓ are the same

(B) due to the error in the measurement of d is twice that due to the error in the measurement of ℓ.

(C) due to the error in the measurement of ℓ is twice that due to the error in the measurement of d.

(D) due to the error in the measurement of d is four times that due to the error in the measurement of ℓ.

7. (A)

$$Y = \frac{4MLg}{\pi \ell d^2}; \quad \frac{\Delta Y}{Y} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta d}{d}$$

Contribution of $\ell \rightarrow \dfrac{\Delta \ell}{\ell}$. Contribution of d $\rightarrow 2\dfrac{\Delta d}{d}$

$$\text{L.C.} = \left(\frac{0.5}{100}\right)$$

$$\Delta \ell = \Delta d = 5 \times 10^{-3}$$

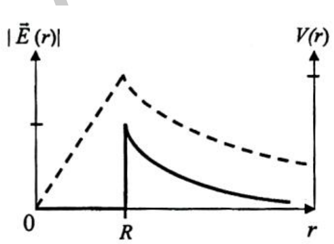
$$\frac{\Delta \ell}{\ell} = \frac{5 \times 10^{-3}}{(0.25)} = 20 \times 10^{-3} = 2 \times 10^{-2}$$

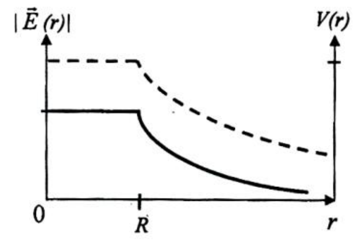
$$\frac{\Delta d}{d} = \frac{5 \times 10^{-3}}{(0.5)} = 10 \times 10^{-3} = 10^{-2}$$

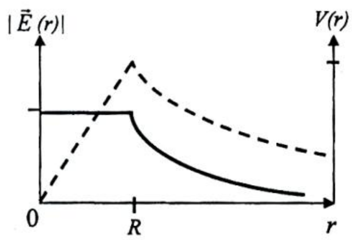
$$\Rightarrow \frac{\Delta \ell}{\ell} = 2\left(\frac{\Delta d}{d}\right)$$

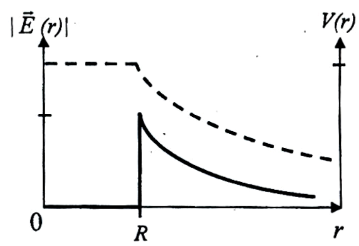
Contribution both will be same

8. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential V(r) with the distance r from the centre, is best represented by which graph?

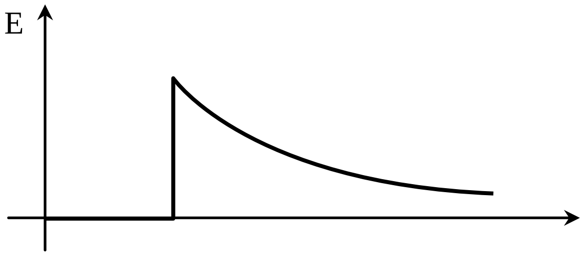
(A)


(B)


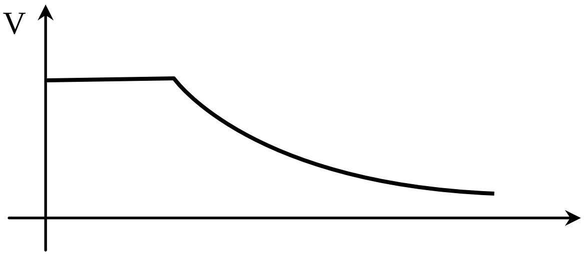
(C)


(D)


8. (D)

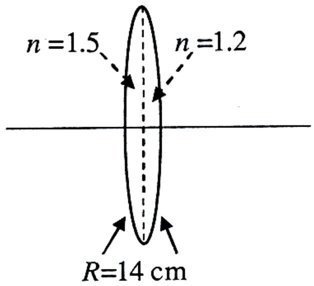


$$E = 0 \quad (r < R)$$
$$= \frac{KQ}{r} \quad (r > R)$$



$$V = \frac{KQ}{R} \quad (r < R) \text{ (Constant)}$$
$$= \frac{KQ}{r} \quad (r > R)$$

9. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature R = 14 cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be



(A) –280.0 cm (B) 40.0 cm
(C) 21.5 cm (D) 13.3 cm

9. (B)

$$f_1 = \frac{R}{(\mu - 1)} = \frac{14}{(1.5 - 1)} = 28 \text{ cm}$$

$$f_2 = \frac{R}{(\mu - 1)} = \frac{14}{(1.2 - 1)} = 70 \text{ cm}$$

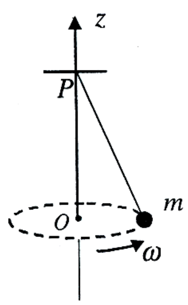
$$\Rightarrow \frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{28} + \frac{1}{70} = \frac{5+2}{140} = \frac{7}{140} = \frac{1}{20}$$

$$f_{eq} = 20 \text{ cm}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{20} - \frac{1}{40} = \left(\frac{1}{40}\right)$$

$$v = 40 \text{ cm}$$

10. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



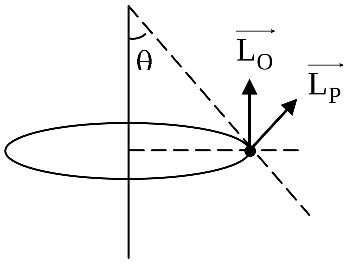
(A) $\vec{L}_O$ and $\vec{L}_P$ **do not** vary with time.

(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant.

(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time.

(D) $\vec{L}_O$ and $\vec{L}_P$ both vary with time.

10. (C)



$\overrightarrow{L_O}$ = $(m\omega r^2)\hat{k}$

direction of $\overrightarrow{L_P}$ varies with time as its direction will be perpendicular to string i.e. changing with time.

$\overrightarrow{L_O}$ = Constant

$\overrightarrow{L_P}$ = Variable

SECTION II : Multiple Correct Answers Type

This section contains **5 multiple choice questions.** Each question has four choices (A) , (B) , (C) and (D) out of which **ONE or MORE** are correct.

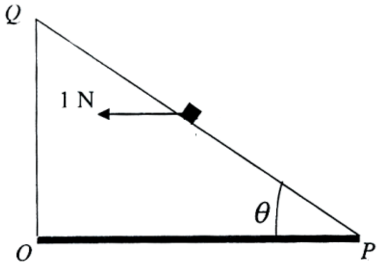
11. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe,

(A) a high-pressure pulse starts traveling up the pipe, if the other end of the pipe is open.
(B) a low-pressure pulse starts traveling up the pipe, if the other end of the pipe is open.
(C) a low-pressure pulse starts traveling up the pipe, if the other end of the pipe is closed.
(D) a high-pressure pulse starts traveling up the pipe, if the other end of the pipe is closed.

11. (B), (D)

If end is open, pressure wave will be reflected in opposite phase when high pressure arrive low pressure will be reflected
(A) is wrong
(B) is correct
If end is closed, pressure wave will be reflected in same phase so high pressure will be reflected back
(C) is wrong
(D) is correct

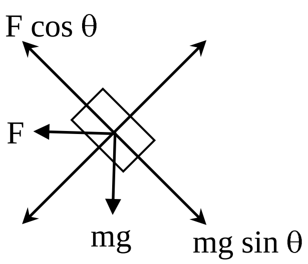
12. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its centre of mass as shown in the figure. The block remains stationary if (take $g = 10\ m/s^2$)

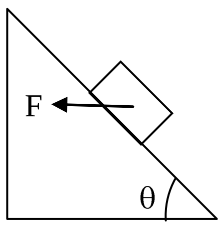


(A) θ = 45°
(B) θ > 45° and a frictional force acts on the block towards P.
(C) θ > 45° and a frictional force acts on the block towards Q.
(D) θ < 45° and a frictional force acts on the block towards Q.

12. (A), (C)

f = 0 (θ = 45°)





So, for block to be at rest

$F\cos\theta = mg\sin\theta$

LHS $F\cos\theta = 1 \times \cos 45° = \frac{1}{\sqrt{2}}$

RHS $mg\sin\theta = 0.1 \times 10 \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

{LHS = RHS)

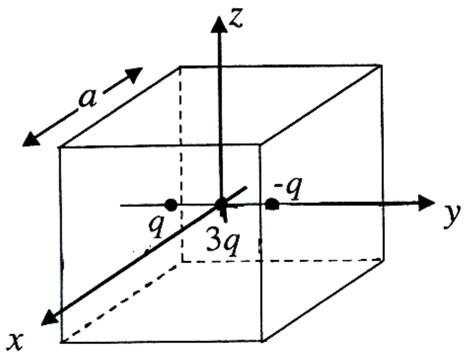
If $\theta > 45°$; $mg\sin\theta > F\cos\theta$

$\therefore$ friction acts towards Q.

If $\theta < 45°$; $F\cos\theta > mg\sin\theta$

$\therefore$ friction acts towards P.

13. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $(0, -a/4, 0)$, $+3q$ at $(0, 0, 0)$ and $-q$ at $(0, +a/4, 0)$. Chooses the correct option(s).



(A) The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$

(B) The net electric flux crossing the plane $y = +a/2$ is more than the net electric flux crossing the plane $y = -a/2$

(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$.

(D) The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x = +a/2$.

13. (A), (C), (D)

As $q_{en} = 3q - q - q = q$

$\therefore$ flux $\phi = \frac{q}{\varepsilon_0}$ (C)

Obtain (A) is correct as four the two given faces charges are symmetrically located.

Option (B) is not correct as for $y = +\frac{a}{2}$ and $y = -\frac{a}{2}$ location of charge is symmetrical so fluxes will be equal.

Option (D) is correct as for $z = +\frac{a}{2}$ and $x = +\frac{a}{2}$ location of charge is symmetrical.

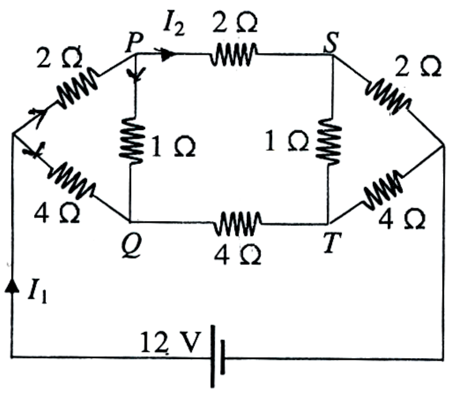
14. For the resistance network shown in the figure, choose the correct option (s).

(A) The current through PQ is zero

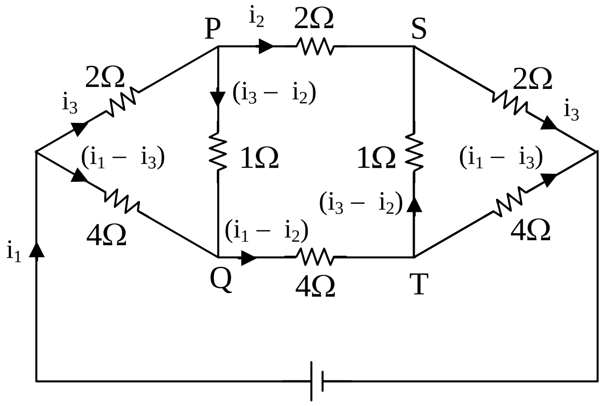
(B) (B) $I_1 = 3A$

(C) The potential at S is less than that at Q

(D) (D) $I_2 = 2A$



14. (A), (B), (C), (D)



i_{ST} $= (i_1 - i_2) - (i_1 - i_3)$

$= (i_3 - i_2)$ (T to S)

$-2i_3 - (i_3 - i_2)\,1 + 4\,(i_1 - i_3) = 0$

$-7i_3 + i_2 + 4i_1 = 0$

$7i_3 = i_2 + 4i_1$ … (1)

$-2i_2 + (i_3 - i_2)\,1 + (i_1 - i_2)\,4 + (i_3 - i_2)\,1 = 0$

$-8i_2 + 2i_3 + 4i_1 = 0$

$i_3 = 4i_2 - 2i_1$ … (2)

$7i_3 = i_2 + 4i_1$

$(i_3 = 4i_2 - 2i_1) \times 7$

$- \quad - \quad +$

$0 = -27i_2 + 18i_1$

$\Rightarrow 2i_1 = 3i_2$

$i_3 = 4i_2 - 2\left(\frac{3i_2}{2}\right)$ (from eq. (2))

$i_3 = 4i_2 - 3i_2$

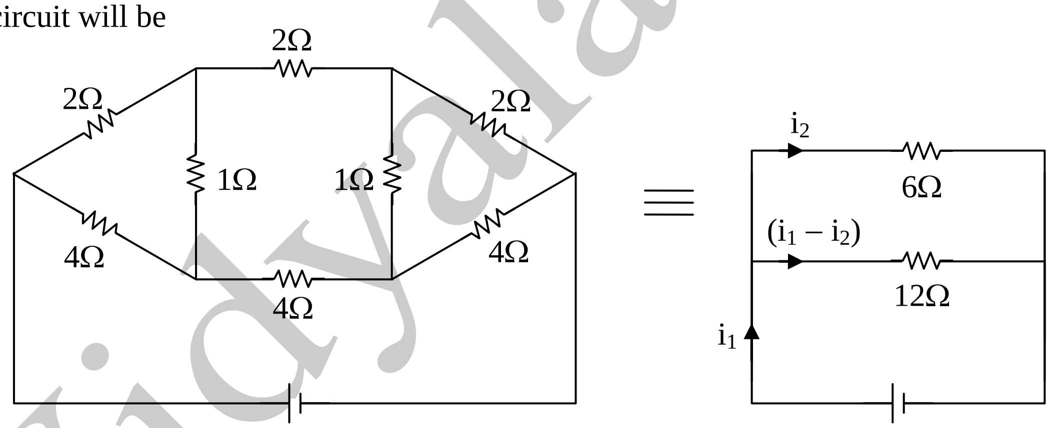
$i_3 = i_2$

$\Rightarrow i_3 - i_2 = 0$

$\Rightarrow i_{PQ} = 0$ … **Ans. (A)**

$\Rightarrow i_{PQ} = i_{ST} = 0$

Eq. circuit will be



$\Rightarrow i_2 = \frac{12}{6} = 2A$ … **Ans. (D)**

$(i_1 - i_2) = \frac{12}{12} = 1A$

$i_1 = 3A$ … **Ans. (B)**

$V_P > V_S$ (as current is going from P to S)

$V_P = V_Q$

$\Rightarrow V_Q > V_S$ … **Ans. (C)**

Alternate Method

This can also be judge directly by symmetry which show current in PQ and ST is 0.

15. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time t = 0, this charge has velocity $\vec{v}$ in the x–y plane, making an angle θ with the x–axis. Which of the following option(s) is (are) correct for time t > 0 ?

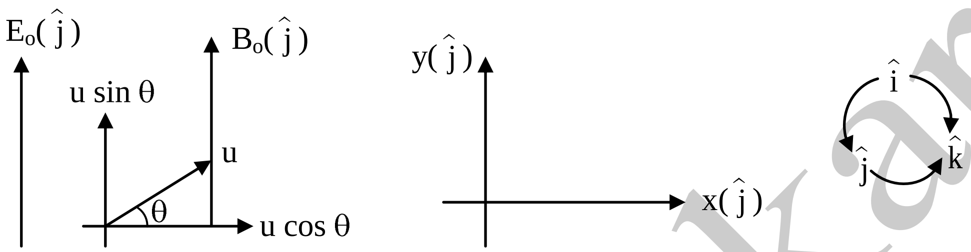
(A) If θ = 0°, the charge moves in a circular path in the x–z plane.

(B) If θ = 0°, the charge undergoes helical motion with constant pitch along the y–axis

(C) If θ = 10°, the charge undergoes helical motion with its pitch increasing with time, along the y–axis

(D) If θ = 90°, the charge undergoes linear but accelerated motion along the y–axis.

15. (C), (D)



For any value θ (≠ 90°)

Partical will move along helical path with increasing pitch (because of acceleration along y due to $\overline{E}$)

If θ = 90° ⇒ motion will be along a straight line and accelerated due to $\overline{E}$ (as magnetic force is zero).

SECTION III : Integer Answer Type

This section contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

---

16. A proton is fired from very far away towards a nucleus with charge Q = 120 e, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is : (take the proton mass, $m_p = (5/3) \times 10^{-27}$kg; $h/e = 4.2 \times 10^{-15}$J.s/C; $\frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\text{m/F}$; 1 fm = 10^{-15}m)

16. [7]

$$\frac{1}{2}mv_\infty^2 = \frac{1}{4\pi\varepsilon_0}\frac{(120)e^2}{(10\text{ fm})} \qquad \text{[By Conservation of energy]}$$

Assuming the nucleus to be considerably massive, we can disregard its motion.

∴ Let momentum of proton be $p = mv_\infty$

$$\therefore \quad \frac{p^2}{2m_p} = \frac{1}{4\pi\varepsilon_0}\frac{(120)e^2}{(10\times10^{-15})}$$

$$\therefore \quad p = 9 \times 10^9 \times 2 \times \frac{5}{3} \times 10^{-27} \times \frac{120\times e^2}{10^{-14}}$$

$$\Rightarrow \quad p = 30 \times 120 \times 10^{9-27+14} \times e^2$$

$$\therefore \quad p = \sqrt{3600\times10^{-4}\times e^2}$$

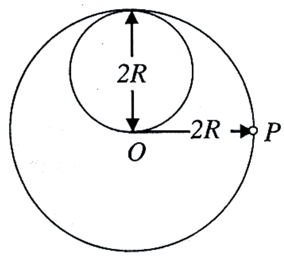
$$\therefore \quad p = 60 \times 10^{-2} \times e$$

$\therefore \quad \lambda = \dfrac{h}{p}$

$= \dfrac{h}{e \times 60 \times 10^5} \qquad = \dfrac{42 \times 10^{-15}}{60 \times 10^{-2}} \qquad = \dfrac{42}{6} \times 10^{-15}$ m

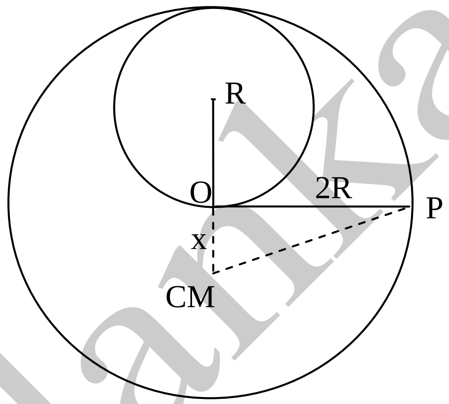
$\therefore \quad \lambda = 7$ fm

17. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\dfrac{I_P}{I_O}$ to the nearest integer is



17. [3]

$\underbrace{I_{OW}}_{\text{Whole}} = \underbrace{I_{OR}}_{\text{Remaining}} + \underbrace{I_{OC}}_{\text{Cut}}$



$I_{OR} = I_{OW} - I_{OC}$

Consider

Whole Mass = M

Remaining Mass = $\dfrac{3M}{4}$

Cut Mass = $\dfrac{M}{4}$

$I_{OR} = \dfrac{M(2R)^2}{2} - \left(\dfrac{M}{4}\dfrac{R^2}{2} + \dfrac{M}{4}R^2\right)$

$I_O = I_{OR} = \dfrac{13}{8} + MR^2$

$I_P = I_{CM} + \dfrac{3M}{4} \; x^2 + (2R)^2$

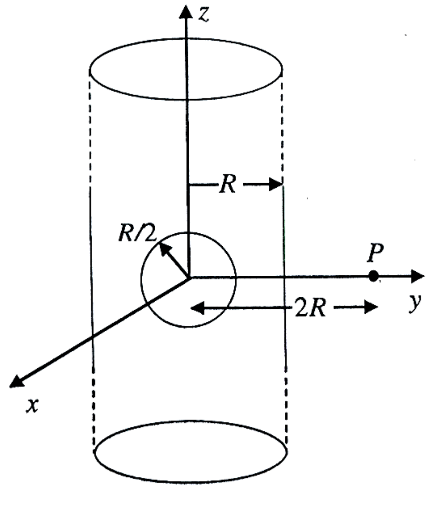
$= \left(I_O - \dfrac{3M}{4}x^2\right) + \dfrac{3M}{4} \; x^2 + 4R^2$

$= \dfrac{13}{8}MR^2 + \dfrac{3M}{4}4R^2 = \dfrac{37}{8}MR^2$

$\therefore \quad \dfrac{I_P}{I_O} = \dfrac{37}{13} \approx 3$

$\therefore$

18. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius R/2 with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance 2R from the axis of the cylinder is given by the expression $\dfrac{23\rho R}{16k\varepsilon_0}$. Then value of k is



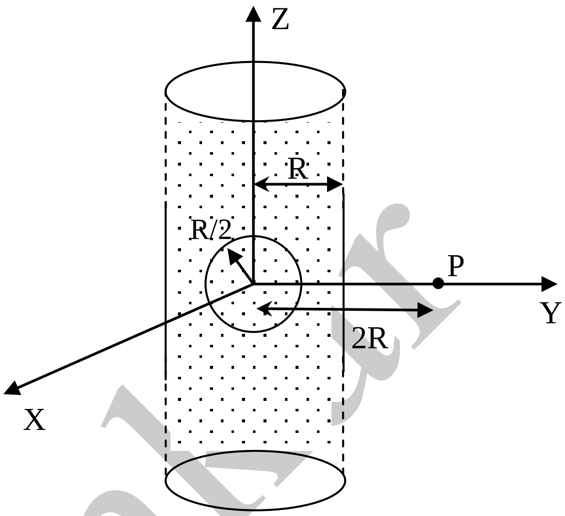
18. [6]

Applying principle of superposition

$$\underset{\text{(Whole)}}{\vec{E}_{P\,\text{whole}}} = \underset{\text{(Remaining)}}{\vec{E}_{P\,\text{shaded}}} + \underset{\text{(Cavity)}}{\vec{E}_{P\,\text{Non shaded}}}$$

$$\therefore \quad \vec{E}_{PS} = \vec{E}_{PW} - \vec{E}_{P\,NS}$$

Applying Gauss Law



For Cylinder, $E\,2\pi\,2R\,\ell = \dfrac{\rho\pi R^2 \ell}{\varepsilon_0}$

$$E_{cy} = \frac{\rho R}{4\varepsilon_0}; \text{ radially outward.}$$

For sphere, $E\,4\pi(2R)^2 = \dfrac{\rho \frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\varepsilon_0}$

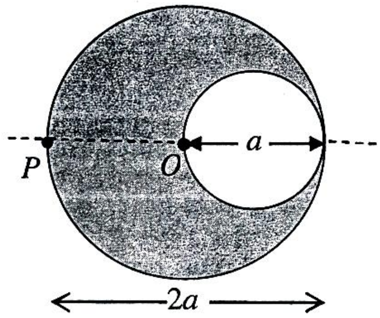
$$E_{sp} = \frac{\rho R}{96\varepsilon_0}$$

$$\therefore \quad \left|\vec{E}_{PS}\right| = \frac{\rho R}{4\varepsilon_0} - \frac{\rho R}{96\varepsilon_0} \quad \because \text{ both the fields are radially outward.}$$

Given, $E = \dfrac{23\rho R}{16K\varepsilon_0}$

$$\therefore \quad \frac{23\rho}{16K\varepsilon_0} = \frac{23\rho R}{96\varepsilon_0} \qquad \therefore \ K = 6.$$

19. A cylindrical cavity of diameter a exists inside a cylinder of diameter 2a as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is



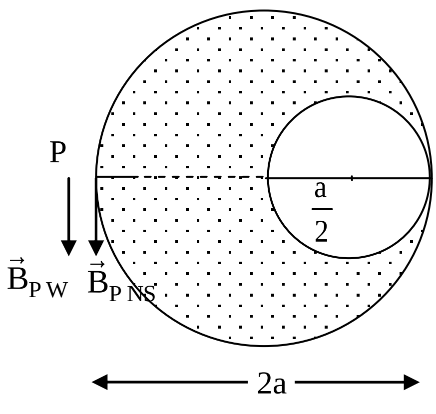
19. [5]

Applying principle of superposition

$$\vec{B}_{P\,\text{Whole}} = \vec{B}_{P\,\text{Shaded}} + \vec{B}_{P\,\text{non shaded}}$$

$$\vec{B}_{P\,Sh} = \vec{B}_{P\,W} + \vec{B}_{P\,NS}$$

$$\left|\vec{B}_{P\,Sh}\right| = \frac{\mu_0 i}{2\pi a} - \frac{\mu_0 i}{3\pi a(4)}$$



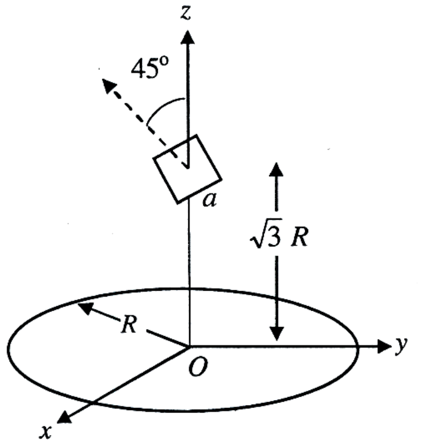
Given $B_{P\,Sh} = \dfrac{N}{12}\mu_0\, a\, J$

$$= \frac{N\mu_0 i}{12\pi a}$$

$$\therefore \quad \frac{N\mu_0 i}{12\pi a} = \frac{5\mu_0 i}{12\pi a}$$

$$N = 5$$

20. A circular wire loop of a radius R is placed in the x–y plane centered at the origin O. A square loop of side a(a <<R) having two turns is placed with its centre at z = $\sqrt{3}R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z–axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2}R}$, then the value of p is



20. [7]

We know,

$\phi \propto i$

$\phi = Mi$

ϕ is the Magnetic flux linked through square loop, while i is the current flow through the circular loop.

$$\phi = B(a^2 \cos 45°)\, N = \frac{\mu_0 i R^2}{2\{R^2 + (\sqrt{3}R)^2\}^{\frac{3}{2}}} \frac{a^2}{\sqrt{2}} 2$$

$$\phi = \frac{\mu_0 i a^2}{\sqrt{2}(4)(2)R} = M\,i$$

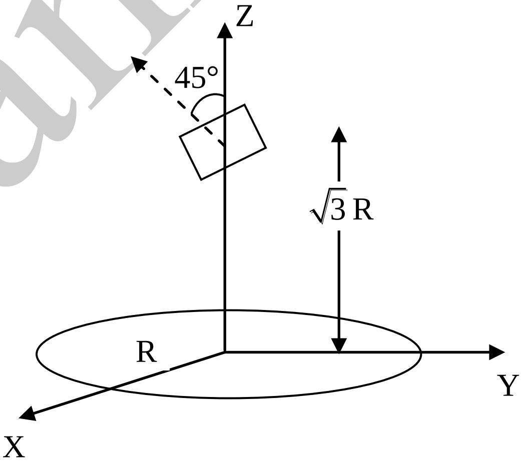
$$\therefore\ M = \frac{\mu_0 a^2}{8\sqrt{2}R}$$

$$M = \frac{\mu_0 a^2}{2^{\frac{P}{2}} R} \quad \ldots \text{(Given)}$$

$$\therefore\ \frac{\mu_0 a^2}{2^{\frac{P}{2}} R} = \frac{\mu_0 a^2}{8\sqrt{2}R}$$

$$\therefore\ 2^{\frac{p}{2}} = 2^{\frac{7}{2}}$$

$$\therefore\ P = 7$$



PART II : CHEMISTRY

SECTION I : Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A) , (B), (C) and (D) out of which **ONLY ONE** is correct.

21. The colour of light absorbed by an aqueous solution of $CuSO_4$ is

(A) orange – red (B) blue – green (C) yellow (D) violet

21. (A)

The $CuSO_4$ solution is blue. The colour that is absorbed complementary to the colour that is observed.

22. Which ordering of compounds is according to the decreasing order of the oxidation state of nitrogen ?

(A) HNO_3 , NO, NH_4Cl, N_2

(B) HNO_3, NO, N_2, NH_4Cl

(C) HNO_3, NH_4Cl, NO, N_2

(D) NO, HNO_3, NH_4Cl, N_2

22. (B)

$HNO_3 \rightarrow +5$

$NO \rightarrow +2$, $N_2 = 0$

$NH_4Cl \rightarrow -3$

23. The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is [a_0 is Bohr radius]

(A) $\dfrac{h^2}{4\pi^2 m a_0^2}$ (B) $\dfrac{h^2}{16\pi^2 m a_0^2}$ (C) $\dfrac{h^2}{32\pi^2 m a_0^2}$ (D) $\dfrac{h^2}{64\pi^2 m a_0^2}$

23. (C)

$$mvr = \frac{nh}{2\pi} \quad \text{and} \quad \frac{mv^2}{r} = \frac{e^2}{r^2} \Rightarrow mv^2 r = e^2$$

$$\Rightarrow V = \frac{e^2 \times 2\pi}{nh} \quad \therefore \left(mvr = \frac{nh}{2\pi}\right)$$

$$\Rightarrow K.E. = \frac{1}{2}mv^2 = \frac{\frac{1}{2} m e^4 \times 4\pi^2}{n^2 h^2} \qquad \ldots(1)$$

$$\text{Expression for } a_0 = \frac{h^2}{4\pi^2 m e^2}$$

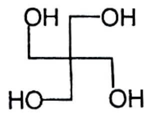
$$\Rightarrow me^2 = \frac{h^2}{4\pi^2 a_0} \qquad \ldots(2)$$

$$\Rightarrow K.E. = \frac{h^2}{8 m a_0^2 \pi^2} \times \frac{1}{n^2} \qquad \ldots(3)$$

For n = 2

$$K.E. = \frac{h^2}{32\pi^2 m a_0^2}$$

24. The number of aldol reaction(s) that occurs in the given transformation is

$CH_3CHO + 4HCHO \xrightarrow{\text{conc. aq. NaOH}}$ 

(A) 1 (B) 2 (C) 3 (D) 4

24. (C)

$$CH_3-\overset{\overset{O}{\|}}{C}-H \xrightarrow[(1)]{HCHO} \overset{OH}{|}CH_2-\overset{\overset{O}{\|}}{C}-H$$

$$\Big\downarrow (2)\ HCHO$$

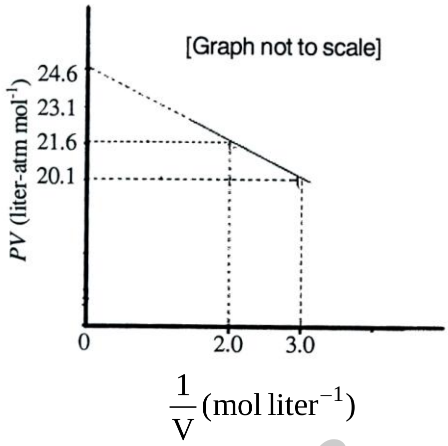
$$H-OCH_2-\underset{\underset{CH_2OH}{|}}{CH}-CHO \xrightarrow[(3)]{HCHO} HOCH_2-\overset{\overset{CH_2OH}{|}}{\underset{\underset{CH_2OH}{|}}{C}}-CHO$$

$$\Big\downarrow (4)\ OH^- \quad \text{Cannizzaro Reaction}$$

$$(HOCH_2)_4C$$

1, 2, 3 are aldol products while 4 is Cannizzaro product.

25. For one mole of a van der Waals gas when b = 0 and T = 300 K, the PV vs. 1/V plot is shown below. The value of the van der Waals constant a (atm. liter^2 mol^{-2}) is



(A) 1.0 (B) 4.5 (C) 1.5 (D) 3.0

25. (C)

b = 0 , T = 300K, n = 1

$$\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$$

$$\left(P+\frac{a}{V^2}\right)(V)=RT \quad\Rightarrow\quad PV+\frac{a}{V}=RT$$

$$PV=-a\times\left(\frac{1}{V}\right)+RT$$

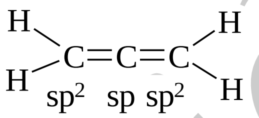
$$y=mx+C \qquad \text{Slope} = -a$$

$$\text{Slope } m=\frac{y_2-y_1}{x_2-x_1}=\frac{20.1-21.6}{3-2}=1.5$$

26. In allene (C_3H_4), the type(s) of hybridization of the carbon atoms is (are)

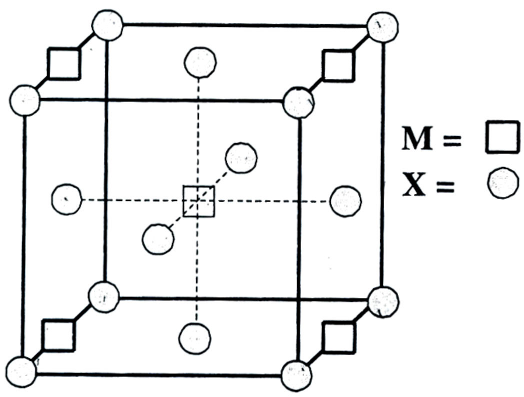
(A) sp and sp^3 (B) sp and sp^2 (C) only sp^2 (D) sp^2 and sp^3

26. (B)



(allene)

27. A compound M_pX_q has cubic close packing (ccp) arrangement of X. Its unit cell structure is shown below. The empirical formula of the compound is



(A) MX (B) MX_2

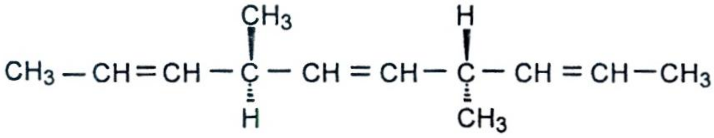
(C) M_2X (D) M_5X_{14}

27. (B)

$$X:8\times\frac{1}{8}+6\times\frac{1}{2}=4$$

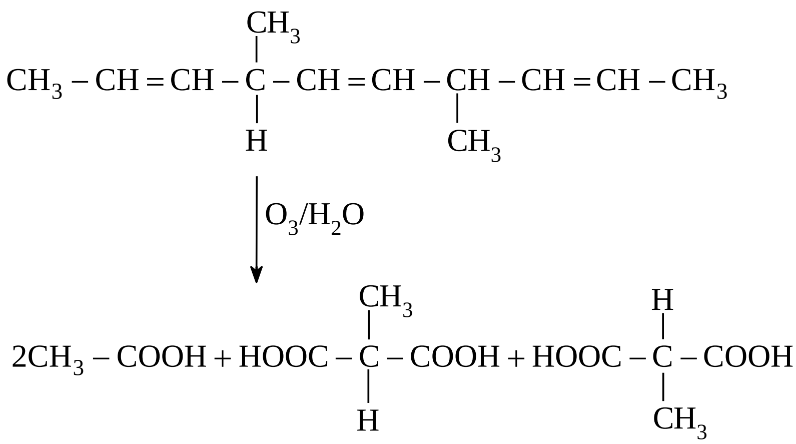
$$M=\frac{1}{4}\times4+1=2\,; \qquad M_2X_4=MX_2$$

28. The number of optically active products obtained from the complete ozonolysis of the given compound is



(A) 0 (B) 1 (C) 2 (D) 4

28. (A)



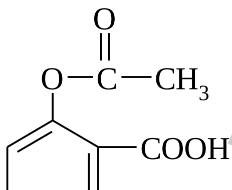
None is optical isomers active due to absence of chiral carbon.

29. As per IUPAC nomenclature, the name of the complex $[Co(H_2O)_4(NH_3)_2]Cl_3$ is
(A) Tetraaquadiaminecobalt (III) chloride
(B) Tetraaquadiaminecobalt (III) chloride
(C) Diaminetetraaquacobalt (III) chloride
(D) Diamminetetraaquacobalt (III) chloride

29. (D)
$Co(H_2O)_4\,(NH_3)_2\;\;Cl_3$
Diamminetetraaquacobalt (III) chloride

30. The carboxyl functional group (–COOH) is present in
(A) picric acid (B) barbituric acid (C) ascorbic acid (D) aspirin

30. (D)

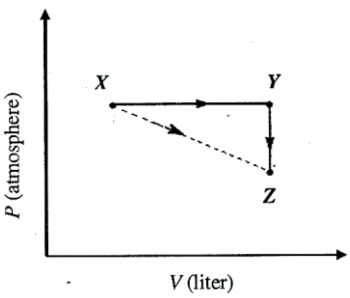


(Aspirin)

SECTION II : Multiple Correct Answers Type

This section contains **5 multiple choice questions.** Each question has four choices (A) , (B) , (C) and (D) out of which **ONE or MORE** are correct.

31. For an ideal gas, consider only P–V work in going from an initial state X to the final state Z. The final state Z can be reached by either of the two paths shown in the figure. Which of the following choice(s) is (are) correct? [take ΔS as change in entropy and W as work done]



(A) $\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}$
(B) $W_{x\to z} = W_{x\to y} + W_{y\to z}$
(C) $W_{x\to y\to z} = W_{x\to y}$
(D) $\Delta S_{x\to y\to z} = \Delta S_{x\to y}$

31. (A),(C)

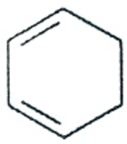
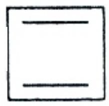
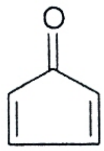
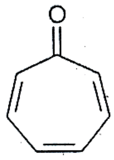
$$W_{x\to z} = W_{x+y} + W_{y\to z}$$

As work done is not a state function so it depends on the path.

$W_{x\to y\to z} = W_{x\to y}$ (as volume is constant) $\Rightarrow$ $W_{y\to z} = 0$

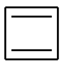
$\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}$ (since ΔS is a state function, it does not depend on the path).

32. Which of the following molecules, in pure form, is (are) **unstable** at room temperature?

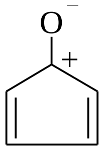
(A)  (B)  (C)  (D) 

32. (B) , (C)

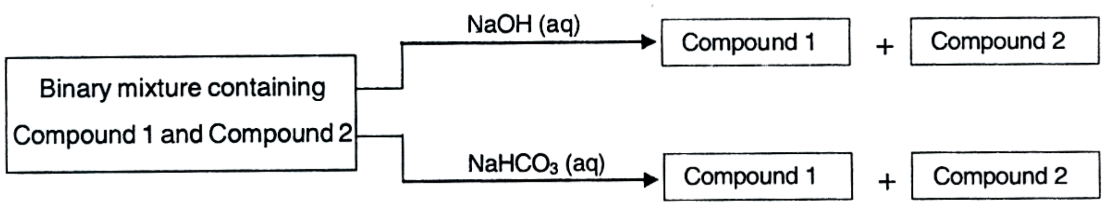
(B)

 - antiaromatic

(C)

 is antiaromatic

33. Identify the binary mixture(s) that can be separated into individual compounds, by differential extraction, as shown in the given scheme.



(A) C_6H_5OH and C_6H_5COOH (B) C_6H_5COOH and $C_6H_5CH_2OH$
(C) $C_6H_5CH_2OH$ and C_6H_5OH (D) $C_6H_5CH_2OH$ and $C_6H_5CH_2COOH$

33. (A), (B), (D)

All of them contain alcoholic–OH group and–COOHgroup

34. Choose the correct reason(s) for the stability of the **lyophobic** colloidal particles.

(A) Preferential adsorption of ions on their surface from the solution
(B) Preferential adsorption of solvent on their surface from the solution
(C) Attraction between different particles having opposite charges on their surface
(D) Potential difference between the fixed layer and the diffused layer of opposite charges around the colloidal particles

34. (A), (D)

(A) $\rightarrow$ Preferential adsorption of ions on surface from the solution.

(C) $\rightarrow$ Attraction between particles having same charges on their surface accounts for the Brownian motion.

(D) $\rightarrow$ Definition of Zeta – Potential (Refer NCERT).

35. Which of the following hydrogen halides react(s) with $AgNO_3$(aq) to given a precipitate that dissolves in $Na_2S_2O_3$(aq)?

(A) HCl (B) HF (C) HBr (D) HI

35. (A), (C), (D)

HCl, HBr, HI gives AgCl, AgBr, AgI, ppt. HF does not react with HF.

Ag^+ forms a water soluble $[Ag(S_2O_3)_2]^{3-}$ ion.

SECTION III : Integer Answer Type

This section contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

36. An organic compound undergoes first–order decomposition. The time taken for its decomposition to $\frac{1}{8}$ and $\frac{1}{10}$ of its initial concentration are $t_{1/8}$ and $t_{1/10}$ respectively. What is the value of $\frac{[t_{1/8}]}{[t_{1/10}]} \times 10$? (take $\log_{10}2 = 0.3$)

36. [9]

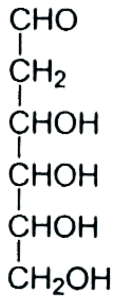
$$K = \frac{2.302}{t}\log\left(\frac{a_0}{a_0 - x}\right)$$

$$K = \frac{2.303}{t_{1/8}}\log\left(\frac{a_0}{\frac{1}{8}a_0}\right)$$

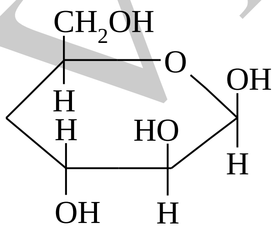
$$K = \frac{2.303}{t_{1/10}}\log\left(\frac{a_0}{\frac{1}{10}a_0}\right) \quad \Rightarrow \frac{t_{1/8}}{t_{1/10}} = \frac{\log 8}{\log 10}$$

$$\Rightarrow \frac{t_{1/8}}{t_{1/10}} \times 10 = \log 8 \times 10 \quad = 3 \log 2 \times 10 \; = 3 \times 0.3 \times 10 = 9$$

37. When the following aldohexose exists in its D–configuration, the total number of stereoisomers in its pyranose form is

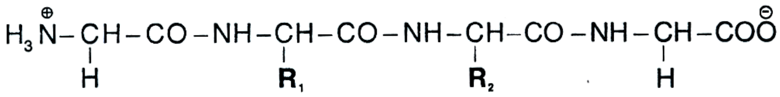


37. [8]



Total No. of stereoisomers = 2^4 = 16 which contains 8D–Configuration and 8–L Configuration

38. The substituents R_1 and R_2 for nine peptides are listed in the table given below. How many of these peptides are positively charged at pH = 7.0?



Peptide	R_1	R_2
I	H	H
II	H	CH_3
III	CH_2COOH	H
IV	CH_2CONH_2	$(CH_2)_4NH_2$
V	CH_2CONH_2	CH_2CONH_2
VI	$(CH_2)_4NH_2$	$(CH_2)_4NH_2$
VII	CH_2COOH	CH_2CONH_2
VIII	CH_2OH	$(CH_2)_4NH_2$
IX	$(CH_2)_4NH_2$	CH_3

38. [4]
Presence of $-NH_2$ group containing substituent for R_1 and R_2 would be positively charged at pH = 7.0 peptides IV, VI, VIII and IX contains $-NH_2$ group.

39. The periodic table consist of 18 groups. An isotope of copper, on bombardment with protons, undergoes a nuclear reaction yielding element X as shown below. To which group, element X belongs in the periodic table?

$${}^{63}_{29}Cu + {}^{1}_{1}H \rightarrow 6\,{}^{1}_{0}n + \alpha + 2\,{}^{1}_{1}H + \mathbf{X}$$

39. [8]
Compound X is ${}_{26}Fe$ and it belongs to group No–8.

40. 29.2% (w/w) HCl stock solution has a density of 1.25 g mL^{-1}. The molecular weight of HCl is 36.5 g mol^{-1}. The volume (mL) of stock solution required to prepare a 200 mL solution of 0.4 M HCl is

40. [8]

$$\text{Molarity} = \frac{n}{V} = \frac{W}{M_w \times v} = \frac{W \times d_{sol}}{M_w \times W_{sol}} = \frac{W}{W_{sol}} \times \frac{100 \times d^{soln}}{M_w \times 100} \times 1000$$

$$= \frac{\frac{W}{W}\% \times d \times 10}{M_w}$$

$$= \frac{29.2 \times 1.25 \times 10}{36.5}$$

$$= 10\,M$$

$$M_1V_1 = M_2V_2$$

$$\Rightarrow\ 10 \times V_1 = 0.4 \times 200$$

$$V = \frac{0.4 \times 200}{10} = 8\,ml$$

PART III - MATHEMATICS

SECTION I : Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A) , (B), (C) and (D) out of which **ONLY ONE** is correct.

41. The ellipse E_1 : $\frac{x^2}{9}+\frac{y^2}{4}=1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

41. (C)

Required ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Given it passes (0, 4)

$\Rightarrow \frac{4^2}{b^2}=1 \Rightarrow b=\pm 4$

Also it passes through (3, 2)

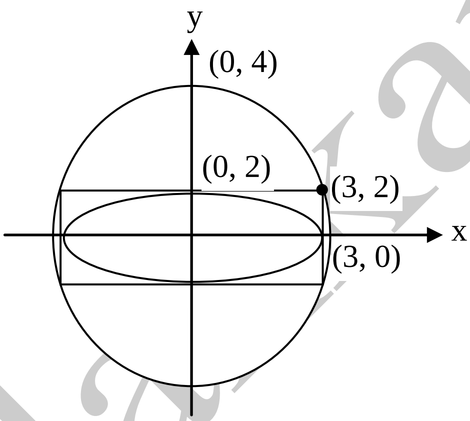
$\Rightarrow \frac{9}{a^2}+\frac{4}{b^2}=1$

$\Rightarrow \frac{9}{a^2}+\frac{4}{16}=1$

$\Rightarrow \frac{9}{a^2}=1-\frac{4}{16} = \frac{12}{16}=\frac{3}{4}$

$\Rightarrow a^2=\frac{9\times 4}{3}=12$

So, $e=\sqrt{1-\frac{a^2}{b^2}} = \sqrt{1-\frac{12}{16}}=\frac{1}{2}$



42. The point P is the intersection of the straight line joining the points Q(2, 3, 5) and R (1, −1, 4) with the plane 5x − 4y − z = 1. If S is the foot of the perpendicular drawn from the point T(2, 1, 4) to QR, then the length of the line segment PS is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

42. (A)

Equation of PQ is $\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-5}{1}$

Let any point P on any line is (r + 2, 4r + 3, r + 5)

As P is intersection with 5x – 4y – z = 1

$\Rightarrow 5(r+2)-4(4r+3)-(r+5)=1$

$\Rightarrow r=-\frac{2}{3}$

So, $P\equiv\left(\frac{4}{3},\frac{1}{3},\frac{13}{3}\right)$

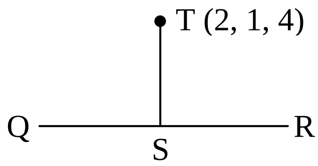
Also we take any point on QR is $S\equiv \ t+2, 4t+3, t+5$

So, drs of line $\perp^r$ to QR (i.e. ST) is
(t, 4t + 2, t + 1)

As $\perp^r \Rightarrow \ell_1 \ell_2 + m_1 m_2 + n_1 n_2 = 0$

$\Rightarrow 1 \times t + 4 \times (4t + 2) + 1 \times (t + 1) = 0$

$\Rightarrow t = -\dfrac{1}{2}$



So, $S \equiv \left(\dfrac{3}{2}, 1, \dfrac{9}{2}\right)$

So, $PS = \sqrt{\left(\dfrac{4}{3}-\dfrac{3}{2}\right)^2 + \left(\dfrac{1}{3}-1\right)^2 + \left(\dfrac{13}{3}-\dfrac{9}{2}\right)^2} = \dfrac{1}{\sqrt{2}}$

43. The integral $\displaystyle\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}}$ dx equals (for some arbitrary constant K)

(A) $-\dfrac{1}{(\sec x + \tan x)^{11/2}}\left\{\dfrac{1}{11} - \dfrac{1}{7}(\sec x + \tan x)^2\right\} + K$

(B) $\dfrac{1}{(\sec x + \tan x)^{11/2}}\left\{\dfrac{1}{11} - \dfrac{1}{7}(\sec x + \tan x)^2\right\} + K$

(C) $-\dfrac{1}{(\sec x + \tan x)^{11/2}}\left\{\dfrac{1}{11} + \dfrac{1}{7}(\sec x + \tan x)^2\right\} + K$

(D) $\dfrac{1}{(\sec x + \tan x)^{11/2}}\left\{\dfrac{1}{11} + \dfrac{1}{7}(\sec x + \tan x)^2\right\} + K$

43. (C)

Let sec x + tan x = t $\quad\Rightarrow\quad \sec x = \dfrac{1}{2}\left(t + \dfrac{1}{t}\right)$

$\sec x \tan x + \sec^2 x = \dfrac{dt}{dx}$

$\sec x (\sec x + \tan x) = \dfrac{dt}{dx}$

$$\int \frac{\left(\frac{1}{2}\left(t + \frac{1}{t}\right)\right)^2}{t^{9/2}} \times \frac{dt}{\left(\frac{1}{2}\left(t + \frac{1}{t}\right)\right)t}$$

$$= \frac{1}{2}\int \frac{t + \frac{1}{t}}{t^{11/2}} dt = \frac{1}{2}\int t^{-9/2} + t^{-13/2} \; dt$$

$$= \frac{1}{2}\left[\frac{t^{-7/2}}{-\frac{7}{2}} + \frac{t^{-11/2}}{-\frac{11}{2}}\right] + K = \frac{1}{2}\left[\frac{(\sec x + \tan x)^{-7/2}}{-\frac{7}{2}} + \frac{(\sec x + \tan x)^{-11/2}}{-\frac{11}{2}}\right] + K$$

44. Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + 1$ is real. Then *a* cannot take the value

(A) -1 (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{3}{4}$

44. (D)

$z = x + iy \;\; ; y \neq 0$

$a = z^2 + z + 1 \in \mathbb{R}$

$a = (x + iy)^2 + (x + iy) + 1$

$= (x^2 - y^2 + x + 1) + i\,(2xy + y) \in \mathbb{R}$

$\Rightarrow \; 2xy + y = 0$

$\Rightarrow \; y\,(2x + 1) = 0$

$x = -\dfrac{1}{2} \qquad \therefore\; y \neq 0$

$\therefore\; a = x^2 - y^2 + x + 1 \; = \dfrac{1}{4} - y^2 - \dfrac{1}{2} + 1$

$a = \dfrac{3}{4} - y^2$

$\because\; y \neq 0 \qquad \Rightarrow y^2 > 0$

$\Rightarrow \; y^2 = \dfrac{3}{4} - a > 0$

$\therefore\; a < \dfrac{3}{4}$

$\therefore\; a \neq \dfrac{3}{4}$

$\therefore$ alternative (D) is correct.

45. Let $f(x) = \begin{cases} x^2\left|\cos\dfrac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in \mathbb{R}$, then f is

(A) differentiable both at x = 0 and at x = 2
(B) differentiable at x = 0 but not differentiable at x = 2
(C) not differentiable at x = 0 but differentiable at x = 2
(D) differentiable neither at x = 0 nor at x = 2

45. (B)

$f(x) = \begin{cases} x^2\left|\cos\dfrac{\pi}{x}\right|; & x \neq 0 \\ 0 \;\;; & x = 0 \end{cases}$

<u>**At x = 0**</u>

$$\text{RHD at } x = 0 \;=\; \underset{h\to 0}{\text{Lt}}\;\frac{f(0+h)-f(0)}{h} \;=\; \underset{h\to 0}{\text{Lt}}\;\frac{h^2\left|\cos\frac{\pi}{h}\right| - 0}{h} = 0$$

$$\text{LHD at } x = 0 \;=\; \underset{h\to 0}{\text{Lt}}\;\frac{f(0-h)-f(0)}{-h}$$

$$= \underset{h\to 0}{\text{Lt}}\;\frac{(-h)^2\left|\cos\left(\frac{\pi}{h}\right)\right|}{-h} = -\underset{h\to 0}{\text{Lt}}\; h\left|\cos\left(\frac{\pi}{h}\right)\right| = 0$$

$\because$ LHD = RHD

$\Rightarrow$ f is differentiable at x = 0

<u>At x = 2</u>

LHD at x = 2 $-\ \underset{h\to 0}{\text{Lt}}\ \dfrac{f(2-h)-f(2)}{h}$

$$= \underset{h\to 0}{\text{Lt}}\ \frac{(-h)^2\cos\left(\dfrac{\pi}{2-h}\right)-0}{h} \qquad \left(\frac{0}{0}\right) \text{ using L'Hospital}$$

$$= \underset{h\to 0}{\text{Lt}}\ \frac{[-2(2-h)]\cdot\cos\left(\dfrac{\pi}{2-h}\right)+(2-h)^2\cdot\left(-\sin\left(\dfrac{\pi}{2-h}\right)\right)\cdot\pi\cdot\dfrac{+1}{(2-h)^2}}{1}$$

$= -\pi$

$\therefore$ LHD is equal to $-\pi$

RHD at x = 2 = $\underset{h\to 0}{\text{Lt}}\ \dfrac{f(2+h)-f(2)}{h}$

$$= \underset{h\to 0}{\text{Lt}}\ \frac{(2+h)^2\cos\left(\dfrac{\pi}{2+h}\right)}{h} \qquad \left(\frac{0}{0}\right)$$

$$= \underset{h\to 0}{\text{Lt}}\ \frac{2(2+h)\cos\left(\dfrac{\pi}{2+h}\right)\cdot+(2+h)^2\left(-\sin\dfrac{\pi}{2+h}\right)(\pi)\cdot\left(\dfrac{-1}{(2+h)^2}\right)}{1} \text{ (using H'Hospital)}$$

$=\ (4)\ (0) + \pi = \pi$

$\therefore$ RHD = π

$\therefore$ LHD $\neq$ RHD $\therefore$ f is not differentiable at x = 2

46. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

(A) 75 (B) 150 (C) 210 (D) 243

46. (B)

Method I

Total no. of ways = $3^5 - {}^3C_1\ 3-1\ ^5 + {}^3C_2\ 3-2\ ^5$

= 243 – 3 × 32 +3 = 246 – 96 = 150

Alternative Method

System I

Boxes	I	II	III
Balls	1	2	2

For this system no. of ways

$$= \left(\frac{5!}{2!2!1!}\times\frac{1}{2!}\right)\times 3! = \left(\frac{5\times4\times3\times2}{2\times2\times2}\right)\times 6 = 90$$

System II

Boxes	I	II	III
Balls	1	3	1

For this system no. of ways

$$= \left(\frac{5!}{3!\times1!\times1!}\times\frac{1}{2!}\right)\times 3! = 10\times 6 = 60$$

Total no. of ways = 90 + 60 = 150

47. If $\lim\limits_{x\to\infty}\left(\dfrac{x^2+x+1}{x+1}-ax-b\right)=4$, then

(A) a = 1, b = 4. (B) a = 1, b = –4 (C) a = 2, b = –3 (D) a = 2, b = 3

47. (B)

$$\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$$

$$\lim_{x\to\infty}\left(\frac{x^2+x+1-ax^2-ax-bx-b}{x+1}\right)=4$$

$$\lim_{x\to\infty}\left(\frac{(1-a)x^2+(1-a-b)x+(1-b)}{x+1}\right)=4$$

Limiting value is finite nonzero so highest degree in numerator and denominator will be same.

Therefore $1 - a = 0 \Rightarrow a = 1$

and $\dfrac{1-a-b}{1}=4$

$\dfrac{1-1-b}{1}=4$

$b = -4 \;\therefore$ answer is (B)

48. The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

(A) one-one and onto. (B) onto but not one-one.
(C) one-one but not onto. (D) neither one-one nor onto.

48. (B)

Given function is $f : [0, 3] \to [1, 29]$

$f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36$

$f'(x) = 6\{x^2 - 5x + 6\}$

$f'(x)$ will change the sign so it is increasing as well as decreasing.

Hence it is not one–one.

$f(x)$ is increasing in $(-\infty, 2] \cup [3, \infty)$ and decreasing in $(2, 3)$

$f(x)$ at $x = 0$ is 1

$f(x)$ at $x = 2$ is 29

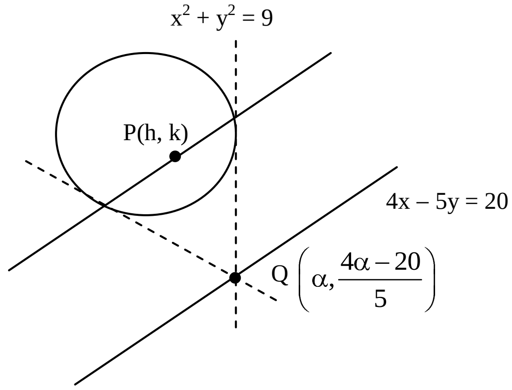
$f(x)$ at $x = 3$ in 27

It is onto. $\therefore$ answer is (B)

49. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is

(A) $20(x^2 + y^2) - 36x + 45y = 0$ (B) $20(x^2 + y^2) + 36x - 45y = 0$
(C) $36(x^2 + y^2) - 20x + 45y = 0$ (D) $36(x^2 + y^2) - 20x - 45y = 0$

49. (A)



$\alpha x + \left(\dfrac{4\alpha-20}{5}\right)y = 9$

$5\alpha x + (4\alpha - 20)y = 45$ ………….. (i)

$hx + ky = h^2 + k^2$ …………... (ii)

$$\frac{5\alpha}{h}=\frac{4\alpha-20}{k}=\frac{45}{h^2+k^2}$$

$\alpha = \dfrac{9h}{h^2+k^2}$ ………….... (iii)

$4\alpha - 20 = \dfrac{45k}{h^2+k^2}$ ……………. (iv)

from (iii) & (iv)

$$\frac{4(9h)}{h^2+k^2} - 20 = \frac{45k}{h^2+k^2}$$

$$\frac{36h}{h^2+k^2} - 20 = \frac{45k}{h^2+k^2}$$

$36h - 20\,(h^2 + k^2) = 45\,k$

$36\,x - 20\,(x^2 + y^2) = 45\,y$

$20\,(x^2 + y^2) - 36x + 45\,y = 0$

$\therefore$ correct answer is (A).

50. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j}a_{ij}$ for $1 \le i, j \le 3$. If the determinant of P is 2, then the determinant of the matrix Q is

(A) 2^{10} (B) 2^{11} (C) 2^{12} (D) 2^{13}

50. (D)

$P = [a_{ij}]$

$Q = [b_{ij}]$

$b_{ij} = 2^{i+j}\,.\,a_{ij}$

$b_{11} = 2^2 a_{11}$ $b_{21} = 2^3\,.\,a_{21}$ $b_{31} = 2^4\,a_{31}$

$b_{12} = 2^3\,a_{12}$ $b_{22} = 2^4\,.\,a_{22}$ $b_{32} = 2^5\,a_{32}$

$b_{13} = 2^4\,a_{13}$ $b_{23} = 2^5\,.\,a_{22}$ $b_{33} = 2^6\,a_{33}$

$$\text{Given } P = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 2$$

$$Q = \begin{vmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{vmatrix}$$

$$Q = 2^2 \cdot 2^3 \cdot 2^4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 2^2 a_{31} & 2^2 a_{32} & 2^2 a_{33} \end{vmatrix}$$

$$Q = 2^2 \cdot 2^3 \cdot 2^4 \cdot 2^2 \cdot 2^1 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$Q = 2^2 \cdot 2^3 \cdot 2^4 \cdot 2^2 \cdot 2^1 \cdot 2^1$$

$$Q = 2^{13}$$

SECTION II : Multiple Correct Answers Type

This section contains **5 multiple choice questions.** Each question has four choices (A) , (B) , (C) and (D) out of which **ONE or MORE** are correct.

51. If y (x) satisfies the differential equation $y' - y\tan x = 2x\sec x$ and y (0) = 0, then

(A) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$ (B) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$

(C) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$ (D) $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

51. (A), (D)

$y' - y\tan x = 2x\sec x$

I.F. = $e^{\int \tan x dx} = e^{\log\cos x} = \cos x$

$\therefore \quad y\cos x = \int 2x\sec x.\cos x dx$

$\Rightarrow y\cos x = x^2 + c$

$\Rightarrow y \cdot \cos x = x^2 \qquad (\because y(0) = 0)$

$\Rightarrow y = x^2 \sec x$

$\therefore \quad y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$ and $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

52. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 And E_3 are functioning. Which of the following is (are) true?

(A) $P\left[X_1^c | X\right] = \frac{3}{16}$

(B) P [Exactly two engines of the ship are functioning |X] = $\frac{7}{8}$

(C) $P\left[X | X_2\right] = \frac{5}{16}$

(D) $P\left[X | X_1\right] = \frac{7}{16}$

52. (B), (D)

(i) $P\left(\frac{X_1^c}{X}\right) = \dfrac{\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}}{\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}} = \frac{1}{8}$

(ii) $P\left(\frac{\text{Exactly two engines are functioning}}{X}\right) = \dfrac{\frac{8}{32} - \frac{1}{4}\times\frac{1}{4}\times\frac{1}{2}}{\frac{8}{32}} = \frac{7}{8}$

(iii) $P\left(\dfrac{X}{X_2}\right) = \dfrac{\dfrac{1}{4}\left(\dfrac{1}{2}\times\dfrac{3}{4}+\dfrac{1}{2}\times\dfrac{1}{4}+\dfrac{1}{2}\times\dfrac{1}{4}\right)}{\dfrac{1}{4}\left(\dfrac{1}{2}\times\dfrac{1}{4}+\dfrac{1}{2}\times\dfrac{3}{4}+\dfrac{1}{2}\times\dfrac{1}{4}+\dfrac{1}{2}\times\dfrac{3}{4}\right)} = \dfrac{5}{8}$

(iv) $P\left(\dfrac{X}{X_1}\right) = \dfrac{\dfrac{1}{2}\left(\dfrac{1}{4}\times\dfrac{1}{4}+\dfrac{1}{4}\times\dfrac{3}{4}+\dfrac{1}{4}\times\dfrac{3}{4}\right)}{\dfrac{1}{2}\left(\dfrac{3}{4}\times\dfrac{3}{4}+\dfrac{1}{4}\times\dfrac{1}{4}+\dfrac{1}{4}\times\dfrac{3}{4}+\dfrac{1}{4}\times\dfrac{3}{4}\right)} = \dfrac{7}{16}$

53. Let $\theta,\ \varphi \in\ 0, 2\pi$ be such that

$$2\cos\theta\ \ 1-\sin\varphi\ \ = \sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi - 1$$

$$\tan\ \ 2\pi-\theta\ \ > 0 \quad \text{and} \quad -1 < \sin\theta < -\frac{\sqrt{3}}{2}$$

Then φ cannot satisfy

(A) $0 < \varphi < \dfrac{\pi}{2}$ (B) $\dfrac{\pi}{2} < \varphi < \dfrac{4\pi}{3}$ (C) $\dfrac{4\pi}{3} < \varphi < \dfrac{3\pi}{2}$ (D) $\dfrac{3\pi}{2} < \varphi < 2\pi$

53. (A), (C), (D)

Conditions :

$-\tan(\theta) > 0 \Rightarrow \tan\theta < 0$

and $-1 < \sin\theta < -\dfrac{\sqrt{3}}{2}$

$\therefore\ \theta \in \left(\dfrac{3\pi}{2}, \dfrac{5\pi}{3}\right) \Rightarrow 0 < \cos\theta < \dfrac{1}{2}$

Also, $2\cos\theta\,(1-\sin\phi) = \sin^2\theta\left(\dfrac{1}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)\cos\phi - 1$

$\Rightarrow\ 2\cos\theta - 2\cos\theta\sin\phi = 2\sin\theta\cos\phi - 1$

$\Rightarrow\ 1 + 2\cos\theta = 2\sin(\theta+\phi)$

$\Rightarrow\ \sin(\theta+\phi) = \dfrac{1}{2} + \cos\theta$

$\Rightarrow\ \dfrac{1}{2} < \sin(\theta+\phi) < 1 \quad \Rightarrow\ \dfrac{\pi}{2} < \phi < \dfrac{4\pi}{3}$

54. Let S be the area of the region enclosed by $y = e^{-x^2}$, y = 0, x = 0, and x = 1. Then

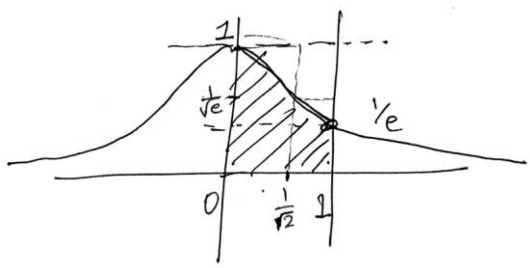
(A) $S \ge \dfrac{1}{e}$ (B) $S \ge 1-\dfrac{1}{e}$

(C) $S \le \dfrac{1}{4}\left(1+\dfrac{1}{\sqrt{e}}\right)$ (D) $S \le \dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{e}}\left(1-\dfrac{1}{\sqrt{2}}\right)$

54. (A), (B), (D)

$y = e^{-x^2}$, y = 0, x = 0, x = 1

$\displaystyle\int_0^1 e^{-x^2}dx$



We have

$0 \le x \le 1$

$\Rightarrow 0 \le x^2 \le x \le 1$

$\Rightarrow 0 \ge -x^2 \ge -x \ge -1$

$\Rightarrow 1 \ge e^{-x^2} \ge e^{-x} \ge e^{-1}$

$$\Rightarrow \int_0^1 e^{-x^2} dx \ge \int_0^1 e^{-x} dx \quad \Rightarrow \int_0^1 e^{-x^2} dx \ge -e^{-x}\Big|_0^1 = 1 - \frac{1}{e}$$

$\therefore S \ge 1 - \frac{1}{e}$

Also $e^{-x^2} \ge \frac{1}{e}$

$$\Rightarrow \int_0^1 e^{-x^2} dx \ge \int_0^1 \frac{1}{e} dx = \frac{1}{e}$$

$\therefore S \ge \frac{1}{e}$

(C) is wrong because (B) is correct.

$$\int_0^1 e^{-x^2} dx \le \frac{1}{\sqrt{2}} \times 1 + \left(1 - \frac{1}{\sqrt{2}}\right) \cdot e^{-\frac{1}{2}}$$

$$\le \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$$

55. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, parallel to the straight line 2x – y = 1. The points of contact of the tangents on the hyperbola are

(A) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ (C) $3\sqrt{3}, -2\sqrt{2}$ (D) $-3\sqrt{3}, 2\sqrt{2}$

55. (A), (B)

Slope of tangent = m = 2

Equation of tangent in slope form is

$y = mx \pm \sqrt{a^2m^2 - b^2}$

$y = 2x \pm 4\sqrt{2}$

and point of contact is $\left(-\frac{ma^2}{c}, \frac{-b^2}{c}\right)$

$$\equiv \left(-\frac{2\times 9}{\pm 4\sqrt{2}}, -\frac{4}{\pm 4\sqrt{2}}\right)$$

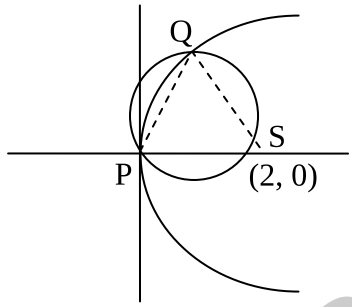
$$\equiv \left(\mp\frac{9}{2\sqrt{2}}, \mp\frac{1}{\sqrt{2}}\right)$$

SECTION III : Integer Answer Type

This section contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

56. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is

56. [4]



Solving $y^2 = 8x$
and $x^2 + y^2 - 2x - 4y = 0$
Simultaneously, we get (2, 4) and (0, 0)

Focus is (2, 0)

$\therefore$ Area = $\frac{1}{2} \times 2 \times 4$ = 4 sq. units.

57. Let p(x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 and p(3) = 2, then p′(0) is

57. [9]

Since least degree hence p(x) is cubic.
Let $p(x) = ax^3 + bx^2 + cx + d$
$p(1) = 6 = a + b + c + d$ …… (1)
$p(3) = 2 = 27a + 9b + 3c + d$ …… (2)
$p'(x) = 3ax^2 + 2bx + c$
$p'(1) = 0 = 3a + 2b + c$ …… (3)
$p'(3) = 0 = 27a + 6b + c$ …… (4)

(4) – (3) $\Rightarrow 24a + 4b = 0 \Rightarrow b = -6a$ …… (5)
(2) – (1) $\Rightarrow -4 = 26a + 8b + 2c$
From (5)
$-4 = 26a - 48a + 2c$
$\Rightarrow c = \frac{-4 + 22a}{2} = 11a - 2$ …… (6)
From (3)
$3a + 2b + c = 0$
$3(a) + 2(-6a) + (11a - 2) = 0$
$3a - 12a + 11a - 2 = 0$
$2a = 2 \Rightarrow a = 1$
$\therefore c = f'(0) = 11a - 2 = 11 - 2 = 9$

58. Let f : IR → IR be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is

58. [5]

$$f(x) = |x| + |x^2 - 1|$$

$$= \begin{cases} x^2 - x - 1 & \text{if} \quad x \le -1 \\ -x^2 - x + 1 & \text{if} \quad -1 \le x < 0 \\ -x^2 + x + 1 & \text{if} \quad 0 < x < 1 \\ x^2 + x - 1 & \text{if} \quad x \ge 1 \end{cases}$$

So, total no. of local maxima and local minima is = 5

59. The value of $6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots}}}\right)$ is

59. [4]

$$6+\log_{3/2}\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots}}}$$

put $x = \sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots}}$

$x^2 = 4-\frac{x}{3\sqrt{2}}$

$3\sqrt{2}x^2 = 12\sqrt{2}-x$

$3\sqrt{2}x^2+x-12\sqrt{2} = 0$

$\Rightarrow\ x = \frac{4\sqrt{2}}{3};\qquad x = -\sqrt{3}/2$ not possible

$6+\log_{3/2}\left(\frac{1}{3\sqrt{2}}\times\frac{4\sqrt{2}}{3}\right)$

$6+\log_{3/2}\left(\frac{4}{9}\right)$

6 – 2 = 4

60. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is

60. [3]

$|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$

$\Rightarrow\ 2-2\ \ \vec{a}\cdot\vec{b}\ \ +2-2\ \ \vec{b}\cdot\vec{c}\ \ +2-2\ \ \vec{c}\cdot\vec{a}\ \ =9$

$\Rightarrow\ -\frac{3}{2}=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ ………… (1)

Now, $|\vec{a}+\vec{b}+\vec{c}|^2\ \ \geq 0$

$\Rightarrow\ 1+1+1+2\ \ \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\ \ \geq 0$

$\Rightarrow\ \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\geq-\frac{3}{2}$ ………… (2)

Equation (1) and (2) are simultaneously true

if $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=-\frac{1}{2}$

Now, $|2\vec{a}+5\vec{b}+5\vec{c}|^2$

$=\ \ 4+25+25+20\ \ \vec{a}\cdot\vec{b}\ \ +50\ \ \vec{b}\cdot\vec{c}\ \ +20(\vec{a}\cdot\vec{c})$

$= 54-10-25-10=9\Rightarrow|2\vec{a}+5\vec{b}+5\vec{c}|=3$

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