MATHEMATICS (51)

Aims:

1. To acquire knowledge and understanding of the terms, symbols, concepts, principles, processes, proofs, etc. of mathematics.
2. To develop an understanding of mathematical concepts and their application to further studies in mathematics and science.
3. To develop skills to apply mathematical knowledge to solve real life problems.
4. To develop the necessary skills to work with modern technological devices such as calculators and computers.
5. To develop drawing skills, skills of reading tables, charts and graphs.
6. To develop an interest in mathematics.

CLASS IX

There will be ***one*** *paper of* ***two and a half*** *hours duration carrying 80 marks and Internal Assessment of 20 marks.*

The paper will be divided into ***two*** *sections, Section I (40 marks), Section II (40 marks).*

Section I: *will consist of compulsory short answer questions.*

Section II: *Candidates will be required to answer* ***four*** *out of* ***seven*** *questions.*

The solution of a question may require the knowledge of more than one branch of the syllabus.

1. Pure Arithmetic

Irrational Numbers

(a) *Rational, irrational numbers as real numbers, their place in the number system. Surds and rationalization of surds.*

(b) *Irrational numbers as non-repeating, non-terminating decimals.*

(c) *Classical definition of a rational number p/q, $p, q \in Z, q \neq 0$.*

Hence, define irrational numbers as what cannot be expressed as above.

(d) *Simplifying an expression by rationalising the denominator.*

2. Commercial Mathematics

(i) Profit and Loss

The meaning of Marked price, selling price and discount, thus giving an idea of profit and loss on day to day dealings. Simple problems related to Profit and Loss and Discount, including inverse working.

(ii) Compound Interest

Compound Interest as a repeated Simple Interest computation with a growing Principal. Use of formula -

$A = P(1+\frac{r}{100})^n$. *Finding CI from the relation CI = A-P. Simple direct problems based on above formulae.*

3. Algebra

(i) Expansions

$(a \pm b)^2$

$(a \pm b)^3$

$(x \pm a)(x \pm b)$

(ii) Factorisation

$a^2 - b^2$

$a^3 \pm b^3$

$ax^2 + bx + c$, *by splitting the middle term.*

(iii) Changing the subject of a formula.

- *Concept that each formula is a perfect equation with variables.*
- *Concept of expressing one variable in terms, of another various operators on terms transposing the terms squaring or taking square root etc.*

(iv) Linear Equations and Simultaneous (linear) Equations

- *Solving algebraically (by elimination as well as substitution) and graphically.*

- *Solving simple problems based on these by framing appropriate formulae.*

(v) Indices/ Exponents

Handling positive, fractional, negative and "zero" indices.

Simplification of expressions involving various exponents

$a^m \times a^n = a^{m+n}, a^m \div a^n = a^{m-n}, (a^m)^n = a^{mn}$

etc use of laws of exponents.

(vi) Logarithms

(a) Logarithmic form vis-à-vis exponential form: interchanging.

(b) Laws of Logarithms and its use

Expansion of expression with the help of laws of logarithm

eg. $y = \frac{a^4 \times b^2}{c^3}$

$\log y = 4 \log a + 3 \log b - 3 \log c$ *etc. .*

4. Geometry

(i) Triangles, Relation between sides and angles of triangles. Types of triangles, Congruent triangles.

(a) Congruency: four cases: SSS, SAS, AAS, RHS. Illustration through cutouts. Simple applications.

(b) Problems based on:

- *Angles opposite equal sides are equal and converse.*
- *If two sides of a triangle are unequal, then the greater angle is opposite the greater side and converse.*
- *Sum of any two sides of a triangle is greater than the third side.*
- *Of all straight lines that can be drawn to a given line from a point outside it, the perpendicular is the shortest.*

Proofs not required.

(ii) Constructions (using ruler and compasses*)*

Constructions of triangles involving 30°, 45°, 60°, 75°, 90°, 120°, 135° angles.

(iii) Mid Point Theorem and its converse, equal intercept theorem

(a) Proof and simple applications of mid point theorem and its converse.

(b) Equal intercept theorem: proof and simple application.

(iv) Similarity, conditions of similar triangles.

(a) As a size transformation.

(b) Comparison with congruency, keyword being proportionality.

(c) Three conditions: SSS, SAS, AA. Simple applications (proof not included).

(d) Applications of Basic Proportionality Theorem.

(v) Pythagoras Theorem

Proof and Simple applications of Pythagoras Theorem and its converse.

(vi) Rectilinear Figures

Rectilinear figures or polygons, Different kinds of polygons and its names interior and exterior angles and their relations. Types of regular polygons parallelograms, conditions of parallelograms, Rhombus, Rectangles. Proof and use of theorems on parallelogram.

(a) Sum of interior angles of a polygon.

(b) Sum of exterior angles of a polygon.

(c) Regular polygons.

(d) Parallelogram:

- *Both pairs of opposite sides equal (without proof).*
- *Both pairs of opposite angles equal.*
- *One pair of opposite sides equal and parallel (without proof).*
- *Diagonals bisect each other and bisect the parallelogram.*
- *Rhombus as a special parallelogram whose diagonals meet at right angles.*
- *In a rectangle, diagonals are equal, in a square they are equal and meet at right angles.*

(e) Quadrilaterals

Construction of quadrilaterals (including parallelograms and rhombus) and regular hexagon using ruler and a pair of compasses only.

(f) Proof and use of area theorems on parallelograms:

- *Parallelograms on the same base and between the same parallels are equal in area.*
- *The area of a triangle is half that of a parallelogram on the same base and between the same parallels.*
- *Triangles between the same base and between the same parallels are equal in area (without proof).*
- *Triangles with equal areas on the same bases have equal corresponding altitudes.*

Note: Proofs of the theorems given above are to be taught unless specified otherwise.

5. Statistics

Introduction, collection of data, presentation of data, Graphical representation of data, Mean, Median of ungrouped data.

(i) Understanding and recognition of raw, arrayed and grouped data.

(ii) Tabulation of raw data using tally-marks.

(iii) Understanding and recognition of discrete and continuous variables.

(iv) Mean, median of ungrouped data

(v) Class intervals, class boundaries and limits, frequency, frequency table, class size for grouped data.

(vi) Grouped frequency distributions: the need to and how to convert discontinuous intervals to continuous intervals.

(vii) Drawing a histogram and frequency polygon.

(viii) Understanding of how a histogram differs from a bar chart.

6. Mensuration

Area and perimeter of a triangle and a quadrilateral. Area and circumference of a circle. Surface area and volume of Cube, Cuboids and Cylinder.

(a) Area and perimeter of triangle (including Heron's formula), square, rhombus, rectangle, parallelogram and trapezium.

(b) (i) Circle: Area and circumference

(ii) Simple direct problems involving inner and outer dimensions and cost.

(c) Surface area and volume of 3-D solids: cube, cuboid and cylinder including problems of type involving:

- *Different internal and external dimensions of the solid.*
- *Cost.*
- *Concept of volume being equal to area of cross-section x height.*
- *Open/closed cubes/cuboids/cylinders.*

7. Trigonometry

(a) Trigonometric Ratios: sine, cosine, tangent of an angle and their reciprocals.

(b) Trigonometric ratios of standard angles- 0, 30, 45, 60, 90 degrees. Evaluation of an expression involving these ratios.

(c) Simple 2-D problems involving one right-angled triangle.

(d) Concept of sine and cosine being complementary with simple, direct application.

8. Co-ordinate Geometry

Cartesian System, Plotting a point in the plane for given coordinates.

(a) Dependent and independent variables.

(b) Ordered pairs, co-ordinates of points and plotting them in the Cartesian Plane.

(c) Graphs of $x=0$, $y=0$, $x=a$, $y=a$, $x=y$, $y= mx+c$ including identification and conceptual understanding of slope and y-intercept.

(d) Recognition of graphs based on the above.

INTERNAL ASSESSMENT

A minimum of three assignments are to be done during the year as prescribed by the teacher.

Suggested Assignments

- Surveys of a class of students - height, weight, number of family members, pocket money, etc.
- Correlation of body weight to body height.
- Planning delivery routes for a postman/milkman.
- Running a tuck shop/canteen.
- Visit one or two stores where sales are being offered to investigate - cost price, marked price, selling price, discount, profit/loss.
- Study ways of raising a loan to buy a car or house, e.g. bank loan or purchase a refrigerator or a television set through hire purchase.

CLASS X

*There will be **one** paper of **two and a half** hours duration carrying 80 marks and Internal Assessment of 20 marks.*

*The paper will be divided into **two** sections, Section I (40 marks), Section II (40 marks).*

***Section I:** Will consist of compulsory short answer questions.*

***Section II:** Candidates will be required to answer **four** out of **seven** questions.*

1. Commercial Mathematics

(i) Compound Interest

(a) Compound interest as a repeated Simple Interest computation with a growing Principal. Use of this in computing Amount over a period of 2 or 3-years.

(b) Use of formula $A = P\ (1+ r/100)^n$. Finding CI from the relation CI = A – P.

- *Interest compounded half-yearly included.*
- *Using the formula to find one quantity given different combinations of A, P, r, n, CI and SI; difference between CI and SI type included.*
- *Rate of growth and depreciation.*

Note: Paying back in equal installments, being given rate of interest and installment amount, **not included**.

(ii) Sales Tax and Value Added Tax

Computation of tax including problems involving discounts, list-price, profit, loss, basic/cost price including inverse cases.

(iii) Banking

(a) Savings Bank Accounts.

Types of accounts. Idea of savings Bank Account, computation of interest for a series of months.

(b) Recurring Deposit Accounts*: computation of interest using the formula:*

$$\text{SI} = P\frac{n(n+1)}{2\times 12}\times\frac{r}{100}$$

(iv) Shares and Dividends

(a) Face/Nominal Value, Market Value, Dividend, Rate of Dividend, Premium.

(b) Formulae

- *Income = number of shares × rate of dividend × FV.*
- *Return = (Income / Investment) ×100.* ***Note:*** *Brokerage and fractional shares* ***not included***

2. Algebra

(i) Linear Inequations

Linear Inequations in one unknown for $x \in N$, W, Z, R. Solving

- *Algebraically and writing the solution in set notation form.*
- *Representation of solution on the number line.*

(ii) Quadratic Equations

(a) Quadratic equations in one unknown. Solving by:

- *Factorisation.*
- *Formula.*

(b) Nature of roots,
Two distinct real roots if $b^2 - 4ac > 0$
Two equal real roots if $b^2 - 4ac = 0$
No real roots if $b^2 - 4ac < 0$

(c) Solving problems.

(iii) Reflection

(a) Reflection of a point in a line:

x=0, y =0, x= a, y=a, the origin.

(b) Reflection of a point in the origin.

(c) Invariant points.

(iv) Ratio and Proportion

(a) *Duplicate, triplicate, sub-duplicate, sub-triplicate, compounded ratios.*

(b) *Continued proportion, mean proportion*

(c) *Componendo and dividendo, alternendo and invertendo properties.*

(d) *Direct applications*.

(v) Factorization

(a) *Factor Theorem.*

(b) *Remainder Theorem.*

(c) *Factorising a polynomial completely after obtaining one factor by factor theorem.*

Note: f (x) not to exceed degree 3.

(vi) Matrices

(a) *Order of a matrix. Row and column matrices.*

(b) *Compatibility for addition and multiplication.*

(c) *Null and Identity matrices.*

(d) *Addition and subtraction of* 2×2 *matrices.*

(e) *Multiplication of a* 2×2 *matrix by*

- *a non-zero rational number*
- *a matrix.*

(vii) Co-ordinate Geometry

Co-ordinates expressed as (x,y) Distance between two points, section, and Midpoint formula, Concept of slope, equation of a line, Various forms of straight lines.

(a) *Distance formula.*

(b) *Section and Mid-point formula (Internal section only, co-ordinates of the centroid of a triangle included).*

(c) *Equation of a line:*

- *Slope –intercept form* $y = mx + c$
- *Two- point form* $(y-y_1) = m(x-x_1)$

  Geometric understanding of 'm' as slope/ gradient/ $\tan\theta$ *where* θ *is the angle the line makes with the positive direction of the x axis.*

  Geometric understanding of c as the y-intercept/the ordinate of the point where the line intercepts the y axis/ the point on the line where x=0.

- *Conditions for two lines to be parallel or perpendicular. Simple applications of all of the above.*

3. Geometry

(i) Symmetry

(a) *Lines of symmetry of an isosceles triangle, equilateral triangle, rhombus, square, rectangle, pentagon, hexagon, octagon (all regular) and diamond-shaped figure.*

(b) *Being given a figure, to draw its lines of symmetry. Being given part of one of the figures listed above to draw the rest of the figure based on the given lines of symmetry (neat recognizable free hand sketches acceptable).*

(ii) Similarity

Axioms of similarity of triangles. Basic theorem of proportionality.

(a) *Areas of similar triangles are proportional to the squares on corresponding sides.*

(b) *Direct applications based on the above including applications to maps and models.*

(iii) Loci

Loci: Definition, meaning, Theorems based on Loci.

(a) *The locus of a point equidistant from a fixed point is a circle with the fixed point as centre.*

(b) *The locus of a point equidistant from two interacting lines is the bisector of the angles between the lines.*

(c) The locus of a point equidistant from two given points is the perpendicular bisector of the line joining the points.

(iv) Circles

(a) *Chord Properties:*

- *A straight line drawn from the center of a circle to bisect a chord which is not a diameter is at right angles to the chord.*
- *The perpendicular to a chord from the center bisects the chord (without proof).*
- *Equal chords are equidistant from the center.*
- *Chords equidistant from the center are equal (without proof).*
- *There is one and only one circle that passes through three given points not in a straight line.*

(b) Arc and chord properties:

- *The angle that an arc of a circle subtends at the center is double that which it subtends at any point on the remaining part of the circle.*
- *Angles in the same segment of a circle are equal (without proof).*
- *Angle in a semi-circle is a right angle.*
- *If two arcs subtend equal angles at the center, they are equal, and its converse.*
- *If two chords are equal, they cut off equal arcs, and its converse (without proof).*
- *If two chords intersect internally or externally then the product of the lengths of the segments are equal.*

(c) Cyclic Properties:

- *Opposite angles of a cyclic quadrilateral are supplementary.*
- *The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle (without proof).*

(d) Tangent Properties:

- *The tangent at any point of a circle and the radius through the point are perpendicular to each other.*
- *If two circles touch, the point of contact lies on the straight line joining their centers.*
- *From any point outside a circle two tangents can be drawn and they are equal in length.*
- *If a chord and a tangent intersect externally, then the product of the lengths of segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.*
- *If a line touches a circle and from the point of contact, a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments.*

Note: Proofs of the theorems given above are to be taught unless specified otherwise.

(v) Constructions

(a) Construction of tangents to a circle from an external point.

(b) Circumscribing and inscribing a circle on a triangle and a regular hexagon.

4. Mensuration

Area and circumference of circle, Area and volume of solids – cone, sphere.

(a) Circle: Area and Circumference. Direct application problems including Inner and Outer area..

(b) Three-dimensional solids - right circular cone and sphere: Area (total surface and curved surface) and Volume. Direct application problems including cost, Inner and Outer volume and melting and recasting method to find the volume or surface area of a new solid. Combination of two solids included.

Note: Frustum is not included.

Areas of sectors of circles other than quarter-circle and semicircle are not included.

5. Trigonometry

(a) Using Identities to solve/prove simple algebraic trigonometric expressions

$sin^2 A + cos^2 A = 1$

$1 + tan^2 A = sec^2 A$

$1+cot^2 A = cosec^2 A;\ 0 \le A \le 90°$

(b) *Trigonometric ratios of complementary angles and direct application:*

$\sin A = \cos(90 - A), \cos A = \sin(90 - A)$

$\tan A = \cot(90 - A), \cot A = \tan(90 - A)$

$\sec A = \mathrm{cosec}(90 - A), \mathrm{cosec}\ A = \sec(90 - A)$

(c) *Heights and distances: Solving 2-D problems involving angles of elevation and* depression using trigonometric tables.

Note: Cases involving more than two right angled triangles excluded.

6. Statistics

Statistics – basic concepts, , Histograms and Ogive, Mean, Median, Mode.

(a) *Graphical Representation. Histograms and ogives.*

- *Finding the mode from the histogram, the upper quartile, lower Quartile and median from the ogive.*
- *Calculation of inter Quartile range.*

(b) *Computation of:*

- *Measures of Central Tendency: Mean, median, mode for raw and arrayed data. Mean*, median class and modal class for grouped data. (both continuous and discontinuous).*

** Mean by all 3 methods included:*

Direct : $\frac{\Sigma fx}{\Sigma f}$

Short-cut : $A + \frac{\Sigma fd}{\Sigma f}$ where $d = x - A$

Step-deviation:

$$A + \frac{\Sigma ft}{\Sigma f} \times i \text{ where } t = \frac{x - A}{i}$$

7. Probability

- *Random experiments*
- *Sample space*
- *Events*
- *Definition of probability*
- *Simple problems on single events*

(tossing of one or two coins, throwing a die and selecting a student from a group)

Note: SI units, signs, symbols and abbreviations

(1) Agreed conventions

(a) Units may be written in full or using the agreed symbols, but no other abbreviation may be used.

(b) The letter 's' is never added to symbols to indicate the plural form.

(c) A full stop is not written after symbols for units unless it occurs at the end of a sentence.

(d) When unit symbols are combined as a quotient, e.g. metre per second, it is recommended that they be written as m/s, or as m s^{-1}.

(e) Three decimal signs are in common international use: the full point, the mid-point and the comma. Since the full point is sometimes used for multiplication and the comma for spacing digits in large numbers, it is recommended that the mid-point be used for decimals.

(2) Names and symbols

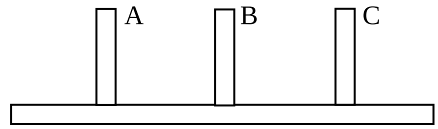
In general			
Implies that	$\Rightarrow$	is logically equivalent to	$\Leftrightarrow$
Identically equal to	$\equiv$	is approximately equal to	>>
In set language			
Belongs to	$\in$	does not belong to	$\notin$
is equivalent to	$\leftrightarrow$	is not equivalent to	$\not\leftrightarrow$
union	$\cup$	intersection	$\cap$
universal set	ξ	is contained in	$\subset$
natural (counting) numbers	N	the empty set	ø
		whole numbers	W
integers	Z	real numbers	R
In measures			
Kilometre	km	Metre	m
Centimetre	cm	Millimetre	mm
Kilogram	kg	Gram	g
Litre	l	Centilitre	cl
square kilometre	km^2	Square meter	m^2
square centimetre	cm^2	Hectare	ha
cubic metre	m^3	Cubic centimetre	cm^3
kilometres per hour	km/h	Metres per second	m/s

INTERNAL ASSESSMENT

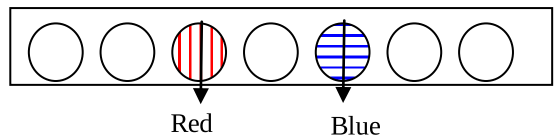
The minimum number of assignments: Three assignments as prescribed by the teacher.

Suggested Assignments

- Comparative newspaper coverage of different items.
- Survey of various types of Bank accounts, rates of interest offered.
- Planning a home budget.
- Cutting a circle into equal sections of a small central angle to find the area of a circle by using the formula $A = \pi r^2$.
- To use flat cut outs to form cube, cuboids, pyramids and cones and to obtain formulae for volume and total surface area.
- To use a newspaper to study and report on shares and dividends.
- Draw a circle of radius r on a ½ cm graph paper, and then on a 2 mm graph paper. Estimate the area enclosed in each case by actually counting the squares. Now try out with circles of different radii. Establish the pattern, if any, between the two observed values and the theoretical value (area = πr^2). Any modifications?
- Set up a dropper with ink in it vertical at a height say 20 cm above a horizontally placed sheet of plain paper. Release one ink drop; observe the pattern, if any, on the paper. Vary the vertical distance and repeat. Discover any pattern of relationship between the vertical height and the ink drop observed.
- You are provided (or you construct a model as shown) - three vertical sticks (size of a pencil) stuck to a horizontal board. You should also have discs of varying sizes with holes (like a doughnut). Start with one disc; place it on (in) stick A. Transfer it to another stick (B or C); this is one move (m). Now try with two discs placed in A such that the large disc is below and the smaller disc is above (number of discs = n=2 now). Now transfer them one at a time in B or C to obtain similar situation (larger disc below). How many moves? Try with more discs (n = 1, 2, 3, etc.) and generalise.



- The board has some holes to hold marbles, red on one side and blue on the other. Start with one pair. Interchange the positions by making one move at a time. A marble can jump over another to fill the hole behind. The move (m) equal 3. Try with 2 (n=2) and more. Find relationship between n and m.



- Take a square sheet of paper of side 10 cm. Four small squares are to be cut from the corners of the square sheet and then the paper folded at the cuts to form an open box. What should be the size of the squares cut so that the volume of the open box is maximum?
- Take an open box, four sets of marbles (ensuring that marbles in each set are of the same size) and some water. By placing the marbles and water in the box, attempt to answer the question: do larger marbles or smaller marbles occupy more volume in a given space?
- An eccentric artist says that the best paintings have the same area as their perimeter (numerically). Let us not argue whether such sizes increases the viewer's appreciation, but only try and find what sides (in integers only) a rectangle must have if its area and perimeter are to be equal (note: there are only two such rectangles).
- Find by construction the centre of a circle, using only a 60-30 setsquare and a pencil.
- Various types of "cryptarithm".

EVALUATION

The assignments/project work are to be evaluated by the subject teacher and by an External Examiner. (The External Examiner may be a teacher nominated by the Head of the school, who could be from the faculty, **but not teaching the subject in the section/class**. For example, a teacher of Mathematics of Class VIII may be deputed to be an External Examiner for Class X, Mathematics projects.)

The Internal Examiner and the External Examiner will assess the assignments independently.

Award of marks (20 Marks)

Subject Teacher (Internal Examiner)	:	10 marks
External Examiner	:	10 marks

The total marks obtained out of 20 are to be sent to the Council by the Head of the school.

The Head of the school will be responsible for the entry of marks on the mark sheets provided by the Council.

INTERNAL ASSESSMENT IN MATHEMATICS- GUIDELINES FOR MARKING WITH GRADES

Criteria	Preparation	Concepts	Computation	Presentation	Understanding	Marks
Grade I	Exhibits and selects a well defined problem. Appropriate use of techniques.	Admirable use of mathematical concepts and methods and exhibits competency in using extensive range of mathematical techniques.	Careful and accurate work with appropriate computation, construction and measurement with correct units.	Presents well stated conclusions; uses effective mathematical language, symbols, conventions, tables, diagrams, graphs, etc.	Shows strong personal contribution; demonstrate knowledge and understanding of assignment and can apply the same in different situations.	4 marks for each criterion
Grade II	Exhibits and selects routine approach. Fairly good techniques.	Appropriate use of mathematical concepts and methods and shows adequate competency in using limited range of techniques.	Commits negligible errors in computation, construction and measurement.	Some statements of conclusions; uses appropriate math language, symbols, conventions, tables, diagrams, graphs, etc.	Neat with average amount of help; assignment shows learning of mathematics with a limited ability to use it.	3 marks for each criterion
Grade III	Exhibits and selects trivial problems. Satisfactory techniques.	Uses appropriate mathematical concepts and shows competency in using limited range of techniques.	Commits a few errors in computation, construction and measurement.	Assignment is presentable though it is disorganized in some places.	Lack of ability to conclude without help; shows some learning of mathematics with a limited ability to use it.	2 marks for each criterion
Grade IV	Exhibits and selects an insignificant problem. Uses some unsuitable techniques.	Uses inappropriate mathematical concepts for the assignment.	Commits many mistakes in computation, construction and measurement.	Presentation made is somewhat disorganized and untidy.	Lack of ability to conclude even with considerable help; assignment contributes to mathematical learning to a certain extent.	1 mark for each criterion
Grade V	Exhibits and selects a completely irrelevant problem. Uses unsuitable techniques.	Not able to use mathematical concepts.	Inaccurate computation, construction and measurement.	Presentation made is completely disorganized, untidy and poor.	Assignment does not contribute to mathematical learning and lacks practical applicability.	0 mark