Q. No. 1 – 5 Carry One Mark Each

1. "India is a country of rich heritage and cultural diversity." Which one of the following facts best supports the claim made in the above sentence?

   (A) India is a union of 28 states and 7 union territories.

   (B) India has a population of over 1.1 billion.

   (C) India is home to 22 official languages and thousands of dialects.

   (D) The Indian cricket team draws players from over ten states.

Answer: C

Exp: Diversity is shown in terms of difference language

2. The value of one U.S. dollar is 65 Indian Rupees today, compared to 60 last year. The Indian Rupee has ____________.

   (A) Depressed (B) Depreciated (C) Appreciated (D) Stabilized

Answer: B

3. 'Advice' is ________________.

   (A) a verb (B) a noun

   (C) an adjective (D) both a verb and a noun

Answer: B

4. The next term in the series 81, 54, 36, 24 … is ________

Answer: 16

Exp: $81-54=27;27\times\frac{2}{3}=18$

$54-36=18;18\times\frac{2}{3}=12$

$36-24=12;12\times\frac{2}{3}=8$

$\therefore 24-8=16$

5. In which of the following options will the expression P < M be definitely true?

   (A) M < R > P > S (B) M > S < P < F

   (C) Q < M < F = P (D) P = A < R < M

Answer: D

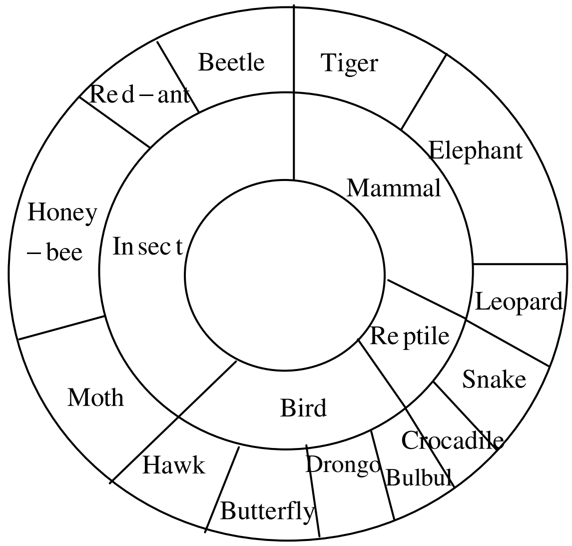
Q. No. 6 – 10 Carry Two Marks Each

6. Find the next term in the sequence: 7G, 11K, 13M, ___

   (A) 15Q (B) 17Q (C) 15P (D) 17P

Answer: B

7. The multi-level hierarchical pie chart shows the population of animals in a reserve forest. The correct conclusions from this information are:



(i) Butterflies are birds

(ii) There are more tigers in this forest than red ants

(iii) All reptiles in this forest are either snakes or crocodiles

(iv) Elephants are the largest mammals in this forest

(A) (i) and (ii) only (B) (i), (ii), (iii) and (iv)

(C) (i), (iii) and (iv) only (D) (i), (ii) and (iii) only

Answer: D

Exp: It is not mentioned that elephant is the largest animal

8. A man can row at 8 km per hour in still water. If it takes him thrice as long to row upstream, as to row downstream, then find the stream velocity in km per hour.

Answer: 4

Exp: 4 km/hr.

Speed of man=8

Left distance =d

Time taken=$\frac{d}{8}$

Upstream:

Speed of stream=s

$\Rightarrow$ speed upstream $= S' = (8-s)$

$t' = \left(\frac{d}{8-s}\right)$

Downstream:

Given speed downstream $= t'' = \frac{d}{8+s}$

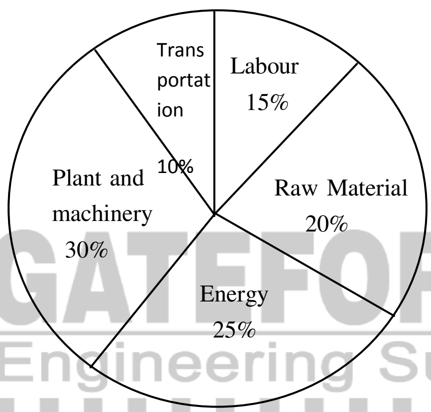
$\Rightarrow 3t' = t''$

$\Rightarrow \frac{3d}{8-s} = \frac{d}{8+s}$

$\Rightarrow \frac{3d}{8-s} = \frac{d}{8+s}$

$\Rightarrow s = 4km/hr$

9. A firm producing air purifiers sold 200 units in 2012. The following pie chart presents the share of raw material, labour, energy, plant & machinery, and transportation costs in the total manufacturing cost of the firm in 2012. The expenditure on labour in 2012 is Rs. 4,50,000. In 2013, the raw material expenses increased by 30% and all other expenses increased by 20%. If the company registered a profit of Rs. 10 lakhs in 2012, at what price (in Rs.) was each air purifier sold?



Answer: 20,000

Exp: Total expenditure$= = \frac{15}{100}x = 4,50,000$

$x = 3\times10^6$

Profit=10 lakhs

So, total selling price =40,00,000 … (1)

Total purifies=200 … (2)

S.P of each purifier=(1)/(2)=20,000

10. A batch of one hundred bulbs is inspected by testing four randomly chosen bulbs. The batch is rejected if even one of the bulbs is defective. A batch typically has five defective bulbs. The probability that the current batch is accepted is __________

Answer: 0.8145

Exp: Probability for one bulb to be non defective is $\frac{95}{100}$

$\therefore$ Probabilities that none of the bulbs is defectives $\left(\frac{95}{100}\right)^4 = 0.8145$

Q.No. 1 – 25 Carry One Mark Each

1. The maximum value of the function f(x) = ln(1 + x)- x (where .x > - 1) occurs at x=______.

Answer: 0

Exp: $f^{1}(x)=0 \Rightarrow \frac{1}{1+x}-1=0$

$\Rightarrow \frac{-x}{1+x}=0 \Rightarrow x=0$

and $f^{11}(x)=\frac{-1}{(1+x)^2}<0$ at $x=0$

2. Which ONE of the following is a linear non-homogeneous differential equation, where x and y are the independent and dependent variables respectively?

(A) $\frac{dy}{dx}+xy=e^{-x}$ (B) $\frac{dy}{dx}+xy=0$

(C) $\frac{dy}{dx}+xy=e^{-y}$ (D) $\frac{dy}{dx}+e^{-y}=e^{-y}=0$

Answer: A

Exp: (A) $\frac{dy}{dx}+xy=e^{-x}$ is a first order linear equation (non-homogeneous)

(B) $\frac{dy}{dx}+xy=0$ is a first order linear equation (homogeneous

(C), (D) are non linear equations

3. Match the application to appropriate numerical method.

Application	Numerical |Method
P1: Numerical integration	M1: Newton-Raphson Method
P2: Solution to a transcendental equation	M2: Runge-Kutta Method
P3: Solution to a system of linear equations	M3: Simpson's 1/3-rule
P4: Solution to a differential equation	M4: Gauss Elimination Method

(A) P1—M3, P2—M2, P3—M4, P4—M1 (B) P1—M3, P2—M1, P3—M4, P4—M2

(C) P1—M4, P2—M1, P3—M3, P4—M2 (D) P1—M2, P2—M1, P3—M3, P4—M4

Answer: B

Exp: $P1-M3, P2-M1, P3-M4, P4-M2$

4. An unbiased coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is

(A) 0.067 (B) 0.073 (C) 0.082 (D) 0.091

Answer: C

Exp: P[fourth head appears at the tenth toss] = P [getting 3 heads in the first 9 tosses and one head at tenth toss]

$$= \left[9_{C_3} \cdot \left(\frac{1}{2}\right)^9 \right] \times \left[\frac{1}{2}\right] = \frac{21}{256} = 0.082$$

5. If z = xyln(xy), then

(A) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$ (B) $y\frac{\partial z}{\partial x} = x\frac{\partial z}{\partial y}$

(C) $x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$ (D) $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$

Answer: C

Exp: $\frac{\partial z}{\partial x} = y\left[x \times \frac{1}{xy} \times y + \ln xy \right] = y(1 + \ln xy)$

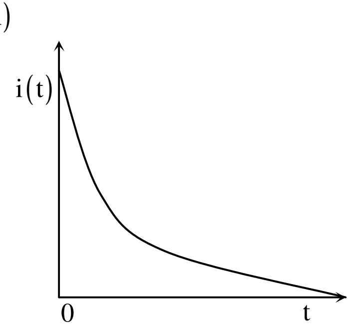
and $\frac{\partial z}{\partial y} = x(1 + \ln xy) \Rightarrow x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$

6. A series RC circuit is connected to a DC voltage source at time t = 0. The relation between the
source voltage V_S, the resistance R, the capacitance C, and the current i(t) is given below:

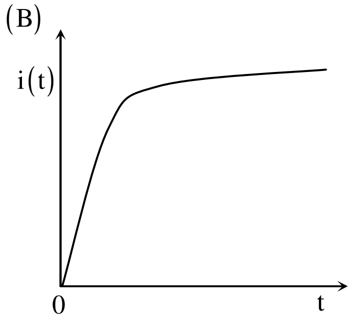
$$V_e = Ri(t) + \frac{1}{c}\int_0^t i(u)\,du$$

Which one of the following represents the current f(t)?

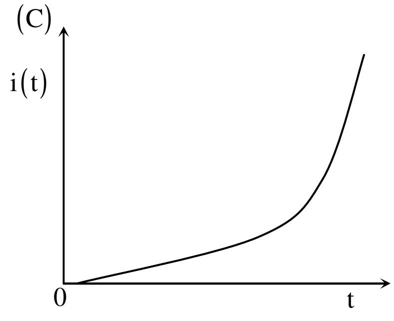
(A)



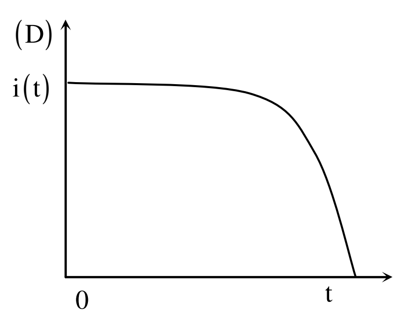
(B)



(C)



(D)

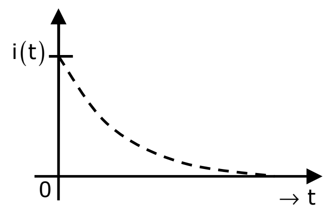


Answer: A

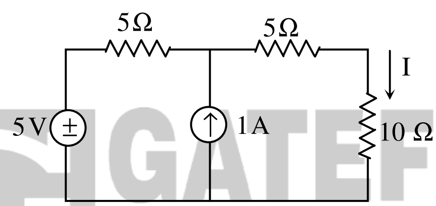
Exp: In a series RC circuit,

$\rightarrow$ Initially at t = 0, capacitor charges with a current of $\frac{V_S}{R}$ and in steady state at $t = \infty$, capacitor behaves like open circuit and no current flows through the circuit

$\rightarrow$ So the current i(t) represents an exponential decay function

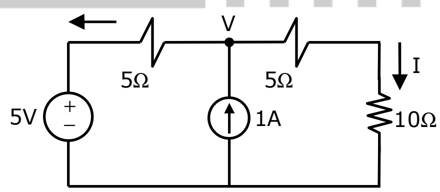


7. In the figure shown, the value of the current I (in Amperes) is __________.



Answer: 0.5

Exp:



Apply KCL at node V, $\frac{V-5}{5} - 1 + \frac{V}{15} = 0$

$$\Rightarrow V = \frac{30}{4} \text{ volts}$$

$$\Rightarrow \text{current } I = \frac{V}{15} \Rightarrow \frac{2}{4} \Rightarrow 0.50 \text{ Amperes}$$

8. In MOSFET fabrication, the channel length is defined during the process of

(A) Isolation oxide growth

(B) Channel stop implantation

(C) Poly-silicon gate patterning

(D) Lithography step leading to the contact pads

Answer: C

9. A thin P-type silicon sample is uniformly illuminated with light which generates excess carriers. The recombination rate is directly proportional to

(A) The minority carrier mobility

(B) The minority carrier recombination lifetime

(C) The majority carrier concentration

(D) The excess minority carrier concentration

Answer: D

Exp: Recombination rate, $R = B\left(n_{n_o} + n_n^{'}\right)\left(P_{n_o} + P_n^{'}\right)$

n_{n_0} & P_{n_0} = Electron and hole concentrations respectively under thermal equilibrium

$n_n^{'}$ & $p_n^{'}$ = Excess elements and hole concentrations respectively

10. At T = 300 K, the hole mobility of a semiconductor $\mu_P = 500\,\text{cm}^2/\text{V}-\text{s}$ and $\dfrac{kT}{q} = 26\,\text{mV}$.

The hole diffusion constant D_P in cm^2/s is ________

Answer: 13

Exp: From Einstein relation,

$$\frac{D_P}{\mu_p} = \frac{kJ}{q}$$

$$\Rightarrow D_P = 26\,\text{mV} \times 500\text{cm}^2/\text{v}-\text{s} = 13\text{cm}^2/\text{s}$$

11. The desirable characteristics of a transconductance amplifier are

(A) High input resistance and high output resistance

(B) High input resistance and low output resistance

(C) Low input resistance and high output resistance

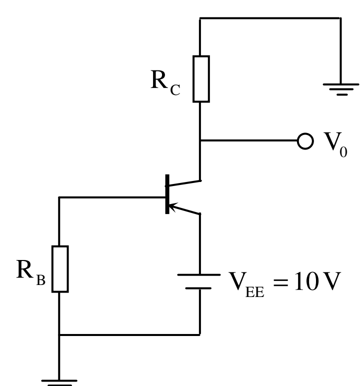
(D) Low input resistance and low output resistance

Answer: A

Exp: Transconductance amplifier must have $z_i = \infty$ and $z_0 = \infty$ ideally

12. In the circuit shown, the PNP transistor has $|V_{BE}| = 0.7$ and $\beta = 50$. Assume that $R_B = 100\text{k}\Omega$

For V_0 to be 5 V, the value of $R_C\,(\text{in k}\Omega)$ ________________



Answer: 1.075

Exp: KVL in base loop gives,

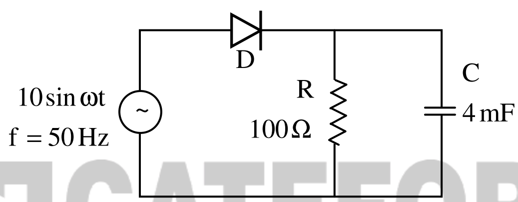
$$I_B = \frac{10-0.7}{100K} = 93\mu A$$

$$\Rightarrow I_C = \beta I_B = 50\times 93\mu A = 4.65\,mA$$

from figure, $V_0 = I_C R_C$

$$\Rightarrow R_C = \frac{V_0}{I_C} = \frac{5V}{4.65\,mA} = 1.075\,\Omega$$

13. The figure shows a half-wave rectifier. The diode D is ideal. The average steady-state current (in Amperes) through the diode is approximately ____________.



Answer: 0.09

Exp: $V_{dc} = V_m - \frac{I_{dc}}{4fc}$

$$I_{dc}R_L = V_m - \frac{I_{dc}}{4fc}$$

$$I_{dc}\left[R_L + \frac{1}{4fc}\right] = V_m$$

$$\Rightarrow I_{dc} = \frac{10}{100 + \frac{1}{4\times 50\times 4\times 10^{-3}}} = 0.09A$$

14. An analog voltage in the range 0 to 8 V is divided in 16 equal intervals for conversion to 4-bit digital output. The maximum quantization error (in V) is _________________

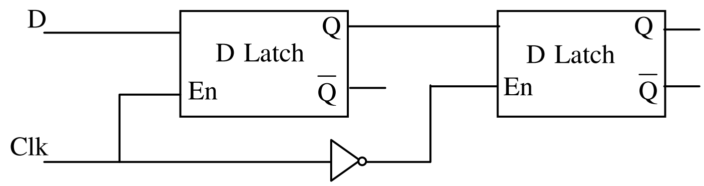
Answer: 0.25

Exp: Maximum quantization error is $\frac{\text{step} - \text{size}}{2}$

$$\text{step} - \text{size} = \frac{8-0}{16} = \frac{1}{2} = 0.5V$$

Quantization error = 0.25 V

15. The circuit shown in the figure is a

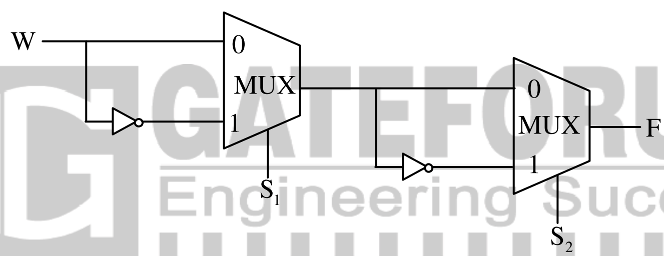


(A) Toggle Flip Flop
(B) JK Flip Flop
(C) SR Latch
(D) Master-Slave D Flip Flop

Answer: D

Exp: Latches are used to construct Flip-Flop. Latches are level triggered, so if you use two latches in cascaded with inverted clock, then one latch will behave as master and another latch which is having inverted clock will be used as a slave and combined it will behave as a flip-flop. So given circuit is implementing Master-Slave D flip-flop

16. Consider the multiplexer based logic circuit shown in the figure.

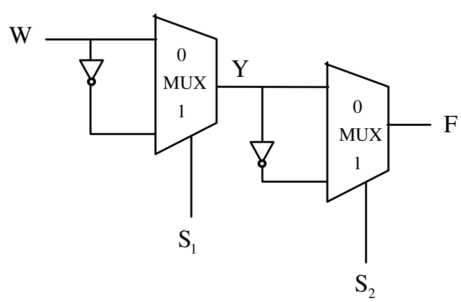


Which one of the following Boolean functions is realized by the circuit?

(A) $F = W\overline{S}_1\overline{S}_2$
(B) $F = WS_1 + WS_2 + S_1S_2$
(C) $F = \overline{W} + S_1 + S_2$
(D) $F = W \oplus S_1 \oplus S_2$

Answer: D

Exp:



Output of first MUX = $w\overline{s}_1 + \overline{w}s_1 = w \oplus s_1$

Let $Y = w \oplus s_1$

Output of second MUX = $Y\overline{s}_2 + \overline{Y}s_2$

$= Y \oplus s_2$

$= w \oplus s_1 + s_2$

17. Let x(t)= $\cos(10\pi t)$ + $\cos(30\pi t)$ be sampled at 20 Hz and reconstructed using an ideal low-pass filter with cut-off frequency of 20 Hz. The frequency/frequencies present in the reconstructed signal is/are

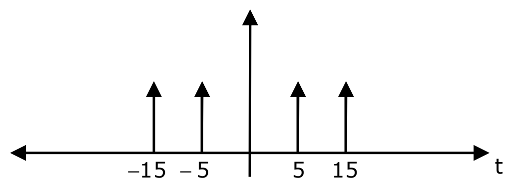
(A) 5 Hz and 15 Hz only (B) 10 Hz and 15 Hz only

(C) 5 Hz, 10 Hz and 15 Hz only (D) 5 Hz only

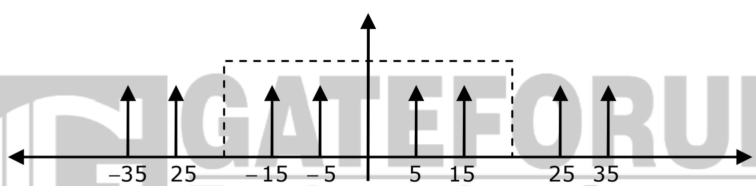
Answer: (A)

Explanation: $x(t) = \cos(10\pi t) + \cos(30\pi t),\ F_s = 20\text{Hz}$

Spectrum of x(t)



Spectrum of sampled version of x(t)



After LPF, signal will contain 5 and 15Hz component only

18. For an all-pass system $H(z) = \dfrac{(z^{-1} - b)}{(1 - az^{-1})}$, where $\left|H\left(e^{-j\omega}\right)\right| = 1$, for all ω.If $\text{Re}(a) \neq 0, \text{Im}(a) \neq 0$, then b equals

(A) a (B) a* (C) 1/a* (D) 1/a

Answer: (B)

Exp: For an all pass system, $\text{pole} = \dfrac{1}{\text{zero}^*}$ or $\text{zero} = \dfrac{1}{\text{pole}^*}$

$$\text{pole} = a$$

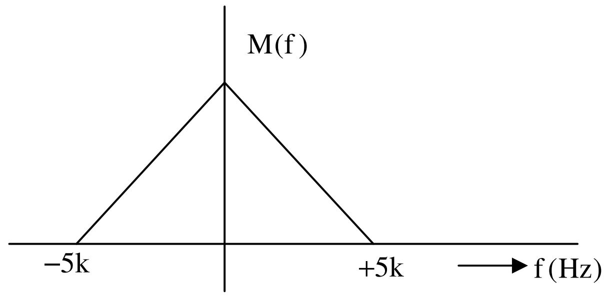
$$\text{zero} = \frac{1}{b}$$

$$\Rightarrow \frac{1}{b} = \frac{1}{a^*} \text{ or } b = a^*$$

19. A modulated signal is y(t) = m.(t)cos(40000πt), where the baseband signal m(t) has frequency components less than 5 kHz only. The minimum required rate (in kHz) at which y(t) should be sampled to recover m(t) is __________________.

Answer: 10 KHz.

Exp: Since m(t) is a base band signal with maximum frequency 5 KHz, assumed spreads as follows:

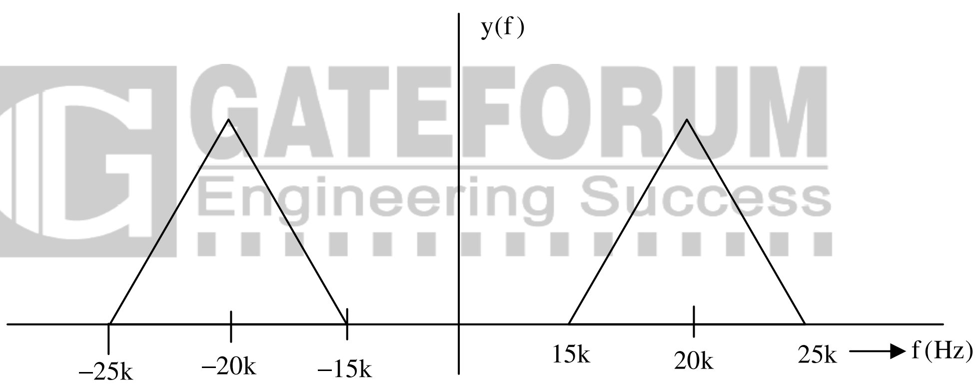


$$m(t) \xrightarrow{f} M(f)$$

$$\because\ y(t) = m(t)\cos(40000\pi t) \xrightarrow{7} m(f)\ {}^{*}\frac{1}{2}\left[\delta(f-20k)+\delta(f+20k)\right]$$

$$\therefore\ y(f) = \frac{1}{2}\left[M(f-20k)+M(f+20k)\right]$$

Thus the spectrum of the modulated signal is as follows:

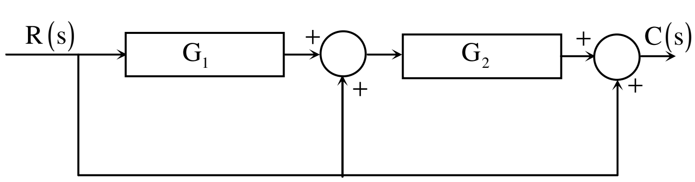


If y(t) is sampled with a sampling frequency 'f_s' then the resultant signal is a periodic extension of successive replica of y(f) with a period 'fs'.

It is observed that 10 KHz and 20 KHz are the two sampling frequencies which causes a replica of M(f) which can be filtered out by a LPF.

Thus the minimum sampling frequency (f_s) which extracts m(t) from g(f) is 10 KHz.

20. Consider the following block diagram in the figure.

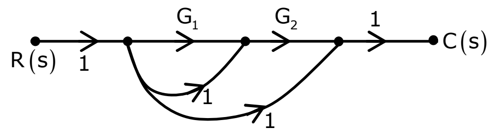


The transfer function $\frac{C(s)}{R(s)}$ is

(A) $\frac{G_1G_2}{1+G_1G_2}$ (B) $G_1G_2+G_1+1$ (C) $G_1G_2+G_2+1$ (D) $\frac{G_1}{1+G_1G_2}$

Answer: C

Exp: By drawing the signal flow graph for the given block diagram



Number of parallel paths are three

Gains $P_1 = G_1G_2,\ P_2 = G_2, P_3 = 1$

By mason's gain formula,

$$\frac{C(s)}{R(s)} = P_1 + P_2 + P_3$$

$$\Rightarrow \boxed{G_1G_2 + G_2 + 1}$$

21. The input $-3e^{2t}u(t)$, where u(t) is the unit step function, is applied to a system with transfer function . $\frac{s-2}{s+3}$. If the initial value of the output is -2, then the value of the output at steady state is__________________.

Answer: 0

Exp: 1

$$\frac{Y(s)}{X(s)} = \frac{S-2}{S+3}$$

$$\Rightarrow SY(s) + 3Y(s) = S\times(s) - 2X(s)$$

Due to initial condition, we can write above equation as

$$Sy(s) - y(0) + 3y(s) = sx(s) - x(0^-) - 2x(s)$$

$$y(0^-) = -2,\ x(0^-) = 0 \quad [x(t) = 3e^{2t}u(t)]$$

$$\Rightarrow Sy(s) + 2 + 3y(s) = (s-2)\left(\frac{-3}{s-2}\right)$$

$$(s+3)y(s) = -3-2 \Rightarrow y(s) = \frac{-5}{5+3}$$

$$\Rightarrow y(t) = -5e^{-3t}u(t)$$

$$y(\infty)(\text{steady sate}) = 0$$

Exp: 2

$$H(s) = \frac{s-2}{s+3}; X(t) = -3e^{2t}.u(t)$$

$$\therefore X(s) = \frac{-3}{s-2} \Rightarrow Y(s) = \frac{-3}{s+3}$$

$$y(t)\big|_{\text{at } t=\infty} \Rightarrow y(\infty) = \lim_{s\to 0} S.y(s) = \lim_{s\to 0} \frac{-3s}{s+3}$$

$$y(\infty) = 0$$

22. The phase response of a passband waveform at the receiver is given by

$$\phi(f) = -2\pi\alpha(f - f_c) - 2\pi\beta f_c$$

Where f_c is the centre frequency, and α and β are positive constants. The actual signal propagation delay from the transmitter to receiver is

(A) $\frac{\alpha-\beta}{\alpha+\beta}$ (B) $\frac{\alpha\beta}{\alpha+\beta}$ (C) α (D) β

Answer: C

Exp: Phase response of pass band waveform

$$\phi(f) = -2\pi\alpha(f - f_c) - 2\pi\beta f_c$$

$$\text{Group delay } t_y = \frac{-d\phi(f)}{2\pi\, df} = \alpha$$

Thus 'α' is actual signal propagation delay from transmitter to receiver

23. Consider an FM signal $f(t) = \cos[2\pi f_c t + \beta_1 \sin 2\pi f_1 t + \beta_2 \sin 2\pi f_2 t.]$. The maximum deviation of the instantaneous frequency from the carrier frequency f_c is

(A) $\beta_1 f_1 + \beta_2 f_2$ (B) $\beta_1 f_2 + \beta_2 f_1$ (C) $\beta_1 + \beta_2$ (D) $f_1 + f_2$

Answer: A

Exp: Instantaneous phase $\phi_i(t) = 2\pi f_c t + \beta_1 \sin 2\pi f_1 + \beta_2 \sin 2\pi f_2 t$

$$\text{Instantaneous frequency } f_i(t) = \frac{d}{dt}\phi_i(t) \times \frac{1}{2\pi}$$

$$= f_c + \beta_1 f_1 \cos 2\pi f_1 t + \beta_2 f_2 \cos 2\pi f_2 t$$

Instantaneous frequency deviation $= \beta_1 f_1 \cos 2\pi f_1 t + \beta_2 f_2 \cos 2\pi f_2 t$

Maximum $\Delta f = \beta_1 f_1 + \beta_2 f_2$

24. Consider an air filled rectangular waveguide with a cross-section of 5 cm × 3 cm. For this waveguide, the cut-off frequency (in MHz) of TE_{21} mode is _________.

Answer: 7810MHz.

Exp: $f_c(TE_{21}) = \frac{C}{2}\sqrt{\left(\frac{2}{9}\right)^2 + \left(\frac{1}{b}\right)^2}$

$$= \frac{3\times10^{10}}{2}\sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{1}{3}\right)^2}$$

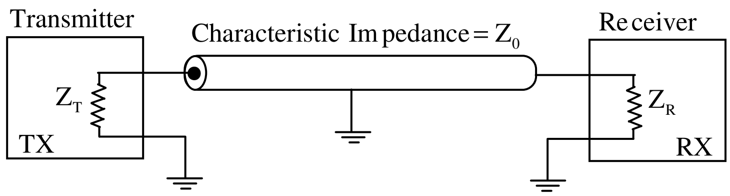
$$= 1.5\times10^{10}\sqrt{0.16 + 0.111}$$

$$= 0.52 \times 1.5\times10^{10}$$

$$= 7.81\,\text{GHz}$$

$$= 7810\,\text{MHz.}$$

25. In the following figure, the transmitter Tx sends a wideband modulated RF signal via a coaxial cable to the receiver Rx. The output impedance Z_T of Tx, the characteristic impedance Z_0 of the cable and the input impedance Z_R of Rx are all real.



Which one of the following statements is TRUE about the distortion of the received signal due to impedance mismatch?

(A) The signal gets distorted if $Z_R \neq Z_0$, irrespective of the value of Z_T

(B) The signal gets distorted if $Z_T \neq Z_0$, irrespective of the value of Z_R

(C) Signal distortion implies impedance mismatch at both ends: $Z_T \neq Z_0$ and $Z_R \neq Z_0$

(D) Impedance mismatches do NOT result in signal distortion but reduce power transfer efficiency

Answer: C

Exp: Signal distortion implies impedance mismatch at both ends. i.e.,

$Z_T \neq Z_0$

$Z_R \neq Z_0$

Q. No. 26 – 55 Carry Two Marks Each

26. The maximum value of $f(x)=2x^3 - 9x^2 + 12x - 3$ in the interval $0 \le x \le 3$ is _______.

Answer: 6

Exp: $f^1(x) = 6x^2 - 18x + 12 = 0 \Rightarrow x = 1, 2 \in [0,3]$

Now $f(0) = -3$; $f(3) = 6$ and $f(1) = 2$; $f(2) = 1$

Hence, f(x) is maximum at $x = 3$ and the maximum value is 6

27. Which one of the following statements is NOT true for a square matrix?

(A) If A is upper triangular, the eigenvalues of A are the diagonal elements of it

(B) If A is real symmetric, the eigenvalues of A are always real and positive

(C) If A is real, the eigenvalues of A and A^T are always the same

(D) If all the principal minors of A are positive, all the eigenvalues of A are also positive

Answer: B

Exp: Consider, $A\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ which is real symmetric matrix

Characteristic equation is $|A-\lambda I|=0 \quad \Rightarrow (1+\lambda)^2-1=0$

$$\Rightarrow \lambda+1=\pm 1$$
$$\therefore \lambda = 0, -2 \quad (\text{not positive})$$

(B) is not true

(A), (C), (D) are true using properties of eigen values

28. A fair coin is tossed repeatedly till both head and tail appear at least once. The average number of tosses required is __________________.

Exp: Let the first toss be Head.

Let x denotes the number of tosses(after getting first head) to get first tail.

We can summarize the even as:

Event (After getting first H)	x	Probability(p(x))
T	1	1/2
HT	2	1/2*1/2=1/4
HHT	3	1/8
and so on………..		

$$E(x)=\sum_{x=1}^{\infty} xp(x) = 1x\frac{1}{2}+2x\frac{1}{4}+3x\frac{1}{8}\cdots$$

$$\text{Let, } S = 1x\frac{1}{2}+2x\frac{1}{4}+3x\frac{1}{8}\cdots \qquad (I)$$

$$\Rightarrow \frac{1}{2}S = \frac{1}{4}+2x\frac{1}{8}+3x\frac{1}{16}\cdots \qquad (II)$$

$(I-II)$ gives

$$\left(1-\frac{1}{2}\right)S=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$$

$$\Rightarrow \frac{1}{2}S=\frac{\frac{1}{2}}{1-\frac{1}{2}}=1$$

$$\Rightarrow S=2$$

$$\Rightarrow E(x)=2$$

i.e. The expected number of tosses (after first head) to get first tail is 2 and same can be applicable if first toss results in tail.

Hence the average number of tosses is 1+2 = 3.

29. Let X_1, X_2, and X_3 be independent and identically distributed random variables with the uniform distribution on [0, 1]. The probability $P\{X_1 + X_2 \le X_3\}$ is ________.

Answer: 0.16

Exp: Given $x_1\ x_2$ and x_3 be independent and identically distributed with uniform distribution on $[0,1]$

Let $z = x_1 + x_2 - x_3$

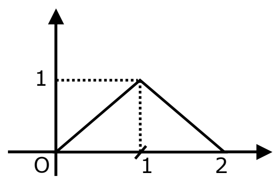
$$\Rightarrow P\{x_1 + x_2 \le x_3\} = P\{x_1 + x_2 - x_3 \le 0\}$$
$$= P\{z \le 0\}$$

Let us find probability density function of random variable z.

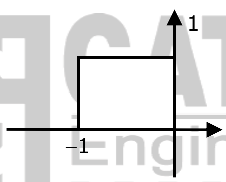
Since Z is summation of three random variable x_1, x_2 and $-x_3$

Overall pdf of z is convolution of the pdf of $x_1\ x_2$ and $-x_3$

pdf of $\{x_1 + x_2\}$ is



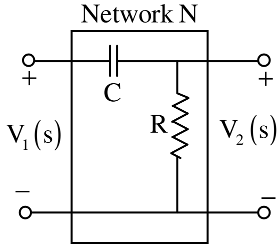
pdf of $-x_3$ is



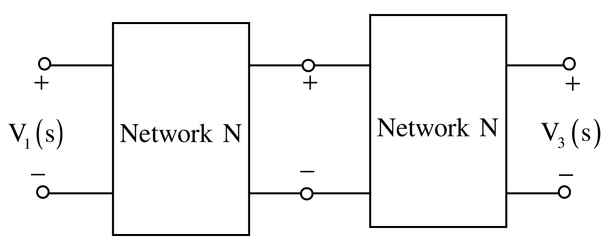
$$P\{z \le 0\} = \int_{-1}^{0} \frac{(z+1)^2}{2} dz = \left.\frac{(z+1)^3}{6}\right|_{-1}^{0} = \frac{1}{6} = 0.16$$

30. Consider the building block called 'Network N' shown in the figure.

Let $C = 100\mu F$ and $R = 10k\Omega$



Two such blocks are connected in cascade, as shown in the figure.

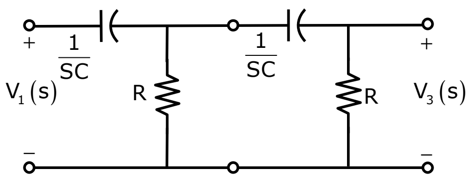


The transfer function $\frac{v_3(s)}{v_1(s)}$ of the cascaded network is

(A) $\frac{s}{1+s}$ (B) $\frac{s^2}{1+3s+s^2}$ (C) $\left(\frac{s}{1+s}\right)^2$ (D) $\frac{s}{2+s}$

Answer: B

Exp: Two blocks are connected in cascade, Represent in s-domain,



$$\frac{V_3(s)}{V_1(s)} = \frac{R\ .\ R}{\frac{1}{sc}\left[R+R+\frac{1}{SC}\right]+R\left[\frac{1}{SC}+R\right]}$$

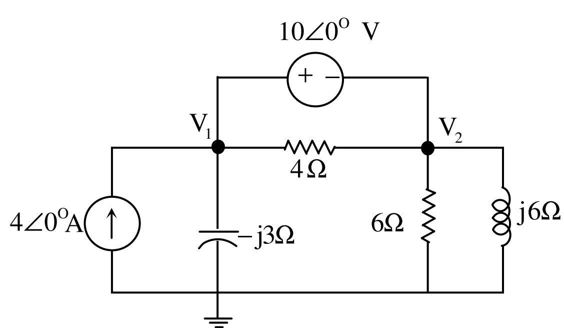
$$= \frac{R\ .\ R}{\frac{1}{SC}\cdot\frac{1}{SC}\left[2R(SC)+1\right]+\frac{R}{SC}\left[1+RSC\right]}$$

$$= \frac{S^2C^2.R.R}{\left[1+2R(SC)\right]+RSC+R^2S^2C^2}$$

$$= \frac{S^2.100\times100\times10^{-6}\times10^{-6}\times10\times10\times10^3\times10^3}{S^2\times100\times10^6\times10^4\times10^{-12}+3S+100\times10^{-6}\times10^4+1}$$

$$\frac{V_3(s)}{V_1(s)} = \frac{S^2}{1+3S+S^2}$$

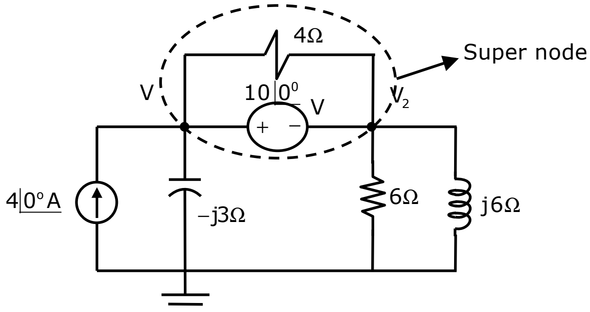
31. In the circuit shown in the figure, the value of node voltage V_2 is



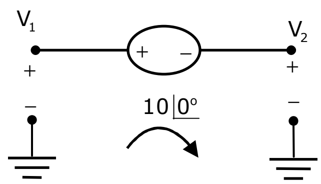
(A) $22+j\,2$ V (B) $2+j\,22$ V (C) $22-j\,2$ V (D) $2-j\,22$ V

Answer: D

Exp:



KVL for V_1 & V_2 :



$$V_1 - V_2 = 10\angle 0^\circ \quad ....(1)$$

$$\boxed{V_1 = V_2 + 10\angle 0^\circ}$$

KCL at super node:

$$-4\angle 0^\circ + \frac{V_1}{-j3} + \frac{V_2}{6} + \frac{V_2}{j6} = 0 \quad ...(2)$$

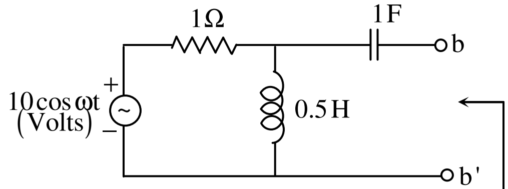
$$\frac{V_1}{-j3} + \frac{V_2}{V_6} + \frac{V_2}{j6} = 4\angle 0^\circ$$

$$\text{from } (1)\,\&\,(2),\ \frac{V_2 + 10\angle 0^\circ}{-j3} + \frac{V_2}{6} + \frac{V_2}{j6} = 4\angle 0^\circ$$

$$V_2\left[\frac{1}{-j3} + \frac{1}{6} + \frac{1}{j6}\right] = 4\angle 0^\circ + \frac{10}{j3}$$

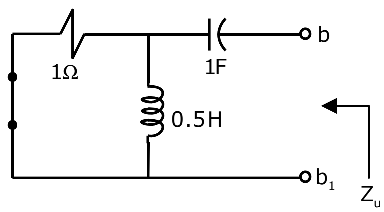
$$\therefore V_2 = (2 - j22)\,\text{Volts}$$

32. In the circuit shown in the figure, the angular frequency ω (in rad/s), at which the Norton equivalent impedance as seen from terminals b-b' is purely resistive, is __________________.



Answer: 2 r/sec

Exp: Norton's equivalent impedance



$$Z_N = \frac{1 * j\omega.\frac{1}{2}}{1 + j\omega.\frac{1}{2}} + \frac{1}{j\omega.1}$$

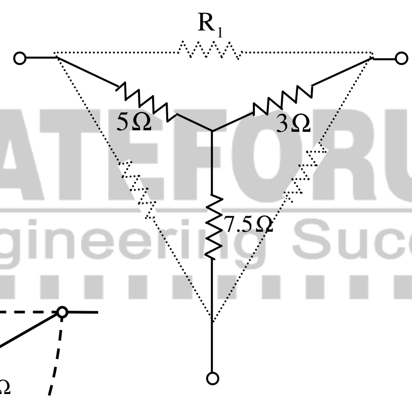
$$= \frac{j\omega}{2 + j\omega} + \frac{1}{j\omega}$$

$$Z_N = \frac{\left(2 - \omega^2\right) + j\omega}{\left[2j\omega - \omega^2\right]} \Rightarrow Z_N = \frac{\left[\left(\omega^2 - 2\right) - j\omega\right].\left[\omega^2 + 2j\omega\right]}{\left[\omega^4 + 4\omega^2\right]}$$

Equating imaginary term to zero i.e., $\omega^3 - 4\omega = 0$

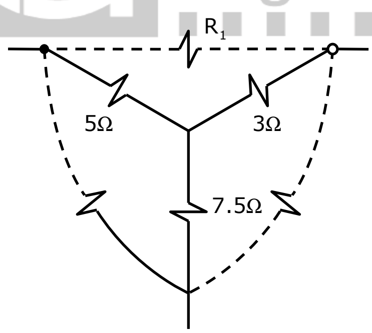
$$\Rightarrow \omega\left(\omega^2 - 4\right) = 0 \Rightarrow \boxed{\omega = 2\,r/\sec}$$

33. For the Y-network shown in the figure, the value of $R_1 (\text{in}\,\Omega)$ in the equivalent Δ-network is ____________________.



Answer: 10Ω

Exp:



$$R_1 = \frac{(7.5)(5) + (3)(5) + (7.5)(3)}{7.5}\Omega$$

$$R_1 = 10\Omega$$

34. The donor and accepter impurities in an abrupt junction silicon diode are 1 x 10^{16} cm^{-3} and 5 x 10^{18} cm^{-3}, respectively. Assume that the intrinsic carrier concentration in silicon n_i = 1.5 x 10^{10} cm^{-3} at 300 K, $\frac{kT}{q} = 26\,mV$ and the permittivity of silicon $\varepsilon_{si} = 1.04 \times 10^{-12}\,F/cm$. The built-in potential and the depletion width of the diode under thermal equilibrium conditions, respectively, are

(A) 0.7 V and 1 x 10^{-4} cm (B) 0.86 V and 1 x 10^{-4} cm

(C) 0.7 V and 3.3 x 10^{-5} cm (D) 0.86 V and 3.3 x 10^{-5} cm

Answer: D

Exp: $V_{bi} = V_T \ln\frac{N_A N_D}{n_i^2} = 26\,\text{mv} \ln\left[\frac{5\times10^{18}\times1\times10^{16}}{\left(1.5\times10^{10}\right)^2}\right]$

$= 0.859\text{V}$

$W = \sqrt{\frac{2\varepsilon_S V_{bi}}{q}\left[\frac{N_A + N_D}{N_A N_D}\right]} = 3.34\times10^{-5}\text{cm}$

35. The slope of the I_D vs V_{GS} curve of an n-channel MOSFET in linear regime is $10^{-3}\Omega^{-1}$ at $V_{DS} = 0.1\text{V}$.. For the same device, neglecting channel length modulation, the slope of the $\sqrt{I_D}$ vs V_{GS} curve $\left(\text{in } \sqrt{A}/V\right)$ under saturation regime is approximately ___________.

Answer: 0.07

Exp: In linear region, $I_D = k\left[\left(V_{GS} - V_T\right)V_{DS} - \frac{V_{DS}^2}{2}\right]$

$\frac{\partial I_D}{\partial V_{GS}} = 10^{-3} = kV_{DS}$ $\quad \because V_{DS}$ is small, $\frac{V_{DS}^2}{2}$ is neglected

$\Rightarrow K = \frac{10^{-3}}{0.1} = 0.01$

In saturation region, $I_D = \frac{1}{2}k\left(V_{GS} - V_T\right)^2$

$\sqrt{I_D} = \sqrt{\frac{k}{2}}\left(V_{GS} - V_T\right)$

$\frac{\partial\sqrt{I_D}}{\partial V_{GS}} = \sqrt{\frac{k}{2}} = \sqrt{\frac{0.01}{2}} = 0.07$

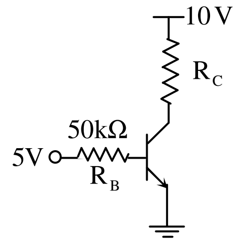
36. An ideal MOS capacitor has boron doping-concentration of 10^{15} cm^{-3} in the substrate. When a gate voltage is applied, a depletion region of width 0.5 μm is formed with a surface (channel) potential of 0.2 V. Given that $\varepsilon_o = 8.854\times10^{-14}$ F/cm and the relative permittivities of silicon and silicon dioxide are 12 and 4, respectively, the peak electric field (in V/μm) in the oxide region is _________________.

Answer: 2.4

Exp: $E_s = \frac{2\times0.2}{0.5} = 0.8\,\text{v}/\mu\text{m}$

$E_{ox} = \frac{E_s}{E_{ox}}E_s = 2.4\,\text{v}/\mu\text{m}$

37. In the circuit shown, the silicon BJT has $\beta = 50$. Assume V_{BE} =0.7 V and $V_{CE(sat)}$ = 0.2 V. Which one of the following statements is correct?



(A) For $R_C = 1\ k\Omega$, the BJT operates in the saturation region

(B) For $R_C = 3\ k\Omega$, , the BJT operates in the saturation region

(C) For R_C =20 kΩ, , the BJT operates in the cut-off region

(D) For R_C =20 kΩ, , the BJT operates in the linear region

Answer: B

Exp: KVL in base loop,

$$5 - I_B(50k) - 0.7 = 0$$

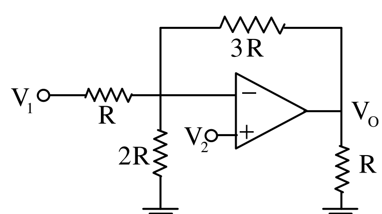
$$I_B = \frac{5-0.7}{50k} = 80\mu A$$

$$\Rightarrow I_C = \beta I_B = 50 \times 86\mu A = 4.3mA$$

$$\therefore R_C = \frac{10 - V_{CE}(sat)}{I_C} = \frac{10-0.2}{4.3mA}$$

$R_C = 2279\,\Omega$ and the BJT is in saturation

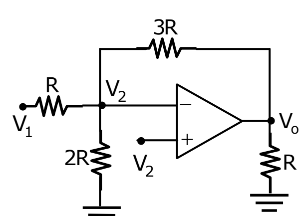
38. Assuming that the Op-amp in the circuit shown is ideal, V_O is given by



(A) $\frac{5}{2}V_1 - 3V_2$ (B) $ZV_1 - \frac{5}{2}V_2$ (C) $-\frac{3}{2}V_1 + \frac{7}{2}V_2$ (D) $-3V_1 + \frac{11}{2}V_2$

Answer: D

Exp: Virtual ground and KCL at inverting terminal gives

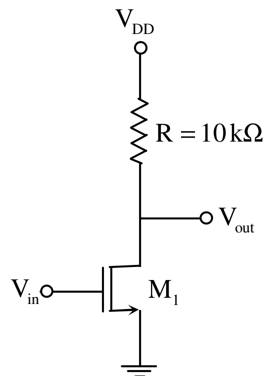


$$\frac{V_2 - V_1}{R} + \frac{V_2}{2R} + \frac{V_2 - V_0}{3R} = 0$$

$$\frac{V_0}{3R} = \frac{V_2}{R} + \frac{V_2}{3R} + \frac{V_2}{2R} - \frac{V_1}{R}$$

$$V_0 = -3V_1 + \frac{11}{2}V_2$$

39. For the MOSFET M_1 shown in the figure, assume W/L = 2, $V_{DD} = 2.0$ V, $\mu_n C_{ox} = 100\mu A/V^2$ and $V_{TH} = 0.5$ V. The transistor M_1 switches from saturation region to linear region when V_{in} (in Volts) is_________________.



Answer: 1.5

Exp: Transistor m_1 switch from saturation to linear

$\Rightarrow V_{DS} = V_{GS} - V_T$; where $V_{DS} = V_0$ and $V_{GS} = V_i$

$\therefore V_{DS} = V_0 = V_i - V_T$

Drain current $I_D = \frac{1}{2}\mu_n \cos\frac{W}{L}(V_{GS} - V_T)^2$

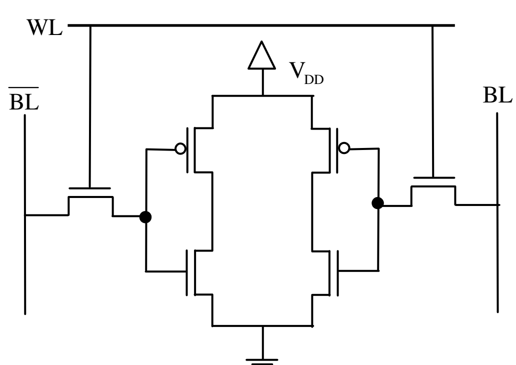
$$\frac{V_{DD} - V_o}{10K} = \frac{1}{2}\times 100\times 10^{-6}\times 2(V_{GS} - 0.5)^2$$

$$\frac{2-(V_i - 0.5)}{10K} = 100\times 10^{-6}(V_i - 0.5)^2$$

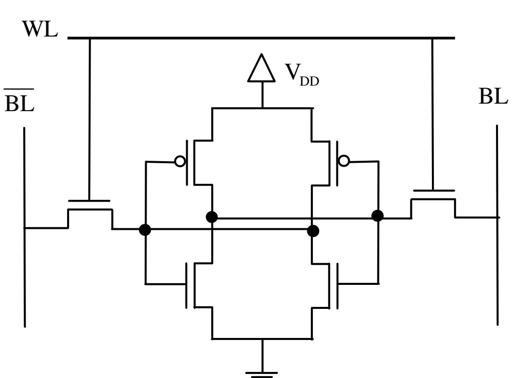
$\Rightarrow V_i = 1.5V$

40. If WL is the Word Line and BL the Bit Line, an SRAM cell is shown in

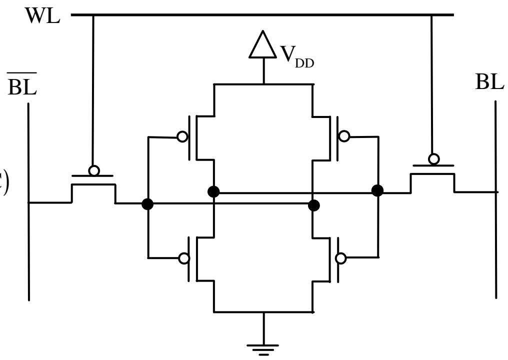
(A)



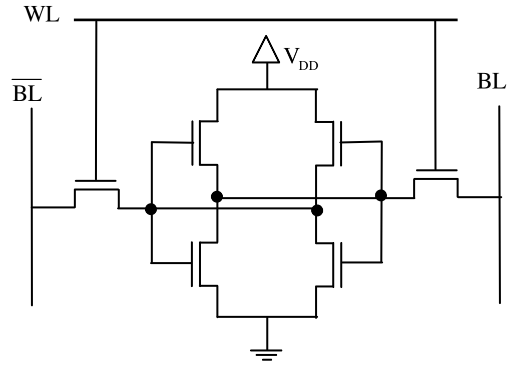
(B)



(C)



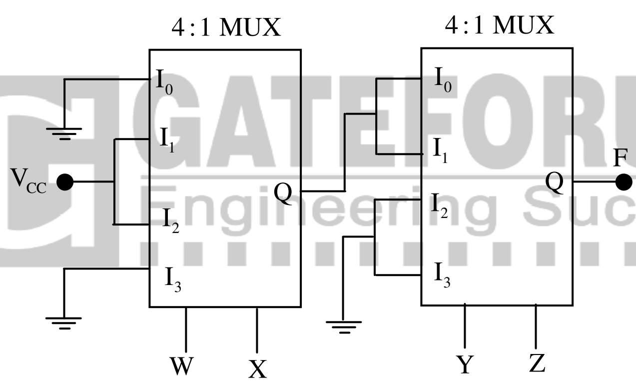
(D)



Answer: B

Exp: For an SRAM construction four MOSFETs are required (2-PMOS and 2-NMOS) with interchanged outputs connected to each CMOS inverter. So option (B) is correct.

41. In the circuit shown, W and Y are MSBs of the control inputs. The output F is given by

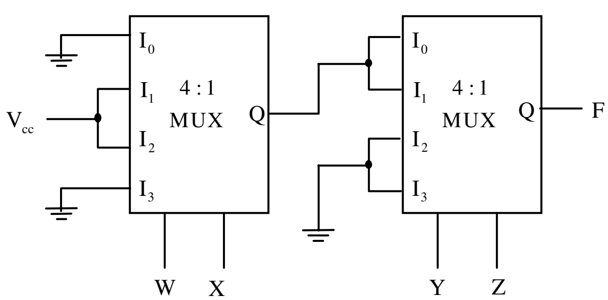


(A) $F = W\overline{X} + \overline{W}X + \overline{Y}\,\overline{Z}$ (B) $F = W\overline{X} + \overline{W}X + \overline{Y}Z$

(C) $F = W\overline{X}\,\overline{Y} + \overline{W}X\overline{Y}$ (D) $F = \left(\overline{W} + \overline{X}\right)\overline{Y}Z$

Answer: C

Exp:



The output of the first MUX = $\overline{W}\times V_{cc} + W\overline{X}.V_{cc}$

$\overline{W}X + W\overline{X}$ $\quad (\because V_{cc} = \text{logic}1)$

$= W \oplus X$

Let $Q = W \oplus X$

The output of the second MUX = $Q.\bar{Y}\bar{Z} + Q.\bar{Y}Z$

$= Q.\bar{Y}\left(\bar{Z}+Z\right)$

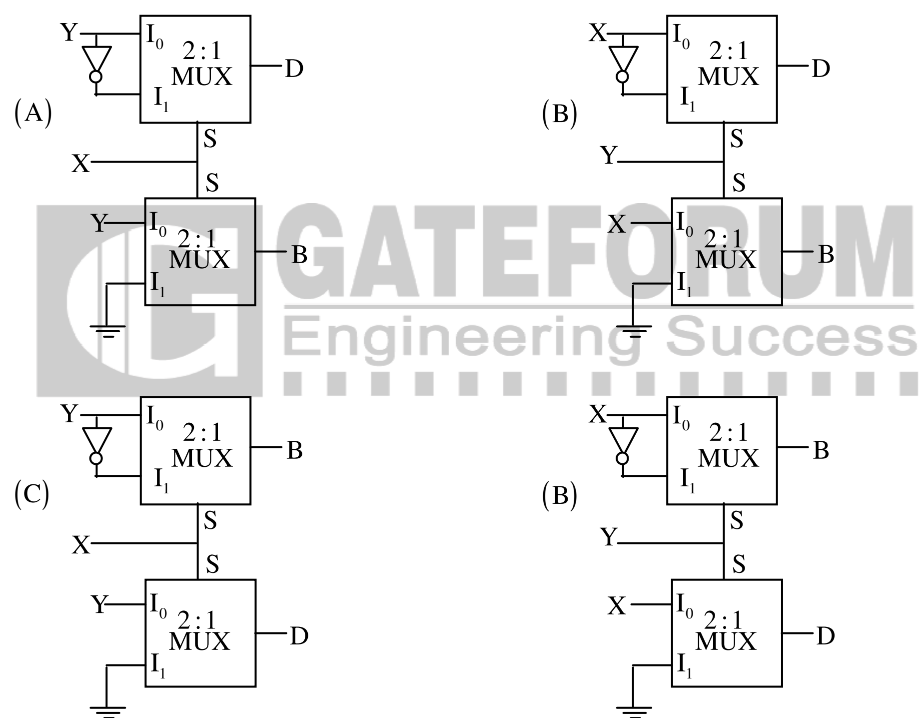
$= Q.\bar{Y}.1 = Q.\bar{Y}$

Put the value of Q in above expression

$= \left(\bar{W}X + W\bar{X}\right).\bar{Y}$

$= \bar{W}X.\bar{Y} + W\bar{X}.\bar{Y}$

42. If X and Y are inputs and the Difference **(D = X – Y)** and the Borrow **(B)** are the outputs, which
one of the following diagrams implements a half-subtractor?

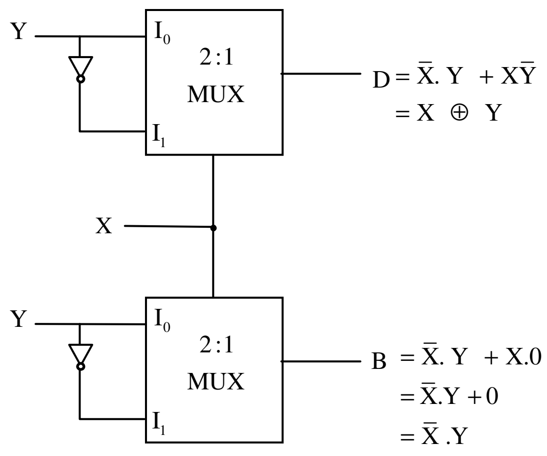


Answer: A

Exp:

X	Y	D	B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

So, $D = X \oplus Y = \bar{X}Y + X\bar{Y}$ and $B = \bar{X}.Y$



43. Let $H_1(z) = \left(1 - pz^{-1}\right)^{-1}, H_2(z) = \left(1 - qz^{-1}\right)^{-1}, H(z) = H_1(z) + rH_2(z)$. The quantities p, q, r are real numbers. Consider $p = \frac{1}{2}, q = \frac{1}{4}, |r| < 1$. If the zero of H(z) lies on the unit circle, then r = ________

Answer: -0.5

Exp: $H_1(z) = \left(1 - Pz^{-1}\right)^{-1}$

$H_2(z) = \left(1 - qz^{-1}\right)^{-1}$

$$H(z) = \frac{1}{1 - Pz^{-1}} + r\frac{1}{\left(1 - qz^{-1}\right)} = \frac{1 - qz^{-1} + r\left(1 - Pz^{-1}\right)}{\left(1 - Pz^{-1}\right)\left(1 - Pz^{-1}\right)} = \frac{(1 + r) - (q + rp)z^{-1}}{\left(1 - Pz^{-1}\right)\left(1 - Pz^{-1}\right)}$$

zero of $H(z) = \frac{q + rp}{1 + r}$

Since zero is existing on unit circle

$$\Rightarrow \frac{q + rp}{1 + r} = 1 \text{ or } \frac{q + rp}{1 + r} = -1$$

$$\frac{-\frac{1}{4} + \frac{r}{2}}{1 + r} = 1 \text{ or } \frac{-\frac{1}{4} + \frac{r}{2}}{1 + r} = -1$$

$$-\frac{1}{4} + \frac{r}{2} = 1 + r \qquad \text{or} \qquad -\frac{1}{4} + \frac{r}{2} = -1 - r$$

$$\Rightarrow r = -\frac{5}{2} \Rightarrow \frac{r}{2} = -\frac{5}{4} \qquad \text{or} \qquad \frac{3}{4} = \frac{-3r}{2} \qquad r = -\tfrac{1}{2} \Rightarrow r = -0.5$$

$r = -\frac{5}{2}$ is not possible

44. Let h(t) denote the impulse response of a causal system with transfer function $\frac{1}{s+1}$. Consider the following three statements.

S1: The system is stable.

S2: $\frac{h(t+1)}{h(t)}$ is independent of t for t 0.

S3: A non-causal system with the same transfer function is stable.

For the above system,

(A) Only S1 and S2 are true

(B) only S2 and S3 are true

(C) Only S1 and S3 are true

(D) S1, S2 and S3 are true

Answer: A

Exp: $h(t) \leftrightarrow H(s) = \frac{1}{s+1} \Rightarrow h(t) = e^{-t}u(t)$

S_1: System is stable (TRUE)

Because h(t) absolutely integrable

S_2: $\frac{h(t+1)}{h(t)}$ is independent of time (TRUE)

$\frac{e^{-(t+1)}}{e^{-t}} \Rightarrow e^{-1}$ (independent of time)

S_3: A non-causal system with same transfer function is stable

$\frac{1}{s+1} \leftrightarrow -e^{-t}u(-t)$ (a non-causal system) but this is not absolutely integrable thus unstable.

Only S_1 and S_2 are TRUE

45. The z-transform of the sequence x[n] is given by $X(z) = \frac{1}{\left(1-2z^{-1}\right)^2}$,, with the region of convergence $|z| > 2$. Then, $x[2]$ is ______.

Answer: 12

Exp(1):

$$X(z) = \frac{1}{\left(1-2z^{-1}\right)^2} = \frac{1}{\left(1-2z^{-1}\right)} \frac{1}{\left(1-2z^{-1}\right)}$$

$$x[n] = 2^n u[n] * 2^n u[n]$$

$$x[n] = \sum_{k=0}^{n} 2^K . 2^{(n-k)}$$

$$\Rightarrow x[2] = \sum_{k=0}^{2} 2^k . 2^{(2-k)} = 2^0.2^2 + 2^1.2^1 + 2^2.2^0 = 4+4+4 = 12$$

Exp(2):

$$X(z)=\frac{1}{\left(1-2Z^{-1}\right)^2}=\frac{Z^2}{(Z-2)^2}$$

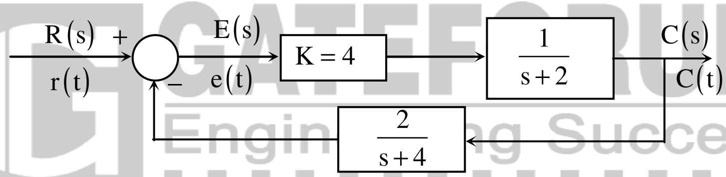
$$X(n)=Z^{-1}\left[\underbrace{\frac{Z}{Z-2}}_{u(z)}\cdot\underbrace{\frac{Z}{Z-2}}_{v(z)}\right]$$

$$=\sum_{m=0}^{n}u_m.V_{n-m}\ \left(\text{using conduction theorem and } u_n=2^n;\ v_n=2^n\right)$$

$$=\sum_{m=0}^{n}2^m.2^{n-m}=2^n(n+1)$$

$$\therefore x(2)=12$$

46. The steady state error of the system shown in the figure for a unit step input is _______.



Answer: 0.5

Exp: Given $G(s)=\frac{4}{s+2};H(s)=\frac{2}{s+4}$

For unit step input,

$$k_p=\lim_{s\to 0}G(s)H(s)$$

$$k_p=\lim_{s\to 0}\left(\frac{4}{s+2}\right)\left(\frac{2}{s+4}\right)$$

$$\boxed{k_p=1}$$

Steady state error $e_{ss}=\frac{A}{1+k_p}$

$$e_{ss}=\frac{1}{1+1}$$

$$e_{ss}=\frac{1}{2}\Rightarrow 0.50$$

47. The state equation of a second-order linear system is given by

$$\dot{x}(t) = Ax(t),\ x(0) = x_0$$

For $x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$ and for $x_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $x(t) = \begin{bmatrix} e^{-t} - e^{-2t} \\ -e^{-t} + 2e^{-2t} \end{bmatrix}$

when $x_0 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $x(t)$ is

(A) $\begin{bmatrix} -8e^{-t} + 11e^{-2t} \\ 8e^{-t} - 22e^{-2t} \end{bmatrix}$ (B) $\begin{bmatrix} 11e^{-t} - 8e^{-2t} \\ -11e^{-t} + 16e^{-2t} \end{bmatrix}$

(C) $\begin{bmatrix} 3e^{-t} - 5e^{-2t} \\ -3e^{-t} + 10e^{-2t} \end{bmatrix}$ (D) $\begin{bmatrix} 5e^{-t} - 3e^{-2t} \\ -5e^{-t} + 6e^{-2t} \end{bmatrix}$

Answer: B

Exp: Apply linearity principle,

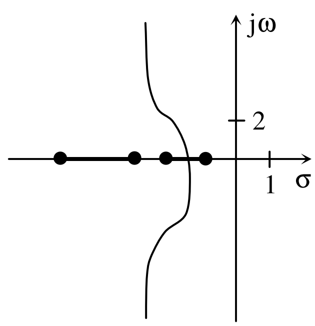
$$\begin{bmatrix} 3 \\ 5 \end{bmatrix} = a\begin{bmatrix} 1 \\ -1 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \end{bmatrix} s$$

$a = 3\,;\ b = 8$

$$\Rightarrow x(t) = 3\begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} + \begin{bmatrix} e^{-t} - e^{-2t} \\ -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$\Rightarrow x(t) = \begin{bmatrix} 11e^{-t} - 8e^{-2t} \\ -11e^{-t} + 16e^{-2t} \end{bmatrix}$$

48. In the root locus plot shown in the figure, the pole/zero marks and the arrows have been removed. Which one of the following transfer functions has this root locus?



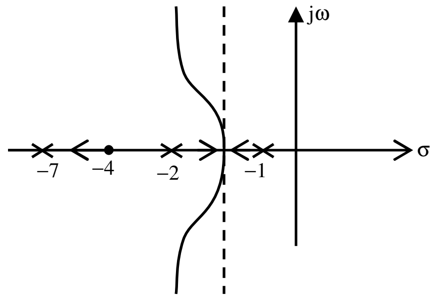
(A) $\dfrac{s+1}{(s+2)(s+4)(s+7)}$ (B) $\dfrac{s+4}{(s+1)(s+2)(s+7)}$

(C) $\dfrac{s+7}{(s+1)(s+2)(s+4)}$ (D) $\dfrac{(s+1)(s+2)}{(s+7)(s+4)}$

Answer: B

Exp:: For transfer function $\dfrac{(s+4)}{(s+1)(s+2)(s+3)}$

From pole zero plot



49. Let X(t) be a wide sense stationary (WSS) random process with power spectral density $S_X(f)$. If Y(t) is the process defined as $Y(t)=X(2t-1)$, the power spectral density $S_Y(f)$ is

(A) $S_Y(f)=\frac{1}{2}S_X\left(\frac{f}{2}\right)e^{-j\pi f}$ (B) $S_Y(f)=\frac{1}{2}S_X\left(\frac{f}{2}\right)e^{-j\pi f/2}$

(C) $S_Y(f)=\frac{1}{2}S_X\left(\frac{f}{2}\right)$ (D) $S_Y(f)=\frac{1}{2}S_X\left(\frac{f}{2}\right)e^{-j2\pi f}$

Answer: C

Exp: Shifting in time domain does not change PSD. Since PSD is Fourier transform of autocorrelation function of WSS process, autocorrelation function depends on time difference.

$$X(t)\leftrightarrow R_x(z)\leftrightarrow S_x(f)$$

$$Y(t)=X(2t-1)\leftrightarrow R_y(2\zeta)\leftrightarrow \frac{1}{2}S_x\left(\frac{f}{2}\right)$$

[time scaling property of Fourier transform]

50. A real band-limited random process X(t) has two-sided power spectral density

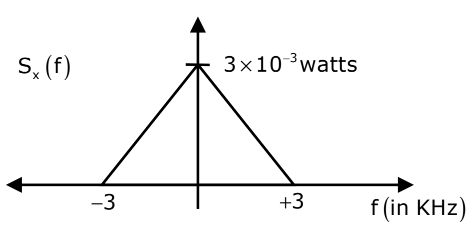
$$S_x(f)=\begin{cases}10^{-6}\left(3000-|f|\right)\text{ Watts/Hz} & \text{for } |f|\le 3\text{kHz}\\ 0 & \text{otherwise}\end{cases}$$

Where f is the frequency expressed in Hz. The signal X(t)modulates a carrier cos16000 πt and the resultant signal is passed through an ideal band-pass filter of unity gain with centre frequency of 8 kHz and band-width of 2 kHz. The output power (in Watts) is _______.

Answer: 2.5

Exp:

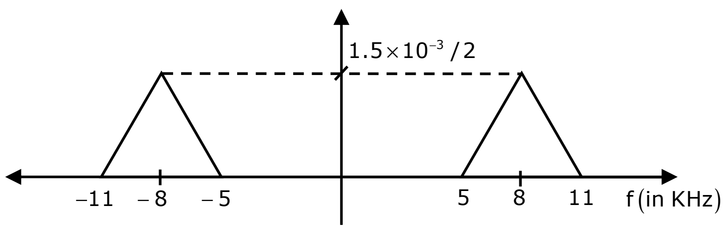
$S_x(f)$



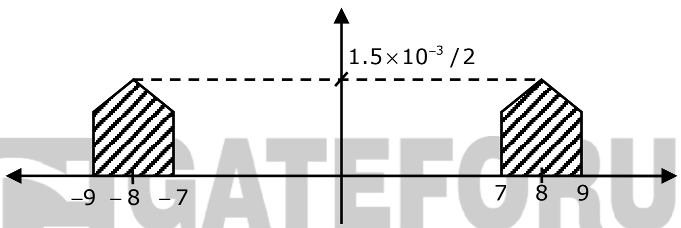
After modulation with $\cos(16000\pi t)$

$$S_y(f) = \frac{1}{4}\left[S_x(f - f_c) + S_x(f + f_c)\right]$$

This is obtain the power spectral density Random process y(t), we shift the given power spectral density random process x(t) to the right by f_c shift it to be the left by f_c and the two shifted power spectral and divide by 4.



After band pass filter of center frequency 8 KHz and BW of 2 kHz



Total output power is area of shaded region

$= 2[\text{Area of one side portion}]$

$= 2[\text{Area of triangle} + \text{Area of rectangle}]$

$$= \frac{2\left[-\frac{1}{2}\times2\times10^{3}\times0.5\times10^{-3} + 2\times10^{3}\times1\times10^{-3}\right]}{2}$$

$= [0.5 + 2] = 2.5\,\text{watts}$

51. In a PCM system, the signal $m(t) = \{\sin(100\pi t) + \cos(100\pi t)\}$ V is sampled at the Nyquist rate. The samples are processed by a uniform quantizer with step size 0.75 V. The minimum data rate of the PCM system in bits per second is _____.

Answer: 200

Exp: Nyquist rate $= 2\times50\,\text{Hz}$

$= 100\ \text{samples / sec}$

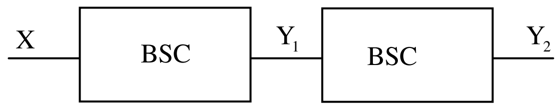
$$\Delta = \frac{m(t)_{max} - m(t)_{min}}{L} \Rightarrow L = \frac{\sqrt{2} - (-\sqrt{2})}{0.75}$$

$$L = \frac{2\sqrt{2}}{0.75} = 3.77 = 4$$

No. of bits required to encode ‘4’ levels = 2 bits/level

Thus data rate $= 2\times100 = 200$ bits / sec

52. A binary random variable X takes the value of 1 with probability 1/3. X is input to a cascade of 2 independent identical binary symmetric channels (BSCs) each with crossover probability 1/2. The outputs of BSCs are the random variables Y_1 and Y_2 as shown in the figure.



The value of $H(Y_1) + H(Y_2)$ in bits is___________.

Answer: 2

Exp: Let $P\{x=2\}=\frac{1}{3}$, $P\{x=0\}=\frac{2}{3}$

to find $H(Y_1)$ we need to know $P\{y_1=0\}$ and $P\{y_2=1\}$

$$P\{Y_1=0\}=P\{Y_1=0/x_1=0\}P\{x_1=0\}+P\{y_1=0/x_1=1\}P\{x_1=1\}$$

$$=\frac{1}{2}.\frac{1}{3}+\frac{1}{2}\times\frac{2}{3}=\frac{1}{2}$$

$$P\{y_1=1\}=\frac{1}{2}$$

$$\Rightarrow H(y_1)=\frac{1}{2}\log_2{}^2+\frac{1}{2}\log_2{}^2=1$$

Similarly

$$P\{y_2=0\}=\frac{1}{2} \text{ and } P\{y_2=1\}=\frac{1}{2}$$

$$\Rightarrow H\{y_2\}=1$$

$$\Rightarrow H\{y_1\}+H\{y_2\}=2 \text{ bits}$$

53. Given the vector $A=(\cos x)(\sin y)\hat{a}_x+(\sin x)(\cos y)\hat{a}_y$, where $\hat{a}_x,\hat{a}_y$ denote unit vectors along x,y directions, respectively. The magnitude of curl of A is ________

Answer: 0

Exp (1):

$$\text{Curl } \vec{A}=\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \cos x\sin y & \sin x\cos y & 0\end{vmatrix}$$

$$=\vec{0}$$

$$\therefore \left|\text{Curl } \vec{A}\right|=0$$

Exp(2):

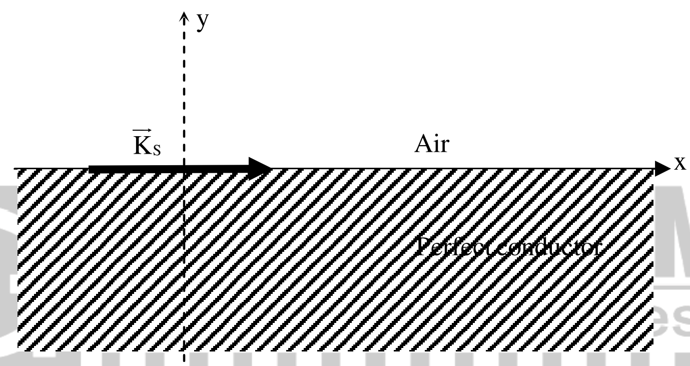
Given $A = \cos x \sin y \hat{a}_x + \sin x \cos y \ \hat{a}_y$

$$\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \cos x \sin y & \sin x \cos y & 0 \end{vmatrix}$$

$$= a_x(0) - a_y(0) + a_z(\cos x \cos y - \cos x \cos y) = 0$$

$$\therefore |\nabla \times A| = 0$$

54. A region shown below contains a perfect conducting half-space and air. The surface current $\vec{K}_s$ on the surface of the perfect conductor is $\vec{K}_s = \hat{x}2$ amperes per meter. The tangential $\vec{H}$ field in the air just above the perfect conductor is



(A) $(\hat{x} + \hat{z})2$ amperes per meter
(B) $\hat{x}2$ amperes per meter
(C) $-\hat{z}2$ amperes per meter
(D) $\hat{z}2$ amperes per meter

Answer: D

Exp: Given medium (1) is perfect conductor

Medium (2) is air

$\therefore H_1 = 0$

From boundary conditions

$(H_1 - H_2) \times a_n = K_S$

$\left. \begin{array}{l} H_1 = 0 \\ a_n = a_y \end{array} \right| K_S = 2\hat{a}_x$

$-H_2 \times a_y = 2\hat{a}_x$

$-(H_x a_x + H_y a_y + H_z a_z) \times a_y = 2a_x$

$-H_x a_z + H_z a_x = 2a_x$

$\therefore H_z = 2$

$\boxed{H = 2a_z}$

55. Assume that a plane wave in air with an electric field $\vec{E} = 10\cos\left(\omega t - 3x - \sqrt{3}z\right)\hat{a}_y$ V/m is incident on a non-magnetic dielectric slab of relative permittivity 3 which covers the region. $Z > 0$ The angle of transmission in the dielectric slab is _________________ degrees.

Answer: 30

Exp: Given $E = 10\cos\left(\omega t - 3x - \sqrt{3}z\right)a_y$

$$E = E_0 e^{-J\beta\left(x\cos\theta_x + y\cos\theta_y + z\cos\theta_z\right)}$$

So, $\beta_x = \beta\cos\theta_x = 3$

$\beta_y = \beta\cos\theta_y = 0$

$\beta_z = \beta\cos\theta_z = \sqrt{3}$

$$\beta_x{}^2 + \beta_y{}^2 + \beta_z{}^2 = \beta^2$$

$$9 + 3 = \beta^2 \Rightarrow \beta = \sqrt{13}$$

$$\beta\cos\theta_z = \sqrt{3} \Rightarrow \cos\theta_z = \sqrt{\frac{3}{13}} \Rightarrow \theta_z = 61.28 = \theta_i$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{E_2}{E_1}} \Rightarrow \frac{\sin 61.28}{\sin\theta_t} = \sqrt{\frac{3}{1}} \Rightarrow \frac{0.8769}{\sqrt{3}} = \sin\theta_t$$

$$\theta_t = 30.4 \Rightarrow \theta_t \simeq 30^{o}$$