Experiments and Measurements

Errors in measurement, accuracy, measurements of length, mass and charge of small and large objects, fundamental constants. Basic knowledge of scientific instruments and their working.

Mathematical Physics

Theory of Systems of linear Equation, Linear algebra and matrices.
Series and their convergence.

Limits and continuity, differentiation and integration, Taylor's expansion, L'Hospital rule, maxima, minima.
Analytical geometry of curves and surfaces.
Ordinary (first and second order) differential equations.
Complex numbers, roots of complex numbers, trigonometric identities, Argand's diagram.
Vector addition and products, gradient, divergence and curl, Gauss and Stokes theorems.
Probability, basic laws of probability, mean, standard deviation.

MODEL QUESTIONS

1. A heavy ball tied to a string spins around the circle. While the ball is spinning, the length of the string is slowly halved. The angular frequency of rotation of the ball is
   a) halved
   b) doubled
   c) quadrupled
   d) unchanged

2. Unpolarized light passes through three polarizing filters. The axis of the second one is at an angle of $+30^0$ with respect to the first, and the axis of the third is at an angle $+30^0$ with respect to the second. The fraction of the original intensity that energes from the third polarizer is
   a) 9/32
   b) 3/8
   c) 2/9
   d) 1/8

3. Two large metal spheres, A and B are near each other. The electrostatic force between them is attractive. Of the three possibilities :
   i) the two spheres are oppositely charged
   ii) one sphere is charged and the other is uncharged
   iii) both spheres are uncharged
      a) only case i) is possible.
      b) Cases i) and ii) are possible, but not iii).
      c) All three cases are possible.
      d) It depends on the size of the spheres compared to their separation.

4. A resistor inductor, and a capacitor are connected in series to an ac voltage source **U**(t) = V cos $[2\pi vt]$. . The peak voltages across the three elements are $V_{R,}$ V_L and V_C respectively. Then
   a) $V_{R,}$ V_L and V_C must be less than V.
   b) $V_{R,}$ must be less than V, *but* V_L and V_C need not.
   c) At any instant, the voltage across the resistor and the voltage from the source must have the same sign.
   d) At any instant, the voltage across the resistor must be smaller in magnitude than the voltage from the source.

5. Two spheres of radius r1 and r2 and at temperatures T1 and T2, are placed in vacuum. The first sphere is a blockbody. The second sphere may absorb more heat from the first than it radiates out if

   a) T1 = T2, but r1 is sufficiently large compared r2.
   b) T1 = T2, but the second sphere is painted, with a colour matching the peak of the radiation from the first.
   c) T1 > T2
   d) None of the above.

MATHEMATICAL SCIENCES

Mathematical Sciences question paper will consist of multiple choice questions for 100 marks from mathematics. There will be negative marking for wrong answer.

SYLLABUS

1. Algebra :

Theory of Equations :
Relations between roots and Coefficients, Newton's identities, Rolle's theorem, Reciprocal Equations, Des Cartes, Rule of Signs, Cubic and quartic equations, Complex numbers and De Moivre's Theorem.
Determinants :
Cofactors, Properties of determinants, Solution of a Linear System, Cramer's Rule.
Inequalities :
AM-GM inequality, Cauchy-Schwarz inequality.
Set Theory :
Relations, Functions, Cardinality.
Algebraic Structures :
Binary Operations, Groups, Rings: Definitions, Examples and Elementary Theorems.
Vector Spaces :
Subspaces, Linear Independence, Bases, Dimension, Linear Transformations, Matrices, Rank, Nullity, Eigen values and Eigen vectors.

2. Geometry :

Two-dimensional Coordinate Geometry :
Conics and their equations in Cartesian and Polar Coordinates, Ellipse, Parabola and Hyperbola.
Three-dimensional Co-ordinate Geometry :
Planes, Lines, Spheres and Cones.

3. Vector Algebra and Vector Calculus :
Vectors, addition, Scalar multiplication, Dot Product, Cross Product, Triple Product, Equations to the Line and the Plane. Grad, Divergence and Curl, Vector Integration, Green's Gauss' and Stokes' Theorems.

4. Calculus and Analysis
Real Number System, Sequence and Series. Continuity, Differentiability, Mean Value Theorems, Indeterminate Value Theorem, L'Hospital Rule, Tangents and Normals, Maxima and Minima, Riemann Integration, Multiple Integrals, Partial differentiations, Lengths, areas and volumes by integration.

5. Differential Equations :
First Order ODE; Method of Separation of Variables; Exact equations; Euler's equation; Orthogonal Family of curves, Second Order Linear ODE : Variation of Parameters.

MODEL QUESTIONS

Four possible answers are provided for each question.
Select the correct answer by making (√) against (A), (B), (C) or (D).

1. Let ρ be a non-trivial relation on a set X. If ρ is symmetric and antisymmetric then ρ is

   (A) reflexive, (B) transitive, (C) an equivalence relation, (D) the diagonal relation.

2. The set $\{5, 15, 25, 35\}$ is a group under multiplication modulo 40. The identity element of this group is

   (A) 5, (B) 15, (C) 25, (D) 35.

3. Let $\mathbb{Z}_n$ be the additive group of integers modulo n. The number of homomorphisms from $\mathbb{Z}_n$ to itself is

   (A) 0, (B) 1, (C) n, (D) n^2.

4. Let $v = (1, 1)$ and $w = (1, -1) \in \mathbb{R}^2$. Then a vector $u = (a, b) \in \mathbb{R}^2$ is in the $\mathbb{R}$-linear span of v and w

   (A) only when $a = b$, (B) always, (C) for exactly one value of (a, b), (D) for at most finitely many values of (a, b).

5. Let A be a 3×3 real matrix. Suppose $A^4 = 0$. Then A has

   (A) exactly two distinct real eigenvalues, (B) exactly one non-zero real eigenvalue, (C) exactly 3 distinct real eigenvalues, (D) no non-zero real eigenvalue.

6. Let a, b, c, d be real numbers and let $f : \mathbb{C} \to \mathbb{C}$ be the map defined by $f(x + iy) := (ax + by) + i(cx + dy)$. Then f is linear over $\mathbb{C}$ if and only if

   (A) $(a, b) = (d, c)$, (B) $(a, b) = (d, -c)$, (C) $(a, b) = (-d, c)$, (D) $(a, b) = (-d, -c)$.

7. The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \max\{1 - |x|, 0\}$ is differentiable

   (A) at all points, (B) at all except one point, (C) at all except three points, (D) nowhere.

8. Let $f : [0, 1] \to \mathbb{R}$ be a continuous function with $f(0) = f(1)$. If f is differentiable on $(0, 1)$ and the derivative f' is continuous on $(0, 1)$ then f' is

   (A) strictly positive in $(0, 1)$, (B) strictly negative in $(0, 1)$, (C) identically zero in $(0, 1)$, (D) zero at some point in $(0, 1)$.

9. A unit normal vector to the curve $\mathbf{C} := \{(x, x^2) : x \in \mathbb{R}\}$ in the plane $\mathbb{R}^2$ at the point $(0, 0)$ is given by

   (A) $(0, -1)$, (B) $(-1, 0)$, (C) $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, (D) $(1, 0)$.

10. The differential equation $\dfrac{d^2y}{d^2x} + 4\dfrac{dy}{dx} + 4y = 0$ has general solution of the form :

    (A) $A\cos 2x + B\sin 2x$, (B) $Ae^{-2x} + Bxe^{-2x}$, (C) $Ae^{2x} + Bxe^{2x}$, (D) $Ae^{2x} + Be^{-2x}$.