Reg. No.

Question Paper Code: 53186

B.E./B.Tech.Degree Examinations, November/December 2010
Regulations 2008

Fourth Semester

Common to ECE and Bio Medical Engineering

MA 2261 Probability and Random Processes

Time: Three Hours | Maximum: 100 Marks

Answer ALL Questions

Part A - (10 x 2 = 20 Marks)

1. A continuous random variable X has probability density function

$$f(x) = \begin{cases} 3x^2, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}.$$

Find k such that $P(X > k) = 0.5$

2. If X is uniformly distributed in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$. Find the pdf of $Y = \tan X$.

3. Let X and Y be continuous random variables with joint probability density function $f_{XY}(x, y) = \frac{x\,(x-y)}{8}$, $0 < x < 2$, $-x < y < x$ and $f_{XY}(x, y) = 0$ elsewhere. Find $f_{Y|X}(y|x)$.

4. State Central Limit Theorem for iid random variables.

5. Consider the random process $X(t) = \cos(t + \phi)$, where ϕ is a random variable with density function $f(\phi) = \frac{1}{\pi}$, $\quad -\frac{\pi}{2} < \phi < \frac{\pi}{2}$.
Check whether or not the process is wide sense stationary.

6. State the postulates of a Poisson process.

7. Find the variance of the stationary process $\{X(t)\}$ whose auto correlation function is given by $R_{XX}(\tau) = 2 + 4e^{-2|\tau|}$.

8. State any two properties of cross correlation function.

9. If $Y(t)$ is the output of an linear time invariant system with impulse response $h(t)$, then find the cross correlation of the input function $X(t)$ and output function $Y(t)$.

10. Define Band-Limited white noise.

Part B - (5 x 16 = 80 Marks)

11. (a) (i) A random variable X has the following probability distribution.

x	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

Find :
(1) The value of K
(2) $P(1.5 < X < 4.5 \mid X > 2)$ and
(3) The smallest value of n for which $P(X \leq n) > \frac{1}{2}$ (8)

(ii) Find the M.G.F. of the random variable X having the probability density function

$$f(x) = \begin{cases} \frac{x}{4}\, e^{-\frac{x}{2}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Also deduce the first four moments about the origin. (8)

OR

11. (b) (i) If X is uniformly distributed in (-1, 1), then find the probability density function of $Y = sin\frac{\pi X}{2}$. (6)

(ii) If X and Y are independent random variables following N (8, 2) and $N(12, 4\sqrt{3})$ respectively, find the value of λ such that

$$P[2X - Y \leq 2\lambda] = P[X + 2Y \geq \lambda]$$

(10)

12. (a) Two random variables X and Y have the joint probability density function given by

$$f_{XY}(x, y) = \begin{cases} k(1 - x^2 y), & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of 'k'

(ii) Obtain the marginal probability density functions of X and Y.

(iii) Also find the correlation coefficient between X and Y.

(16)

OR

12. (b) (i) If X and Y are independent continuous random variables, show that the pdf of $U = X + Y$ is given by $h(u) = \int_{-\infty}^{\infty} f_x(v) f_y(u - v) dv$. (8)

(ii) If V_i, $i = 1, 2, 3 \cdots 20$ are independent noise voltages received in an adder and V is the sum of the voltages received, find the probability that the total incoming voltage V exceeds 105, using the central limit theorem. Assume that each of the random variables V_i is uniformly distributed over $(0, 10)$. (8)

13. (a) (i) The process $\{X(t)\}$ whose probability distribution under certain condition is given by

$$P\{X(t) = n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1, 2.. \\ \dfrac{at}{1+at}, & n = 0 \end{cases}$$

Find the mean and variance of the process. Is the process first-order stationary? (8)

(ii) If the WSS process $\{X(t)\}$ is given by $X(t) = 10\cos(100t + \theta)$, where θ is uniformly distributed over $(-\pi, \pi)$, prove that $\{X(t)\}$ is correlation ergodic. (8)

OR

13. (b) (i) If the process $\{X(t); t \geq 0\}$ is a Poisson process with parameter λ, obtain $P[X(t) = n]$. Is the process first order stationary? (8)

(ii) Prove that a random telegraph signal process $Y(t) = \alpha X(t)$ is a Wide Sense Stationary Process when α is a random variable which is independent of $X(t)$, assumes values -1 and $+1$ with equal probability and $R_{XX}(t_1, t_2) = e^{-2\lambda|t_1 - t_2|}$

(8)

14. (a) (i) If $\{X(t)\}$ and $\{Y(t)\}$ are two random processes with auto correlation function $R_{XX}(\tau)$ and $R_{YY}(\tau)$ respectively then prove that $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0) R_{YY}(0)}$. Establish any two properties of auto correlation function $R_{XX}(\tau)$ (8)

(ii) Find the power spectral density of the random process whose auto correlation function is $R(\tau) = \begin{cases} 1-|\tau|, & \text{for} |\tau| \leq 1 \\ 0, & \text{elsewhere} \end{cases}$. (8)

OR

14. (b) (i) State and prove Weiner-Khintchine Theorem. (8)

(ii) The cross-power spectrum of real random processes $\{X(t)\}$ and $\{Y(t)\}$ is given by

$$S_{xy}(\omega) = \begin{cases} a + bj\omega, & \text{for } |\ \omega\ | < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the cross correlation function. (8)

15. (a) (i) Show that if the input $\{X(t)\}$ is a WSS process for a linear system then output $\{Y(t)\}$ is a WSS process. Also find $R_{XY}(\tau)$. (8)

(ii) If $X(t)$ is the input voltage to a circuit and $Y(t)$ is the output voltage. $\{X(t)\}$ is a stationary random process with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-2|\tau|}$. Find the mean μ_Y and power spectrum $S_{YY}(\omega)$ of the output if the system transfer function is given by $H(\omega) = \dfrac{1}{\omega + 2i}$. (8)

OR

15. (b) (i) If $Y(t) = A\cos(w_0 t + \theta) + N(t)$, where A is a constant, θ is a random variable with a uniform distribution in $(-\pi, \pi)$ and $\{N(t)\}$ is a band-limited Gaussian white noise with power spectral density

$$S_{NN}(w) = \begin{cases} \dfrac{N_0}{2}, & \text{for } |w - w_0| < w_B \\ 0, & \text{elsewhere} \end{cases}.$$

Find the power spectral density $\{Y(t)\}$. Assume that $\{N(t)\}$ and θ are independent. (10)

(ii) A system has an impulse response $h(t) = e^{-\beta t} U(t)$, find the power spectral density of the output $Y(t)$ corresponding to the input $X(t)$. (6)
