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Reg. No. : ....

Name : ....

M.Sc. Final Degree Examination, July 2009
(I.D.E.)
Branch : MATHEMATICS
MM 1209 : Complex Analysis – I
(Prior to 2006 admission)

Time : 3 Hours Max. Marks : 85

Instructions : *i) Answer either Part* ***A*** *or Part* ***B*** *of* ***each*** *question.*
ii) ***All*** *questions carry* ***equal*** *marks.*

I. A) i) Show that C_∞ can be represented as the unit sphere in R^3. For each of the points 0, 1 + i, 3 + i in C give the corresponding points of the unit sphere. **8**

ii) Define an analytic function. $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ have radius of convergence R > 0. Show that f is infinitely differentiable on B (a ; R) and that $a_n = \frac{1}{n!} f^{(n)}(a)$. **9**

B) i) Show that f (z) = $|z|^2$ has a derivative only at the origin. **5**

ii) If f : G $\to \mathbb{C}$ is analytic then show that f preserves angles at each point z_0 of G where $f'(z_0) \neq 0$. **7**

iii) If z_2, z_3, z_4 are distinct points in C_∞ and w_2, w_3, w_4 are also distinct points in C_∞ then show that there is one and only one Mölius transformation s such that $sz_2 = w_2$, $sz_3 = w_3$ and $sz_4 = w_4$. **5**

P.T.O.

II. A) i) If $r : [a, b] \to \mathbb{C}$ is piece wise smooth then show that r is of bounded variation and $v(r) = \int_a^b |r'(t)|\, dt$. **8**

ii) Show that $\int_0^{2\pi} \frac{e^{is}}{e^{is} - z}\, ds = 2\pi$, if $|z| < 1$. **4**

iii) State and prove Lioville's theorem. **5**

B) i) Let G be a connected open set and let $f : G \to \mathbb{C}$ be an analytic function. Show that the following are equivalent (1) $f \equiv 0$ (2) There is a point a in G such that $f^n(a) = 0$ for all $n \geq 0$ (3) $\{z \in G : f(z) = 0\}$ has a limit point in G. **9**

ii) Let f be analytic in B (a ; R). Show that $f(z) = \sum_{n=0}^{\infty} a_n (z - a)^n$ for $|z - a| < R$ where $a_n = \frac{1}{n!} f^n(a)$ and that this series has radius of convergence $\geq R$. **8**

III. A) i) Let G be an open subset of the plane and $f : G \to \mathbb{C}$ an analytic function. If $r_1, \ldots\ldots, r_m$ are closed rectifiable curves in G such that $\sum_{i=1}^{m} n(r_i, w) = 0$, for all w in $\mathbb{C} - G$ then show that $\sum_{k=1}^{m} \int_{r_k} f = 0$. **10**

ii) State and prove Morera's theorem. **7**

B) i) If r_0 and r_1 are rectifiable curves in G from a to b and r_0 and r_1 are FEP homotopic then show that $\int_{r_0} f = \int_{r_1} f$ for any function f analytic in G. **7**

ii) If G is simply connected and $f : G \to \mathbb{C}$ is analytic in G, then show that f has a primitive in G. **5**

iii) Evaluate $\int_r \frac{dZ}{Z^2 + 1}$ where $r(\theta) = 2|\cos 2\theta|\, e^{i\theta}$, for $0 \leq \theta \leq 2\pi$. **5**

IV. A) i) State and prove Goursat's theorem. **10**

ii) Show that if f has an isolated singularity at a then z = a is a removable singularity if and only if $\lim_{z \to a} (z - a) f(z) = 0$. **7**

B) i) State and prove the theorem on Laurent series development. **9**

ii) Find the Laurent series expansion of $f(z) = \frac{1}{z(z-1)(z-2)}$ in

(1) ann (0 ; 0, 1) (2) ann (0 ; 1, 2). **8**

V. A) i) Show that for a > 1, $\int_0^{\pi} \frac{d\theta}{a + \cos\theta} = \frac{\pi}{\sqrt{a^2 - 1}}$. **7**

ii) Show that $\int_0^{\infty} \frac{\log x}{1 + x^2} = 0$. **10**

B) i) State and prove the Residue theorem. **6**

ii) State and prove Rouche's theorem. **5**

iii) State and prove Schwarz's lemma. **6**

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