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Reg. No. : ....

Name : ....

M.Sc. (Final) Degree Examination, July 2009
Branch : MATHEMATICS (I.D.E.)
MM 1209 – Complex Analysis – I
(2006 admn. onwards)

Time: 3 Hours Max. Marks: 85

***Instruction** : Answer either **A** or **B** of **each** question. Notations as in the text.*

I. A. a) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{Z^n}{n!}$. **5**

b) Define an analytic function. Give an example with justification. Suppose G is an open connected set and $f : G \to \mathbb{C}$ is differentiable with $f'(z) = 0$ for all z in G. Prove that f is a constant. **6**

c) Let $r : [a, b] \to \mathbb{C}$ be of bounded variation and let $f : [a, b] \to \mathbb{C}$ be continuous. Suppose $a = t_0 < t_1 < \ldots < t_n = b$. Prove that

$$\int_a^b f dr = \sum_{k=1}^{n} \int_{t_{k-1}}^{t_k} f dr$$ **6**

B. a) Find the radius of convergence of the power series $1 + z + z^2 + \ldots$ **4**

b) Define sin z. Is it analytic ? Justify. **3**

c) Let G be open in $\mathbb{C}$ and let r be a rectifiable path in G with initial and end points α and β respectively. If $f : G \to \mathbb{C}$ is a continuous function with primitive $F : G \to \mathbb{C}$, then prove that $\int_r f = F(\beta) - F(\alpha)$. **10**

P.T.O.

II A. a) Prove that an analytic function is infinitely differentiable. **6**

b) Define an entire function. Prove that an entire function f has a power series expansion of the form $f|z| = \sum_{n=0}^{\infty} a_n z^n$ with infinite radius of convergence. **6**

c) For a closed rectifiable curve r and a $\notin$ {r}, prove that the winding number of r with respect to a is always an integer. **5**

B. a) Suppose f is analytic in the disc B (a : R) and suppose r is a closed rectifiable curve in B (a, R). Prove that $\int_r f = 0$. **5**

b) Suppose f is analytic in an open connected set G and f is not identically zero in G. Prove that zeros of f has finite multiplicity. **6**

c) Give an example of a closed rectifiable curve r in $\mathbb{C}$ such that for any integer k there is a point a {r} with n (r ; a) = k. **6**

III. A. a) State and prove the first version of Cauchy's integral formula. **7**

b) State the third version of Cauchy's theorem. Using this deduce that if r is a closed rectifiable curve in a region G such that r ~ 0 , then n (r ; w) = 0 for all w $\in \mathbb{C} \setminus G$. **5**

c) State and prove the open mapping theorem for a non-constant analytic functions. **5**

B. a) State and prove the second version of Cauchy's Integral Formula. **6**

b) Using the open mapping theorem prove that the inverse of a bijective analytic function is analytic. **5**

c) State Goursat's theorem is the converse true. Justify. **6**

IV. A. a) Define the residue at an isolated singularity of a function. **2**

b) Prove that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$. **6**

c) Define a menomorphic function. Give an example with justification. **2**

d) State and prove the argument principle. **7**

B. a) Define the three types of singularities. Give an example for each. **6**

b) For a > 1, prove that $\int_0^\pi \frac{d\theta}{a+\cos\theta} = \frac{\pi}{\sqrt{a^2-1}}$. **6**

c) State and prove Rouche's theorem. **5**

V. A. a) Describe the spherical representation of the extended plane. **7**

b) Define conformal mapping. Prove that $f(z) = e^z$ is a conformal thought out $\mathbb{C}$. **6**

c) State the second version of the maximum modulus theorem. What happens if the boundedness condition is dropped ? Justify. **4**

B. a) Describe how the extended plane $\mathbb{C}_\infty$ is identified with the unit sphere in $\mathbb{R}^3$. **7**

b) Define a linear transformation and a Mobius transformation. Prove that a Mobius transformation maps circles on to circles. **5**

c) State and prove Schwarz's lemma. **5**

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