Roll No. ....

Total No. of Questions : 9] [Total No. of Pages : 03

B.Tech. (Sem. - 1st/2nd)

ENGINEERING MATHEMATICS - I

SUBJECT CODE : AM - 101 (2K4 & ONWARDS)

Paper ID : [A0111]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours **Maximum Marks : 60**

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Five** questions from Section - B & C.
3) Select atleast two questions from Section - B & C.

Section - A

Q1) *[Marks : 2 each]*

a) Test for the convergence of the series $\sum\left(\frac{n}{n+1}\right)^{n^2}$.

b) Using double integration, find area enclosed between the curves $y^2 = x^3$ and $x = y$.

c) If $u = x^3 + xy$ and $v = xy$. Find $\frac{\partial(u,v)}{\partial(x,y)}$.

d) Prove $\Gamma(n+1) = n\Gamma(n)$, where $n > 0$.

e) Find the curvature of curve $y^2 = x^3 + 8$ at the point (1, 3).

f) Find the cube roots of unity.

g) Evaluate $\int_0^2\int_1^2\int_0^{yz} xyz\, dxdydz$.

h) Define homogeneous function with an example.

i) Find the centre and the radius of the sphere $x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$.

j) Expand $\tan x$ in powers of x upto x^3.

Section - B

[Marks : 8 Each]

Q2) (a) State and prove Euler's theorem.

(b) If $z=\sqrt{x^2+y^2}$ and $x^3 + y^3 + 3axy = 5a^2$, find the value of $\frac{dz}{dx}$, when $x = y = a$.

Q3) (a) Trace the curve $a^2y^2 = x^2(a^2 - x^2)$.

(b) If ρ_1, ρ_2 be the radii of curvature at the extremities of the chord of the cardioid $r = a(1 + \cos\theta)$ which pass through the pole, show that $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$.

Q4) (a) Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ using Taylor's theorem.

(b) Discuss maxima and minima of $x^3y^2(1 - x - y)$.

Q5) (a) Find the moment, about x-axis of arc of parabola $y = \sqrt{x}$, lying between (0, 0) & (4, 2).

(b) Find root mean square of $\sin x$ over the range $x = 0$ to $\pi/2$.

Section - C

[Marks : 8 Each]

Q6) (a) Show that the two circles

$x^2 + y^2 + z^2 - 2x + 3y + 4z - 5 = 0, 5y + 6z + 1 = 0$

$x^2 + y^2 + z^2 - 3x - 4y + 5z - 6 = 0, x + 2y - 7z = 0$

lie on the same sphere and find its equations.

(b) Find the equation of cone whose vertex is at the points (1, 1, 3) and which passes through the ellipse $4x^2 + z^2 = 1, y = 4$.

Q7) (a) Change the order of integration $\int_0^1\int_{x^2}^{2-x} xy\, dxdy$ and hence evaluate the integral.

(b) Prove that $\int_1^0 \frac{x\,dx}{\sqrt{1-x^5}} = \frac{1}{5}\beta\left(\frac{2}{5},\frac{1}{2}\right)$.

Q8) (a) Test the convergence of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+..........$

(b) Show that the series $\sum_{n=1}^{\infty}\frac{\sin(x^2+nx)}{n(n+2)}$ for all real x, is uniformly convergent.

Q9) (a) Separate $\tan^{-1}(x+iy)$ into real and imaginary parts.

(b) Solve the equation $x^4 - x^3 + x^2 - x + 1 = 0$, using De Moivre's theorem.

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