Reg. No. : | | | | | | | | | | | |

Question Paper Code :

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2011.

Fourth Semester

Computer Science and Engineering

MA 2262 — PROBABILITY AND QUEUEING THEORY

(Common to Information Technology)

(Regulation 2008)

Time : Three hours Maximum : 100 marks

(Normal tables be permitted in the examination hall)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A continuous random variable X that can assume any value between $x = 2$ and $x = 5$ has a density function given by $f(x) = k(1 + x)$. Find $P(X < 4)$.

2. Give the probability law of Poisson distribution and also its mean and variance.

3. The joint pdf of the RV (X, Y) is given by $f(x, y) = Kxy\, e^{-(x^2+y^2)}$, $x > 0, y > 0$. Find the value of K.

4. Given the RV X with density function

   $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$ Find the pdf of $Y = 8X^3$.

5. Define transition probability matrix.

6. Define markov process.

7. Draw the state transition diagram for $M/M/1$ queueing model.

8. What do the letters in the symbolic representation $(a/b/c):(d/e)$ of a queueing model represent?

9. Write the Pollaczek-Khintchine formula.

10. Define series queues.

PART B — (5 × 16 = 80 marks)

11. (a) (i) The DF of a continuous random variable X is given by

$$F(x) = 0, x < 0$$
$$= x^2, 0 \le x < 1/2$$
$$= 1 - 3/25\,(3 - x^2);\ 1/2 \le x < 3$$
$$= 1 \qquad ;\ x \ge 3.$$

Find the pdf of X and evaluate $P(|X| \le 1)$ and $P\left(1/3 < X < 4\right)$ using both the pdf and PDF.

(ii) A random variable X has the following probability distribution

x :	–2	–1	0	1	2	3
$p(x)$:	0.1	k	0.2	2k	0.3	3k

(1) Find k, (2) Evaluate $P(X < 2)$ and $P(-2 < X < 2)$, (3) Find the PDF of X and (4) Evaluate the mean of X.

Or

(b) (i) The probability function of an infinite discrete distribution is given by $P(X = j) = 1/2\, j$; $j = 1, 2, ..., \infty$. Verify that the total probability is 1 and find the mean and variance of the distribution. Find also $P(X$ is even), $P(X \ge 5)$ and $P(X$ is divisible by 3).

(ii) Define Gamma distribution and find its mean and variance.

12. (a) (i) The joint probability mass function of (X, Y) is given by $P(x, y) = K(2x + 3y), x = 0, 1, 2;\ y = 1, 2, 3$. Find all the marginal and conditional probability distributions.

(ii) State and prove central limit theorem.

Or

(b) (i) If X and Y are independent RVs with pdf's $e^{-x}, x \ge 0$, and $e^{-y}, y \ge 0$, respectively, find the density functions of $U = \dfrac{X}{X+Y}$ and $V = X + Y$. Are U and V independent?

(ii) Find the correlation coefficient for the following data :

X	1	1	1	2	2	3

:	0	4	8	2	6	0
Y	1	1	2	6	3	3
:	8	2	4		0	6

13. (a) (i) Define Poisson process and derive the Poisson probability law.

(ii) A man either drives a car (or) catches a train to go to office each day. He never goes two days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find (1) the probability that he takes a train on the third day and (2) the probability that he drives to work in the long run.

Or

(b) (i) Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is wide-sense stationary, if A and ω_0 are constants and θ is a uniformly distributed RV in $(0, 2\pi)$.

(ii) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is (1) more than 1 min. (2) between 1 min and 2 min and (3) 4 min (or) less.

14. (a) Find the mean number of customers in the queue, system, average waiting time in the queue and system of $M/M/1$ queueing model.

Or

(b) There are three typists in an office. Each typist can type an average of
6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour,

(i) What fraction of the time all the typists will be busy?

(ii) What is the average number of letters waiting to be typed?

(iii) What is the average time a letter has to spend for waiting and for being typed?

(iv) What is the probability that a letter will take longer than 20 min. waiting to be typed and being typed?

15. (a) Discuss $M/G/1$ queueing model and derive Pollaczek-Khinchine formula.

Or

(b) Discuss open and closed networks.