# MATHEMATICS

**1.** If $A^2 - A + I = O$, then the inverse of $A$ is

(a) $A - I$ (b) $I - A$ (c) $A + I$ (d) $A$

**2.** If $C$ is the mid-point of $AB$ and $P$ is any point outside $AB$, then

(a) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = \vec{0}$ (b) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec{0}$

(c) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$ (d) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$

**3.** If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$, then its roots are

(a) 0, 1 (b) –1, 1 (c) 0, –1 (d) –1, 2

**4.** Let $f : R \to R$ be a differentiable function having $f(6) = 6$, $f'(6) = \left(\frac{1}{48}\right)$.

Then $\lim_{x \to 2} \int_6^{f(x)} \frac{4t^3}{x - 2} dt$ equals

(a) 12 (b) 18 (c) 24 (d) 36

**5.** If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(a) $\alpha = a^2 + b^2,\ \beta = 2ab$ (b) $\alpha = a^2 + b^2,\ \beta = a^2 - b^2$

(c) $\alpha = 2ab,\ \beta = a^2 + b^2$ (d) $\alpha = a^2 + b^2,\ \beta = ab$

**6.** The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if

(a) $k = 1$ or $-1$ (b) $k = 0$ or $-3$

(c) $k = 3$ or $-3$ (d) $k = 0$ or $-1$

**7.** If the coefficients of $r$th, $(r+1)$th and $(r+2)$th terms in the binomial expansion of $(1+y)^m$ are in arithmetic progression, then $m$ and $r$ satisfy the equation

(a) $m^2 - m(4r+1) + 4r^2 - 2 = 0$ (b) $m^2 - m(4r-1) + 4r^2 + 2 = 0$

(c) $m^2 - m(4r-1) + 4r^2 - 2 = 0$ (d) $m^2 - m(4r+1) + 4r^2 + 2 = 0$

**8.** In $\Delta ABC$, $\tan\frac{A}{2} = \frac{5}{6}$, $\tan\frac{C}{2} = \frac{2}{5}$, then

(a) $a, c, b$ are in A.P. (b) $a, b, c$ are in A.P.

(c) $b, a, c$ are in A.P. (d) $a, b, c$ are in G.P.

**9.** The equation of the chord joining two points $(x_1, y_1)$ and $(x_2, y_2)$ on the rectangular hyperbola $xy = c^2$ is

(a) $\dfrac{x}{x_1+x_2}+\dfrac{y}{y_1+y_2}=1$ (b) $\dfrac{x}{x_1-x_2}+\dfrac{y}{y_1-y_2}=1$

(c) $\dfrac{x}{y_1+y_2}+\dfrac{y}{x_1+x_2}=1$ (d) $\dfrac{x}{y_1-y_2}+\dfrac{y}{x_1-x_2}=1$

**10.** *A* and *B* are two independent events such that $P(A)=\dfrac{1}{5}$, $P(A\cup B)=\dfrac{7}{10}$. Then $P(\overline{B})$ is

(a) $\dfrac{3}{8}$ (b) $\dfrac{2}{7}$ (c) $\dfrac{7}{9}$ (d) none of these

**11.** If a circle passes through the point $(a, b)$ and cuts the circle $x^2+y^2=p^2$ orthogonally, then the equation of the locus of its centre is

(a) $x^2+y^2-2ax-3by+(a^2-b^2-p^2)=0$ (b) $2ax+2by-(a^2+b^2+p^2)=0$

(c) $x^2+y^2-3ax-4by+(a^2+b^2-p^2)=0$ (d) $2ax+2by-(a^2-b^2+p^2)=0$

**12.** Let $z$, $w$ be complex numbers such that $\bar{z}+i\,\overline{w}=0$ and $\arg zw=\pi$. Then $\arg z$ equals

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{3\pi}{4}$ (d) $\dfrac{5\pi}{4}$

**13.** Let $T_r$ be the $r$th term of an arithmetic progression, whose first term is $a$ and common difference is $d$. If for some positive integers $m$, $n$, $m \neq n$, $T_m=\dfrac{1}{n}$ and $T_n=\dfrac{1}{m}$, then $a-d$ equals

(a) 0 (b) 1 (c) $\dfrac{1}{mn}$ (d) $\dfrac{1}{m}+\dfrac{1}{n}$

**14.** Let $\dfrac{d}{dx}F(x)=\left(\dfrac{e^{\sin x}}{x}\right)$, $x>0$. If $\displaystyle\int_1^4 \frac{3}{x}e^{\sin x^3}\,dx=F(k)-F(1)$, then one of the possible value of $k$, is

(a) 16 (b) 63 (c) 64 (d) 15

**15.** The equation of the common tangent touching the circle $(x-3)^2+y^2=9$ and the parabola $y^2=4x$ above the $x$-axis is

(a) $\sqrt{3}y=3x+1$ (b) $\sqrt{3}y=-(x+3)$

(c) $\sqrt{3}y=x+3$ (d) $\sqrt{3}y=-(3x+1)$

**16.** If the letters of the word SACHIN arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number

(a) 603 (b) 602 (c) 601 (d) 600

**17.** If $\cos^{-1}x-\cos^{-1}\dfrac{y}{2}=\alpha$, then $4x^2-4xy\cos\alpha+y^2$ is equal to

(a) $4\sin^2\alpha$ (b) $-4\sin^2\alpha$ (c) $2\sin 2\alpha$ (d) 4

**18.** If $x\frac{dy}{dx} = y(\log y - \log x + 1)$ then the solution of the equation is

(a) $\log\left(\frac{y}{x}\right) = cx$ (b) $\log\left(\frac{x}{y}\right) = cy$ (c) $y\log\left(\frac{x}{y}\right) = cx$ (d) $x\log\left(\frac{y}{x}\right) = cy$

**19.** Consider the following statements:

(1) Mode can be computed from histogram

(2) Median is not independent of change of scale

(3) Variance is independent of change of origin and scale

Which of these is/are correct?

(a) only (1) (b) only (2) (c) only (2) (d) (1), (2) and (3)

**20.** A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five given questions. The number of choices available to him is

(a) 196 (b) 280 (c) 346 (d) 140

**21.** The upper $\frac{3}{4}$th portion of a vertical pole subtends an angle $\tan^{-1}\frac{3}{5}$ at a point in the horizontal plane through its foot and at a distance 40 m form the foot. A possible height of the vertical pole is

(a) 40 m (b) 60 m (c) 80 m (d) 20 m

**22.** The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having its centre at (0, 3) is

(a) 4 (b) 3 (c) $\sqrt{12}$ (d) $7/2$

**23.** The interior angles of a complex polygon are in arithmetic progression with common difference 5°. If smallest angle is $\frac{2\pi}{3}$, then number of sides are

(a) 9 (b) 7 (c) 16 (d) 20

**24.** The number of values of '$a$' for which $(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a^2 - 4 = 0$ is an identity in $x$, is

(a) 0 (b) 1 (c) 2 (d) 3

**25.** For a non-zero complex number $z$, $\left|\frac{|\bar{z}^2|}{z\,\bar{z}}\right|$ is equal to

(a) $\left|\frac{\bar{z}}{z}\right|$ (b) 2 (c) $|\bar{z}|$ (d) none of these

**26.** Let $f(x) = \max.\{x, 2 - x\}$ for all $x \in R$. Then

(a) $f(x)$ is not continuous everywhere (b) $f(x)$ is differentiable everywhere

(c) $f(x)$ is continuous at $x = 1$ but not differentiable there

(d) $f(x)$ is neither continuous nor differentiable at $x = 1$

**27.** Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $f(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)g(x)\,dx$, is

(a) $e + \frac{e^2}{2} - \frac{3}{2}$ (b) $e - \frac{e^2}{2} - \frac{3}{2}$ (c) $e + \frac{e^2}{2} + \frac{5}{2}$ (d) $e - \frac{e^2}{2} - \frac{5}{2}$

**28.** Let $z_1$ and $z_2$ be two roots of the equation $z^2 + az + b = 0$, $z$ being complex. Further, assume that the origin $z_1$ and $z_2$ form an equilateral triangle. Then

(a) $a^2 = 2b$ (b) $a^2 = 3b$ (c) $a^2 = 4b$ (d) $a^2 = b$

**29.** If 1, $\frac{1}{2}\log_3(3^{1-x} + 2)$, $\log_3(4.3^x - 1)$ are in A.P., then $x$ equals

(a) $\log_3 4$ (b) $1 - \log_3 4$ (c) $1 - \log_4 3$ (d) $\log_4 3$

**30.** *A* and *B* play a game where each is asked to select a number from 1 to 25. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial is

(a) 1/25 (b) 24/25 (c) 2/25 (d) none of these

**31.** $\lim_{n\to\infty}\frac{1}{n}\left(\sqrt[n]{e} + \sqrt[n]{e^2} + \sqrt[n]{e^3} + \ldots\ldots\ldots + \sqrt[n]{e^n}\right)$ is equal to

(a) $e + 1$ (b) $e - 1$ (c) $e$ (d) none of these

**32.** Given the statement: If $x$ is an integer and $x^2$ is even, then $x$ is also even

(a) false (b) true (c) unpredictable (d) none of these

**33.** The value of $a$ for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is

(a) 3 (b) 2 (c) 1 (d) 0

**34.** A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm$^3$/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is

(a) $\frac{1}{54\pi}$ cm/min. (b) $\frac{5}{6\pi}$ cm/min. (c) $\frac{1}{36\pi}$ cm/min. (d) $\frac{1}{18\pi}$ cm/min.

**35.** A circle touches the $x$-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is

(a) a hyperbola (b) a parabola (c) an ellipse (d) a circle

**36.** Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

(a) $\frac{8}{9}$ (b) $\frac{7}{9}$ (c) $\frac{2}{9}$ (d) $\frac{1}{9}$

**37.** If the angle $\theta$ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is such that $\sin\theta=\frac{1}{3}$ the value of $\lambda$ is

(a) $\frac{3}{4}$ (b) $-\frac{4}{3}$ (c) $\frac{5}{3}$ (d) $-\frac{3}{5}$

**38.** The range of the function $f(x)={}^{7-x}P_{x-3}$ is

(a) $\{1, 2, 3\}$ (b) $\{1, 2, 3, 4, 5, 6\}$
(c) $\{1, 2, 3, 4\}$ (d) $\{1, 2, 3, 4, 5\}$

**39.** Let $A=\begin{pmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{pmatrix}$ and $(10)B=\begin{pmatrix}4 & 2 & 2\\ -5 & 0 & \alpha\\ 1 & -2 & 3\end{pmatrix}$. If $B$ is the inverse of matrix $A$, then $\alpha$ is

(a) –2 (b) –1 (c) 2 (d) 5

**40.** If $\int_0^{\pi} x\, f(\sin x)\,dx = A\int_0^{\pi/2} f(\sin x)\,dx$, then $A$ is

(a) 0 (b) $\pi$ (c) $\frac{\pi}{4}$ (d) $2\pi$