# ANSWERS TO PAPER – II
## MATHEMATICS

**1.** **(a)** $(fog)(x) = f(g(x)) = f(3x-1)$

$= 3(3x-1)^2 + 2 = 3(9x^2 - 6x + 1) + 2 = 27x^2 - 18x + 5$

$(gof)(x) = g(f(x)) = g(3x^2 + 2) = 3(3x^2 + 2) - 1 = 9x^2 + 5$

**2.** **(a)**

**3.** **(a)**

**4.** **(a)**

**5.** **(c)**

**6.** **(d)**

**7.** **(c)** Let $\alpha$ be the common root of the given equation. Then

$a\alpha^2 + 2c\alpha + b = 0$ and $a\alpha^2 + 2b\alpha + c = 0$

$\Rightarrow \quad 2\alpha(c-b) + (b-c) = 0$

$\Rightarrow \quad \alpha = \frac{1}{2} \qquad [\because\ b \neq c]$

Putting $\alpha = \frac{1}{2}$ in $a\alpha^2 + 2c\alpha + b = 0$, we get $a + 4b + 4c = 0$

**8.** **(a)** Let $u$ be a variable such that $x + y + z + u = n$

Then, $x + y + z \leq n \quad \Rightarrow \quad u \geq 0$

The number of non-negative integral solutions of the given inequality is same as the number of non-negative integral solutions of $x + y + z + u = n$.

Hence, the required number of solutions is ${}^{n+4-1}C_{4-1} = {}^{n+3}C_3$

**9.** **(c)**

**10.** **(b)**

**11.** **(a)**

**12.** **(a)**

**13.** **(b)**

**14.** **(a)**

**15.** **(b)** $|\omega| = \left|\dfrac{1-iz}{z-i}\right| = 1 \quad \Rightarrow \quad |1-iz|^2 = |z-i|^2$

$\Rightarrow \quad (1-iz)(1+i\bar{z}) = (z-i)(\bar{z}+i) \quad \Rightarrow \quad 2i(z-\bar{z}) = 0$

$\Rightarrow \quad 2i(2iy) = 0 \Rightarrow \; y = 0$, which is the equation of the real axis.

Hence $z$ lies on the real axis.

**16.** **(c)**

**17.** **(d)**

**18.** **(b)** Let $\dfrac{x}{c} = z \quad \Rightarrow \quad \dfrac{1}{c}dx = dz$, when $x \to ac$, then $z \to a$ and $x \to bc$, then $z \to b$

given integral = $\displaystyle\int_a^b f(z)\,dz = \int_a^b f(x)\,dx$

**19.** **(a)**

**20.** **(c)** $\dfrac{x^2 - bx}{ax - c} = \dfrac{\lambda - 1}{\lambda + 1}$

$\Rightarrow \quad (\lambda+1)(x^2 - bx) = (\lambda-1)(ax - c) \quad \Rightarrow \quad (\lambda+1)x^2 - [(\lambda+1)b + a(\lambda-1)]x + c(\lambda-1) = 0$

If $\alpha$, $\beta$ are the roots of the equation then $\alpha + \beta = \dfrac{b(\lambda+1) + a(\lambda-1)}{(\lambda+1)} = 0$

$\therefore \quad (a+b)\lambda = a - b \quad \Rightarrow \quad \lambda = \dfrac{a-b}{a+b}$

**21.** **(a)** $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 0 & a-b & a^2 - bc - b^2 + ca \\ 0 & b-c & b^2 - ca - c^2 + ab \\ 1 & c & c^2 - ba \end{vmatrix}$ Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

Expanding along first column, we get the value of determinant is equal to zero.

**22.** **(d)**

**23.** **(b)**

**24.** **(c)**

**25.** **(b)** $\frac{dx}{dt} = a\left(1 - \frac{1}{t^2}\right) = a\left(\frac{t^2 - 1}{t^2}\right)$

$\frac{dy}{dt} = a\left(1 + \frac{1}{t^2}\right) = a\left(\frac{t^2 + 1}{t^2}\right)$; $\quad \therefore \; \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{t^2 + 1}{t^2 - 1}$

**26.** **(d)**

**27.** **(c)** Total number of mappings = $n^n$

Favourable cases (mapping is one-one) = ${}^nP_n = n!$

**28.** **(c)**

**29.** **(c)** $\frac{s-a}{s} = \frac{s-c}{s-b} \quad \Rightarrow \quad (s-a)(s-b) = s(s-c) \;\Rightarrow\; \sin^2\frac{C}{2} = \cos^2\frac{C}{2} \quad \Rightarrow \quad C = 90°$

**30.** **(c)** Two conic are confocal if they have same foci

Foci of the ellipse is $S(\pm kae_1, 0)$ and the foci of the hyperbola is $S(\pm ae_2, 0)$

$e_1 = \sqrt{1 - \frac{a^2}{k^2a^2}} = \frac{\sqrt{k^2 - 1}}{k}, \quad e_2 = \sqrt{1 + \frac{a^2}{a^2}} = \sqrt{2}$

For conics to be confocal, $kae_1 = ae_2 \quad \Rightarrow \quad k = \pm\sqrt{3}$

**31.** **(a)**

**32.** **(b)** $\vec{a}.(\vec{b} \times \vec{c}) = 0 \Rightarrow \begin{vmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 3 & 2 & p \end{vmatrix} = 0 \;\Rightarrow\; 1(-p-8) - 3(2p-12) - 2(4+3) = 0$

$\Rightarrow \quad -p - 8 - 6p + 36 - 14 = 0 \;\Rightarrow\; -7p + 14 = 0$

$\therefore \quad p = 2$

**33.** **(d)** $A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 3A$

**34.** **(b)** $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$ , Take $C_1 = C_1 + C_2 + C_3$, $\begin{vmatrix} x+1 & 1 & 1 \\ x+1 & x-1 & 1 \\ x+1 & 1 & x-1 \end{vmatrix} = 0$

Take $R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$\begin{vmatrix} x+1 & 1 & 1 \\ 0 & x-2 & 0 \\ 0 & 0 & x-2 \end{vmatrix} = 0$ i.e. $(x+1)(x-2)(x-2) = 0 \ \Rightarrow\ x = -1, 2$

**35.** **(c)** $A^{-1} = \frac{1}{|A|} adjA = \frac{1}{1}\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

**36.** **(d)** $|kA| = k^n |A|$

**37.** **(a)** $A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \Rightarrow A^3 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix}$, $\therefore\ A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

**38.** **(b)** $A = 45^o, \frac{b}{\sin B} = \frac{a}{\sin A}$ *i.e.* $b = \frac{a \sin B}{\sin A} = \frac{2\sin 60^o}{\sin 45^o} = \frac{2\left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{\sqrt{2}}} = \sqrt{6}$

**39.** **(b)** $\cos^{-1}\frac{2}{\sqrt{5}} + \tan^{-1}\frac{1}{3} = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \tan^{-1}\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}} = \tan^{-1}(1) = \frac{\pi}{4}$

**40.** **(a)** $(1+i)^5 + (1-i)^5 = (\sqrt{2})^5 2\cos\frac{5\pi}{4} = -4\sqrt{2}.2\cos\frac{\pi}{4} = -8$

**41.** **(c)** $\cot^{-1} x = \theta \ \Rightarrow\ \sin\theta = \frac{1}{\sqrt{1+x^2}}$ ; $\tan^{-1}(\sin\theta) = \tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \alpha$

$\tan\alpha = \frac{1}{\sqrt{1+x^2}}$, $\therefore\ \cos\alpha = \sqrt{\frac{1+x^2}{2+x^2}}$

**42.** **(a)** $\theta = \tan^{-1}\left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \tan^{-1}\left|\frac{2\sqrt{25/4+6}}{1}\right|$

$= \tan^{-1} 2\sqrt{\frac{49}{4}} = \tan^{-1}(7)$

**43.** **(a)** $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ i.e., $2(-1)(4) + 2\frac{3}{2}(-3) = k - 7$

$\Rightarrow \quad -8 - 9 = k - 7, \quad \therefore \quad k = -10$

**44.** **(b)** $S(a+h, k) = (2, 0)$

$\therefore \quad a + h = 2$ and $k = 0$ $x = -3$ is $x = -a + h = -3$; $\therefore \quad h = \frac{-1}{2}, a = \frac{5}{2}$

Equation of the parabola is $(y-0)^2 = 4\left(\frac{5}{2}\right)\left(x + \frac{1}{2}\right)$ i.e., $y^2 = 10x + 5$

**45.** **(b)** $\frac{x^2}{36} + \frac{y^2}{9} = 1$ $a^2 = 36, a = 6$, $\therefore \quad SP + S'P = 2a = 12$

**46.** **(d)** $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x} = \int_0^{\pi/2} \frac{\sec^2 x\, dx}{a^2 + b^2 \tan^2 x}$

$= \frac{1}{b}\int_0^{\infty} \frac{dt}{a^2 + t^2} = \frac{1}{b}\left[\frac{1}{a}\tan^{-1}\frac{t}{a}\right]_0^{\infty}$. Put $b\tan x = t \Rightarrow b\sec^2 x\, dx = dt$,

when $x \to 0$, $t \to 0$ and $x \to \frac{\pi}{2}, t \to \infty$

$= \frac{1}{ab}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{2ab}$

**47.** **(a)** $I = \int_0^{\pi/2} \log \sin x\, dx$ …(i)

$I = \int_0^{\pi/2} \log \cos x\, dx$ …(ii)

$\therefore \quad 2I = \int_0^{\pi/2} \log \sin x \cos x\, dx = \int_0^{\pi/2} \log\left(\frac{\sin 2x}{2}\right) dx$

$= \int_0^{\pi/2} \log \sin 2x\, dx = \int_0^{\pi/2} \log 2 dx = I_1 - (\log 2)\frac{\pi}{2}$ …(iii),

where $I_1 = \int_0^{\pi/2} \log \sin 2x\, dx = \int_0^{\pi} \frac{1}{2} \log \sin y\, dy$, put $2x = y \Rightarrow \quad dx = \frac{1}{2}dy$

$\Rightarrow x \to 0, y \to 0$ and $x \to \frac{\pi}{2}, y \to \pi$

$$I_1 = \frac{1}{2}\int_0^{\pi} \log \sin y\, dy = \frac{1}{2} 2\int_0^{\pi/2} \log \sin y\, dy = I \quad (\text{Because } \log \sin(\pi - y) = \log \sin y)$$

So, $2I = I - \frac{\pi}{2}\log 2;\ I = -\frac{\pi}{2}\log 2 = \frac{\pi}{2}\log\frac{1}{2}$

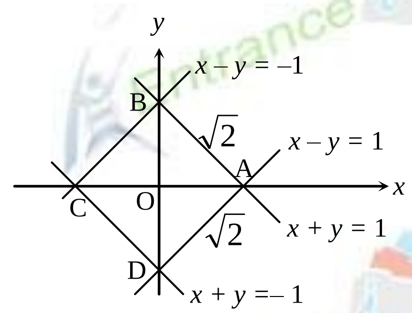
**48.** **(a)** $\lim\limits_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\ldots\text{to } n \text{ terms}\right]$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n+r} = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{1+\frac{r}{n}} = \int_0^1 \frac{1}{1+x}\,dx = [\log(1+x)]_0^1 = \log 2$$

**49.** **(b)** $A(1, 0), B(0, 1), C(-1, 0), D(0, -1)$

So, $AB = \sqrt{2}$

$\Rightarrow$ area = 2sq. units




**50.** **(c)** $y = x^2 - 7x + 10$ put $y = 0, x^2 - 7x + 10 = 0$ i.e., $(x-2)(x-5) = 0$

$\therefore \quad x = 2, x = 5$

$$\text{Area} = \int_2^5 y\,dx = \int_2^5 (x^2 - 7x + 10)dx = \left[\frac{x^3}{3} - \frac{7x^2}{2} + 10x\right]_2^5$$

$$= \left[\frac{125}{3} - \frac{175}{2} + 50\right] - \left(\frac{8}{3} - 14 + 20\right) = \frac{117}{3} - \frac{175}{2} + 44 = \frac{-9}{2}$$

$\therefore \quad \text{Area} = \frac{9}{2}$

**51.** **(b)** $\vec{a} + \vec{b} + \vec{c} = 2\hat{i} + 2\hat{j} + 2\hat{k} = 2(\hat{i} + \hat{j} + \hat{k})$

unit vector parallel to this vector $= \frac{2(\hat{i}+\hat{j}+\hat{k})}{\sqrt{12}} = \frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$

**52.** **(c)** $\lim\limits_{n\to\infty}\left[1+\frac{2}{n}\right]^{2n}$ put $n = 2N$ when $n \to \infty, N \to \infty$

$$\lim_{n\to\infty}\left[1+\frac{1}{N}\right]^{4N} = e^4$$

**53.** **(a)** $x^y = e^{x-y}$. Taking logarithms on both sides, $y \log x = (x - y)\log e$

i.e. $y \log x = x - y$, i.e., $y + y \log x = x$ or $y = \frac{x}{1+\log x}$

$$\therefore \frac{dy}{dx} = \frac{(1+\log x)1 - x\frac{1}{x}}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$$

**54.** **(d)** $x^m y^n = a^{m+n}$

$m \log x + n \log y = (m+n)\log a$

$\frac{m}{x} + \frac{n}{y}.\frac{dy}{dx} = 0$, $\therefore \frac{dy}{dx} = -\frac{my}{nx}$

$\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\frac{my_1}{nx_1}$

length subtangent $= \left|\frac{y_1}{-\frac{my_1}{nx_1}}\right| = \left|-\frac{nx_1}{m}\right|$. So length is $= \left|\frac{nx_1}{m}\right|$

**55.** **(b)** $xy = 2$ …(i)

$y^2 = 4x$ …(ii)

solving we get

$x = 1, y = 2$

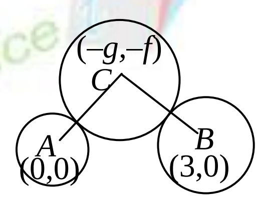
The point of intersection is (1, 2)

By (i), $\frac{dy}{dx} + y = 0 \therefore \frac{dy}{dx} = \frac{-y}{x}; \left(\frac{dy}{dx}\right)_{(1,2)} = \frac{-2}{1} = m_1$

By (ii), $2y\frac{dy}{dx} = 4 \therefore \frac{dy}{dx} = \frac{2}{y}; \left(\frac{dy}{dx}\right)_{(1,\,2)} = 1 = m_2$

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \tan^{-1}\left|\frac{-2-1}{1-2}\right| = \tan^{-1}(3)$$

**56.** **(a)** Take the given circles as




$x^2 + y^2 = 4$ (centre (0, 0) radius $= r_2 = 2$) …(i)

$(x-3)^2 + y^2 = 1$ (centre (3, 0), radius $= r_2 = 1$) …(ii)

Let the circle $x^2 + y^2 + 2gx + 2fy - 1 = 0$ …(iii)

Touch the circle (i) and (ii). Let the radius be $r$.

$AC^2 = g^2 + f^2 = (2+r)^2$ ...(iv)

$BC^2 = (g+3)^2 + f^2 = (1+r)^2$ ...(v)

(iv) – (v) gives $-6g - 9 = 3 + 2r$ i.e. $-(3g+6) = r$

Square both sides $9g^2 + 36 + 36g = r^2$

i.e. $9g^2 + 36 + 36g = g^2 + f^2 + 1$

i.e. $8g^2 - f^2 + 36g + 35 = 0$

The locus of the centre $(-g, -f)$ is $8x^2 - y^2 - 36x + 35 = 0,$ which is a hyperbola.

**57.** **(c)** The eccentricity of rectangular hyperbola is $\sqrt{2}$ because $a = b.$

**58.** **(c)** This problem is done by using mathematical induction. Here it is enough if we proceed as follows,

when $n = 1,$ LHS = 1 . 1! = 2! – 1

when $n = 2,$ LHS = 1 . 1! + 2 . 2! = 5 = 3! – 1

when $n = 3,$ LHS = 1 . 1! + 2 . 2! + 3 . 3! = 1 + 4 + 18 = 4! –1

$\therefore$ for $n = n,$ LHS $= (n+1)! - 1$

**59.** **(d)** Here $(m+2)$ parallel lines of one type cut $(m+2)$ parallel lines of another type

$\therefore$ Number of parallelograms formed $= {}^{m+2}C_2 \times {}^{m+2}C_2$

$= \dfrac{(m+2)(m+1)}{2.1} \times \dfrac{(m+2)(m+1)}{2.1} = \dfrac{(m+2)^2(m+1)^2}{4}$

**60.** **(c)** $6^2 \equiv 6(\text{mod}10)$

$6^3 \equiv 6(\text{mod}10).$ Similarly $6^{500} \equiv 6(\text{mod}10)$

$\therefore$ Unit digit is 6.

**61.** **(c)**

**62.** **(c)** $(2a)^2 + (3b)^2 + (4c)^2 - (2a)(3b) - (3b)(4c) - (4c)(2a) = 0$

$\Rightarrow (2a - 3b)^2 + (3b - 4c)^2 + (4c - 2a)^2 = 0$

$\Rightarrow 2a = 3b = 4c \Rightarrow \dfrac{a}{\frac{1}{2}} = \dfrac{b}{\frac{1}{3}} = \dfrac{c}{\frac{1}{4}}$

63. **(b)** AM ≥ GM for positive numbers. So, $\dfrac{4^x + \dfrac{4}{4^x}}{2} \ge \sqrt{4^x \cdot \dfrac{4}{4^x}} = 2$.

$4^x + 4^{1-x} \ge 4$

64. **(b)**

65. **(d)**

66. **(b)**

67. **(b)** Expression = $(1+i)2i + (-2i)^3$.

68. **(a)** $\dfrac{1+z}{z+\bar{z}} = \dfrac{z\bar{z}+z}{1+\bar{z}}$ $(\because |z|=1 \Rightarrow |z|^2=1 \Rightarrow z\bar{z}=1)$ = $\dfrac{z(\bar{z}+1)}{1+\bar{z}} = z$.

69. **(c)**

$$\begin{array}{cccccc} & \times & \times & \times & \times & \times \\ \text{Ways} & 9 & 9 & 9 & 9 & 9 \end{array}$$

0 cannot be placed in the first place. In the next place any digit except the one used in the first place can be used, etc.

70. **(b)** The candidate is unsuccessful if he fails in 9 or 8 or 7 or 6 or 5 papers.

$\therefore$ the number of ways to be unsuccessful

$= {}^9C_9 + {}^9C_8 + {}^9C_7 + {}^9C_6 + {}^9C_5 = {}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + {}^9C_4$

$= \dfrac{1}{2}({}^9C_0 + {}^9C_1 + ..... + {}^9C_9) = \dfrac{1}{2} \cdot 2^9 = 2^8$

71. **(c)** $\Delta = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & -yz & 1 \\ y & -zx & 1 \\ z & -xy & 1 \end{vmatrix}$

$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} - \dfrac{1}{xyz} \begin{vmatrix} x^2 & xyz & x \\ y^2 & zxy & y \\ z^2 & xyz & z \end{vmatrix}$ etc.

72. **(c)**

**73.** **(c)**

**74.** **(b)**

**75.** **(a)**

**76.** **(a)**

**77.** **(c)** Here, $\cos x = \dfrac{2\cos(x-y)\cos(x+y)}{\cos(x-y)+\cos(x+y)} = \dfrac{\cos^2 x - \sin^2 y}{\cos x.\cos y}$

or $\cos^2 x(1-\cos y) = \sin^2 y$

or $\cos^2 x = 2\cos^2 \dfrac{y}{2}$ or $\left(\cos x \cdot \sec \dfrac{y}{2}\right)^2 = 2$

**78.** **(c)** $\cos\alpha = \dfrac{1}{2}\left(x + \dfrac{1}{x}\right)$

$\Rightarrow$ $x = \cos\alpha \pm i \sin\alpha$. Similarly, $y = \cos\beta \pm i \sin\beta$

$\therefore$ $\dfrac{x}{y} = \cos(\alpha-\beta) \pm i\sin(\alpha-\beta)$.

**79.** **(d)**

**80.** **(d)** $\dfrac{1}{\sqrt{2}}\left(\cos\dfrac{9\pi}{10} - \sin\dfrac{9\pi}{10}\right) = \cos\left(\dfrac{\pi}{4} + \dfrac{9\pi}{10}\right)$

$= \cos\dfrac{23\pi}{20} = \cos\left(2\pi - \dfrac{23\pi}{20}\right) = \cos\dfrac{17\pi}{20}$

$\therefore$ the value $= \cos^{-1}\left(\cos\dfrac{17\pi}{20}\right) = \dfrac{17\pi}{20}$.

**81.** **(c)** $|4-5x| > 2^2 = 4$

$\Rightarrow$ $\left|\dfrac{5x}{4} - 1\right| > 1$

$\Rightarrow$ $\dfrac{5x}{4} - 1 > 1$ or $\dfrac{5x}{4} - 1 < -1$

$\therefore$ $x > \dfrac{8}{5}$ or $x < 0$.

So, the solution set = $(-\infty, 0) \cup \left(\dfrac{8}{5}, \infty\right)$.

**82.** **(c)** $2s(2s - 2c) = ab$ or $\frac{s(s-c)}{ab} = \frac{1}{4}$

or $\cos^2 \frac{C}{2} = \frac{1}{4}$ or $\cos \frac{C}{2} = \frac{1}{2}$ $\left(\because \frac{C}{2} \text{ must be acute}\right)$

**83.** **(a)**

**84.** **(b)**

**85.** **(c)**

**86.** **(d)** Let the point be $(t, t)$

So, $\left| \frac{\frac{t}{4} + \frac{t}{3} - 1}{\sqrt{\frac{1}{4^2} + \frac{1}{3^2}}} \right| = 4 \Rightarrow \left(\frac{t}{4} + \frac{t}{3}\right) = 1 \pm 4 \cdot \sqrt{\frac{1}{4^2} + \frac{1}{3^2}}$

**87.** **(a)**

**88.** **(c)**

**89.** **(a)**

**90.** **(c)** The tangents to the parabola $y^2 = 4ax$ at the points $(a, 2a)$, $(a, -2a)$ are $y = x + a$ and $y = -x - a$.
The third side of the triangle is $x = a$.
Clearly, these lines form a right-angled triangle whose two sides are equal.

**91.** **(a)** The distance between foci = $2ae = 4$ and $e = \frac{2}{3}$.

$\Rightarrow a = 3$. So, $b^2 = a^2(1 - e^2) = 9 \cdot \left(1 - \frac{4}{9}\right) = 5$.

**92.** **(b)** For the ellipse, $a^2 = 16; b^2 = a^2(1 - e^2)$

$\Rightarrow e = \frac{\sqrt{16 - b^2}}{4} \Rightarrow ae = \sqrt{16 - b^2}$.

For the hyperbola, $a^2 = \frac{144}{25}, b^2 = \frac{81}{25}; b^2 = a^2(e^2 - 1)$

$\Rightarrow \; e = \frac{5}{4} \Rightarrow ae = 3.$ $\qquad \therefore \; \sqrt{16 - b^2} = 3.$

**93.** **(a)**

**94.** **(d)** $x + n - [x + n]$ has the period 1 and $\tan\frac{\pi x}{2}$ has the period $\frac{\pi}{\frac{\pi}{2}}$,

i.e., 2, LCM of 1, 2 is 2.

**95.** **(c)** The function $f(x) = x - [x]$ is same as $f(x) = \{x\}$, which is many-one and, therefore, inverse function is not defined.

**96.** **(c)** $x = e^{y+x} \Rightarrow \log x = y + x;$

$\therefore \; \frac{1}{x} = \frac{dy}{dx} + 1.$

**97.** **(a)** $\lim_{x \to 0} \frac{\log_e(1+x) + x^2 - x}{x^2} = \lim_{x \to 0} \frac{\left(x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots\right) + x^2 - x}{x^2} = \frac{1}{2}.$

**98.** **(a)** $f(1+0) = \lim_{x \to 1} (|1 + h - 1| - [1 + h]) = \lim_{h \to 0} (h - 1) = -1$

$f(1-0) = \lim_{h \to 0} (|1 - h - 1| - [1 - h]) = \lim_{h \to 0} (h - 0) = 0$

**99.** **(b)** $h(x) = \min\{x, x^2\} = x, x \le 0,\ x^2, 0 < x < 1, x, x \ge 1.$

As $x,\ x^2$ are polynomial functions, they are continuous and differentiable in their respective intervals of definition. So, the only doubtful points are 0 and 1. Check continuity and differentiability at $x = 0, 1$.

**100.** **(d)** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^t(\cos t - \sin t)}{e^t(\sin t + \cos t)}$ $\quad \therefore \quad \left.\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = 0$.