Quantitative Ability

46. Let's calculate this from the last.

A	B	C	D
x	x	x	x

So after 'C' gave 1/5 of her share to D she is left with 'x'.

$\Rightarrow$ Before she gave to D she had $\frac{5}{4}x$.

A	B	C	D
x	x	$\frac{5}{4}x$	$\frac{3}{4}x$

So this is the situation before C gave to D (or) after B gave to C.
So B is left with 'x' after she gave $1/4^{th}$ of his chocolates to C.

$\Rightarrow$ Before B gave to C, she had $\frac{4}{3}x$.

A	B	C	D
x	$\frac{4}{3}x$	$\frac{11}{12}x$	$\frac{3}{4}x$

This is the situation after A gave $1/3^{rd}$ of his share to B.

$\Rightarrow$ Initially 'A' had $\frac{3}{2}x$

$\therefore$

A	B	C	D
$\frac{3}{2}x$	$\frac{5}{6}x$	$\frac{11}{12}x$	$\frac{3}{4}x$

Given A – B = 80

$\Rightarrow \frac{4}{6}x = 80 \Rightarrow x = 120$ and C – D = $\frac{1}{6}x = 20$

Alternative solution:

Considering only the transactions A $\to$ B and B $\to$ C, A will have $\frac{2}{3}A$ and B will have $\frac{3}{4}\left(B + \frac{A}{3}\right)$. But it is given that A and B finally had an equal number. So we get $\frac{2}{3}A = \frac{3}{4}\left(B + \frac{A}{3}\right)$ --- (1). Also, given that A – B = 80 ---- (2).

Now, from (1) and (2) we get A = 180 and B = 100 and also that everyone has 120 chocolates with them in the end.

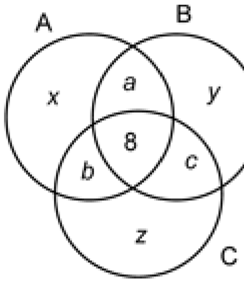
Now considering the transactions B $\to$ C and C $\to$ D, we get

$$\frac{4}{5}\left(\frac{C + \left(100 + \frac{80}{3}\right)}{4}\right) = \frac{1}{5}\left(C + \frac{\left(100 + \frac{180}{3}\right)}{4}\right) + D = 120$$

$\Rightarrow$ C = 110 and D = 90

$\Rightarrow$ C – D = 20 Choice (1)

47.



Given that $a + b + c + 8 = 85$

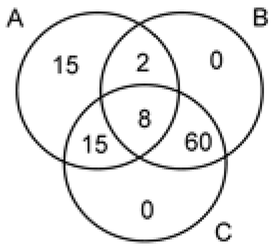
$\Rightarrow a + b + c = 77$

For a to be minimum $b + c$ should be maximum.

The maximum value of $b + c$ is 75 ($\because n(c) = 83$)

$\therefore$ The minimum value of a is 2.

The following values satisfy all the constraints.



Choice (2)

48. First give away two marbles to each brother. Then the problem is equivalent to the situation where the remaining 17 marbles need to be distributed among four brothers, such that each brother receives at least 1 marble.

This can be done in $^{17-1}C_{4-1} = {}^{16}C_3 = \frac{16 \times 15 \times 14}{1 \times 2 \times 3} = 560$ ways. Choice (4)

49. The smallest cube that can be constructed with the blocks of dimension 4 cm × 3 cm × 2 cm will have an edge equal to LCM (4, 3, 2) i.e., 12 cm. Hence its volume is 12^3 i.e., 1728 cu.cm. Choice (2)

50. Let the length and breadth of each congruent rectangle be l and b respectively.

Clearly, 2l = 3b (from the figure, by observation)

$\therefore$ Length of big rectangle = $l + 2b = \frac{7b}{2}$

Breadth of big rectangle = 2l = 3b

Ratio of length and breadth of big rectangle = 7 : 6

As perimeter of the bigger rectangle 130 cm, its length = 35 cm and breadth = 30 cm.

The required area = 35 × 30 = 1050 sq.cm. Choice (4)

51. Let the capacities of the three taps be r, s and t.

Given $r + s + t = \frac{1}{2}$ and $s = 4t$

$\Rightarrow s + t = 5t = \frac{1}{2} - r$

Further $(r)(x) + (5t)(y) = 1$ and $x + y = 4$

Hence $(r)(x) + \left(\frac{1}{2} - r\right)(4 - x) = 1$

$\Rightarrow 4rx - 8r - x + 2 = 0$

$\Rightarrow (x - 2)(4r - 1) = 0$

$\Rightarrow x = 2$ or $r = \frac{1}{4}$, but $x \neq y$.

Hence $r = \frac{1}{4}$ and $t = \frac{1}{5}\left(\frac{1}{2} - r\right) = \frac{1}{20}$

Hence T alone takes 20 hours.

Alternative solution:

Let the capacities of the three taps be 'a' units, 4 units and 1 unit per hour. Now total capacity of the tank is $(a + 4 + 1) \times 2$ units. Also, $[(a \times x) + (4 + 1)(y)]$ will be the capacity of the tank. Hence $(a + 5) \times 2 = [ax + 5(4 - x)]$.

$\Rightarrow 2a + 10 = (a - 5)x + 20$

$\Rightarrow a(2 - x) = 5(2 - x)$, but since $x \neq y \neq 2$, we get $a = 5$ and the capacity of tank $= 2(a + 5) = 20$ units.

$\Rightarrow$ T alone can fill the tank in 20 hours. Choice (3)

52. $PR^2 = PQ^2 + QR^2 - 2(PQ)(QR)\cos\angle PQR = p^2 + q^2 + pq$

$PR^2 = PS^2 + SR^2 - 2(PS)(SR)\cos\angle PSR$

$= s^2 + r^2 - 2sr\cos(180° - \angle PQR)$

$p^2 + q^2 + pq = s^2 + r^2 - sr$

Adding pq both sides, $(p + q)^2 = s^2 + r^2 - sr + pq$

$= s^2 + r^2 - sr + 3sr = (s + r)^2$

$p + q = s + r$

$s = p + q - r$ Choice (3)

53. Let the entire continued fraction be x.

$\therefore x = \frac{1}{1 + x}$

$\Rightarrow x^2 + x - 1 = 0 \quad \Rightarrow x = \frac{-1 \pm \sqrt{5}}{2}$

As $x > 0$, $x = \frac{\sqrt{5} - 1}{2}$ Choice (1)

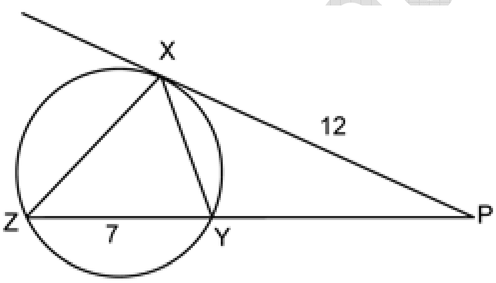
54. Since the given base is 5, the remainder rule for 4 is similar to the remainder rule for 9 that we have in our base 10.
When a number is expressed in base 5, the four's remainder of the number is equal to the fours remainder of the sum of the digits of the number.
Given N = 2323……23 (100 digits)
Sum of the digits in our base of 10 is 250.
The 4's remainder of 250 is 2. [We can verify that if 250 is expressed in base 5, i.e. $(2000)_5$ the sum of the digits (i.e. 2) has the same 4's remainder]
This means when N itself is divided by 4, the remainder is 2.
$\therefore N^{4231}$ leaves a remainder of 0 when divided by 4.

Alternative solution:

The number N is of the form
$2 \times 5^{99} + 3 \times 5^{98} + 2 \times 5^{97} + \ldots\ldots\ldots 3 \times 5^2 + 2 \times 5^1 + 3 \times 5^0$
$= 2(5^{99} + 5^{97} + \ldots.. 5^1) + 3(5^{98} + 5^{96} + \ldots\ldots. 5^0)$
$= 2 \times$ (even number) + 3(even number).
Hence N is an even number. Any even number raised to a power of 2 or more will definitely be divisible by 4. Choice (1)

55. In the given options other than 1423, all other numbers are divisible by 3. 1423 leaves a remainder 1 when divided by 3.
Hence $(1423)^{2143}$ leaves remainder 1 when divided by 3.
$\therefore (1423)^{2143}$ is not oven.
Note: In order to check out the remainders of each option, first convert them into base 10 and apply the remainder rule for three.
For example $(1423)_5 = (238)_{10}$
Clearly 238 divided by 3 leaves a remainder if 1 and hence $(1423)^{2143}$ leaves remainder 1 when divided by 3. Choice (2)

56.



We have $PX^2 = PY \times PZ$
$\Rightarrow 144 = x(x + 7)$
$x = 9$
Also $\angle PXY = \angle XZY$ [Alternate segment Theorem]
$\therefore \Delta PXY$ is similar to ΔPZX
$\Rightarrow$ Perimeters of ΔPXY and ΔPZX will be in the ratio PX : PZ i.e. 3 : 4
Since perimeter of PXY is 27 cm perimeter of PXZ is 36 cm. Choice (1)

57. Let the number be *abcd*
Reversing the number, we get *dcba*
$abcd - dcba = 7083$ ----- (1)
$(1000a + 100b + 10c + d) - (1000d + 100c + 10b + a) = 7083$
$999(a - d) + 90(b - c) = 7083$ i.e.,
$111(a - d) + 10(b - c) = 787$ ------ (2)
(1) implies that $a > d$
$-9 \leq b - c \leq 9$
$\therefore 697 \leq 111(a - d) \leq 877$
Only possible value of $a - d = 7$
From (2), $b - c = 1$
(a, d) can be (9, 2), (8, 1) or (7, 0)
(b, c) can be (9, 8), (8, 7), …… or (1, 0)
$\therefore (a, d)$ has 3 possibilities and (b, c) has 9 possibilities.
$\therefore (a, b, c, d)$ has 27 possibilities. Choice (3)

58. We can use the symbol P and T for the length of the platform and train respectively.
The data is tabulated below

$P_1 = 2p$	$T_1 = 2t$	$S_1 = 4u = 72$ kmph
$P_2 = 3p$	$T_2 = 3t$	$S_2 = 3u = 54$ kmph

The ratio of the times taken

$$= \frac{P_1 + T_1}{S_1} \frac{S_2}{P_2 + T_2} = \frac{2p + 2t}{3p + 3t} \frac{3}{4} = \frac{1}{2}$$

Choice (3)

59. The point P is not important here, but AB is a chord of length = $4\sqrt{6}$ cm (given).

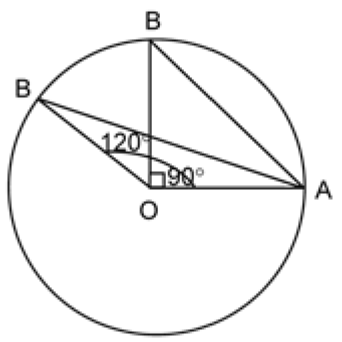
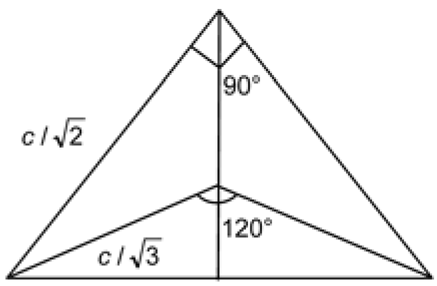


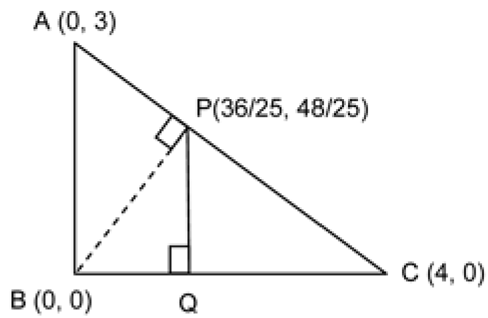
Fig A



For a circle of a given radius *r*, as the angle subtended by a chord at the centre increases from 90° to 120°, the chord length increases from $\sqrt{2}r$ to $\sqrt{3}r$.
For a chord of a given length *c*, as the angle subtended at the centre increases from 90° to 120°, the radius decreases from $c/\sqrt{2}$ to $c/\sqrt{3}$. As $c = 4\sqrt{6}$, we get $\frac{4\sqrt{6}}{\sqrt{3}} < r < \frac{4\sqrt{6}}{\sqrt{2}}$ i.e. $4\sqrt{2} < r < 4\sqrt{3}$ i.e. $5.66 < r < 6.93$
Among the options, 6 is the only possible value. Choice (3)

60. First a black square can be selected in 32 ways. For every black square selected there will be 4 white squares in that row and 4 white square in that column and hence we will have 8 ways to choose a white square.
$\therefore 32 \times 8 = 256$ ways. Choice (1)

61.



Let P(x, y) be the point such that BP $\perp$ AC
Let Q be the foot of the perpendicular from P on BC.

Clearly $\frac{1}{2}$ AB $\times$ BC = $\frac{1}{2}$ AC $\times$ BP = Area of ΔABC

$\Rightarrow$ BP = $\frac{12}{5}$ and ΔBPQ is similar to ΔABC ($\because \angle$PBQ = $\angle$CAB and $\angle$BPQ = $\angle$BCA)

Hence BQ = $\frac{BP}{AC} \times AB = \frac{36}{25}$ and PQ = $\frac{BP}{AC} \times BC = \frac{48}{25}$

$\Rightarrow$ The co-ordinates of the point P are $\left(\frac{36}{25}, \frac{48}{25}\right)$ Choice (4)

62. $4^{\log_2 \log_3(4x+1)} = 2^{2\log_2 \log_3(4x+1)}$
$= 2^{\log_2 [\log_3(4x+1)]^2} = [\log_3(4x+1)]^2$
$\therefore [\log_3(4x+1)]^2 - 6[\log_3(4x+1)] + 8 = 0$
$\therefore \log_3(4x+1) = 2$ or 4.
$\therefore x = 2$ or 20. As $x > 4$, $x = 20$
$\therefore \log_4(x-4) = 2$. Choice (1)

63. Let there be b boys and g girls.
$\Rightarrow b - 1 = 3s$ and $b = 4(s - 1)$
$\Rightarrow 3s + 1 = 4s - 4 \Rightarrow s = 5$ and $b = 16$
$\Rightarrow s + b = 21$

Alternative solution:

Since I have thrice as many brother as sisters, the total number of siblings I have will be a multiple of 4. Hence, total children that my parents have (including me) will be of the form $4k + 1$. From the choices only option (3) is possible. Choice (3)

64. Let the negative marking per mistake for the first twenty mistakes be n_1 and for all the subsequent mistakes let it be n_2.
A attempted 160 questions and got only 80 correct and B attempted 150 questions and got only 100 correct.
So, $80(1) - 20(n_1) - 60(n_2) = 55$
$100(1) - 20(n_1) - 30(n_2) = 85$
$\Rightarrow 20 + 30n_2 = 30$
$\Rightarrow n_2 = 1/3$ Choice (2)

65. The sum becomes k times itself in 16 years and it becomes $2k$ times in 40 years both at SI.
$\Rightarrow$ The sum will earn an interest of k times itself in (40 – 16) i.e., 24 years, or an interest of $2kp$ in 48 years.
$\therefore$ It requires 40 + 2(24) i.e., 88 years for the sum to become $4k$ times itself.

Choice (2)