## PART 01 — MATHEMATICS

(Common to all candidates)

(Answer ALL questions)

1. The unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$ is

   1. $-i + 2j + 2\bar{k}$
   2. $\frac{1}{3}(-i + 2j + 2\bar{k})$
   3. $\frac{1}{3}(i - 2j + 2\bar{k})$
   4. $i - 2j - 2\bar{k}$

2. If $r = \sqrt{x^2 + y^2 + z^2}$, then $\nabla\left(\frac{1}{r}\right)$ is equal to

   1. $\frac{\bar{r}}{r^3}$
   2. $\frac{\bar{r}}{r^2}$
   3. $\frac{-\bar{r}}{r^2}$
   4. $\frac{-r}{r^3}$

3. If $\bar{A} = x^2zi - 2y^3z^2\bar{j} + xy^2z\bar{k}$, then $div\bar{A}$ at $(1, -1, 1)$ is

   1. 0
   2. –3
   3. 3
   4. 1

4. If $\bar{A} = x^2yi - 2xz\bar{j} + 2yz\bar{k}$, then $curl curl \bar{A}$ is

   1. $(x + 2)\bar{j}$
   2. $(2x + 2)\bar{j}$
   3. $(2x + 1)\bar{j}$
   4. $(2x + 2y)\bar{j}$

5. If $\bar{V} = (x + 2y + az)i + (bx - 3y - z)\bar{j} + (4x + cy + 2z)\bar{k}$ is irrotational, then

   1. $a = 4, b = -1, c = 2$
   2. $a = 2, b = -1, c = 4$
   3. $a = 4, b = 2, c = -1$
   4. $a = 4, b = -2, c = 1$

6. Which of the following is a factor of the determinant?

$$\begin{vmatrix} a + b + 2c & a & b \\ c & b + c + 2a & b \\ c & a & c + a + 2b \end{vmatrix}$$

   1. $a$
   2. $a - b$
   3. $a + b$
   4. $a + b + c$

7. If $a + b + c = 0$, one root of $\begin{vmatrix} a - x & c & b \\ c & b - x & a \\ b & a & c - x \end{vmatrix} = 0$ is

   1. $x = 1$
   2. $x = 2$
   3. $x = a^2 + b^2 + c^2$
   4. $x = 0$

8. If $A$ is a $4\times4$ matrix. **A** second order minor of A has its value as $0$. Then the rank of $A$ is

1. $< 2$
2. $= 2$
3. $> 2$
4. anything

9. Given $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix}$, then the determinant value of $A^{-1}$ is

1. 32
2. $\frac{1}{32}$
3. $\frac{1}{64}$
4. 64

10. If $\begin{pmatrix} 3 & 1 \\ 4 & 1 \end{pmatrix} X = \begin{pmatrix} 5 & -1 \\ 2 & 3 \end{pmatrix}$, then

1. $X = \begin{pmatrix} -3 & 4 \\ 14 & 13 \end{pmatrix}$
2. $X = \begin{pmatrix} 3 & -4 \\ -14 & 13 \end{pmatrix}$
3. $X = \begin{pmatrix} -3 & 4 \\ 14 & -13 \end{pmatrix}$
4. $X = \begin{pmatrix} -3 & -4 \\ -14 & 13 \end{pmatrix}$

11. $C-R$ equations for a function $w = P(r,\theta) + iQ(r,\theta)$ to be analytic, in polar form are

1. $\frac{\partial P}{\partial r} = \frac{1}{r}\frac{\partial Q}{\partial \theta}, \frac{\partial Q}{\partial r} = \frac{-1}{r}\frac{\partial P}{\partial \theta}$
2. $\frac{\partial Q}{\partial \theta} = \frac{1}{r}\frac{\partial P}{\partial r}, \frac{\partial P}{\partial \theta} = \frac{1}{r}\frac{\partial Q}{\partial r}$
3. $\frac{\partial P}{\partial r} = \frac{-1}{r}\frac{\partial Q}{\partial \theta}, \frac{\partial Q}{\partial r} = \frac{1}{r}\frac{\partial P}{\partial \theta}$
4. $\frac{\partial P}{\partial \theta} = \frac{1}{r}\frac{\partial Q}{\partial r}, \frac{\partial Q}{\partial \theta} = \frac{-1}{r}\frac{\partial P}{\partial r}$

12. If $f(z) = u + iv$ is an analytic function and $u$ and v are harmonic, then u and v will satisfy

1. one dimensional wave equation
2. one dimensional heat equation
3. Laplace equation
4. Poisson equation

13. In the analytic function $f(z) = u + iv$, the curves $u(x,y) = c_1$ and $v(x,y) = c_2$ are orthogonal if the product of the slopes $m_1$ and $m_2$ are

1. $m_1 m_2 = 0$
2. $m_1 m_2 = -\pi$
3. $m_1 m_2 = \frac{-\pi}{2}$
4. $m_1 m_2 = -1$

14. If the imaginary part of the analytic function $f(z) = u + iv$ is constant, then

1. $u$ is not a constant
2. $f(z)$ is not a complex constant
3, $f(z)$ is equal to zero
4. $u$ is a constant

15. If $f(z) = P(r,\theta) + iQ(r, 8)$ is analytic, then $f'(z)$ is equal to

1. $e^{i\theta}\left(\frac{\partial P}{\partial r} + i\frac{\partial Q}{\partial \theta}\right)$
2. $e^{-i\theta}\left(\frac{\partial P}{\partial r} + i\frac{\partial Q}{\partial \theta}\right)$
3. $e^{-i\theta}\left(\frac{\partial P}{\partial r} + i\frac{\partial Q}{\partial r}\right)$
4. $e^{+i\theta}\left(\frac{\partial P}{\partial r} + i\frac{\partial Q}{\partial r}\right)$

16. The formula for the radius of curvature in cartesian coordinate is

1. $\dfrac{\left(1+(y')^2\right)^{1/2}}{y''(x)}$

2. $\dfrac{\left(1+(y')^2\right)^{3/2}}{y''(x)}$

3. $\dfrac{\left(1+(y')^2\right)^{3/2}}{(y'')^2}$

4. $\dfrac{\left(1+(y')^2\right)^{1/2}}{(y''(x))^2}$

17. The condition for the function $z = f(x, y)$ to have a extremum at $(a, a)$ is $\dfrac{\partial z}{\partial x} = 0$ and $\dfrac{\partial z}{\partial y} = 0$. $A = \dfrac{\partial^2 z}{\partial x^2}, B = \dfrac{\partial^2 z}{\partial x \partial y}, C = \dfrac{\partial^2 z}{\partial y^2}$. $A = AC - B^2$. Then the function $z$ has a maximum value at $(a, a)$ if

1. $\Delta > 0, A < 0$
2. $\Delta > 0, A = 0$
3. $\Delta < 0, A < 0$
4. $\Delta > 0, A > 0$

18. The stationary point of $f(x, y) = x^2 - xy + y^2 - 2x + y$ is

1. $(0, 1)$
2. $.(1, 0)$
3. $(-1, 0)$
4. $(1, -1)$

19. $\displaystyle\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ is

1. $\dfrac{\pi}{2}$
2. $\pi$
3. $\dfrac{\pi}{4}$
4. $2\pi$

20. $\int x \cos x \, dx$ is

1. $x \sin x - \cos x$
2. $x \sin x + \cos x$
3. $x \sin x - x \cos x$
4. $x \sin x + x \cos x$

21. For the following data :

| | | | | |
|---|---|---|---|---|
| $x$ : | 0 | 2 | 4 | 6 |
| $y$ : | $-1$ | 3 | 7 | 11 |

the straight line $y = mx + c$ by the method of least square is

1. $y = -2x - 1$
2. $y = x - 1$
3. $y = 1 - 2x$
4. $y = 2x - 1$

22. The velocity $v$ (km/min) of a train which starts from rest, is given at fixed intervals of time $t$ (min) as follows :

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| t : | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| v : | 10 | 18 | 25 | 29 | 32 | 20 | 11 | 5 | 2 | 0 |

The approximate distance covered by Simpson's 1/3 rule is

1. 306.3
2. 309.3
3. 310.3
4. 307.3

23. Find the cubic polynomial by Newton's forward difference which takes the following

| | | | | |
|---|---|---|---|---|
| x: | 0 | 1 | 2 | 3 |
| $f(x)$: | 1 | 2 | 1 | 10 |

Then f(4) is

1. 40
2. 41
3. 39
4. 42

24. The first derivative $\frac{dy}{dx}$ at $x=0$ for the given data

| | | | | |
|---|---|---|---|---|
| x : | 0 | 1 | 2 | 3 |
| $f(x)$: | 2 | 1 | 2 | 5 |

is

1. 2
2. –2
3. –1
4. 1

25. Error in Simpson's $\frac{1}{3}$ rule is of the order

1. $-h^2$
2. $h^3$
3. $h^4$
4. $\frac{2h^3}{3}$

26. A lot consists of ten good articles, four with minor defects and two with major defects. Two articles are chosen from the lot at random (without replacement). Then the probability that neither of them good is

1. $\frac{5}{8}$
2. $\frac{7}{8}$
3. $\frac{3}{8}$
4. $\frac{1}{8}$

27. If A, $B, C$ are any three events such that $P(A)=P(B)=P(C)=\frac{1}{4}$;

$P(A\cap B)=P(B\cap C)=0, \quad P(C\cap A)=\frac{1}{8}$.

Then the probability that atleast one of the events A, B, $C$ occurs, is

1. $\frac{1}{32}$
2. $\frac{3}{32}$
3. $\frac{7}{8}$
4. $\frac{5}{8}$

28. To establish the mutual independence of n events, the equations needed are

1. $2^n+n+1$
2. $n^2+n+1$
3. $2^n-(n+1)$
4. $2^n+2(n+1)$

29. If atleast one child in a family with two children is a boy, then the probability that both children are boys is

1. 3/4
2. 1/3
3. 1/4
4. 1/2

30. A discrete random variable X takes the values $a, ar, ar^2, \cdots, ar^{n-1}$ with equal probability. Then Arithmetic Mean (A.M) is

1. $a(1-r^n)$
2. $\frac{1}{n}a(1-r^n)$
3. $\frac{a}{n}\frac{(1-r^n)}{1-r}$
4. $\frac{a}{n}\frac{(r^n-1)}{1-r}$

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