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## ANSWERS & HINTS

### for

## WBJEE - 2011

### by Aakash Institute & Aakash IIT-JEE

### MULTIPLE CHOICE QUESTIONS

### SUB : MATHEMATICS

1. The eccentricity of the hyperbola  $4x^2 - 9y^2 = 36$  is

(A)  $\frac{\sqrt{11}}{3}$       (B)  $\frac{\sqrt{15}}{3}$       (C)  $\frac{\sqrt{13}}{3}$       (D)  $\frac{\sqrt{14}}{3}$

Ans : (C)

Hints :  $\frac{x^2}{9} - \frac{y^2}{4} = 1$

$a = 3, b = 2$

$\therefore e = \sqrt{\frac{a^2 + b^2}{a^2}} = \sqrt{\frac{13}{9}} = \frac{\sqrt{13}}{3}$

2. The length of the latus rectum of the ellipse  $16x^2 + 25y^2 = 400$  is

(A)  $5/16$  unit      (B)  $32/5$  unit      (C)  $16/5$  unit      (D)  $5/32$  unit

Ans : (B)

Hints : Length of latus rectum =  $2 \frac{b^2}{a} = \frac{2 \times 16}{5} = \frac{32}{5}$

$16x^2 + 25y^2 = 400$

$\frac{x^2}{25} + \frac{y^2}{16} = 1;$

$a^2 = 25; b^2 = 16$

3. The vertex of the parabola  $y^2 + 6x - 2y + 13 = 0$  is

$(y - 1)^2 = -6x - 12$

$(y - 1)^2 = -6(x + 2) = 4 \left( \frac{-6}{4} \right) (x + 2)$

Vertex  $\rightarrow (-2, 1)$

(A)  $(1, -1)$       (B)  $(-2, 1)$       (C)  $\left( \frac{3}{2}, 1 \right)$       (D)  $\left( -\frac{7}{2}, 1 \right)$

**Ans : (B)**

**Hints :**

4. The coordinates of a moving point p are  $(2t^2 + 4, 4t + 6)$ . Then its locus will be a  
 (A) circle (B) straight line (C) parabola (D) ellipse

**Ans : (C)**

**Hints :**  $x = 2t^2 + 4, y = 4t + 6, y = 4t + 6 \rightarrow t = \left(\frac{y-6}{4}\right)$

$$x = 2\left(\frac{y-6}{4}\right)^2 + 4 \Rightarrow \frac{(y-6)^2}{8} = x - 4$$

$$(y-6)^2 = 4(2)(x-4)$$

5. The equation  $8x^2 + 12y^2 - 4x + 4y - 1 = 0$  represents  
 (A) an ellipse (B) a hyperbola (C) a parabola (D) a circle

**Ans : (A)**

**Hints :**  $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

represents ellipse if  $h^2 - ab < 0$

$$3x^2 + 12y^2 - 4x + 4y - 1 = 0$$

$$h = 0, a = 3, b = 12$$

$$h^2 - ab < 0$$

6. If the straight line  $y = mx$  lies outside of the circle  $x^2 + y^2 - 20y + 90 = 0$ , then the value of m will satisfy  
 (A)  $m < 3$  (B)  $|m| < 3$  (C)  $m > 3$  (D)  $|m| > 3$

**Ans : (B)**

**Hints :**  $x^2 + m^2x^2 - 20mx + 90 = 0$

$$x^2(1+m^2) - 20mx + 90 = 0$$

$$D < 0$$

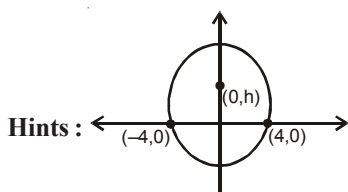
$$400m^2 - 4 \times 90(1+m^2) < 0$$

$$40m^2 < 360$$

$$m^2 < 9 ; |m| < 3$$

7. The locus of the centre of a circle which passes through two variable points  $(a, 0), (-a, 0)$  is  
 (A)  $x=1$  (B)  $x+y=a$  (C)  $x+y=2a$  (D)  $x=0$

**Ans : (D)**



Centre lies on y-axis locus  $x=0$

8. The coordinates of the two points lying on  $x + y = 4$  and at a unit distance from the straight line  $4x + 3y = 10$  are  
 (A)  $(-3, 1), (7, 11)$  (B)  $(3, 1), (-7, 11)$  (C)  $(3, 1), (7, 11)$  (D)  $(5, 3), (-1, 2)$

Ans : (B)

Hints : Let  $p(h, 4 - h)$

$$\left| \frac{4h + 3(4 - h) - 10}{5} \right| = 1$$

$$|h + 2| = 5$$

$$h = 3, -7; \quad p = 1, 1$$

$$(3, 1), (-7, 11)$$

9. The intercept on the line  $y = x$  by the circle  $x^2 + y^2 - 2x = 0$  is AB. Equation of the circle with AB as diameter is

(A)  $x^2 + y^2 = 1$

(B)  $x(x - 1) + y(y - 1) = 0$

(C)  $x^2 + y^2 = 2$

(D)  $(x - 1)(x - 2) + (y - 1)(y - 2) = 0$

Ans : (B)

Hints :  $2x^2 - 2x = 0 \quad x(x + 1) = 0 \quad x = 0, 1; \quad y = 0, 1$

$(0, 0), (1, 1)$  as diametric ends

$$(x - 0)(x - 1) + (y + 0)(y - 1) = 0$$

$$x^2 + y^2 - x - y = 0$$

10. If the coordinates of one end of a diameter of the circle  $x^2 + y^2 + 4x - 8y + 5 = 0$ , is  $(2, 1)$ , the coordinates of the other end is

(A)  $(-6, -7)$

(B)  $(6, 7)$

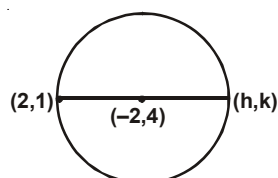
(C)  $(-6, 7)$

(D)  $(7, -6)$

Ans : (C)

Hints :  $x^2 + y^2 + 9x - 8y + 5 = 0$

Centre circle  $(-2, 4)$



$$\frac{h + 2}{2} = -2$$

$$h = -4 - 2 = -6$$

$$\frac{k + 1}{2} = 4 \Rightarrow k = 7$$

$$(h, k) \rightarrow (-6, 7)$$

11. If the three points  $A(1, 6), B(3, -4)$  and  $C(x, y)$  are collinear then the equation satisfying by  $x$  and  $y$  is

(A)  $5x + y - 11 = 0$

(B)  $5x + 13y + 5 = 0$

(C)  $5x - 13y + 5 = 0$

(D)  $13x - 5y + 5 = 0$

Ans : (A)

Hints: 
$$\begin{vmatrix} 1 & 1 & 6 \\ 1 & 3 & -4 \\ 1 & x & y \end{vmatrix} = 0$$

$\Rightarrow 1(3y + 4x) - (y - 6x) + 1(-4 - 18) = 0 \Rightarrow 3y + 4x - y + 6x - 12 = 0$

$\Rightarrow 2y + 10x - 12 = 0$

$y + 5x = 11$

12. If  $\sin\theta = \frac{2t}{1+t^2}$  and  $\theta$  lies in the second quadrant, then  $\cos\theta$  is equal to

- (A)  $\frac{1-t^2}{1+t^2}$                       (B)  $\frac{t^2-1}{1+t^2}$                       (C)  $\frac{-|1-t^2|}{1+t^2}$                       (D)  $\frac{1+t^2}{|1-t^2|}$

Ans: (C)

Hints:  $\theta$  in 2nd quad  $\cos\theta < 0$

$|\cos\theta| = \frac{|1-t^2|}{1+t^2} = \frac{|1-t^2|}{1+t^2}$

$\cos\theta = -\frac{|1-t^2|}{1+t^2}$

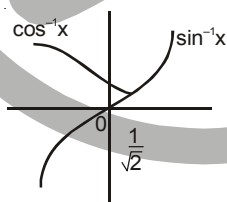
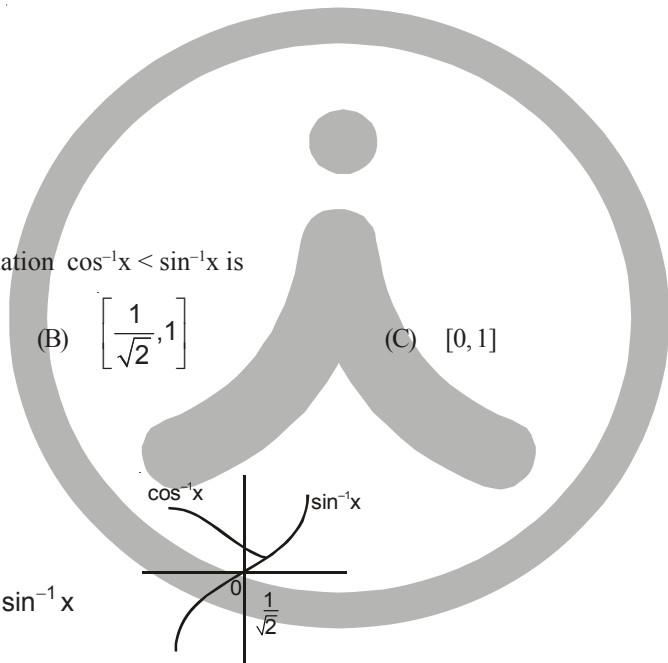
13. The solutions set of inequation  $\cos^{-1}x < \sin^{-1}x$  is

- (A)  $[-1, 1]$                       (B)  $\left[\frac{1}{\sqrt{2}}, 1\right]$                       (C)  $[0, 1]$                       (D)  $\left[\frac{1}{\sqrt{2}}, 1\right]$

Ans: (D)

Hints:  $\cos^{-1}x < \sin^{-1}x$

$x \in \left[\frac{1}{\sqrt{2}}, 1\right], \cos^{-1}x < \sin^{-1}x$



14. The number of solutions of  $2\sin x + \cos x = 3$  is

- (A) 1                      (B) 2                      (C) infinite                      (D) No solution

Ans: (D)

Hints:  $\sqrt{5} < 3$  No solution

15. Let  $\tan\alpha = \frac{a}{a+1}$  and  $\tan\beta = \frac{1}{2a+1}$  then  $\alpha + \beta$  is

- (A)  $\frac{\pi}{4}$                       (B)  $\frac{\pi}{3}$                       (C)  $\frac{\pi}{2}$                       (D)  $\pi$

Ans: (A)

Hints :  $\tan\alpha = \frac{a}{a+1}$ ,  $\tan\beta = \frac{1}{2a+1}$

$$\tan(\alpha + \beta) = \frac{\frac{a}{a+1} + \frac{1}{2a+1}}{1 - \frac{a}{(a+1)(2a+1)}} = \frac{\frac{a(2a+1) + a+1}{(a+1)(2a+1)}}{\frac{(a+1)(2a+1) - a}{(a+1)(2a+1)}} = \frac{2a^2 + 2a + 1}{2a^2 + 2a + 1} = 1$$

$$\alpha + \beta = \frac{\pi}{4}$$

16. If  $\theta + \phi = \frac{\pi}{4}$ , then  $(1 + \tan\theta)(1 + \tan\phi)$  is equal to

- (A) 1                                      (B) 2                                      (C) 5/2                                      (D) 1/3

Ans : (B)

Hints :  $(1 + \tan\theta) \left( 1 + \frac{(1 - \tan\theta)}{1 + \tan\theta} \right)$

$$= (1 + \tan\theta) \frac{2}{1 + \tan\theta} = 2$$

17. If  $\sin\theta$  and  $\cos\theta$  are the roots of the equation  $ax^2 - bx + c = 0$ , then a, b and c satisfy the relation

- (A)  $a^2 + b^2 + 2ac = 0$                                       (B)  $a^2 - b^2 + 2ac = 0$   
 (C)  $a^2 + c^2 + 2ab = 0$                                       (D)  $a^2 - b^2 - 2ac = 0$

Ans : (B)

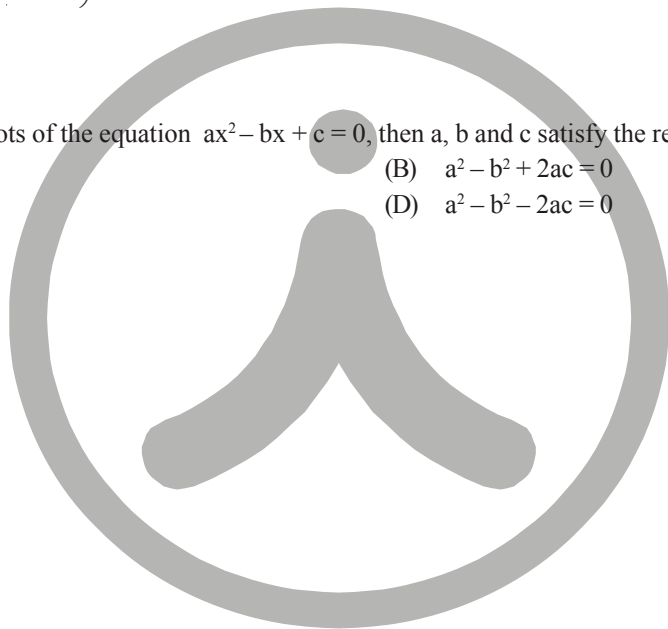
Hints :  $\sin\theta + \cos\theta = \frac{b}{a}$

$$\sin\theta \cdot \cos\theta = \frac{c}{a}$$

$$\left(\frac{b}{a}\right)^2 = 1 + \frac{2c}{a}$$

$$b^2 = a^2 + 2ac$$

$$a^2 - b^2 + 2ac = 0$$



18. If A and B are two matrices such that A+B and AB are both defined, then

- (A) A and B can be any matrices                                      (B) A, B are square matrices not necessarily of the same order  
 (C) A, B are square matrices of the same order                                      (D) Number of columns of A = number of rows of B

Ans : (C)

Hints : Addition is defined if order of A is equal to order of B

A B  
 $n \times m \quad n \times m$  is defined if  $m = n$

$\Rightarrow$  A, B are square matrices of same order

19. If  $A = \begin{pmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{pmatrix}$  is a symmetric matrix, then the value of x is

- (A) 4                                      (B) 3                                      (C) -4                                      (D) -3

Ans : (C)

Hints :  $A = A^T$

$$\begin{pmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{pmatrix} = \begin{pmatrix} 3 & 2x+3 \\ x-1 & x+2 \end{pmatrix}$$

$$\Rightarrow x-1 = 2x+3 \text{ or } x = -4$$

20. If  $z = \begin{pmatrix} 1 & 1+2i & -5i \\ 1-2i & -3 & 5+3i \\ 5i & 5-3i & 7 \end{pmatrix}$  then ( $i = \sqrt{-1}$ )

(A)  $z$  is purely real

(B)  $z$  is purely imaginary

(C)  $z + \bar{z} = 0$

(D)  $(z - \bar{z})i$  is purely imaginary

Ans : (A)

Hints :  $z = \begin{vmatrix} 1 & 1+2i & -5i \\ 1-2i & -3 & 5+3i \\ 5i & 5-3i & 7 \end{vmatrix} = 1(-21-64) - ((1-2i)(7(1+2i) + 5i(5-3i))) + 5i((1+2i)(5+3i) - 15i)$

= Real

21. The equation of the locus of the point of intersection of the straight lines  $x \sin \theta + (1 - \cos \theta) y = a \sin \theta$  and  $x \sin \theta - (1 + \cos \theta) y + a \sin \theta = 0$  is

(A)  $y = ax$

(B)  $x = \pm ay$

(C)  $y^2 = 4x$

(D)  $x^2 + y^2 = a^2$

Ans : (D)

Hints :  $y = a \sin \theta$

$x = a \cos \theta$ .

$$x^2 + y^2 = a^2$$

22. If  $\sin \theta + \cos \theta = 0$  and  $0 < \theta < \pi$ , then  $\theta$

(A) 0

(B)  $\frac{\pi}{4}$

(C)  $\frac{\pi}{2}$

(D)  $\frac{3\pi}{4}$

Ans : (D)

Hints :  $\sin \theta + \cos \theta = 0$

$$\Rightarrow \tan \theta = -1$$

$$\theta = \frac{3\pi}{4}$$

23. The value of  $\cos 15^\circ - \sin 15^\circ$  is

(A) 0

(B)  $\frac{1}{\sqrt{2}}$

(C)  $-\frac{1}{\sqrt{2}}$

(D)  $\frac{1}{2\sqrt{2}}$

Ans : (B)

Hints :  $\cos 15^\circ - \sin 15^\circ = \sqrt{2} \cos 60^\circ = \frac{1}{\sqrt{2}}$

24. The period of the function  $f(x) = \cos 4x + \tan 3x$  is

(A)  $\pi$

(B)  $\frac{\pi}{2}$

(C)  $\frac{\pi}{3}$

(D)  $\frac{\pi}{4}$

Ans : (A)

Hints :  $\text{LCM} \left( \frac{2\pi}{4}, \frac{\pi}{3} \right) = \pi$

25. If  $y = 2x^3 - 2x^2 + 3x - 5$ , then for  $x = 2$  and  $\Delta x = 0.1$  value of  $\Delta y$  is  
 (A) 2.002 (B) 1.9 (C) 0 (D) 0.9

Ans : (B)

Hints :  $\frac{dy}{dx} = 6x^2 - 4x + 3$   $\Delta y = \left(\frac{dy}{dx}\right)_{x=2} \Delta x = 1.9$

26. The approximate value of  $\sqrt[5]{33}$  correct to 4 decimal places is  
 (A) 2.0000 (B) 2.1001 (C) 2.0125 (D) 2.0500

Ans : (C)

Hints :  $y = x^{1/5}$   $\Delta y = \left(\frac{dy}{dx}\right) \Delta x = \frac{1}{80} \times 1$

$y = 2 + \frac{1}{80}$

27. The value of  $\int_{-2}^2 (x \cos x + \sin x + 1) dx$  is  
 (A) 2 (B) 0 (C) -2 (D) 4

Ans : (D)

Hints :  $\int_{-2}^2 (x \cos x + \sin x + 1) dx = \int_{-2}^2 dx = 4$

28. For the function  $f(x) = e^{\cos x}$ , Rolle's theorem is  
 (A) applicable when  $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$  (B) applicable when  $0 \leq x \leq \frac{\pi}{2}$   
 (C) applicable when  $0 \leq x \leq \pi$  (D) applicable when  $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$

Ans : (A)

Hints :  $f\left(\frac{\pi}{2}\right) = f\left(\frac{3\pi}{2}\right)$

29. The general solution of the differential equation  $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$  is  
 (A)  $(A + Bx)e^{5x}$  (B)  $(A + Bx)e^{-4x}$  (C)  $(A + Bx^2)e^{4x}$  (D)  $(A + Bx^4)e^{4x}$

Ans : (B)

Hints :  $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$

auxiliary equation  $m^2 + 8m + 16 = 0 \Rightarrow m = -4$

Solution  $y = (ax + b)e^{-4x}$

30. If  $x^2 + y^2 = 4$ , then  $y \frac{dy}{dx} + x =$   
 (A) 4 (B) 0 (C) 1 (D) -1

Ans : (B)

Hints :  $x + y \frac{dy}{dx} = 0$

31.  $\int \frac{x^3 dx}{1+x^8} =$

- (A)  $4 \tan^{-1} x^3 + c$       (B)  $\frac{1}{4} \tan^{-1} x^4 + c$       (C)  $x + 4 \tan^{-1} x^4 + c$       (D)  $x^2 + \frac{1}{4} \tan^{-1} x^4 + c$

Ans : (B)

Hints :  $\int \frac{x^3 dy}{1+(x^4)^2} = \frac{1}{4} \tan^{-1}(x^4)$

32.  $\int_{\pi}^{16\pi} |\sin x| dx =$

- (A) 0      (B) 32      (C) 30      (D) 28

Ans : (C)

Hints :  $15 \int_0^{\pi} \sin x dx = 15(-\cos x)_0^{\pi} = 30$

33. The degree and order of the differential equation  $y = x \left( \frac{dy}{dx} \right)^2 + \frac{dy}{dx}$  are respectively

- (A) 1,1      (B) 2,1      (C) 4,1      (D) 1,4

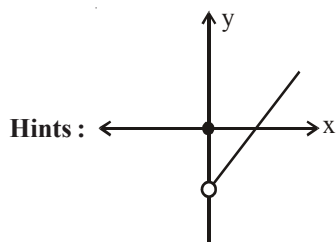
Ans : (C)

Hints :  $y \left( \frac{dy}{dx} \right)^2 = x \left( \frac{dy}{dx} \right)^4 + 1$

34.  $f(x) = \begin{cases} 0 & , x=0 \\ x-3 & , x>0 \end{cases}$  The function f(x) is

- (A) increasing when  $x \geq 0$   
 (B) strictly increasing when  $x > 0$   
 (C) Strictly increasing at  $x = 0$   
 (D) not continuous at  $x = 0$  and so it is not increasing when  $x > 0$

Ans : (B)



35. The function  $f(x) = ax + b$  is strictly increasing for all real x if

- (A)  $a > 0$       (B)  $a < 0$       (C)  $a = 0$       (D)  $a \leq 0$

Ans : (A)

Hints :  $f'(x) = a$   
 $f'(x) > 0 \Rightarrow a > 0$

36.  $\int \frac{\cos 2x}{\cos x} dx =$

- (A)  $2 \sin x + \log |\sec x + \tan x| + C$       (B)  $2 \sin x - \log |\sec x - \tan x| + c$   
 (C)  $2 \sin x - \log |\sec x + \tan x| + C$       (D)  $2 \sin x + \log |\sec x - \tan x| + C$



Ans : (C)

Hints :  $\int \frac{2\cos^2 x - 1}{\cos x} dx = 2\sin x - \log|\sec x + \tan x|$

37.  $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$

- (A)  $-\frac{1}{2}\sin 2x + C$       (B)  $\frac{1}{2}\sin 2x + C$       (C)  $\frac{1}{2}\sin x + C$       (D)  $-\frac{1}{2}\sin x + C$

Ans : (A)

Hints :  $\int (\sin^2 x - \cos^2 x) dx = -\int \cos^2 x dx = -\frac{1}{2}\sin 2x + C$

38. The general solution of the differential equation  $\log_e \left( \frac{dy}{dx} \right) = x + y$  is

- (A)  $e^x + e^{-y} = C$       (B)  $e^x + e^y = C$       (C)  $e^y + e^{-x} = C$       (D)  $e^{-x} + e^{-y} = C$

Ans : (A)

Hints :  $\frac{dy}{dx} = e^x \cdot e^y \Rightarrow \int e^{-y} dy = \int e^x dx \Rightarrow e^x + e^{-y} = C$

39. If  $y = \frac{A}{x} + Bx^2$ , then  $x^2 \frac{d^2y}{dx^2} =$

- (A)  $2y$       (B)  $y^2$       (C)  $y^3$       (D)  $y^4$

Ans : (A)

Hints :  $\frac{x^2 d^2y}{dx^2} = 2 \left( \frac{A}{x} + Bx^2 \right) = 2y$

40. If one of the cube roots of 1 be  $\omega$ , then

$$\begin{vmatrix} 1 & 1+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega & -1 \end{vmatrix} =$$

- (A)  $\omega$       (B)  $i$       (C)  $1$       (D)  $0$

Ans : (D)

Hints :  $C_2 \rightarrow C_2 - C_3$

$C_3 \rightarrow C_3 + C_2$

$C_3 \rightarrow C_3 + \omega C_1$

$C_2 \rightarrow C_2 - C_1$

41. 4 boys and 2 girls occupy seats in a row at random. Then the probability that the two girls occupy seats side by side is

- (A)  $\frac{1}{2}$       (B)  $\frac{1}{4}$       (C)  $\frac{1}{3}$       (D)  $\frac{1}{6}$

Ans : (C)

Hints :  $n(e) = |5 \cdot 2|$

$n(s) = |6|$

$p = \frac{|5 \cdot 2|}{|6|} = \frac{2}{6} = \frac{1}{3}$

42. A coin is tossed again and again. If tail appears on first three tosses, then the chance that head appears on fourth toss is

- (A)  $\frac{1}{16}$                       (B)  $\frac{1}{2}$                       (C)  $\frac{1}{8}$                       (D)  $\frac{1}{4}$

**Ans : (B)**

**Hints :**  $p = 1.1.1. \frac{1}{2} = \frac{1}{2}$

43. The coefficient of  $x^n$  in the expansion of  $\frac{e^{7x} + e^x}{e^{3x}}$  is

- (A)  $\frac{4^{n-1} - (-2)^{n-1}}{n}$                       (B)  $\frac{4^{n-1} - 2^{n-1}}{n}$                       (C)  $\frac{4^n - 2^n}{n}$                       (D)  $\frac{4^n + (-2)^n}{n}$

**Ans : (D)**

**Hints :**  $\frac{e^{7x} + e^x}{e^{3x}} = e^{4x} + e^{-2x}$

Co-efficient of  $x^n$

$$\frac{(4)^n}{n!} + \frac{(2)^n}{n!} (-1)^n = \frac{4^n + (-2)^n}{n!}$$

44. The sum of the series  $\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4} - \dots \infty$  is

- (A)  $2\log_e 2 + 1$                       (B)  $2\log_e 2$                       (C)  $2\log_e 2 - 1$                       (D)  $\log_e 2 - 1$

**Ans : (C)**

**Hints :**  $s = \frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4} - \dots \infty$

$$= \left(\frac{1}{1} - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) - \dots$$

$$= \frac{1}{1} - \frac{1}{2} - \frac{1}{2} + \frac{1}{3} + \frac{1}{3} - \frac{1}{4} - \frac{1}{4} + \frac{1}{5} - \dots$$

$$= \frac{1}{1} - \frac{2}{2} + \frac{2}{3} - \frac{2}{4} + \frac{2}{5} - \dots$$

$$= 2 \left[ \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots \right] - 1 = 2 \log_e 2 - 1$$

45. The number  $(101)^{100} - 1$  is divisible by

- (A)  $10^4$                       (B)  $10^6$                       (C)  $10^8$                       (D)  $10^{12}$

**Ans : (A)**

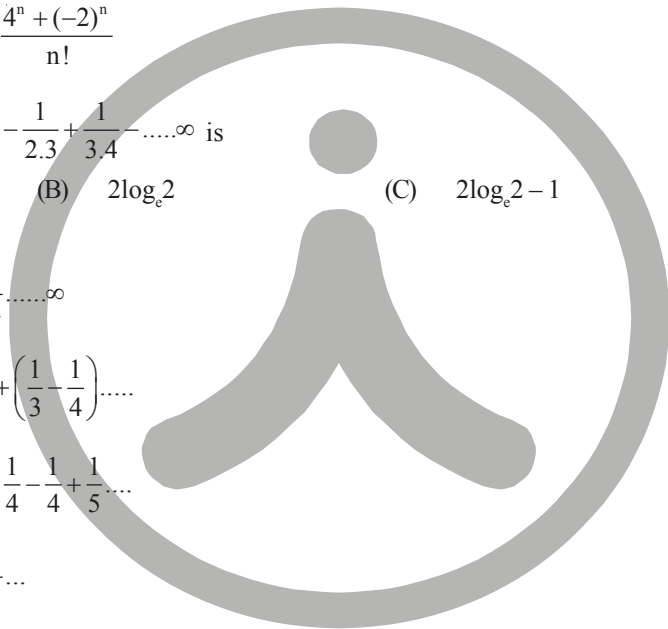
**Hints :**  $(101)^{100} - 1 = {}^{100}C_1 100 + {}^{100}C_2 100^2 + {}^{100}C_3 100^3 + \dots + {}^{100}C_{100} 100^{100}$   
 $= 100^2 [ 1 + {}^{100}C_2 + {}^{100}C_3 100 + \dots ]$   
 $= (10^4)$

46. If A and B are coefficients of  $x^n$  in the expansions of  $(1+x)^{2n}$  and  $(1+x)^{2n-1}$  respectively, then A/B is equal to

- (A) 4                      (B) 2                      (C) 9                      (D) 6

**Ans : (B)**

**Hints :**  $A = {}^{2n}C_n$   
 $B = {}^{2n-1}C_n$



$$\frac{A}{B} = \frac{{}^{2n}C_n}{{}^{2n-1}C_n} = \frac{2n}{n} = 2$$

47. If  $n > 1$  is an integer and  $x \neq 0$ , then  $(1+x)^n - nx - 1$  is divisible by  
 (A)  $nx^3$  (B)  $n^3x$  (C)  $x$  (D)  $nx$

**Ans : (C)**

**Hints :**  $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \dots$   
 $= 1 + nx + x^2({}^nC_2 + {}^nC_3x + \dots)$   
 $(1+x)^n - nx - 1 = x^2({}^nC_2 + {}^nC_3x + \dots)$

48. If  ${}^nC_4, {}^nC_5$  and  ${}^nC_6$  are in A.P., then  $n$  is  
 (A) 7 or 14 (B) 7 (C) 14 (D) 14 or 21

**Ans : (A)**

**Hints :**  ${}^nC_4, {}^nC_5, {}^nC_6$  are in AP  
 $2 \cdot {}^nC_5 = {}^nC_4 + {}^nC_6$

$$\frac{2}{5(n-5)} = \frac{1}{(n-4)} + \frac{1}{30}$$

by solving  $n = 14$  or  $7$

49. The number of diagonals in a polygon is 20. The number of sides of the polygon is  
 (A) 5 (B) 6 (C) 8 (D) 10

**Ans : (C)**

**Hints :**  ${}^nC_2 - n = 20$   
 $n = 8$

50.  ${}^{15}C_3 + {}^{15}C_5 + \dots + {}^{15}C_{15} =$   
 (A)  $2^{14}$  (B)  $2^{14} - 15$  (C)  $2^{14} + 15$  (D)  $2^{14} - 1$

**Ans : (B)**

**Hints :**  ${}^{15}C_3 + {}^{15}C_5 + \dots + {}^{15}C_{15} = 2^{14} - {}^{15}C_1 = 2^{14} - 15$

51. Let  $a, b, c$  be three real numbers such that  $a + 2b + 4c = 0$ . Then the equation  $ax^2 + bx + c = 0$   
 (A) has both the roots complex (B) has its roots lying within  $-1 < x < 0$

- (C) has one of roots equal to  $\frac{1}{2}$  (D) has its roots lying within  $2 < x < 6$

**Ans : (C)**

**Hints :**  $\frac{1}{4}a + \frac{1}{2}b + c = 0$

$$\left(\frac{1}{2}\right)^2 a + \left(\frac{1}{2}\right)b + c = 0$$

$$\therefore x = \frac{1}{2}$$

52. If the ratio of the roots of the equation  $px^2 + qx + r = 0$  is  $a : b$ , then  $\frac{ab}{(a+b)^2} =$

- (A)  $\frac{p^2}{qr}$  (B)  $\frac{pr}{q^2}$  (C)  $\frac{q^2}{pr}$  (D)  $\frac{pq}{r^2}$

**Ans : (B)**

**Hints :** Let roots are  $a\alpha$  and  $b\alpha$

$$\Rightarrow (a+b)\alpha = \frac{-q}{p}$$

$$ab\alpha^2 = \frac{r}{p}$$

$$\frac{ab\alpha^2}{(a+b)^2\alpha^2} = \frac{r}{p} \cdot \frac{p^2}{q^2}$$

$$\frac{ab}{(a+b)^2} = \frac{rp}{q^2}$$

53. If  $\alpha$  and  $\beta$  are the roots of the equation  $x^2 + x + 1 = 0$ , then the equation whose roots are  $\alpha^{19}$  and  $\beta^7$  is  
 (A)  $x^2 - x - 1 = 0$       (B)  $x^2 - x + 1 = 0$       (C)  $x^2 + x - 1 = 0$       (D)  $x^2 + x + 1 = 0$

**Ans : (D)**

**Hints :**  $\alpha$  and  $\beta$  are the roots of  $x^2 + x + 1 = 0$

$$\begin{aligned} \alpha &= \omega & \beta &= \omega^2 \\ \alpha^{19} &= \omega & \beta^7 &= \omega^2 \\ x^2 - (\alpha^{19} + \beta^7)x + \alpha^{19}\beta^7 &= 0 \\ \text{Thou,} \\ x^2 - (\omega + \omega^2)x + \omega \cdot \omega^2 &= 0 \\ x^2 + x + 1 &= 0 \end{aligned}$$

54. For the real parameter  $t$ , the locus of the complex number  $z = (1 - t^2) + i\sqrt{1 + t^2}$  in the complex plane is  
 (A) an ellipse      (B) a parabola      (C) a circle      (D) a hyperbola

**Ans : (B)**

**Hints :** Given  $z = (1 - t^2) + i\sqrt{1 + t^2}$

$$\begin{aligned} \text{Let } z &= x + iy \\ x &= 1 - t^2 \\ y^2 &= 1 + t^2 \\ \text{Thus, } x + y^2 &= 2 \\ y^2 &= 2 - x \\ y^2 &= -(x - 2) \\ \text{Thus parabola} \end{aligned}$$

55. If  $x + \frac{1}{x} = 2 \cos \theta$ , then for any integer  $n$ ,  $x^n + \frac{1}{x^n} =$   
 (A)  $2 \cos n\theta$       (B)  $2 \sin n\theta$       (C)  $2i \cos n\theta$       (D)  $2i \sin n\theta$

**Ans : (A)**

**Hints :**  $x + \frac{1}{x} = 2 \cos \theta$

$$\text{Let } x = \cos \theta + i \sin \theta$$

$$\frac{1}{x} = \cos \theta - i \sin \theta$$

$$\text{Thus } x^n + \frac{1}{x^n} = 2 \cos n\theta$$

56. If  $\omega \neq 1$  is a cube root of unity, then the sum of the series  $S = 1 + 2\omega + 3\omega^2 + \dots + 3n\omega^{3n-1}$  is

- (A)  $\frac{3n}{\omega - 1}$       (B)  $3n(\omega - 1)$       (C)  $\frac{\omega - 1}{3n}$       (D) 0

**Ans : (A)**

**Hints :**  $s = 1 + 2\omega + 3\omega^2 + \dots + 3n\omega^{3n-1}$   
 $s\omega = \omega + 2\omega^2 + \dots + (3n-1)\omega^{3n} + 3n\omega^{3n}$   
 $s(1-\omega) = 1 + \omega + \omega^2 + \dots + \omega^{3n-1} - 3n\omega^{3n}$   
 $= 0 - 3n$   
 $s = \frac{-3n}{1-\omega} = \frac{3n}{\omega-1}$

57. If  $\log_3 x + \log_3 y = 2 + \log_3 2$  and  $\log_3(x+y) = 2$ , then  
 (A)  $x=1, y=8$  (B)  $x=8, y=1$  (C)  $x=3, y=6$  (D)  $x=9, y=3$

**Ans : (C)**

**Hints :**  $\log_3 x + \log_3 y = 2 + \log_3 2$   
 $\Rightarrow x \cdot y = 18$   
 $\log(x+y) = 2 \Rightarrow x+y=9$   
 we will get  $x=3$  and  $y=6$

58. If  $\log_7 2 = \lambda$ , then the value of  $\log_{49} (28)$  is  
 (A)  $(2\lambda + 1)$  (B)  $(2\lambda + 3)$  (C)  $\frac{1}{2}(2\lambda + 1)$  (D)  $2(2\lambda + 1)$

**Ans : (C)**

**Hints :**  $\log_{49} 28 = \log_{7^2} 4 \times 7$   
 $= \frac{1}{2} [2 \log_7 2 + \log_7 7] = \frac{1}{2} [2\lambda + 1]$

59. The sequence  $\log a, \log \frac{a^2}{b}, \log \frac{a^3}{b^2}, \dots$  is  
 (A) a G.P. (B) an A.P. (C) a H.P. (D) both a G.P. and a H.P.

**Ans : (B)**

**Hints :**  $\log a \cdot (2 \log a - \log b) (3 \log a - 2 \log b)$   
 $= T_2 - T_1 = \log a - \log b$   
 $= T_3 - T_2 = \log a - \log b$

60. If in a triangle ABC,  $\sin A, \sin B, \sin C$  are in A.P., then  
 (A) the altitudes are in A.P. (B) the altitudes are in H.P.  
 (C) the angles are in A.P. (D) the angles are in H.P.

**Ans : (B)**

**Hints :**  $\frac{1}{2} ap_1 = \frac{1}{2} bp_2 = \frac{1}{2} cp_3 = \Delta$   
 $a = \frac{2\Delta}{p_1} \mid b = \frac{2\Delta}{p_2} \mid c = \frac{2\Delta}{p_3}$   
 H.P.

61.  $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} =$   
 (A) 0 (B) -1 (C) 1 (D) 2

**Ans : (A)**

**Hints :**  $c_1 \rightarrow c_1 + c_2 + c_3$

62. The area enclosed between  $y^2 = x$  and  $y = x$  is

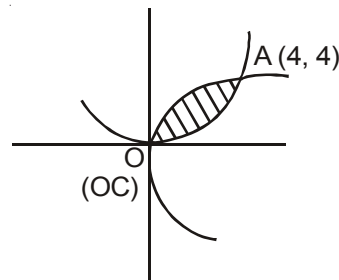
- (A)  $\frac{2}{3}$  sq. units      (B)  $\frac{1}{2}$  units      (C)  $\frac{1}{3}$  units      (D)  $\frac{1}{6}$  units

Ans : (D)

Hints :  $A = \int_0^1 (\sqrt{x} - x) dx$

$$= \frac{2}{3} (x^{3/2})_0^1 - \frac{1}{2} (x^2)_0^1 = \frac{2}{3} [1 - 0] - \frac{1}{2} [1 - 0]$$

$$= \frac{2}{3} - \frac{1}{2} = \frac{4 - 3}{6} = \frac{1}{6}$$



63. Let  $f(x) = x^3 e^{-3x}$ ,  $x > 0$ . Then the maximum value of  $f(x)$  is

- (A)  $e^{-3}$       (B)  $3e^{-3}$       (C)  $27e^{-9}$       (D)  $\infty$

Ans : (A)

Hints :  $f(x) = x^3 \cdot e^{-3x}$   
 $= f'(x) = 3x^2 e^{-3x} + x^3 e^{-3x} (-3)$   
 $= x^2 3e^{-3x} [1 - x] = 0, x = 1$

Maximum at  $x = 1$

$$f(1) = e^{-3}$$

64. The area bounded by  $y^2 = 4x$  and  $x^2 = 4y$  is

- (A)  $\frac{20}{3}$  sq. unit      (B)  $\frac{16}{3}$  sq. unit      (C)  $\frac{14}{3}$  sq. unit      (D)  $\frac{10}{3}$  sq. unit

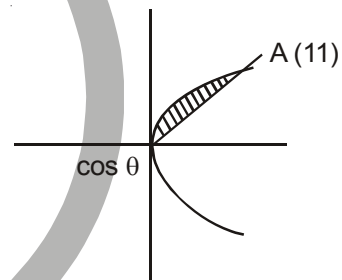
Ans : (B)

Hints :  $A = \int_0^4 \left( \sqrt{4x} - \frac{x^2}{4} \right) dx$

$$= 2 \cdot \frac{2}{3} (x^{3/2})_0^4 - \frac{1}{4 \cdot 3} (x^3)_0^4$$

$$= \frac{4}{3} [4^{3/2} - 0] - \frac{1}{12} [4^3 - 0]$$

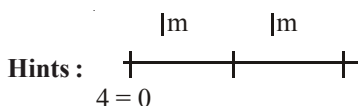
$$= \frac{4^{5/2}}{3} - \frac{16}{3} = \frac{32 - 16}{3} = \frac{16}{3}$$



65. The acceleration of a particle starting from rest moving in a straight line with uniform acceleration is  $8m/sec^2$ . The time taken by the particle to move the second metre is

- (A)  $\frac{\sqrt{2}-1}{2}$  sec      (B)  $\frac{\sqrt{2}+1}{2}$  sec      (C)  $(1+\sqrt{2})$  sec      (D)  $(\sqrt{2}-1)$  sec

Ans : (A)



$$\begin{array}{l|l} S = ut + \frac{1}{2}at^2 & S = uT + \frac{1}{2}aT^2 \\ 1 = \frac{1}{2} \cdot 8 \cdot t^2 & 2 = \frac{1}{2} \cdot 8 \cdot T^2 \\ \frac{1}{4} = t^2 & T^2 = \frac{1}{2} \\ t = \frac{1}{2} & T = \frac{1}{\sqrt{2}} \end{array}$$

$$\text{Time} = \frac{1}{\sqrt{2}} - \frac{1}{2} = \frac{2}{2} - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}-1}{2}$$

66. The solution of

$$\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x} \text{ is}$$

- (A)  $x = c \sin(y/x)$       (B)  $x = c \sin(xy)$       (C)  $y = c \sin(y/x)$       (D)  $xy = c \sin(x/y)$

Ans : (A)

Hints :  $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$

Put  $\frac{y}{x} = \theta, y = \theta x$

$$\frac{dy}{dx} = \theta + \frac{x d\theta}{dx}$$

$$\theta + x \cdot \frac{d\theta}{dx} = \theta + \tan \theta, \frac{d\theta}{\tan \theta} = \frac{dy}{x}$$

$$\int \cot \theta d\theta = \int \frac{dx}{x}$$

$$\log \sin \theta = \log x + \log c$$

$$\sin \theta = x \cdot c, \sin \frac{y}{x} = x \cdot c$$

$$x = c \cdot \sin \frac{y}{x}$$



67. Integrating Factor (I.F.) of the differential equation

$$\frac{dy}{dx} - \frac{3x^2y}{1+x^3} = \frac{\sin^2(x)}{1+x} \text{ is}$$

- (A)  $e^{1+x^3}$       (B)  $\log(1+x^3)$       (C)  $1+x^3$       (D)  $\frac{1}{1+x^3}$

Ans : (D)

Hints : If  $e^{\int p dx} = e^{-\int \frac{3x^2 dx}{1+x^3}} = e^{-\log(1+x^3)} = e^{\log(1+x^3)^{-1}}$

$$= (1+x^3)^{-1} = \frac{1}{1+x^3}$$

68. The differential equation of  $y = ae^{bx}$  (a & b are parameters) is

- (A)  $yy_1 = y_2^2$                       (B)  $yy_2 = y_1^2$                       (C)  $yy_1^2 = y_2$                       (D)  $yy_2^2 = y_1$

**Ans : (B)**

**Hints :**  $y = a.e^{bx}$  ..... (i)

$y_1 = abe^{bx}$

$y_1 = by$  ..... (ii)

$y_2 = by_1$  ..... (iii)

Dividing (ii) & (iii)  $\frac{y_1}{y_2} = \frac{y}{y_1} \Rightarrow y_1^2 = yy_2$

69. The value of

$\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{r^3}{r^4 + n^4}$  is

- (A)  $\frac{1}{2} \log_e(1/2)$                       (B)  $\frac{1}{4} \log_e(1/2)$                       (C)  $\frac{1}{4} \log_e 2$                       (D)  $\frac{1}{2} \log_e 2$

**Ans : (C)**

**Hints :**  $\lim_{n \rightarrow \infty} \sum \frac{n^3 \left(\frac{r}{n}\right)^3}{n^4 \left[\left(\frac{r}{n}\right)^4 + 1\right]}$

$= \frac{1}{4} \int_0^1 \frac{x^3}{1+x^4} dx = \frac{1}{4} [\log(1+x^4)]_0^1$

$= \frac{1}{4} (\log 2 - \log 1) = \frac{1}{4} \log 2$

70. The value of  $\int_0^{\pi} \sin^{50} x \cos^{49} x dx$  is

- (A) 0                                      (B)  $\pi/4$                                       (C)  $\pi/2$                                       (D) 1

**Ans : (A)**

**Hints :**  $I = \int_0^{\pi} \sin^{50} x \cdot \cos^{49} x dx \quad \int_0^a f(x) = \int_0^a f(a-x)$

$I = \int_0^{\pi} \sin^{50} x (-\cos^{49}(x)) = -\int_0^{\pi} \sin^{50} x \cdot \cos^{49} x$

$= I = -I$

$I = 0$

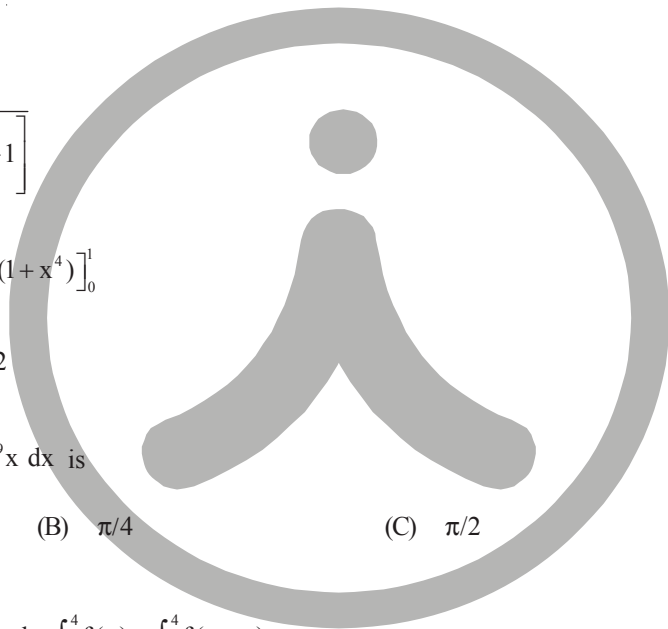
71.  $\int 2^x (f'(x) + f(x) \log 2) dx$  is

- (A)  $2^x f'(x) + C$                       (B)  $2^x f(x) + C$                       (C)  $2^x (\log 2) f(x) + C$                       (D)  $(\log 2) f(x) + C$

**Ans : (B)**

**Hints :**  $I = \int 2^x f'(x) dx + \int 2^x f(x) \log 2 dx$

$= 2^x f(x)$





72. Let  $f(x) = \tan^{-1}x$ . Then  $f'(x) + f''(x)$  is  $= 0$ , when  $x$  is equal to  
 (A) 0 (B) +1 (C) i (D) -i

**Ans : (B)**

**Hints :**  $f(x) = \tan^{-1}x$

$$f'(x) = \frac{1}{1+x^2}$$

$$f''(x) = \frac{-1}{(1+x^2)^2} \cdot 2x, \quad \frac{1}{1+x^2} = \frac{2x}{(1+x^2)^2}$$

$$1+x^2=2x, (x-1)^2=0$$

$$x=1$$

73. If  $y = \tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$ , then  $y'(1) =$   
 (A) 1/4 (B) 1/2 (C) -1/4 (D) -1/2

**Ans : (A)**

**Hints :**  $y = \tan^{-1} \left( \frac{\sqrt{1+x^2}-1}{x} \right)$

Put  $x = \tan\theta$

$$= \tan^{-1} \left( \frac{\sec\theta - 1}{\tan\theta} \right) = \tan^{-1} \left( \frac{1 - \cos\theta}{\sin\theta} \right)$$

$$= \tan^{-1} \left( \frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \right) = \tan^{-1} \tan \frac{\theta}{2}$$

$$= \frac{\theta}{2} = \frac{1}{2} \cdot \tan^{-1}x, \quad y' = \frac{1}{2(1+x^2)}$$

$$y'(1) = \frac{1}{2 \cdot 2} = \frac{1}{4}$$

74. The value of  $\lim_{x \rightarrow 1} \frac{x + x^2 + \dots + x^n - n}{x - 1}$  is  
 (A) n (B)  $\frac{n+1}{2}$  (C)  $\frac{n(n+1)}{2}$  (D)  $\frac{n(n-1)}{2}$

**Ans : (C)**

**Hints :**  $\lim_{x \rightarrow 1} \frac{(x-1) + (x^2-1) + (x^3-1) + \dots + (x^n-1)}{x-1}$

$$= 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

75.  $\lim_{x \rightarrow 0} \frac{\sin(\pi \sin^2 x)}{x^2} =$   
 (A)  $\pi^2$  (B)  $3\pi$  (C)  $2\pi$  (D)  $\pi$

**Ans : (D)**

**Hints :** 
$$= \lim_{x \rightarrow 0} \frac{\sin(\pi \sin^2 x)}{x^2} = \pi \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} = \pi$$
  

$$= \pi$$

76. If the function

$$f(x) = \begin{cases} \frac{x^2 - (A+2)x + A}{x-2} & \text{for } x \neq 2 \\ 2 & \text{for } x = 2 \end{cases}$$

is continuous at  $x = 2$ , then

- (A)  $A = 0$                       (B)  $A = 1$                       (C)  $A = -1$                       (D)  $A = 2$

**Ans : (A)**

**Hints :** 
$$\frac{4 - (A+2)2 + A}{0} = \frac{-A}{0}$$
                      Put  $A = 0$ .

77. 
$$f(x) = \begin{cases} [x] + [-x], & \text{when } x \neq 2 \\ \lambda & \text{when } x = 2 \end{cases}$$

If  $f(x)$  is continuous at  $x = 2$ , the value of  $\lambda$  will be

- (A)  $-1$                       (B)  $1$                       (C)  $0$                       (D)  $2$

**Ans : (A)**

**Hints :** 
$$\text{LHL} = \lim_{h \rightarrow 0} [2-h] + [-(2-h)]$$

$$= \lim_{h \rightarrow 0} 1 + (-2+h) = 1 - 2 = -1$$

$$\text{RML} = \lim_{h \rightarrow 0} [2+h] + [-(2+h)]$$

$$= 2 + (-2-h) = 2 - 3 = -1$$

$$\lambda = -1$$

78. The even function of the following is

(A)  $f(x) = \frac{a^x + a^{-x}}{a^x - a^{-x}}$

(B)  $f(x) = \frac{a^x + 1}{a^x - 1}$

(C)  $f(x) = x \cdot \frac{a^x - 1}{a^x + 1}$

(D)  $f(x) = \log_2(x + \sqrt{x^2 + 1})$

**Ans : (C)**

**Hints :** 
$$f(-x) = (-x) \frac{a^{-x} - 1}{a^{-x} + 1}$$

$$= (-x) \frac{1 - a^x}{1 + a^x}$$

$$= x \frac{(a^x - 1)}{(a^x + 1)} = f(x)$$

79. If  $f(x+2y, x-2y) = xy$ , then  $f(x, y)$  is equal to

(A)  $\frac{1}{4}xy$

(B)  $\frac{1}{4}(x^2 - y^2)$

(C)  $\frac{1}{8}(x^2 - y^2)$

(D)  $\frac{1}{2}(x^2 + y^2)$

**Ans : (C)**

$$x + 2y = a$$

$$x - 2y = b$$

**Hints :**  $2x = a + b$

$$4y = a - b$$

$$f(a, b) = \left(\frac{a+b}{2}\right)\left(\frac{a-b}{4}\right) = \frac{a^2 - b^2}{8}$$

80. The locus of the middle points of all chords of the parabola  $y^2 = 4ax$  passing through the vertex is  
 (A) a straight line                      (B) an ellipse                      (C) a parabola                      (D) a circle

**Ans : (C)**

**Hints :**  $2h = x, 2k = y$

$$y^2 = 4ax$$

$$k^2 = 2ah$$

$$y^2 = 2ax$$



**DESCRIPTIVE TYPE QUESTIONS**  
**SUB : MATHEMATICS**

1. The harmonic mean of two numbers is 4. Their arithmetic mean A and the geometric mean G satisfy the relation  $2A + G^2 = 27$ . find the numbers.

**Ans.** (3, 6)

**Sol :** Let the number be a, b

$$\boxed{A.H=G^2}$$

$$\boxed{H=4}$$

$$\Rightarrow G^2 = 4A$$

$$2A + G^2 = 27$$

$$\Rightarrow 2A + 4A = 27$$

$$\Rightarrow A = \frac{27}{6} = \frac{9}{2}$$

$$\Rightarrow G^2 = 18 \Rightarrow G = \sqrt{18}$$

$$\Rightarrow a.b = 18$$

$$a + b = 9$$

$$\Rightarrow \begin{matrix} a=6 & \text{or} & a=3 \\ b=3 & & b=6 \end{matrix}$$

2. If the area of a rectangle is 64 sq. unit, find the minimum value possible for its perimeter.

**Ans.** 32

**Sol.** Let the dimensions be a, b

$$\text{Area} = ab$$

$$\text{Perimeter} = 2(a + b)$$

$$\text{We have } ab = 64 \Rightarrow b = \frac{64}{a}$$

Perimeter as function of a

$$P(a) = 2\left(a + \frac{64}{a}\right)$$

for maxima or minimum

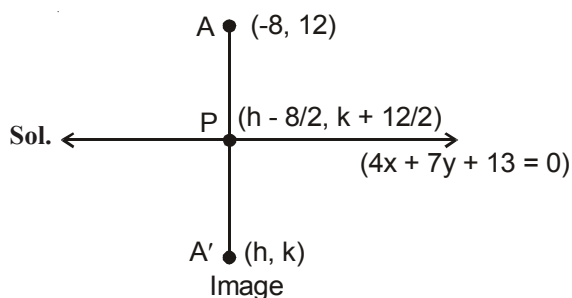
$$P'(a) = 2\left(1 - \frac{64}{a^2}\right) = 0 \Rightarrow a = \pm 8 = 8$$

$$P''(a) = 2 \times \frac{64}{a^3} = \frac{2 \times 64}{8^3} > 0$$

P(8) is minimum

$$\text{Minimum } P(8) = 2(8 + 8) = 32$$

3. Find the image of the point  $(-8, 12)$  with respect to the line  $4x + 7y + 13 = 0$   
**Ans.**  $(-16, -2)$



$$4\left(\frac{h-8}{2}\right) + 7\left(\frac{k+12}{2}\right) + 13 = 0$$

$$2h - 16 + \frac{7k}{2} + 42 + 13 = 0$$

$$4h + 7k + 78 = 0$$

$$\boxed{4h + 7k = -78} \dots\dots (i)$$

2nd equation, we can get

Slope of  $AA' = \frac{7}{4}$

$$\frac{k-12}{h+8} = \frac{7}{4}$$

$$\Rightarrow 4k - 48 = 7h + 56$$

$$4k - 7h = 104 \dots\dots (ii)$$

Solving (i) & (ii)

Equation (i)  $\times 7$  + Equation (ii)  $\times 4$

$$\begin{aligned} 28h + 49k &= -546 \\ \Rightarrow \frac{-28h + 16k}{65k} &= \frac{416}{-130} \end{aligned}$$

$$\boxed{k = -2}$$

$$\boxed{h = -16}$$

$A'(-16, -2)$  is the image of  $(-8, 12)$

4. How many triangles can be formed by joining 6 points lying on a circle ?

**Ans.** 20

**Sol.** Number of triangle

$${}^6C_3 = \frac{6!}{3!3!} = 20$$

5. If  $r^2 = x^2 + y^2 + z^2$ , then prove that

$$\tan^{-1}\left(\frac{yz}{rx}\right) + \tan^{-1}\left(\frac{zx}{ry}\right) + \tan^{-1}\left(\frac{xy}{rz}\right) = \frac{\pi}{2}$$

**A.**  $\theta = \tan^{-1}\left(\frac{S_1 - S_3}{1 - S_2}\right)$



$$1 - S_2 = 1 - \left( \frac{z^2}{r^2} + \frac{x^2}{r^2} + \frac{y^2}{r^2} \right) = 0 \Rightarrow \theta = \frac{\pi}{2}$$

6. Determine the sum of imaginary roots of the equation

$$(2x^2 + x - 1)(4x^2 + 2x - 3) = 6$$

**Ans.**  $-\frac{1}{2}$

**Sol.** Put  $2x^2 + x = y$

$$\Rightarrow (4 - 1)(24 - 3) = 6, \text{ on solving}$$

$$\Rightarrow 2x^2 + x + \frac{1}{2} = 0$$

$$\boxed{\alpha + \beta = -\frac{1}{2}}$$

7. If  $\cos A + \cos B + \cos C = 0$ , prove that

$$\cos 3A + \cos 3B + \cos 3C = 12 \cos A \cos B \cos C$$

**A.** L.H.S =  $\sum 4 \cos^3 A - 3 \cos A$

$$= 4 \sum \cos^3 A - 3 \sum \cos A$$

$$= 12 \cos A \cdot \cos B \cdot \cos C$$

8. Let  $\mathbb{R}$  be the set of real numbers and  $f : \mathbb{R} \rightarrow \mathbb{R}$  be such that for all  $x, y \in \mathbb{R}$ ,

$$|f(x) - f(y)| \leq |x - y|^3. \text{ Prove that } f \text{ is a constant function.}$$

**A.**  $\lim_{x \rightarrow y} \left| \frac{f(x) - f(y)}{x - y} \right| \leq |x - y|^3$

$$= |f'(x)| \leq |0| \Rightarrow |f'(x)| = 0$$

$$\Rightarrow f(x) = \text{constant}$$

9. Find the general solution of

$$(x + \log y) dy + y dx = 0$$

**Ans.**  $xy + y \ln y - y = 0$

**Sol.**  $x dy + y dx + \log y dy = 0$

$$\int d(xy) + \int \log y dy = 0$$

$$xy + y \ln y - y = 0$$

10. Prove that  $I = \int_0^{\pi/2} \frac{\sqrt{\sec x}}{\sqrt{\cos \operatorname{csc} x} + \sqrt{\sec x}} dx = \frac{\pi}{4}$

**A.**  $I = \int_0^{\pi/2} \frac{\sqrt{\sec x}}{\sqrt{\cos \operatorname{csc} x} + \sqrt{\sec x}} dx = \int_0^{\pi/2} \frac{\sqrt{\cos \operatorname{csc} x}}{\sqrt{\cos \operatorname{csc} x} + \sqrt{\sec x}} dx$

$$2I = \int_0^{\pi/2} dx \Rightarrow \boxed{I = \frac{\pi}{4}}$$

