How liquid is the CDS market?*

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Abstract

A common belief is to qualify the credit default swap(CDS) market as very liquid. However, looking at intra-daily CDS data on individual firms from a major inter-dealer broker, we find only limited support for this view. In fact, bid-ask spreads and daily number of trades in our CDS data are more comparable to corporate bond markets than to equity markets. To dig deeper in our data set, we estimate a state-space model of CDS bid and ask quotes on our data. Our model allows for price discreteness, data-errors, heterogeneity of the quotes, jumps in the efficient spreads and intra-daily patterns both in the volatility of the efficient CDS premium and proportional transaction costs. We estimate the model using particle filtering and the Monte Carlo EM algorithm. The volatility of the efficient premium and transaction costs exhibit the usual J-shaped intra-daily pattern observed in equity markets. Also, volatility is much lower during overnight periods and transaction costs much higher.

Keywords : Credit Default Swap, Liquidity, Stochastic Transaction Costs, Inter-dealer Market.

*We thank GFI for providing the data. All errors or omissions are ours.

JEL Classification Numbers:

1 Introduction

Credit Default Swaps (CDS) are arguably one of the most successful financial innovations of the last decade. It is a well-established fact that they provide a more up-to-date picture of creditworthiness than corporate bonds (Blanco, Brennan, and Marsh (2005)) or credit ratings (Hull, Predescu, White (2005)). Also they facilitate taking relatively large long and short positions in the credit markets, improving its efficiency. As a result, it is tempting to go one step further and treat CDS premia as a pure measure of credit risk, by-and-large free of the influence of market microstructure phenomena that bedevil the life of any investigator dealing with the corporate bond markets. In this paper we take a microscopic view of the CDS market using high-frequency data from GFI, a major CDS inter-dealer broker (IDB) and try to check whether the latter view is warranted.

Our data consists of three representative US names (Ford Credit, GMAC, and Sears Acceptance) and one European name (France Telecom). The dataset we analyze contains a complete record of bid and ask quotes, and transaction prices in 2004, 2005, and 2006. We do not observe the volume traded, or the depth posted, or the ID of participants. The access in the IDB is limited to dealers, who trade large amounts. We find that the most active CDS name, Ford, trades on average 4 times a day, while the other three CDSs trade less frequently. Since US corporate bonds trade on average 1.9 per day, and European corporate bond trade on average 4 time a day, the trading frequency of reputed liquid CDSs is quite similar to the corporate bond market.[1] Then we examine the tightness of the market and we find that the cost of turning around a position is around 40 basis points for an investment with a notional of $100. In comparison, Biais and Declerck (2007) find that the costs of a round trip in the European corporate bond market is 30 cents.

As a further step in our analysis we build a dynamic model of discrete bid and ask quotes, based on Hasbrouck (2003). The bid and ask quotes are rounding transformation of an implicit efficient premium, and stochastic market-making costs, where the mean proportional costs can be time-varying. The dynamics of the efficient premium allows for deterministic time-varying volatility and jumps. Further, we allow for data errors to deal with outliers. To solve the resulting non-linear filtering task we employ particle filters and

[1] Though the average trade size is much higher in the CDS market

we use the Monte Carlo EM algorithm to estimate the model parameters.[2]

With this modeling framework we can compare intradaily patterns in the trading activity, volatility of the efficient price, and average transaction costs. We find that the number of quotes per unit of time leads the number of trades per unit of time, which is consistent with either a price discovery process and/or price competition among dealers in the IDB system. The analysis reveals that the volatility has a J-shape during business hours (between 7:30 to 3:30 New York GMT), and is very low during overseas trading. Turning to transaction costs, our results show that the mean costs attain their highest level before the jump in trades, and then declines sharply when the trading frequency picks up, which is consistent with price competition among dealers. Then, transaction costs increase 50% during overseas hours when trading is thin. Overall we find much stronger support in the IDB for price discovery than for the presence of dealers with an inventory motive.

Our results point towards a view that while CDS markets may be much deeper than corporate bond markets, transaction costs are at least as high. Also, looking at intradaily patterns, we find some preliminary evidence for the presence of informed traders. All this suggest that CDS markets are relatively efficient informationally, but in a sense the relatively high transaction costs are the price to be paid for this efficiency. This view is consistent with several recent studies using daily data. Blanco, Brennan, and Marsh (2005) find that the CDS market leads the bond market in determining the price of credit risk, suggesting that informed traders trade first in the CDS market. Acharya and Johnson (2006) find prima facie evidence that informed traders play in the CDS market. Besides, Hull, Predescu, White (2005) provide evidence on the informational efficiency of this market, by showing that CDS premium changes predict rating changes. Our study complements this literature by adding a piece of evidence using high-frequency data.

We also add to the nascent literature on IDB markets. Extant empirical literature (Reiss and Werner (1998),Bjonnes and Rime (2005), Reiss and Werner (2004)) find support for dealers who primarily use the IDB to rebalance their inventories at the end of the day. Interestingly, we don't find much evidence for the inventory motive in our data. In contrast, our findings are consistent with either a price discovery process or competition among dealers.

Last, the econometric methodology we work with could be used to measure transaction costs for illiquid assets. This can be useful for investigating the microstructure of illiquid

[2]The estimation methodology draws heavily on Duan and Fulop (2007a,b)

stocks or derivatives.

2 Overview of the CDS market

2.1 Main characteristics

Credit derivatives are traded over the counter (OTC), which means that there is no physical location and no central organized exchange where orders are matched. Instead, the CDS secondary market operates 24 hours a day through an electronic network of banks, hedge funds and other institutional investors. Quotes are posted by professional intermediaries, as bank dealers. However, dealers are not required explicit continuous presence. Nor do they face rules limiting the size of the bid-ask spread they choose to post, or limiting changes in their prices (unlike the Specialist on the NYSE). Thus the CDS secondary market is organized as a decentralized dealer market. Currently there is a lack of complete post-trade infrastructure for such OTC credit derivatives. In the EU, there is no central counterparty clearing house. In the US, since 2006, there is at least a central warehouse (the Depository Trust and Clearing Corporation).

Dealers trade directly with clients, and among themselves. Trading is conducted in a bilateral non-anonymous communication over the telephone. However, indirect trading via interdealer brokers (IDBs) plays an increasing role. Interdealer brokers collect and post dealer quotes. They also execute trades between dealers by matching buyers and sellers. Only dealers have access to IDB systems, and they are not required to submit quotes to IDBs. Interdealer brokerage involves voice-based systems, and/or electronic platforms, and represents 34% of trades in 2004, according to the International Swaps and Derivatives Association (ISDA) report.

2.2 Trading CDSs in the interdealer market

Trading organization. Interdealer trading in the CDS market can be direct (non anonymous) or brokered (anonymous). There are major differences between the direct and the brokered market: (i) the level of fragmentation of both systems, (ii) the degree of anonymity, (iii) the level of transparency of both organizations. All these key dimensions have an impact on the liquidity of the market. The source of our data are the anonymous centralized electronic IDB market.

The direct interdealer market follows the organization of the secondary OTC market of CDSs, being a fragmented price-driven market. Trading is initiated by one dealer calling another for a bilateral quote, i.e., the quote is not available to other dealers, which distinguishes it from a quote posted to an IDB. This means that trading in that market potentially involves search costs to find the best price.

On the other hand, while indirect IDBs are not costless (there is the interdealer broker' s fee to pay), they allow to decrease search costs and make the matching process between buyers and sellers more efficient. IDB systems are centralized books for brokers that can better match buyers and sellers with a larger vision of the market and of the order flow. Brokers collect dealers' quote and depth information, and post the data into the system that are provided anonymously to the other dealers. This transparency should reduce search costs. Also, the organization of these platforms should enhance the speed of execution.

Since trades are executed anonymously in IDBs market, they enable dealers to trade in size. Moreover, the size of limit orders is undisclosed in some IDBs.[3] Anonymity of quotes and trades is known to lower volatility and enhance liquidity (see Foucault, Moinas, and Theissen, 2007)).

Trading motives. Why do dealers usually trade in anonymous IDBs? As Huang, Cai and Wang (2002) point out, trading in IDBs may be visualized as the second step of a two-stage game. The first stage concerns trades between dealers and clients in the opaque, fragmented quote-driven market. Then, dealers trade in the interdealer market either to manage their inventory positions (i.e. to unwind or to build a position) as suggested by Ho and Stoll (1983), or/and to trade on private information. Given the anonymity of IDBs platforms, the level of information-based trading could be high in these facilities: IDBs could be the favored trading venue of informed interdealer trades (see, for instance, Barclay and Hendershott (2004)).

However, the few empirical studies that exist on IDB's [4] seem to find more evidence for the inventory motive. First, in the London Stock Exchange, Reiss and Werner (1998)

[3]The anonymity of IDB systems is a key factor of these platforms. Dealers choose to trade anonymously because their identity is an additional information that could be exploited by competitors.

[4]There are few studies of interdealer broker markets. The well known electronic interdealer systems are SelectNet and SuperSOES for the Nasdaq before its move to SuperMontage, GovPX for the US Treasury bond market, EBS and Reuters 2000-2 for the FX market.

show that interdealer markets are mainly used by equity dealers to share inventory risks. Bjonnes and Rime (2005) also find evidence that, in the FX market, IDBs are used by dealers to manage positions. Second, Reiss and Werner (2004) find that IDBs in the LSE attract uninformed order-flow, whereas the direct non-anonymous inter-dealer market attracts informed inter-dealer trades, contrarily to standard information-based microstructure theories. Reiss and Werner observe that when adverse selection is perceived to be high, limit order are cancelled in IDB platforms, and liquidity dries up. Besides when a dealer who possesses a time-sensitive private information posts an informed limit order in the IDB it may not be executed, and information may leak out.

In light of these studies, a priori, one expects to observe intraday patterns in IDBs for CDSs that are consistent with the inventory paradigm and less affected by information asymmetries. First, trading activity and the number of trades should pick at the end of the business day, to reflect the desire to close open positions before the overnight hours. Second, if adverse selection is indeed not of major importance on the CDS IDB market, one should not observe an especially high level of transaction costs and volatility at the beginning of the day.

3 Data

3.1 A preliminary look at the data

We use intra-daily bid and ask quotes from GFI, a major CDS interdealer broker. GFI is a hybrid voice-electronic execution platforms for CDSs. Dealer may be providers or consumers of liquidity. Retail and institutional investors (the "buy-side") are not eligible to use the platform directly. The platform has a minimum trade size of $1 million. Only the broker can see the depth of the market; however, dealers can observe the last trade (price, volume, and direction). Bid and ask prices posted on the platform are firm. According to GFI, it is the leading broker since it would represent 60% of the interdealer brokerage activity. As a broker, GFI earns commissions directly from trade counterparts.

The dataset that GFI provided us contains CDS quote and trade price for 4 entities (3 from US and 1 from Europe) whose CDS contracts are reputed very liquid. The 4 names are Ford, GMAC, Sears Acceptance and France Telecom, all of them are on a 5-year contracts on senior debt. Ford and GMAC were downgraded from investment to speculative bond on May, 5 2005. France Telecom is graded as investment (see Table 1, Panel A).

Our sample runs from January 2004, through December 2006. The sample consists of bid, ask quotes, and trade prices expressed in basis point. Our data is time stamped down to a minute. Our local time benchmark is New York GMT. There are no identifiers of the dealers who post quotes or who trade. No information about the depth of quotes, nor information about the size of each transaction is provided.[5]

We examine two measures of trading activity: the number of bid or/and ask quotes per unit of time (the quoting frequency), and the number of trades per unit of time (the trade frequency).

As Figure 1 illustrates, quotes and trades are irregularly spaced . Although trading could take place round-the-clock, the trading activity (number of quotes and trades) of the 3 US entities mainly occur during the New York business hours (7:30 am to 5:30 pm), as Figures 2, 3, and 4 illustrate.[6] However, Ford and GMAC exhibit some weak but significant trading activity when London open at 7:30 am (2:30 am New York), unlike Sears Acceptance. In contrast, the main peak for the trading activity is in London for the European name. Trading of the European CDS France Telecom starts at 7:30 am London. Then it passes to New York at 12:30 am London (7:30 am New York), and continues there until 5:30 pm (see Figure 5).

It is worth noticing, first, that, during New York business hours, we observe a J-shape pattern for the number of trades for the 3 US names. The number of trades does not spike upward at the end of the day, as it is usually the case in other markets (U-shaped pattern for volume). This intraday pattern is quite unexpected, since it seems to go against the hypothesis of IDBs as inventory management tools. Indeed, in case IDBs would have been used by dealers to close or hedge open inventory position before the overnight period, we should observe a jump upward at the end of the business day.

Second, we find that for the 4 names, the peak of the number of trades follows the busiest period in terms of number of quotes (lead-lag pattern between quotes and trades). This pattern may reflect either the price competition among dealers in the IDB platform

[5]From interviews with traders, it seems that the minimum trade size is 5-10 million in Europe and in US: CDS contracts are only traded in relatively large slot (”granularity” of CDS contracts). For instance, a trade on a $20 - 30$ million for the most liquid European CDS is quite standard. Note also that the most common practice is to trade on a 5-year contract.

[6]The reasons may be that news concerning these US firms are probably announced during New York trading hours, and holders of these CDS contracts are presumably US banks or hedge funds.

to fill the book before trading happens, or/and a price discovery process similar to what we observe during the preopening periods in regulated equity markets.[7]

3.2 Descriptive statistics

Table 1 (Panel B) reports some descriptive statistics on the trading activity. For the 4 CDS contracts reputed to be very liquid, we compute the average number of trades per day. France Telecom trades on average 1.42 a day. Concerning the 3 US entities, GMAC and Ford trade on average 4.46 a day, and 2.57 a day respectively. Sears Acceptance trades less (0.78 a day). The trading is low compared to their counterparts in the equity market, but similar to their counterparts in the corporate bond market. Edwards, Nimalendran and Piwowar (2006) find that the sample of the more active US corporate bonds trades in average 3.7 times per day, whereas Edwards, Harris and Piwowar (2007) find a lower figure (1.9) for their sample that covers also less active bonds. Biais and Declerck (2007) find that Euro-denominated corporate bonds trade on average 4 times per day.[8]

Table 1 (Panel C) reports also some descriptive statistics on quotes. First, even for these reputed liquid CDS contracts, the number of quotes (bid or ask) is quite low, compared to the equity market. For instance, GMAC and Ford are active CDSs with more than 1400 bid/ask pairs during three years, whereas we observe only 568 bid/ask pairs for Sears Acceptance. Thus, it is important to use as much information in the data as possible, in particular, to use the individual bid or ask observations.[9]

Second, the level of CDS spreads seems to be related to the rating of the entity. Average quoted b/a spreads are around 10 bp for CDS of speculative-grade entities (the 3 US names), and lower than 3 bp for the European CDS of the investment-grade bond France Telecom.

To measure the cost of a round-trip in the CDS market, one cannot simply use the quoted b/a spread as is the case in the equity or the bond market. The reason is that entering a long position and unwinding it leaves the investor with a negative cash-flow stream with a frequency and a duration equal to the frequency and maturity of the CDS

[7] Equity markets have regulated trading hours. There is a clear distinction between the overnight hours, and the trading hours, unlike OTC 24-hours markets

[8] Even if the trading frequency is similar between the bond market and the CDS market, overall trading should be much higher in the CDS as the size of individual trade is muchi higher.

[9] In contrast, Huang et al. (2002) and others filter out one-sided quotes when using GovPX.

contract, where the amount of the individual cash-flows is equal to the b/a spread. So, to compute the cost of turning around a position, one needs to take the present value of this cash-flow stream. For instance, the cost of a round trip for an investment in GMAC with a notional of \$100 is 42 cents. Analyzing a sample of Euro-denominated corporate bonds, Biais and Declerck (2007) find an average quoted spread around 30 cents. Edwards, Harris and Piwowar find that the effective cost is 20 cents for large trade on US corporate bonds.

These numbers suggest that b/a spreads and transaction costs on the CDS market are not lower than on the corporate bond market. Thus, microstructure phenomena (trading frictions) are likely to be important in this market and one needs to be careful in interpreting CDS premia as a pure measure of credit risk.

4 Econometric Model

To be able to dig further in our data set we link the quotes to an unobserved efficient price and cast the resulting estimation task as a missing-data problem. In our data, in any minute, we keep the last recorded bids and asks and label an observation a joint observation if they have the same time stamp. We discard all joint observations where the ask is smaller than or equal to the bid. Then, filtering and estimation in the resulting non-linear state-space system is solved using simulation-based methods. This modeling framework allows us to estimate intra-daily patterns of the efficient CDS premium and intra-daily patterns of transaction costs. Also, as a by-product we can estimate the salient features of the efficient premium evolution and come up with a filtered estimate of the CDS premium at any time point.

We consider a model conditional on the arrival of bids and asks of CDS spreads. Let $\tau_i, i = 0, ..., T$ denote the joint arrivals of these data points. Let $\mathcal{D}_i$ denote the information set up to τ_i. We either have a bid or ask observation at τ_i denoted by B_{τ_i}, A_{τ_i} respectively, or we may have both. Similarly to Hasbrouck (2003) we assume a latent variable model for our data. In particular, we link all observables to a theoretical efficient log spread, $m_{\tau_i}(= log(M_{\tau_i}))$, which follows

$$m_{\tau_i} = m_{\tau_{i-1}} + \sigma_{u,f(\tau_{i-1},\tau_i)}\sqrt{\Delta\tau_i}u_i + \sum_{h=0}^{\Delta N_i} J_{i,h} \quad (1)$$

where $\Delta\tau_i = \tau_i - \tau_{i-1}$. u_i are iid standard normal and model the effect of public news on the efficient price M_{τ_i}. $\sigma_{u,f(\tau_{i-1},\tau_i)}$ is a volatility coefficient allowing for time-varying volatility. The last term is there to allow for the arrival for jumps in the efficient CDS price. Here ΔN_i is a Poisson random variable with $\lambda \Delta \tau_i$ as its parameter and $J_{i,h}$ are independent normally distributed jump sizes with mean μ_J and variance σ_J^2.

Dealers are assumed to be subject to nonnegative costs of market making, denoted $c_{i,A}(= log(C_{i,A}))$ and $c_{i,B}(= log(C_{i,B}))$, as follows.

$$\begin{aligned} c_{i,A} &\sim N(\mu_{\tau_i}, \sigma_{c,A}^2) \\ c_{i,B} &\sim N(\mu_{\tau_i}, \sigma_{c,B}^2) \end{aligned}$$

We assume that in the absence of tick restrictions a market maker would bid $B_{\tau_i} = M_{\tau_i} - C_{i,B} = e^{m_{\tau_i} - (c_{i,B} + q_i^A \varepsilon_i^A)}$ and offer(ask) $A_{\tau_i} = M_{\tau_i} + C_{i,A} = e^{m_{\tau_i} + c_{i,A} + q_i^B \varepsilon_i^B}$. Further, we also assume that we only observe cases when the bid is smaller than the ask. This means that when we have both a bid and an ask observation, we need to condition on the event $\{c_{i,A} + q_i^A \varepsilon_i^A + c_{i,B} + q_i^B \varepsilon_i^B > 0\}$.

The expressions $q_i^A \varepsilon_i^A$ and $q_i^B \varepsilon_i^B$ represent data errors where q_i^A, q_i^B are iid Bernoullis that take the value 1 with probability p and ε_i^A and ε_i^B are iid normal with mean zero and variance σ_ε^2.

If the tick size is K, the discrete bid and ask prices are given by

$$B_{\tau_i} = Floor\left(e^{m_{\tau_i} - (c_{i,B} + q_i^B \varepsilon_i^B)}, K\right) \tag{2}$$

$$A_{\tau_i} = Ceiling\left(e^{m_{\tau_i} + c_{i,A} + q_i^A \varepsilon_i^A}, K\right) \tag{3}$$

where the floor and ceiling functions round down and up respectively to the next multiple of K.[10]

5 Filtering and ML estimation

5.1 Particle Filter for the model

First, we describe the filtering algorithm for the case when we only have an ask observation. Assume that the time index of the new observation is τ_i. Denote the noisy ask observation

[10]Duan and Fulop (2007b) applies a similar model to high-frequency stock market transaction data. However, their focus is on the efficient price process not microstructure phenomena. Accordingly, their efficient-price model is richer, but their microstructure model is simpler.

before discretization by $a_{\tau_i} = m_{\tau_i} + c_{i,A} + q_i^A \varepsilon_i^A$. Our algorithm is based on the following filtering density/distribution

$$\begin{aligned}
& f(a_{\tau_i}, m_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, c_i^A \mid \mathcal{D}_i) \\
= \; & f(c_i^A \mid a_{\tau_i}, m_{\tau_i}, q_i^A) f(m_{\tau_i} \mid a_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A) \\
\times \; & f(a_{\tau_i} \mid A_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A) f(m_{\tau_{i-1}}, \Delta N_i, q_i^A, \mid \mathcal{D}_i) \\
\propto \; & f(c_i^A \mid a_{\tau_i}, m_{\tau_i}, q_i^A) f(m_{\tau_i} \mid a_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A) f(a_{\tau_i} \mid A_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A) \\
\times \; & p(A_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A) p(\Delta N_i) p(q_i^A) f(m_{\tau_{i-1}} \mid \mathcal{D}_{i-1}) \qquad (4)
\end{aligned}$$

The last expression in (4) suggests a way to sample from the filtering distribution, given a sample of particles representing $f\left(m_{\tau_{i-1}} \middle| \mathcal{D}_{i-1}\right)$. First, extend the state-space to include jumps in the system by simulating from some importance sampler $g_1(\Delta N_i) g_2(q_i^A)$. Then, perform resampling to obtain the particle $(m_{\tau_{i-1}}, q_i^A, \Delta N_i)$ based on the weights $w = \frac{p(A_{\tau_i} | m_{\tau_{i-1}}, \Delta N_i, q_i^A) p(\Delta N_i) p(q_i^A)}{g_1(\Delta N_i) g_2(q_i^A)}$. Finally, sample $(c_i^A, J_i, m_{\tau_i}, a_{\tau_i})$ according to $f(c_i^A \mid a_{\tau_i}, m_{\tau_i}, q_i^A) f(m_{\tau_i} \mid a_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A) f(a_{\tau_i} \mid A_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A)$.

Assume that we have M particles, $m_{\tau_{i-1}}^{(m)}$ representing $f(m_{\tau_{i-1}} \mid \mathcal{D}_{i-1})$. Then our localized particle filter with M particles consists of the following steps:

- **Step 1:** Enlarge the state-space by the jumps in the system by sampling from ΔN_i and q_i^A. Jumps can be rare events so we use stratified sampling to ensure that our simulated sample always contains jumps by putting an equal probability of $\frac{1}{2}$ on having and not having jumps. I.e. we use the importance sampling distributions

$$g_1(\Delta N_i) = \frac{1}{2} p(\Delta N_i \mid \Delta N_i > 0) + \frac{1}{2} 1_{\{\Delta N_i = 0\}} \qquad (5)$$

$$g_2(q_i^A) = \frac{1}{2} 1_{\{q_i^A = 0\}} + \frac{1}{2} 1_{\{q_i^A = 1\}} \qquad (6)$$

To arrive at an empirical representation of $f(m_{\tau_{i-1}}, \Delta N_i, q_i^A, \mid \mathcal{D}_i)$ attach to each of the particles $(m_{\tau_{i-1}}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)})$ the importance weights

$$w_i^{(m)} = p(A_{\tau_i} \mid m_{\tau_{i-1}}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)}) p((q_i^A)^{(m)}) [1_{\Delta N_i^{(m)} > 0} p(\Delta N_i > 0) + 1_{\Delta N_i^{(m)} = 0} p(\Delta N_i = 0)]$$

The likelihood value for the observed ask can be computed as

$$p(A_{\tau_i} \mid \mathcal{D}_{i-1}) \approx \frac{1}{M} \sum_{m=1}^{M} w_i^{(m)}$$

- **Step 2:** Resample the particle set according to the probability $\pi_i^{(m)} = \frac{w_i^{(m)}}{\sum_{m=1}^M w_i^{(m)}}$ to yield M equal-weighted particles denoted by $(m_{\tau_{i-1}|i}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)})$. This equal-weighted particle set is again an empirical representation of $f(m_{\tau_{i-1}}, \Delta N_i, q_i^A, \mid \mathcal{D}_i)$.

- **Step 3:** Corresponding to each particle $(m_{\tau_{i-1}|i}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)})$, sample from the truncated normal density $f(a_{\tau_i} \mid A_{\tau_i}, m_{\tau_{i-1}|i}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)})$ to generate the particle $(a_{\tau_i}^{(m)}, m_{\tau_{i-1}|i}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)})$. The corresponding particle set represents $f(a_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, \mid \mathcal{D}_i)$.

- **Step 4:** Using conditional normality sample from

$$
\begin{aligned}
m_{\tau_i}^{(m)} &\sim f(m_{\tau_i} \mid a_{\tau_i}^{(m)}, m_{\tau_{i-1}|i}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)}) \\
(c_i^A)^{(m)}) &\sim f(c_i^A \mid a_{\tau_i}^{(m)}, m_{\tau_i}^{(m)}, (q_i^A)^{(m)}))
\end{aligned}
$$

  This yields M particles, $(a_{\tau_i}^{(m)}, m_{\tau_i}^{(m)}, m_{\tau_{i-1}|i}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)}, (c_i^A)^{(m)})$ which represent $f(a_{\tau_i}, m_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, c_i^A \mid \mathcal{D}_i)$. One can then proceed to marginalize m_{τ_i} to represent the filtering distribution $f(m_{\tau_i} \mid \mathcal{D}_i)$

The necessary quantities to implement the algorithm are described in detail in Appendix A. The case of only a bid observation at τ_i is very similar to the situation described above, so to conserve space we do not spell out its details. On the other hand, the case of a paired observation with both a bid and an ask, the situation is a bit different. First, both the bid and the ask contains information on the new fundamental price, m_{τ_i}. Second, we need to deal with sample selection as a result of the restriction $\mathcal{S} = \{a_{\tau_i} > b_{\tau_i}\}$ where

$$
\begin{aligned}
b_{\tau_i} &= m_{\tau_i} - (c_{i,B} + q_i^B \varepsilon_i^B) \\
a_{\tau_i} &= m_{\tau_i} + c_{i,A} + q_i^A \varepsilon_i^A
\end{aligned}
$$

The particle filter used to deal with this case is described in Appendix B.

6 Monte Carlo EM algorithm

We now address the issue of computing the maximum likelihood (ML) estimates for the model parameters. The particle filtering algorithm described in the preceding section can generate the log-likelihood function for any fixed parameter values. However, it is ill-suited for finding the ML estimates because the log-likelihood function is inherently irregular with respect to the parameters even with the use of common random numbers. This irregularity arises from the resampling step required for any particle filter. Thus we adopt an indirect approach to the ML estimation via the EM algorithm of Dempster, *et al* (1977). The EM algorithm is an alternative way of obtaining the ML estimate for the incomplete data model, where incomplete data refers to the situation that the model contains some random variable(s) without corresponding observations. The EM algorithm involves two steps - expectation and maximization – and hence its name. One first writes down the complete-data log-likelihood function. Since it is not observable, one needs to compute its expected value by conditioning on the observed data in conjunction with some assumed parameter values. This completes the expectation step. In the maximization step, one finds the new parameter values that maximize the expected complete-data log-likelihood function. The updated parameter values are then used to repeat the E- and M-step until convergence. Interestingly, the EM algorithm will converge to the ML estimate under some regularity conditions.

For our ML estimation, the E-step due to its complexity will have to be computed using the particle filter, which means that we are using the Monte Carlo EM (MCEM) algorithm.[11] Casting optimization as an EM algorithm problem effectively circumvents the irregularity induced by the particle filter, because the E-step ensures that the expected complete-data log-likelihood function is smooth with respect to the model parameters that define the complete-data log-likelihood function. Even though the function is still inherently irregular in relation to the assumed parameter values used in computing the expectation, it becomes immaterial as far as optimization is concerned. In effect, one has decoupled optimization from filtering in each iteration.

In general, the complete-data representation of the model is not unique. We choose a specification that allows for relatively easy M-steps and where the expected complete-data loglikelihood can be written as a combination of sufficient statistics of the latent data and the model parameters. This latter property is important because it means that we need to run the particle filter only once per iteration of the EM algorithm.

First, we need a complete data representation of the fundamental innovations. We include the innovation due to the diffusion part, $Z_{i,u} = \sigma_{u,f(\tau_{i-1},\tau_i)}\sqrt{\Delta\tau_i}u_i$, the number of jumps, ΔN_i and the innovation due to jumps, $Z_{i,J} = \sum_{h=0}^{\Delta N_i} J_{i,h}$. The loglikelihood of the

[11] For a general introduction to the MCEM algorithm, see for instance Wei and Tanner (1990).

number of jumps is

$$L_i^1 = \Delta N_i \ln(\lambda \Delta \tau_i) - \lambda \Delta \tau_i \tag{7}$$

Conditional on the number of jumps, the loglikelihood of the spread innovations is

$$L_i^2 = -\frac{Z_{i,u}^2}{2\sigma^2_{u,f(\tau_{i-1},\tau_i)}\Delta\tau_i} - \frac{\ln\left(\sigma^2_{u,f(\tau_{i-1},\tau_i)}\right)}{2} + 1_{\{\Delta N>0\}} \times \left[-\frac{(Z_{i,J}-\mu_J\Delta N_i)^2}{2\sigma_J^2(\Delta N_i + 1_{\{\Delta N=0\}})} - \frac{\ln\left(\sigma_J^2\right)}{2}\right] \tag{8}$$

Second, we need the complete data representation of the microstructure noises and data errors. We include the microstructure errors $c_{i,A}, c_{i,B}$, the data error indicators $q_{i,A}, q_{i,B}$ and the data errors $\varepsilon_{i,A}, \varepsilon_{i,B}$. Then, if the observation is an ask we have

$$\begin{aligned} L_i^{3,A} &= -\frac{(c_{i,A}-\mu_{\tau_i})^2}{2\sigma^2_{c,A}} - \frac{\ln(\sigma^2_{c,A})}{2} + q_{i,A}\left[-\frac{\varepsilon^2_{i,A}}{2\sigma^2_\varepsilon} - \frac{\ln(\sigma^2_\varepsilon)}{2}\right] \\ &+ q_{i,A}\ln p + (1-q_{i,A})\ln(1-p) \end{aligned} \tag{9}$$

If the observation is a bid we have

$$\begin{aligned} L_i^{3,B} &= -\frac{(c_{i,B}-\mu_{\tau_i})^2}{2\sigma^2_{c,B}} - \frac{\ln(\sigma^2_{c,B})}{2} + q_{i,B}\left[-\frac{\varepsilon^2_{i,B}}{2\sigma^2_\varepsilon} - \frac{\ln(\sigma^2_\varepsilon)}{2}\right] \\ &+ q_{i,B}\ln p + (1-q_{i,B})\ln(1-p) \end{aligned} \tag{10}$$

When we have both a bid and an ask observation, the situation is more involved because of the conditioning on the event $\{c_{i,A} + q_i^A\varepsilon_i^A + c_{i,B} + q_i^B\varepsilon_i^B > 0\}$. Fortunately the EM setup is ideally suited to deal with such situations. In particular, complete the space with draws of the vector $(c_{i,A}, q_i^A, \varepsilon_i^A, c_{i,B}, q_i^B, \varepsilon_i^B)_j$ that were thrown away because of not satisfying the condition. Denote the corresponding loglikelihoods $L_{i,j}^{3,A}$ and $L_{i,j}^{3,B}$. Let k_i denote the number of such draws. Then the complete data loglikelihood is

$$L_i^{3,AB} = L_i^{3,A} + L_i^{3,B} + \sum_{j=0}^{k_i}(L_{i,j}^{3,A} + L_{i,j}^{3,B}) \tag{11}$$

Then the MCEM algorithm can be summarized as follows: (1) Set some initial parameter values, $\theta^{(0)}$; (2) Repeat the following E- and M-steps until convergence.

- **E-step:** Get the conditional expectation of the sufficient statistics in the complete data loglikelihood function of the previous set of parameters. In particular we need to approximate quantities of the form

$$E(X \mid \mathcal{D}_T, \theta^{(k-1)})$$

where X_i is some function of the hidden random states, $\mathcal{D}_T$ is the observed data and θ^{k-1} is the parameter vector at the $(k-1)^{th}$ iteration. The particle filter described in section 5.1 can be used to compute these quantities. We run the filter using the parameters $\theta^{(k-1)}$ to generate the particle set that represents the smoothed distribution for X_i. The m-th particle is denoted by $X_{i|T}^{(m)}$. Thus, the expectation can be approximated by the sample average as follows:

$$E(X_i \mid \mathcal{D}_T, \theta^{(k-1)}) \approx \frac{1}{M}\sum_{m=1}^{M} X_{i|T}^{(m)}$$

When the sample size T is large, undesirable Monte-Carlo noise will be introduced by the use of the smoothed distribution. Intuitively, the particle filter always adapts to the newest observation, and thus its representation of the distant past is bound to be poor. Cappe and Moulines (2005) suggest to use the information only up to $i+L$ when computing any quantity that involves the unobserved state variable at time i. The rationale is the forgetting property expected of the dynamic system; that is, for large enough L, the distribution for the unobserved state variable at time i conditional on the information up to $i+L$ will be almost identical to that conditional on the entire sample. Cappe and Moulines (2005) thus propose to use fixed-lag smoothing by using information only up to $i+L$. Adopting fixed-lag smoothing leads to our approximation as follows:

$$\begin{aligned} & E(X_i \mid \mathcal{D}_T, \theta^{(k-1)}) \\ \approx\ & E(X_i \mid \mathcal{D}_{(i+L)\wedge T}, \theta^{(k-1)}) \\ \approx\ & \frac{1}{M}\sum_{m=1}^{M} X_{i|(i+L)\wedge T}^{(m)} \end{aligned}$$

- **M-step:** Maximize the conditional expected value of the complete-data log-likelihood function obtained in the E-step. In particular, denote the i^{th} complete-data loglikelihood by $L_i(\theta)$

$$L_i(\theta) = L_i^1(\theta) + L_i^2(\theta) + L_i^3(\theta) \tag{12}$$

where $L_i^3(\theta)$ is equal to $L_i^{3,A}(\theta)$, $L_i^{3,B}(\theta)$ or $L_i^{3,AB}(\theta)$ depending on whether the observation is an ask, a bid or an ask-bid pair. Then, in general the M-step can be written as follows

$$\theta^{(k)} = arg\max_{\theta}\sum_{i=1}^{T} E(L_i(\theta) \mid \mathcal{D}_T, \theta^{(k-1)})$$

A detailed description of the M-step is in Appendix C.

The usual way to compute the asymptotic standard error for the maximum likelihood estimate is to use the negative Hessian matrix or the inner product of the individual scores. But in our case, either one is not directly computable because the individual log-likelihood function, $\ln f(s_i; i = 1, \cdots, n \mid \theta)$ is highly irregular with respect to θ due to using the particle filter. An alternative estimator proposed by Duan and Fulop (2007a) can be applied to our setting, however, which uses the smoothed individual scores to compute the asymptotic error.

7 Empirical Results

In this section we investigate intraday patterns of volatility and transaction costs. We allow both the diffusion volatility and the proportional costs to vary during the day. For the 3 US entities, the parameter values are estimated during 5 time periods in New-York GMT: 5:30-7:30 (1), 7:30-9:30 (2), 9:30-14:30 (3), 14:30-16:30 (4) and the overnight period 16:30-05:30 (5). For the European name, the parameter values are estimated during 6 time periods (NY GMT): 5:30-7:30 (1), 7:30-9:30 (2), 9:30-14:30 (3), 14:30-16:30 (4) and the overnight period 16:30-05:30 (5). Table 2 shows the parameter estimates and the asymptotic standard errors.

For the 3 US entities, the volatility parameter exhibits the usual J-shaped pattern during the business hours (5:30 am-5:30 pm) (see Hasbrouck (1999) and others). The volatility is at the lowest level during off hours trading, when almost no trade or quote happens. This pattern may signal the price discovery process and/or price competition among traders. However, it is not likely that portfolio re-balancing is the main motive of the dealers.

The intraday pattern of the mean cost of the US entities, μ, is noteworthy. The mean cost attains its highest level prior to the New York business hours (5:30-7:30), when there are few trades, but when the quoting activity is high in the IDB. Then μ declines sharply from 7:30 to 9:30, when the number of trades picks up. During the late afternoon, μ increases slightly for Ford and GMAC, and it picks up (increase of 50%) during the off-hours period (17:30 - 5:30), when trading is thin. Note that we do not have enough data to estimate the mean costs of Sears during the overnight period. Thus, we observe that the number of trades and the mean costs are inversely related in our data, which is consistent with the theory of liquidity externalities.

8 Conclusion

In this paper, we use a new high-frequency data set for 4 credit default swaps, provided by GFI, one of the main interdealer brokers.

First, we find that bid-ask spreads and roundtrip costs in our data are not substantially lower than their counterparts in the corporate bond markets. This suggests that just like corporate bonds, CDS prices are not likely to be free from microstructure phenomena. So one should be careful when interpreting CDS premia as a pure measure of credit risk.

Second, as we have intra-day observations, we are able to estimate an econometric model allowing for intradaily patterns in transaction costs and volatility. In the model, bid and ask quotes arise from an implicit efficient price and stochastic market-making costs. Further, our framework allows for data errors, discreteness and jumps in the efficient price. We estimate the model using particle filtering and the Monte Carlo EM algorithm. The parameter estimates show a J-shape pattern for the volatility of the efficient price, and for transaction costs during New York business hours for the 3 US names. Patterns are different for the European name whose trading activity and volatility pick up during London business hours. For all the four names, we find that volatility is low and transaction costs are higher when trading is thinner (off hours trading).

These results suggest that this IDB does not seem to be used so much by dealers to manage their inventory position. We do not observe in our data any patterns usually consistent with the inventory paradigm, (for instance, we don't find a peak in the number of trades, or volatility at the end of the day). In contrast, the J-shape pattern of volatility and transaction costs may signal the presence of the price discovery process. If this is indeed the case, we have found an example of an IDB where anonymity promotes informed trades. However, further research is needed on this issue, especially to disentangle the asymmetric information story from price competition among dealers.

Appendix A:

The necessary quantities for executing the particle filtering algorithm when the observation is an ask are described below. For the jump probabilities, we have

$$
\begin{aligned}
p(q_i^A = 1) &= p \text{ and } p(q_i^A = 0) = 1 - p \\
p(\Delta N_i = k) &= \frac{(\lambda \Delta \tau_i)^k}{k!} e^{-\lambda \Delta \tau_i} \text{ for } k = 0, 1, 2, \cdots
\end{aligned}
$$

The conditional distribution of $f(a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A)$ is normal with mean and variance

$$
\begin{aligned}
E[a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A] &= m_{\tau_{i-1}} + \Delta N_i \mu_J + \mu_{\tau_i} \\
Var[a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A] &= \sigma^2_{u, f(\tau_{i-1}, \tau_i)} \Delta \tau_i + \Delta N_i \sigma_J^2 + \sigma_{c,A}^2 + q_i^A \sigma_\varepsilon^2
\end{aligned}
$$

Corresponding to A_{τ_i} it must be that $a_{\tau_i} \in (\ln(A_{\tau_i} - K), \ln A_{\tau_i}]$. Thus we can compute

$$
\begin{aligned}
& p(A_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, (q_i^A)) \qquad (13) \\
= \; & \Phi\left(\frac{\ln A_{\tau_i} - E[a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]}{\sqrt{Var[a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]}} \right) - \Phi\left(\frac{\ln(A_{\tau_i} - K) - E[a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]}{\sqrt{Var[a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]}} \right)
\end{aligned}
$$

The conditional distribution $f(m_{\tau_i} \mid a_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A)$ is again normal with mean and variance

$$
\begin{aligned}
E[m_{\tau_i} \mid a_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A] &= E[m_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A] \\
&+ \frac{Cov[m_{\tau_i}, a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]}{Var[a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]} \times (a_{\tau_i} - E[a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]) \\
Var[m_{\tau_i} \mid a_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A] &= Var[m_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A] - \frac{Cov[m_{\tau_i}, a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]^2}{Var[a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]}
\end{aligned}
$$

where

$$
\begin{aligned}
E[m_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A] &= m_{\tau_{i-1}} + \Delta N_i \mu_J \\
Var[m_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A] &= \sigma^2_{u, f(\tau_{i-1}, \tau_i)} \Delta \tau_i + \Delta N_i \sigma_J^2 \\
Cov[m_{\tau_i}, a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A] &= \sigma^2_{u, f(\tau_{i-1}, \tau_i)} \Delta \tau_i + \Delta N_i \sigma_J^2
\end{aligned}
$$

Similarly, $f(c_i^A \mid a_{\tau_i}, m_{\tau_i}, q_i^A)$ is normal with mean and variance

$$
\begin{aligned}
E[c_i^A \mid a_{\tau_i}, m_{\tau_i}, q_i^A] &= \mu_{\tau_i} + \frac{\sigma_{c,A}^2}{\sigma_{c,A}^2 + q_i^A \sigma_\varepsilon^2} \times (a_{\tau_i} - m_{\tau_i} - \mu_{\tau_i}) \\
Var[c_i^A \mid a_{\tau_i}, m_{\tau_i}, q_i^A] &= \sigma_{c,A}^2 - \frac{\sigma_{c,A}^4}{\sigma_{c,A}^2 + q_i^A \sigma_\varepsilon^2}
\end{aligned}
$$

Appendix B: Particle Filter for the case of a bid and an ask observation

This appendix contains the details of the particle filtering algorithm for the case when we have both a bid and an ask observation. The algorithm differs from the case of only one observation for two reasons. First, both the bid and the ask contains information on the new fundamental price, m_{τ_i}. Second, we need to deal with sample selection as a result of the restriction $\mathcal{S} = \{a_{\tau_i} > b_{\tau_i}\}$ where

$$\begin{aligned} b_{\tau_i} &= m_{\tau_i} - (c_{i,B} + q_i^B \varepsilon_i^B) \\ a_{\tau_i} &= m_{\tau_i} + c_{i,A} + q_i^A \varepsilon_i^A \end{aligned}$$

which also ensures that $A_{\tau_i} - B_{\tau_i} \geq K$.

Our algorithm now is based on the following filtering density/distribution

$$\begin{aligned} & f(a_{\tau_i}, b_{\tau_i}, m_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B, c_i^A, c_i^B \mid \mathcal{D}_i, \mathcal{S}) \\ = \; & f(c_i^A \mid a_{\tau_i}, m_{\tau_i}, q_i^A) f(c_i^B \mid b_{\tau_i}, m_{\tau_i}, q_i^B) f(m_{\tau_i} \mid a_{\tau_i}, b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B) \\ \times \; & f(a_{\tau_i} \mid A_{\tau_i}, b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B, \mathcal{S}) f(b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^b \mid \mathcal{D}_i, \mathcal{S}) \\ \text{where} \quad & f(b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^b \mid \mathcal{D}_i, \mathcal{S}) \\ \propto \; & p(A_{\tau_i}, \mathcal{S} \mid b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B) f(b_{\tau_i} \mid B_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^B) \\ \times \; & p(B_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^B) p(\Delta N_i) p(q_i^A) p(q_i^B) f(m_{\tau_{i-1}} \mid \mathcal{D}_{i-1}) \end{aligned}$$

Assume that we have M particles, $m_{\tau_{i-1}}^{(m)}$ representing $f(m_{\tau_{i-1}} \mid \mathcal{D}_{i-1})$. Then our localized particle filter with M particles consists of the following steps:

- **Step 1:** Enlarge the state-space by the jumps in the system by sampling from $\Delta N_i, q_i^A, q_i^b$. Jumps can be rare events so we use stratified sampling to ensure that our simulated sample always contains jumps by putting an equal probability of $\frac{1}{2}$ on having and not having jumps. I.e. we use the importance sampling distributions

$$g_1(\Delta N_i) = \frac{1}{2} p(\Delta N_i \mid \Delta N_i > 0) + \frac{1}{2} 1_{\{\Delta N_i = 0\}} \tag{14}$$

$$g_2(q_i^A) = \frac{1}{2} 1_{\{q_i^A = 0\}} + \frac{1}{2} 1_{\{q_i^A = 1\}} \tag{15}$$

$$g_3(q_i^B) = \frac{1}{2} 1_{\{q_i^B = 0\}} + \frac{1}{2} 1_{\{q_i^B = 1\}} \tag{16}$$

  This results in particles $m_{\tau_{i-1}}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)}, (q_i^B)^{(m)}$.

Also, sample b_{τ_i} from the truncated normal density $f(b_{\tau_i} \mid B_{\tau_i}, m_{\tau_{i-1}}^{(m)}, \Delta N_i^{(m)}, (q_i^B)^{(m)})$. This results in an M-particle set $(b_{\tau_i}^{(m)}, m_{\tau_{i-1}}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)}, (q_i^B)^{(m)}), m = 1, \ldots, M$. Then, to arrive at an empirical representation of $f(b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^b \mid \mathcal{D}_i, \mathcal{S})$ attach to each of these particles the importance weights

$$\begin{aligned} w_i^{(m)} &= p(A_{\tau_i}, \mathcal{S} \mid b_{\tau_i}^{(m)}, m_{\tau_{i-1}}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)}, (q_i^B)^{(m)}) \\ &\times \; p(B_{\tau_i} \mid m_{\tau_{i-1}}^{(m)}, \Delta N_i^{(m)}, (q_i^B)^{(m)}) \\ &\times \; p((q_i^A)^{(m)}) p((q_i^B)^{(m)}) [1_{\Delta N_i^{(m)} > 0} p(\Delta N_i > 0) + 1_{\Delta N_i^{(m)} = 0} p(\Delta N_i = 0)] \end{aligned}$$

The likelihood value for the observed bid and ask can be computed as

$$p(A_{\tau_i}, B_{\tau_i} \mid \mathcal{D}_{i-1}, \mathcal{S}) = \frac{p(A_{\tau_i}, B_{\tau_i}, \mathcal{S} \mid \mathcal{D}_{i-1})}{p(\mathcal{S})} \approx \frac{\frac{1}{M} \sum_{m=1}^M w_i^{(m)}}{p(\mathcal{S})}$$

- **Step 2:** Resample the particle set according to the probability $\pi_i^{(m)} = \frac{w_i^{(m)}}{\sum_{m=1}^M w_i^{(m)}}$ to yield M equal-weighted particles denoted by $(b_{\tau_i}^{(m)}, m_{\tau_{i-1}|i}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)}, (q_i^B)^{(m)})$. This equal-weighted particle set is again an empirical representation of $f(b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^b \mid \mathcal{D}_i, \mathcal{S})$.

- **Step 3:** Corresponding to each particle $(b_{\tau_i}^{(m)}, m_{\tau_{i-1}|i}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)}, (q_i^B)^{(m)})$, sample from the truncated normal density $f(a_{\tau_i} \mid A_{\tau_i}, b_{\tau_i}^{(m)}, m_{\tau_{i-1}|i}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)}, (q_i^B)^{(m)}, \mathcal{S})$ to generate the particle $(a_{\tau_i}^{(m)}, b_{\tau_i}^{(m)}, m_{\tau_{i-1}|i}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)}, (q_i^B)^{(m)})$. The corresponding particle set represents $f(a_{\tau_i}, b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^b \mid \mathcal{D}_i, \mathcal{S})$.

- **Step 4:** Using conditional normality sample from

$$\begin{aligned} m_{\tau_i}^{(m)} &\sim f(m_{\tau_i} \mid a_{\tau_i}^{(m)}, b_{\tau_i}^{(m)}, m_{\tau_{i-1}|i}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)}, (q_i^B)^{(m)}) \\ (c_i^A)^{(m)} &\sim f(c_i^A \mid a_{\tau_i}^{(m)}, m_{\tau_i}^{(m)}, (q_i^A)^{(m)}) \\ (c_i^B)^{(m)} &\sim f(c_i^B \mid b_{\tau_i}^{(m)}, m_{\tau_i}^{(m)}, (q_i^B)^{(m)}) \end{aligned}$$

  This yields M particles, $(a_{\tau_i}^{(m)}, b_{\tau_i}^{(m)}, m_{\tau_i}^{(m)}, m_{\tau_{i-1}|i}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)}, (q_i^B)^{(m)}, (c_i^A)^{(m)}, (c_i^B)^{(m)})$ which represent $f(a_{\tau_i}, b_{\tau_i}, m_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B, c_i^A, c_i^B \mid \mathcal{D}_i, \mathcal{S})$. One can then proceed to marginalize m_{τ_i} to represent the filtering distribution $f(m_{\tau_i} \mid \mathcal{D}_i)$

The necessary quantities for executing this algorithm are described below. The conditional distribution of $f(b_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^b)$ is normal with mean and variance

$$\begin{aligned} E[b_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^B] &= m_{\tau_{i-1}} + \Delta N_i \mu_J - \mu_{\tau_i} \\ Var[b_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^B] &= \sigma_{u,f(\tau_{i-1},\tau_i)}^2 \Delta\tau_i + \Delta N_i \sigma_J^2 + \sigma_{c,b}^2 + q_i^B \sigma_\varepsilon^2 \end{aligned}$$

Corresponding to B_{τ_i} it must me that $b_{\tau_i} \in [\ln B_{\tau_i}, \ln(B_{\tau_i} + K)$. Thus we can compute

$$\begin{aligned} & p(B_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, (q_i^B)) \qquad (17) \\ = \; & \Phi\left(\frac{\ln(B_{\tau_i} + K) - E[b_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^B]}{\sqrt{Var[B_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^B]}}\right) - \Phi\left(\frac{\ln B_{\tau_i} - E[b_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^B]}{\sqrt{Var[b_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^B]}}\right) \end{aligned}$$

The conditional distribution $f(a_{\tau_i} \mid b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B)$ is normal with mean and variance

$$\begin{aligned} E[a_{\tau_i} \mid b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B] &= E[a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A] \\ &+ \frac{Cov[a_{\tau_i}, b_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A, a_{\tau_i} \mid b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B]}{Var[b_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^B]} \\ &\times (b_{\tau_i} - E[b_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^B]) \\ Var[a_{\tau_i} \mid b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B] &= Var[a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A] - \frac{Cov[a_{\tau_i}, b_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B]]^2}{Var[b_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^B]} \\ \text{where} & \\ Cov[a_{\tau_i}, b_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B] &= \sigma^2_{u,f(\tau_{i-1},\tau_i)} \Delta\tau_i + \Delta N_i \sigma_J^2 \end{aligned}$$

$A_{\tau_i}, \mathcal{S}$ means that $a_{\tau_i} \in (\max[\ln(A_{\tau_i} - K), b_{\tau_i}], \ln A_{\tau_i})$ [12] . Then we have the following

$$\begin{aligned} & p(A_{\tau_i}, \mathcal{S} \mid b_{\tau_i}^{(m)}, m_{\tau_{i-1}}^{(m)}, \Delta N_i^{(m)}, (q_i^A)^{(m)}, (q_i^B)^{(m)}) \qquad (18) \\ = \; & \Phi\left(\frac{\ln A_{\tau_i} - E[a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]}{\sqrt{Var[a_{\tau_i} \mid b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B]}}\right) - \Phi\left(\frac{\max[\ln(A_{\tau_i} - K), b_{\tau_i}] - E[a_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]}{\sqrt{Var[a_{\tau_i} \mid b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B]}}\right) \end{aligned}$$

The last quantity we need is the conditional normal distribution of $f(m_{\tau_i} \mid a_{\tau_i}, b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B)$.

[12] This is enough because $b_{\tau_i} < \ln A_{\tau_i}$

Its mean and variance are

$$
\begin{aligned}
& E[(m_{\tau_i} \mid a_{\tau_i}, b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B] \\
= \; & E[(m_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B] \\
+ \; & Cov[m_{\tau_i}, \begin{pmatrix} a_{\tau_i} \\ b_{\tau_i} \end{pmatrix} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]' Var[\begin{pmatrix} a_{\tau_i} \\ b_{\tau_i} \end{pmatrix} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]^{-1} \\
\times \; & \left[\begin{pmatrix} a_{\tau_i} \\ b_{\tau_i} \end{pmatrix} - E[\begin{pmatrix} a_{\tau_i} \\ b_{\tau_i} \end{pmatrix} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A] \right] \\
& Var[(m_{\tau_i} \mid a_{\tau_i}, b_{\tau_i}, m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B] \\
= \; & Var[(m_{\tau_i} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A, q_i^B] \\
- \; & Cov[m_{\tau_i}, \begin{pmatrix} a_{\tau_i} \\ b_{\tau_i} \end{pmatrix} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]' Var[\begin{pmatrix} a_{\tau_i} \\ b_{\tau_i} \end{pmatrix} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]^{-1} \\
\times \; & Cov[m_{\tau_i}, \begin{pmatrix} a_{\tau_i} \\ b_{\tau_i} \end{pmatrix} \mid m_{\tau_{i-1}}, \Delta N_i, q_i^A]
\end{aligned}
$$

Note that the particle filter provides a sample on the entire past of the system up to τ_i. Any quantity of interest based on the past particles can be computed and carried forward alongside with m_{τ_i}. This is true because at any time t_i, m_{τ_i} is sufficient for moving the algorithm forward. Denote by I_i the quantity whose distribution is of interest; for example, one may be interested in $I_i = (m_{\tau_0} + m_{\tau_1} + \cdots + m_{\tau_i})/(i+1)$. Then, in all of the preceding derivations one can use the vector (I_i, m_{τ_i}) in place of m_{τ_i}. Conditional on m_{τ_i}, the system's forward evolution has nothing to do with I_i, and thus the algorithm remains unchanged. However, the output of the filter at any time τ_i will be a set of particles representing the joint filtering distribution of (I_i, m_{τ_i}), i.e., $f\,(I_i, m_{\tau_i}, | \mathcal{D}_i)$.

Appendix C: M-step of the MCEM algorithm

This appendix describes the parameter updates in the M-step of our MCEM algorithm, parameter-by-parameter. First, let us introduce some notation. Let $l_{X_i}^{|k-1}$ denote the conditional expectation of a variable X_i given the observed data, $\mathcal{D}_T$ and the parameter vector at the $(k-1)^{th}$ iteration, θ^{k-1}

$$l_{X_i}^{|k-1} = E(X_i \mid \mathcal{D}_T, \theta^{(k-1)})$$

As discussed in the main text, the particle filter described in section 5.1 can be used to compute this quantity jointly with fixed-lag smoothing.

Similarly denote by $l_{X_i}^{|k-1,\mathcal{S}^C}$ the expectation of X_i conditionally on θ^{k-1} and the complementer set of $\mathcal{S}$

$$l_{X_i}^{|k-1,\mathcal{S}^C} = E(X_i \mid \theta^{(k-1)}, \mathcal{S}^C)$$

These quantities are computed analytically in our model.

Now we are in the position to write parameter updates.

$$\begin{aligned}
\lambda^{(k)} &= \frac{\sum_{i=1}^{T} l_{\Delta N_i}^{|k-1}}{\tau_T - \tau_0} \\
\theta_{\sigma_u}^{(k)} &= arg\max_{\theta_{\sigma_u}} \sum_{i=1}^{n} -\frac{l_{Z_{i,u}^2}^{|k-1}}{2\sigma_{u,f(\tau_{i-1},\tau_i)}^2 \Delta\tau_i} - \frac{\ln\left(\sigma_{u,f(\tau_{i-1},\tau_i)}^2 \Delta\tau_i\right)}{2} \\
\mu_J^{(k)} &= \frac{\sum_{i=1}^{n} l_{X_{i,1}}^{|k-1}}{\sum_{i=1}^{n} l_{X_{i,2}}^{|k-1}} \\
(\sigma_J^2)^{(k)} &= \frac{\sum_{i=1}^{n} \left[l_{X_{i,3}}^{|k-1} - 2\mu_J^{(k)} l_{X_{i,1}}^{|k-1} + (\mu_J^{(k)})^2 l_{X_{i,2}}^{|k-1}\right]}{\sum_{i=1}^{n} l_{1_{\{\Delta N_i>0\}}}^{|k-1}}
\end{aligned}$$

where

$$\begin{aligned}
X_{i,1} &= 1_{\{\Delta N_i>0\}} \frac{Z_{i,J}\Delta N_i}{\Delta N_i + 1_{\{\Delta N_i=0\}}} \\
X_{i,2} &= 1_{\{\Delta N_i>0\}} \frac{(\Delta N_i)^2}{\Delta N_i + 1_{\{\Delta N_i=0\}}} \\
X_{i,3} &= 1_{\{\Delta N_i>0\}} \frac{Z_{i,J}^2}{\Delta N_i + 1_{\{\Delta N_i=0\}}}
\end{aligned}$$

$$\begin{aligned}
\theta_\mu^{(k)} &= arg\max_{\theta_\mu} \sum_{i\in I_A\bigcup I_{AB}} -\frac{-2l_{c_{i,A}}^{|k-1}\mu_{\tau_i} + \mu_{\tau_i}^2}{2(\sigma_{c,A}^{(k-1)})^2} + \sum_{i\in I_B\bigcup I_{AB}} -\frac{-2l_{c_{i,B}}^{|k-1}\mu_{\tau_i} + \mu_{\tau_i}^2}{2(\sigma_{c,B}^{(k-1)})^2} \\
&+ \sum_{i\in I_{AB}} l_{k_i}^{|k-1} \times \left[-\frac{-2l_{c_{i,B}}^{|k-1,\mathcal{S}^C}\mu_{\tau_i} + \mu_{\tau_i}^2}{2(\sigma_{c,B}^{(k-1)})^2} - \frac{-2l_{c_{i,A}}^{|k-1,\mathcal{S}^C}\mu_{\tau_i} + \mu_{\tau_i}^2}{2(\sigma_{c,A}^{(k-1)})^2}\right]
\end{aligned}$$

$$\sigma_{c,A}^{(k)} = \sqrt{\frac{\sum_{i\in I_A\bigcup I_{AB}} \left[l_{c_{i,A}^2}^{|k-1} - 2l_{c_{i,A}}^{|k-1}\mu_{\tau_i}^{(k)} + (\mu_{\tau_i}^{(k)})^2\right] + \sum_{i\in I_{AB}} l_{k_i}^{|k-1}\left[l_{c_{i,A}^2}^{|k-1,\mathcal{S}^C} - 2l_{c_{i,A}}^{|k-1,\mathcal{S}^C}\mu_{\tau_i}^{(k)} + (\mu_{\tau_i}^{(k)})^2\right]}{T_A + T_{AB} + \sum_{i\in I_{AB}} l_{k_i}^{|k-1}}}$$

$$\sigma_{c,B}^{(k)} = \sqrt{\frac{\sum_{i\in I_B\bigcup I_{AB}} \left[l_{c_{i,B}^2}^{|k-1} - 2l_{c_{i,B}}^{|k-1}\mu_{\tau_i}^{(k)} + (\mu_{\tau_i}^{(k)})^2\right] + \sum_{i\in I_{AB}} l_{k_i}^{|k-1}\left[l_{c_{i,B}^2}^{|k-1,\mathcal{S}^C} - 2l_{c_{i,B}}^{|k-1,\mathcal{S}^C}\mu_{\tau_i}^{(k)} + (\mu_{\tau_i}^{(k)})^2\right]}{T_B + T_{AB} + \sum_{i\in I_{AB}} l_{k_i}^{|k-1}}}$$

$$\sigma_{\varepsilon}^{(k)} = \sqrt{\frac{\sum_{i \in I_B \bigcup I_{AB}} l^{|k-1}_{q_{i,B} \varepsilon^2_{i,B}} + \sum_{i \in I_A \bigcup I_{AB}} l^{|k-1}_{q_{i,A} \varepsilon^2_{i,A}} + \sum_{i \in I_{AB}} l^{|k-1}_{k_i} \left[l^{|k-1,\mathcal{S}^C}_{q_{i,B} \varepsilon^2_{i,B}} + l^{|k-1,\mathcal{S}^C}_{q_{i,A} \varepsilon^2_{i,A}} \right]}{\sum_{i \in I_B \bigcup I_{AB}} l^{|k-1}_{q_{i,B}} + \sum_{i \in I_A \bigcup I_{AB}} l^{|k-1}_{q_{i,A}} + \sum_{i \in I_{AB}} l^{|k-1}_{k_i} \left[l^{|k-1,\mathcal{S}^C}_{q_{i,B}} + l^{|k-1,\mathcal{S}^C}_{q_{i,A}} \right]}}$$

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Table 1
Characteristics of bid and ask quotes and trade price.

Panel A
Characteristics

	Entity			
	Ford	**GMAC**	**Sears Acceptance**	**France Telecom**
Currency	USD	USD	USD	EUR
Country	United States	United States	United States	France
Sector	Automobile	Automobile	Retail	Telecommunication
Grade on 2006/12/31	B	BB+	BB+	A-

Panel B
Descriptive statistics from the raw dataset

Data is between 2004-2006

	Entity			
	Ford	**GMAC**	**Sears Acceptance**	**France Telecom**
Daily Average # bid	*5,81*	*6,65*	*2,49*	*4,32*
N bid	*4265*	*4936*	*1356*	*3078*
Average bid	332	344	106,23	41,27
Daily Average # ask	*5,12*	*6,18*	*2,07*	*4,24*
N ask	*3759*	*4589*	*1127*	*3017*
Average ask	341	352	114,65	42,78
Daily Average # trade	*2,57*	*4,46*	*0,78*	*1,42*
N trade	*1891*	*3313*	*425*	*997*
Average trade price	367	393	113,46	42,38

Panel C
Results from the dataset used for estimation

	Entity			
	Ford	**France Telecom**	**GMAC**	**Sears Acceptance**
N bid	*2611*	*1640*	*2991*	*714*
Average bid	335	40,49	351,45	107,34
N ask	*2149*	*1583*	*2681*	*509*
Average ask	343	40,95	359,25	115,59
N bid/ask pair	*1420*	*1276*	*1656*	*568*
Average midpoint(*)	331	43,61	331,185	109,42
Average b/a spread(*)	10,08	2,87	9,7	9,26
Average cost of a round-trip(**)	43,64	12,43	42,00	40,09
Minimum tick, K	1	0,5	0,5	1

Data is between 2004-2006
At most 1 bid and ask observation has been kept for an identical time stamp (this is per minute)
Only observations where the ask is higher than the bid has been kept

(*) These statistics have been computed using the observation with both a bid and an ask

Table 2

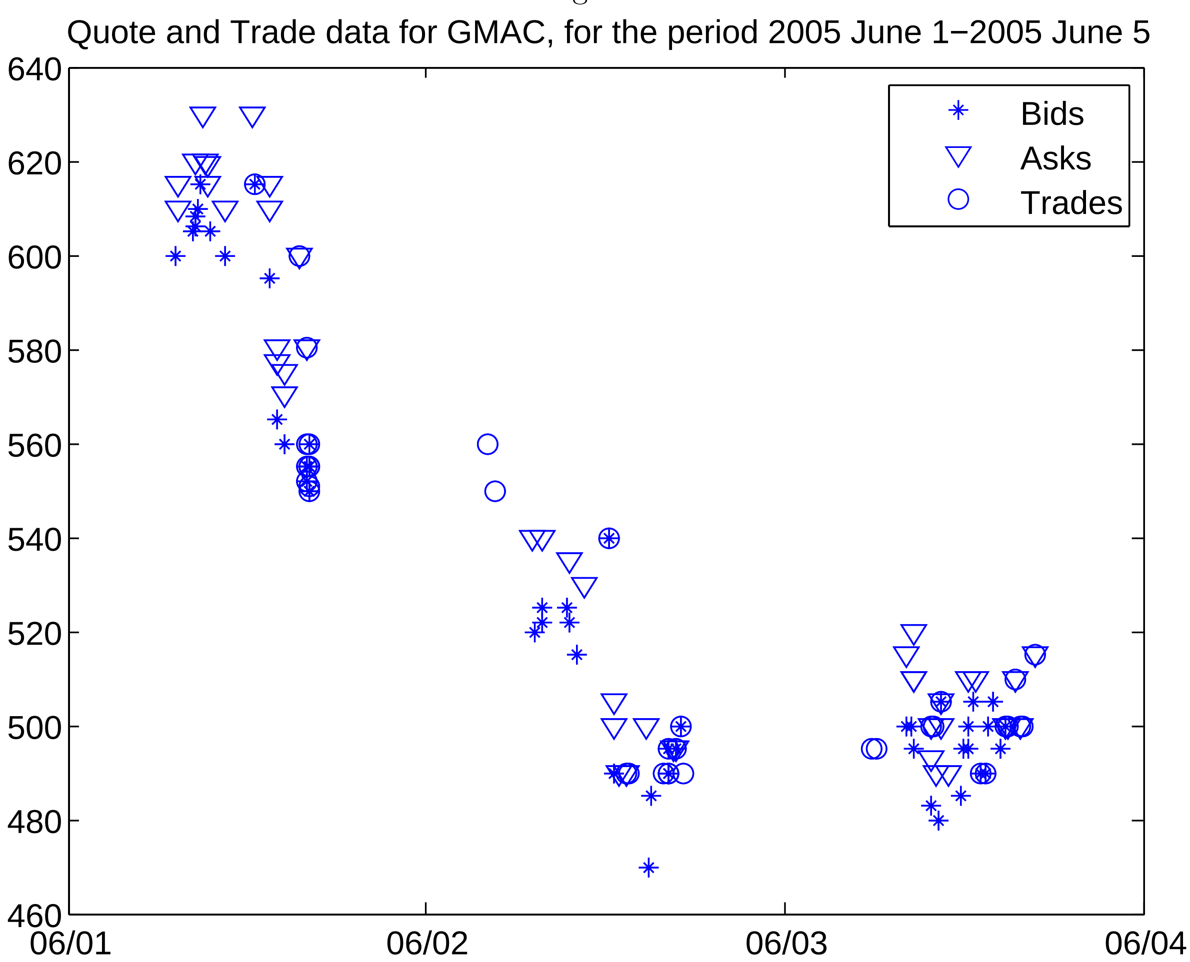
Panel A: US names (*Assymptotic standard errors in italics*)

	Ford		**GMAC**		**Sears Acceptance**	
Time	σ_u	μ	σ_u	μ	σ_u	μ
05:30-07:30	1,0689	0,0146	1,2458	0,0157	0,3848	0,0437
	0,0530	*0,0002*	*0,0569*	*0,0003*	*0,3420*	*0,0016*
07:30-09:30	0,9733	0,0092	1,0961	0,0106	1,2522	0,0328
	0,0355	*0,0003*	*0,0412*	*0,0003*	*0,1322*	*0,0008*
09:30-14:30	0,8436	0,0100	0,8441	0,0098	0,7877	0,0257
	0,0266	*0,0002*	*0,0305*	*0,0002*	*0,0911*	*0,0012*
14:30-16:30	0,7089	0,0103	0,9717	0,0110	0,8085	0,0236
	0,0644	*0,0004*	*0,0512*	*0,0004*	*0,2753*	*0,0028*
16:30-05:30	0,0466104	0,0107857	0,0847063	0,013508	0,0213781	-9,73E-05
	0,0271	*0,0006*	*0,0615*	*0,0006*	*0,1566*	*0,0162*
Parameters						
$\sigma_{c,A}$	0,0105		0,0113		0,0276	
	0,0002		*0,0002*		*0,0009*	
$\sigma_{c,B}$	0,0099		0,0113		0,0287	
	0,0002		*0,0002*		*0,0008*	
σ_{ϵ}	0,0782		0,0727		0,1815	
	0,0073		*0,0069*		*0,0327*	
λ	47,5301		60,7998		22,3496	
	7,5655		*6,7575*		*5,7466*	
μ_J	0,0017		-0,0033		0,0203	
	0,0056		*0,0071*		*0,0167*	
σ_J	0,0586		0,0777		0,1065	
	0,0035		*0,0027*		*0,0142*	

Panel B: European name (*Assymptotic standard errors in italics)*

	France Telecom	
Time	σ_u	μ
00:30-02:30	0,6902	0,0331
	0,1106	*0,0008*
02:30-04:30	1,1769	0,0242
	0,0705	*0,0004*
04:30-07:30	0,5748	0,0197
	0,0890	*0,0007*
07:30-10:30	0,6971	0,0198
	0,0689	*0,0007*
10:30-14:30	0,3150177	0,0169186
	0,1105	*0,0010*
14:30-00:30	0,0005	0,0227
	0,0097	*0,0029*
Parameters		
$\sigma_{c,A}$	0,0196	
	0,0004	
$\sigma_{c,B}$	0,0201	
	0,0004	
σ_{ϵ}	0,0727	
	0,0131	
λ	26,8219	
	7,0712	
μ_J	0,0056	
	0,0093	
σ_J	0,0635	
	0,0090	

Figure 1:



Quote and Trade data for GMAC, for the period 2005 June 1−2005 June 5
640
620
600
580
560
540
520
500
480
460
06/01
06/02
06/03
06/04
Bids
Asks
Trades

Figure 2:

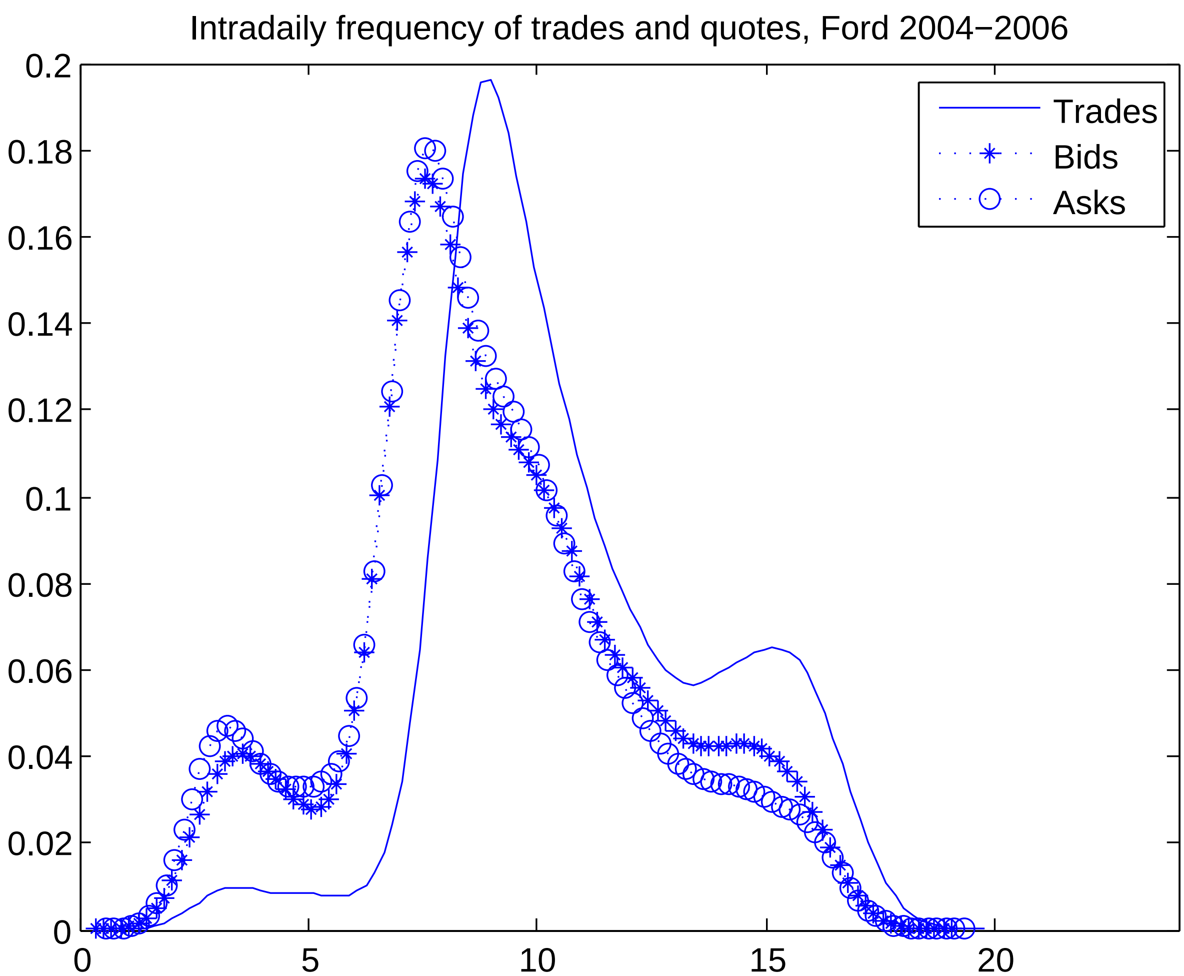
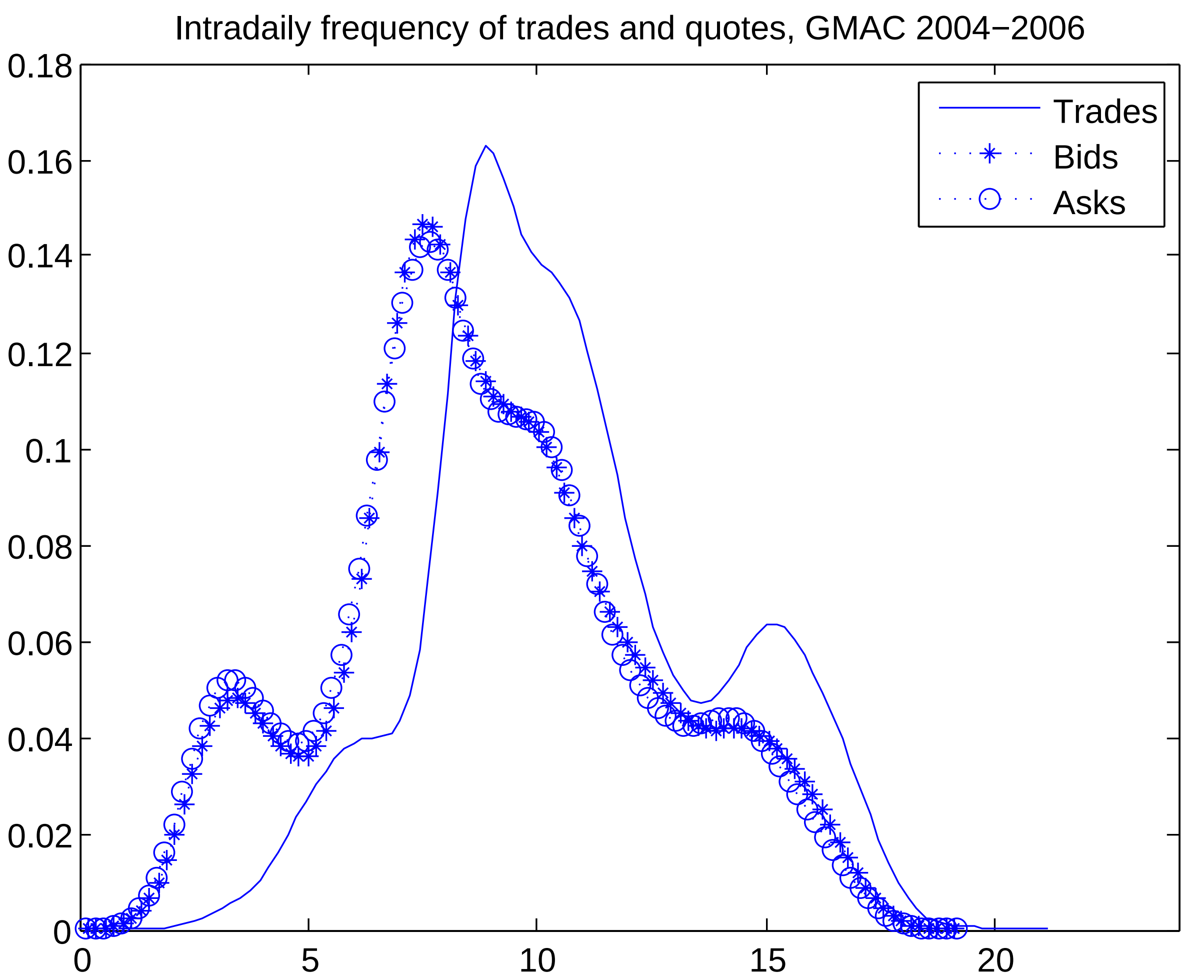

Intradaily frequency of trades and quotes, Ford 2004−2006
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
Trades
Bids
Asks

Figure 3:



Intradaily frequency of trades and quotes, GMAC 2004−2006
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
Trades
Bids
Asks

Figure 4:

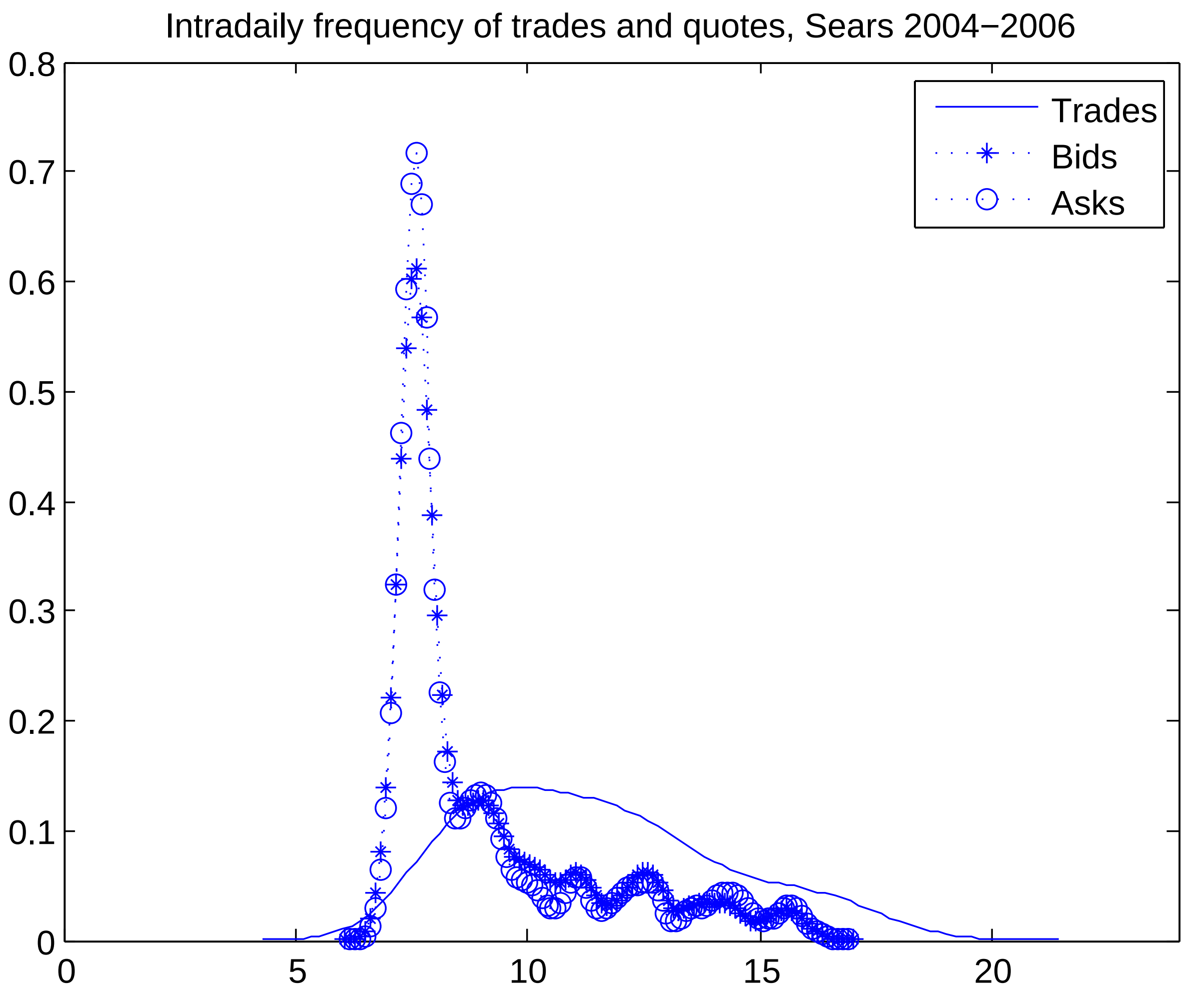

Intradaily frequency of trades and quotes, Sears 2004−2006
Trades
Bids
Asks
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
5
10
15
20

Figure 5:

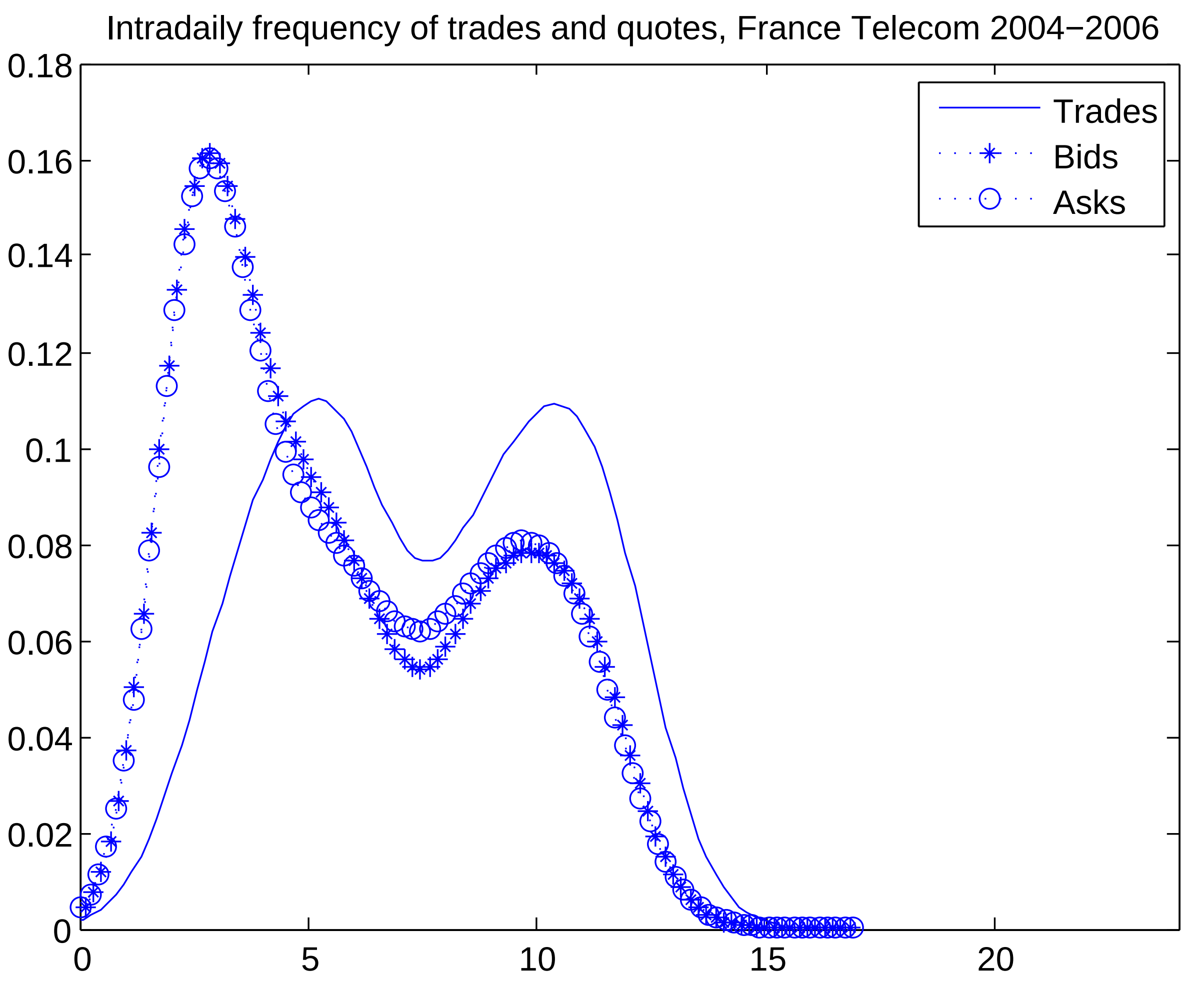

Intradaily frequency of trades and quotes, France Telecom 2004−2006
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
Trades
Bids
Asks

Figure 6:

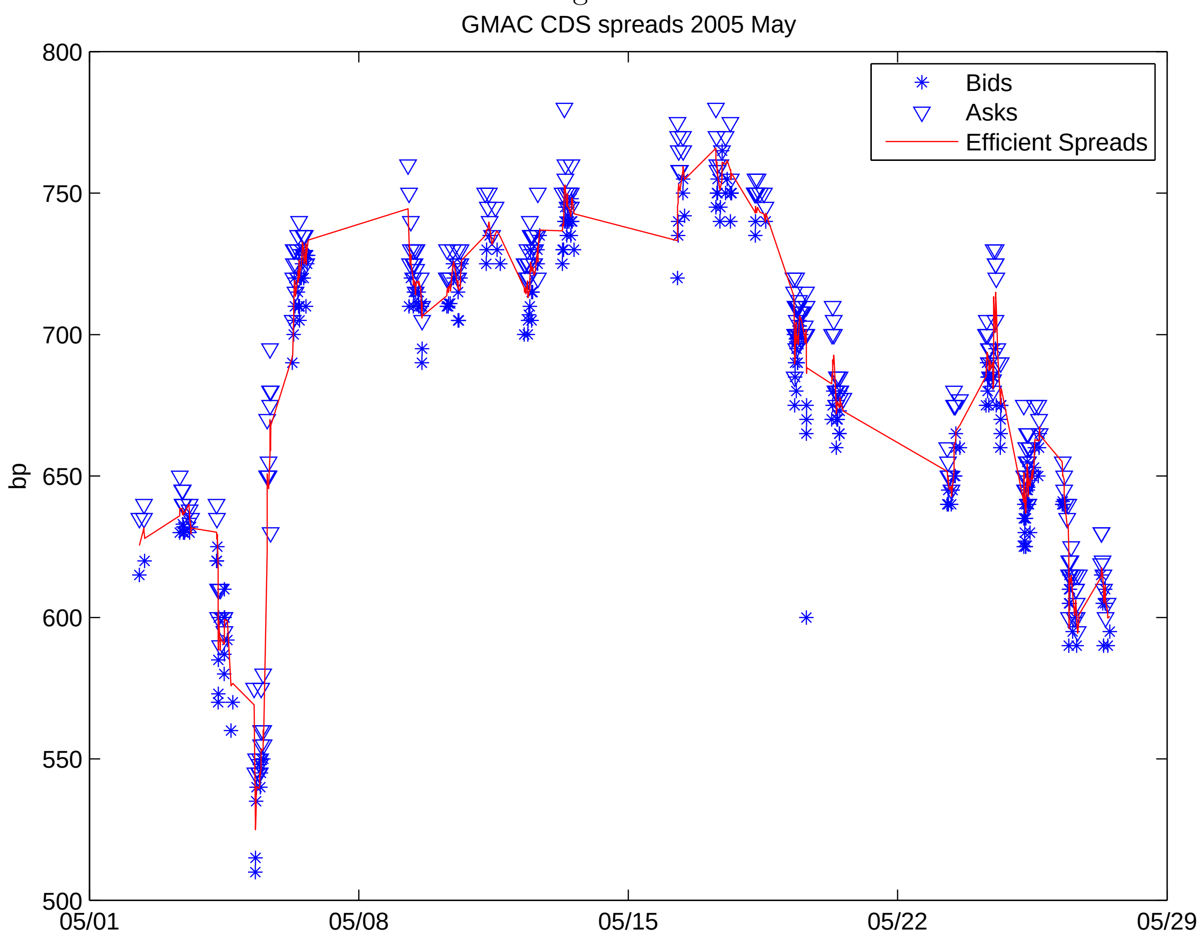

GMAC CDS spreads 2005 May
bp
800
750
700
650
600
550
500
05/01
05/08
05/15
05/22
05/29
Bids
Asks
Efficient Spreads

Figure 7:

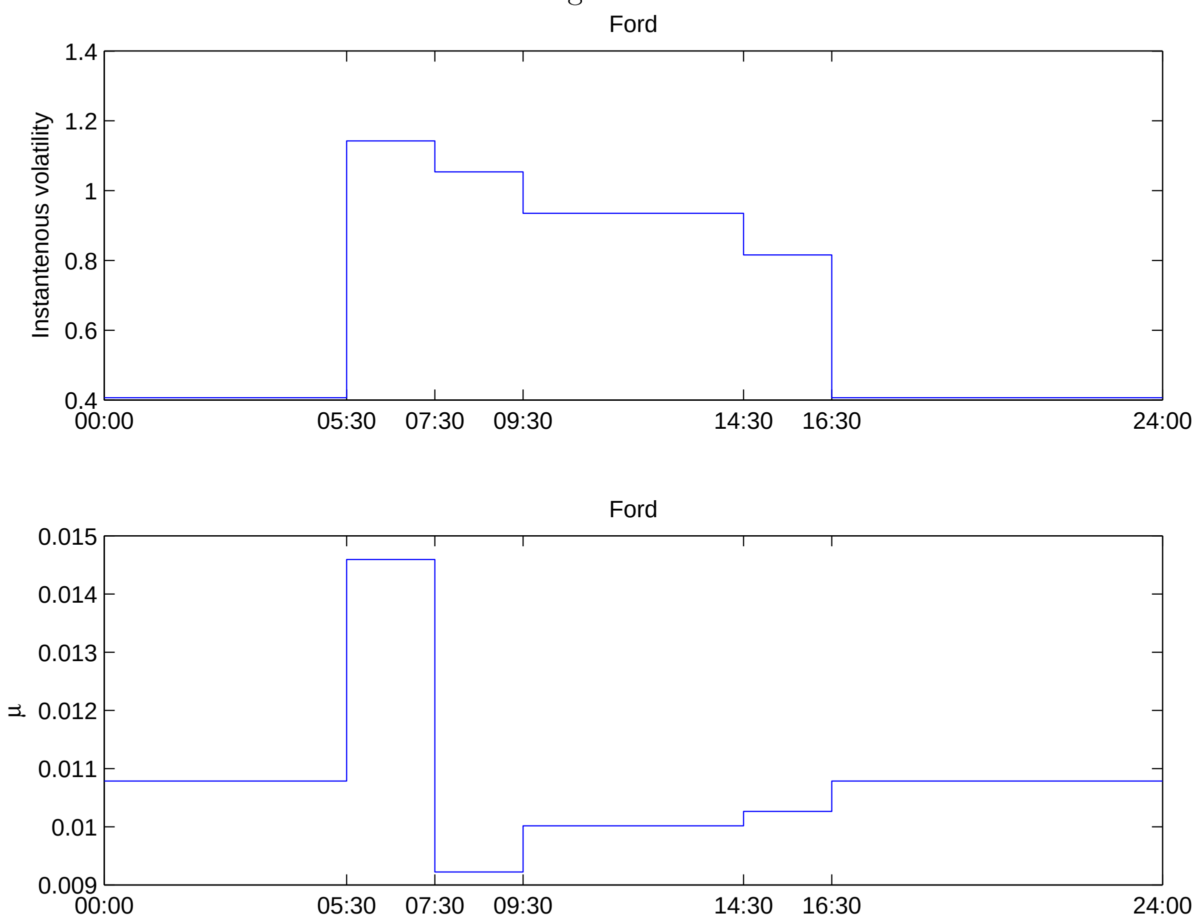

Ford
Instantenous volatility
1.4
1.2
1
0.8
0.6
0.4
00:00
05:30
07:30
09:30
14:30
16:30
24:00
Ford
μ
0.015
0.014
0.013
0.012
0.011
0.01
0.009
00:00
05:30
07:30
09:30
14:30
16:30
24:00

Figure 8:

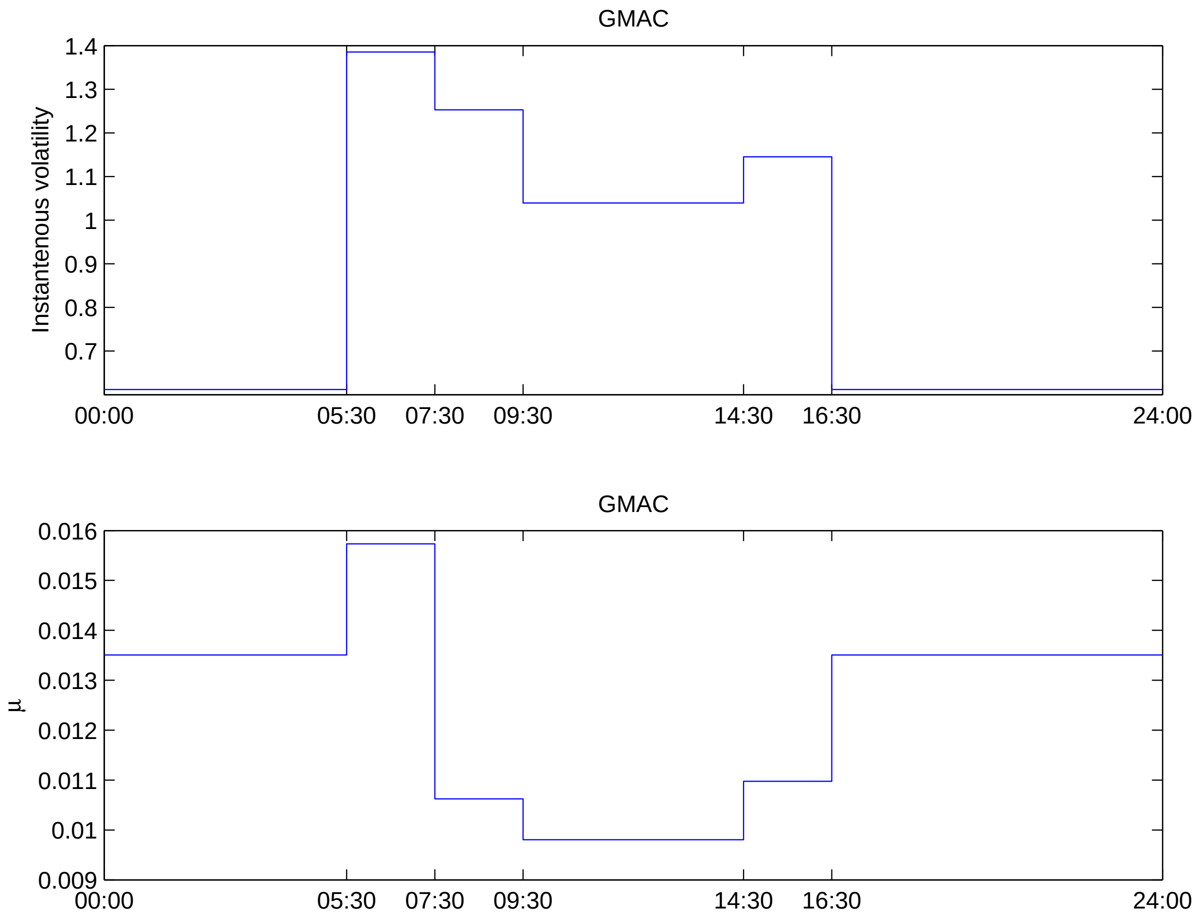

GMAC
Instantenous volatility
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
00:00
05:30
07:30
09:30
14:30
16:30
24:00
GMAC
μ
0.016
0.015
0.014
0.013
0.012
0.011
0.01
0.009
00:00
05:30
07:30
09:30
14:30
16:30
24:00

Figure 9:

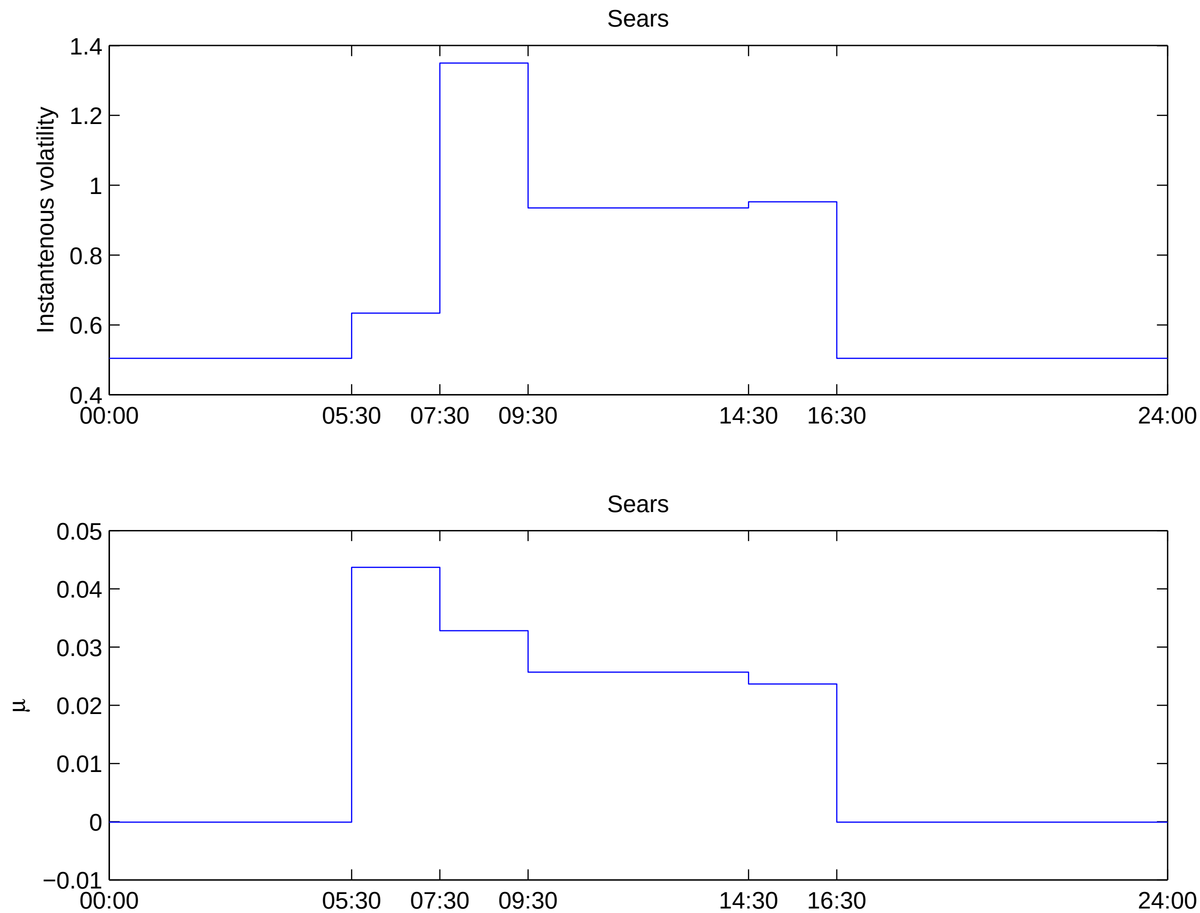

Sears
Instantenous volatility
1.4
1.2
1
0.8
0.6
0.4
00:00
05:30
07:30
09:30
14:30
16:30
24:00
Sears
μ
0.05
0.04
0.03
0.02
0.01
0
−0.01
00:00
05:30
07:30
09:30
14:30
16:30
24:00

Figure 10:

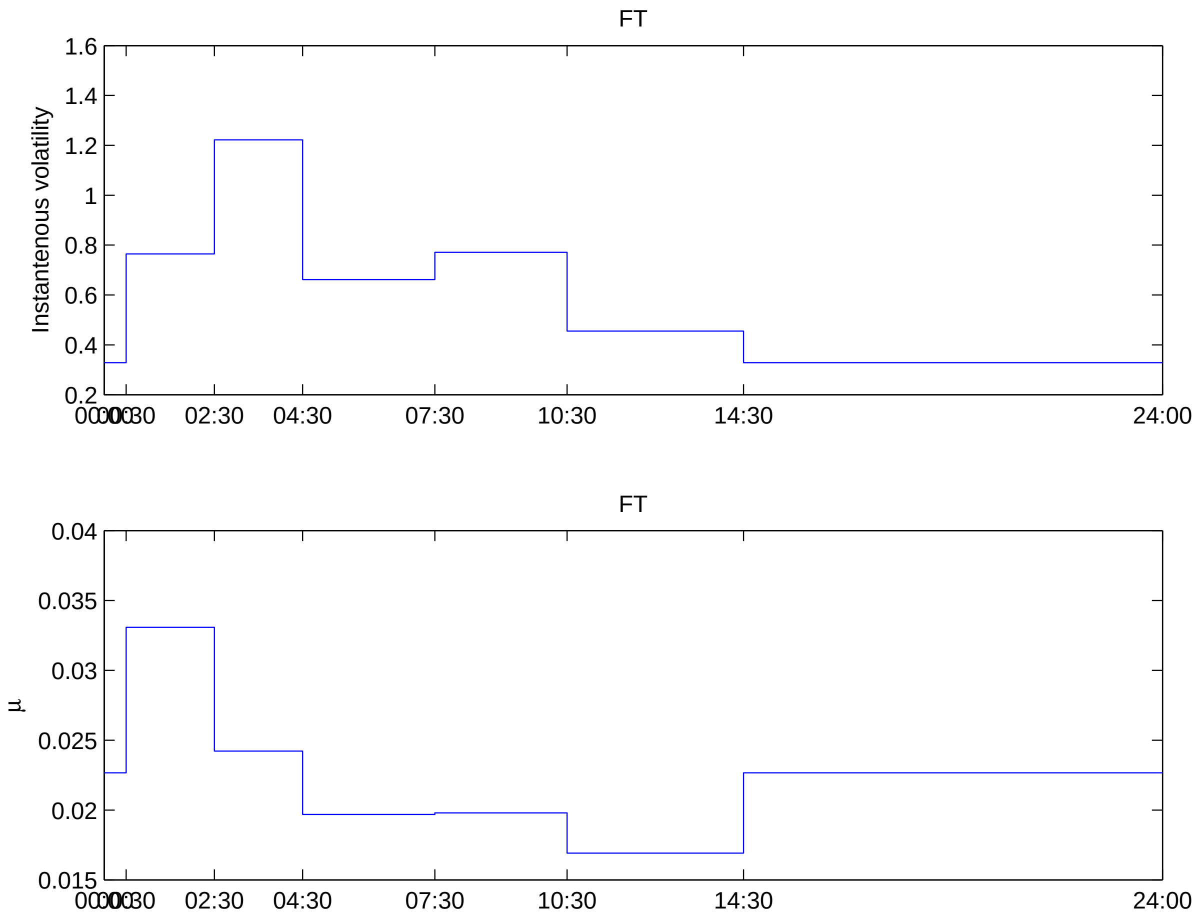
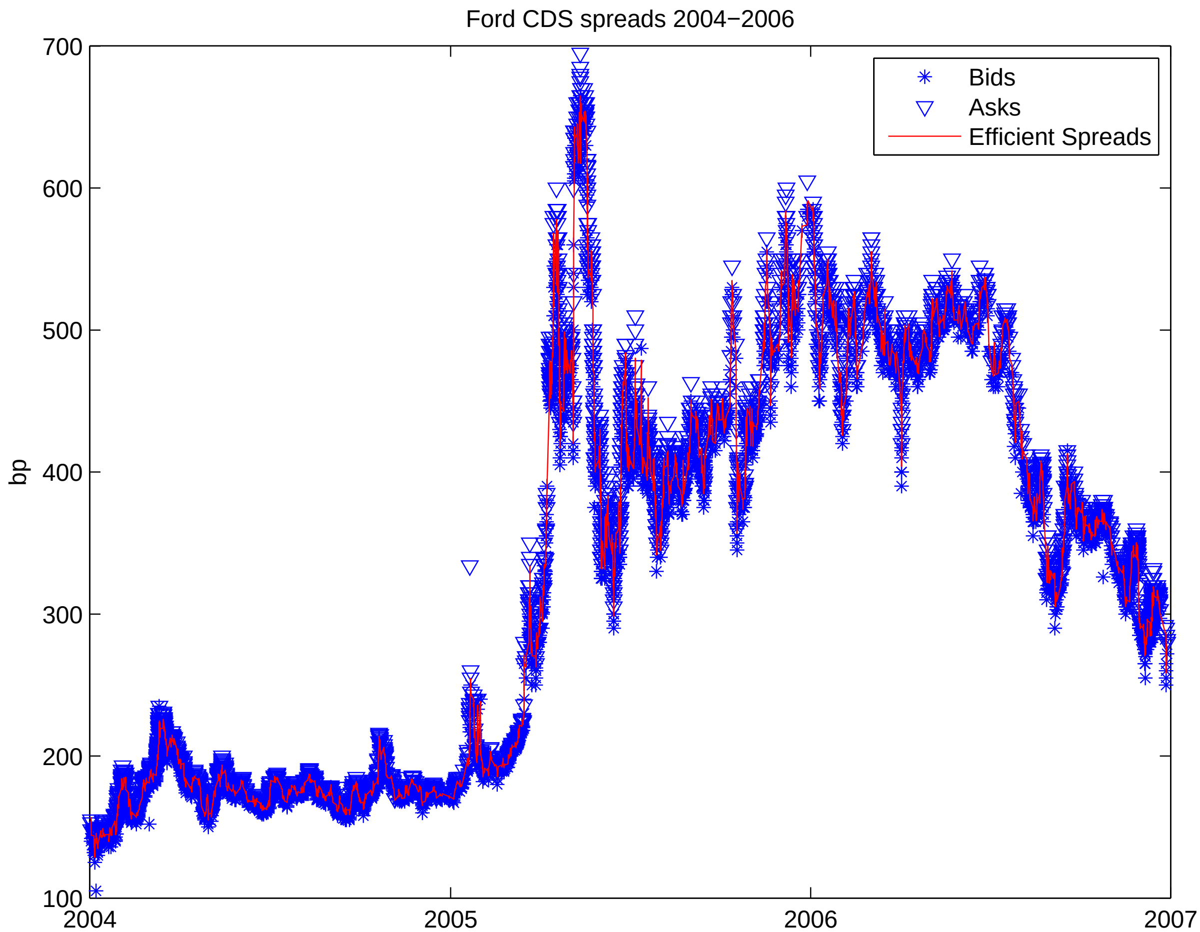

FT
Instantenous volatility
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
00:00
00:30
02:30
04:30
07:30
10:30
14:30
24:00
FT
μ
0.04
0.035
0.03
0.025
0.02
0.015
00:00
00:30
02:30
04:30
07:30
10:30
14:30
24:00

Figure 11:



Ford CDS spreads 2004−2006
700
600
500
400
300
200
100
bp
2004
2005
2006
2007
Bids
Asks
Efficient Spreads

Figure 12:

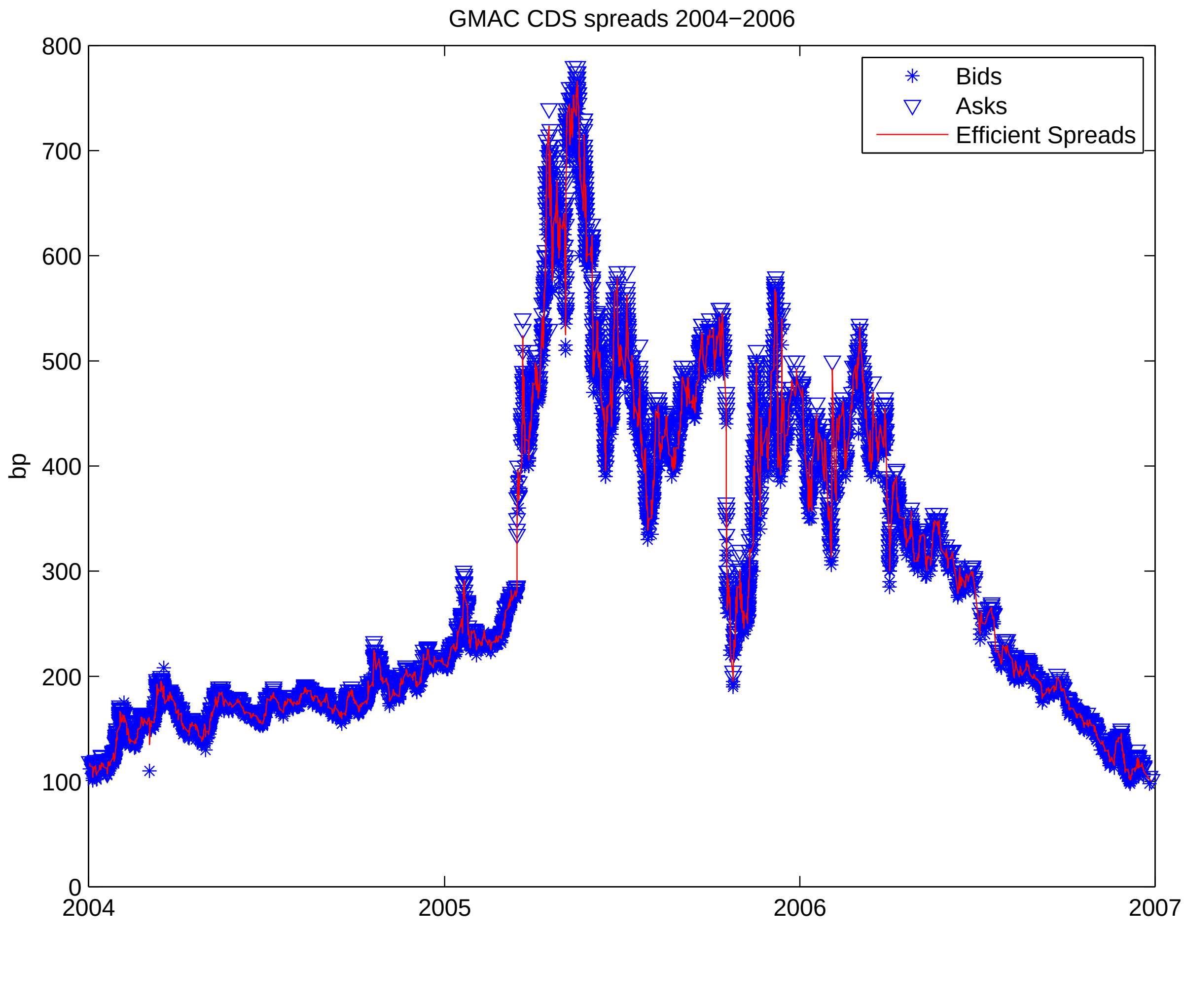
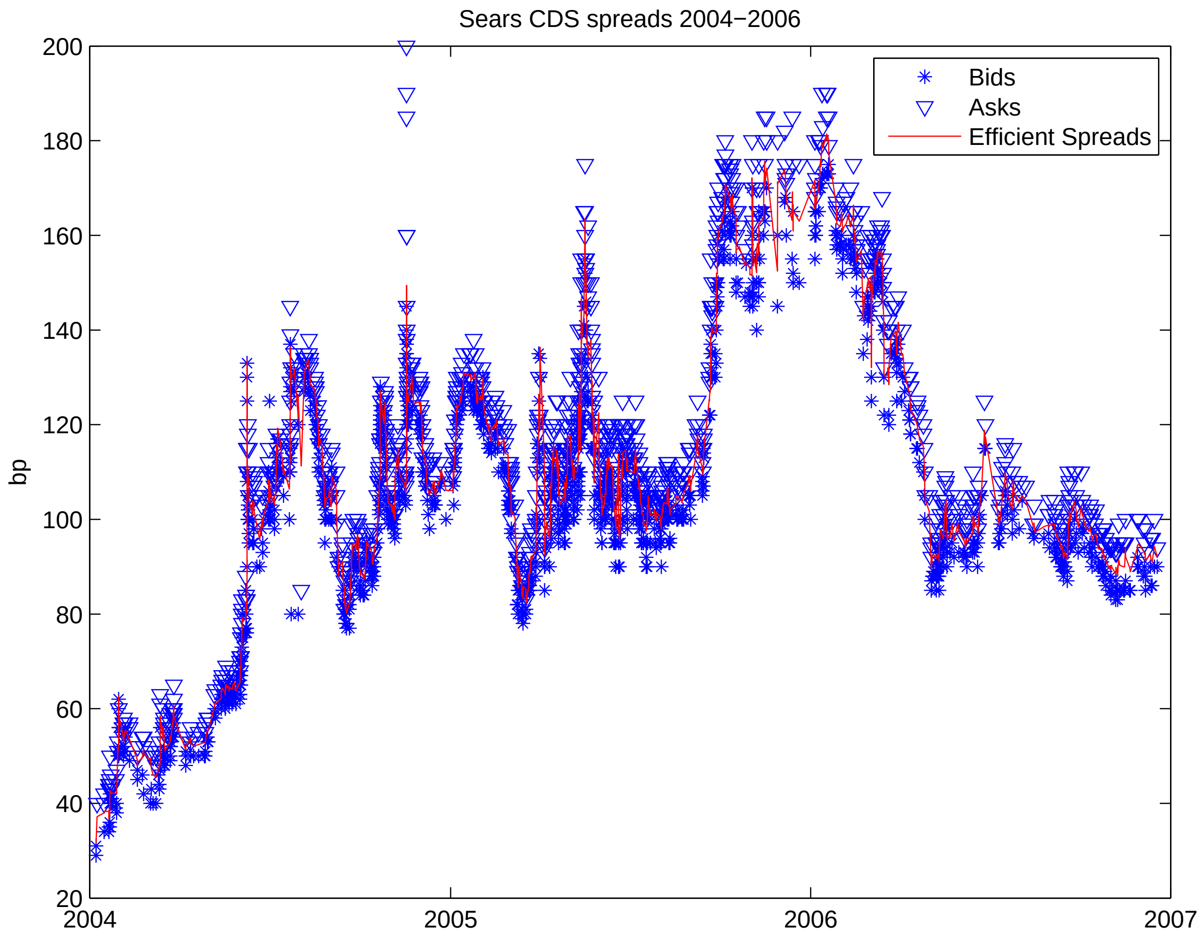
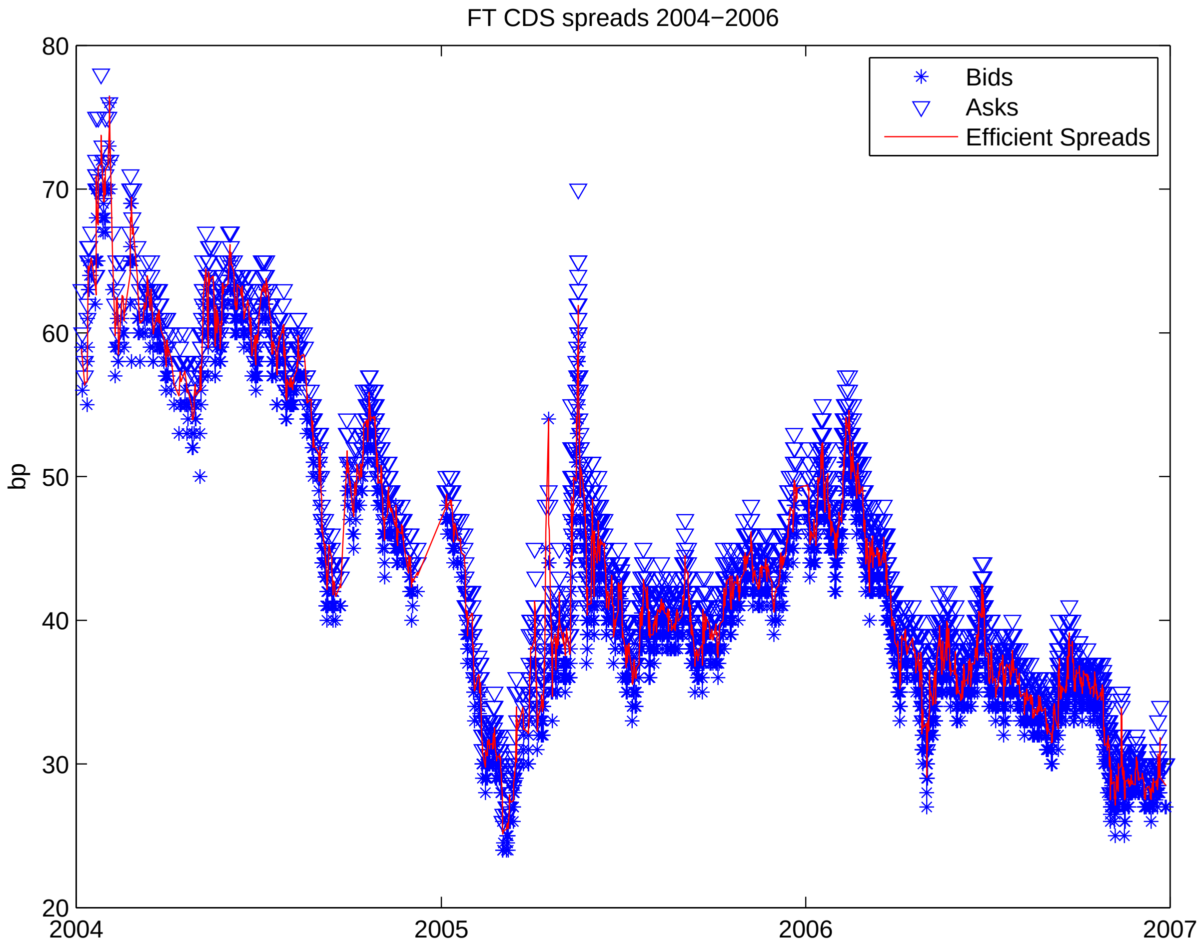

GMAC CDS spreads 2004−2006
Bids
Asks
Efficient Spreads
bp
0
100
200
300
400
500
600
700
800
2004
2005
2006
2007

Figure 13:



Sears CDS spreads 2004−2006
bp
200
180
160
140
120
100
80
60
40
20
2004
2005
2006
2007
Bids
Asks
Efficient Spreads

Figure 14:


FT CDS spreads 2004−2006
bp
80
70
60
50
40
30
20
2004
2005
2006
2007
Bids
Asks
Efficient Spreads