B. Tech

BSCM 2201

Third Semester Examination – 2008

MATHEMATICS – III

Full Marks – 70

Time : 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following questions precisely : 2×10

   (a) Write $\nabla^2 u = u_{xx} + u_{yy} + u_{zz}$ in spherical coordinates.

(b) Classify the partial differential equation

$$4u_{xx} - 4u_{xy} + u_{yy} = 0.$$

(c) Test the function :

$$f(z) = \begin{cases} \frac{xy^2}{x^2 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

for differentiability at $z = 0$.

(d) Find the locus of the points satisfying the equation $\left|\frac{z}{z-1}\right| = 2$.

(e) Express $1 + i$ in polar form.

(f) Is there any analytic function $f(z)$ whose imaginary part is $v(x, y) = x^2 + y^2$ and why ?

(g) Find the poles of the function $f(z) = \frac{1}{(z-1)(z-2)^2}$ with respective order.

(h) Find the value of $\int_{|z|=3} \left(\frac{z}{z-1}\right) dz$.

(i) Find the value of $\int_{|z|=1} \frac{dz}{z-3}$.

(j) Find the residue of $f(z) = \frac{3}{(z-4)(z-1)}$ at $z = 1$.

2. A string is stretched and fastened to two end points π apart. If the motion is started by displacing the string in the form $u(x, 0) = a(\sin(x) - \sin(2x))$ from which it is released at $t = 0$, then find the displacement $u(x, t)$ in

the string at a distance x from the left most end at time t. 10

3. An insulated rod of length l has its ends A and B maintained at $0°C$ and $100°C$ respectively until steady state condition prevail. If the end B is suddenly reduced to $80°C$ and the end A is suddenly raised to $20°C$, then find the temperature distribution in the rod at time t. 10

4. A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the

temperature distribution $u(x, y)$ along the edge $y = 0$ is given by :

$$u(x, 0) = \begin{cases} 20x, & 0 < x \le 5 \\ 20(10 - x), & 5 \le x < 10 \end{cases}$$

while the other three edges are kept at 0°C, then find the temperature distribution at any point of the plate. 10

5. Solve the following partial differential equations by Laplace transform :

(a) $u_x + 2xu_t = 2x$ with $u(x, 0) = 1$ and $u(0, t) = 1$ where subscript denotes the partial derivative with respect to that variable. 5

(b) $u_{xx} = u_{tt}$ with $u(x, 0) = u_t(x, 0) = u(\infty, t) = 0$ and $u(0, t) = g(t)$ where subscript denotes the partial derivative with respect to that variable. 5

6. Write the answer according to the instruction :

(a) Find the analytic function

$$f(z) = u(x, y) + iv(x, y)$$

where $u(x, y) = 2xy$. 5

(b) Find the linear fractional transform which maps left half plane $\Re(z) \leq 0$ into the unit disk. 5

7. Answer as per the instruction :

(a) Integrate $f(z) = \bar{z}$ along the simple curve Γ consists of straight lines joining the

points $z_0 = (0, 0)$ to $z_1 = (4, 0)$ and $z_1 = (4, 0)$ to $z_2 = (4, 2)$. 5

(b) Find the Laurent series of the function $f(z) = \dfrac{3}{(z-1)(z-4)}$ which is valid in the region $1 < |z-4| < 2$. 5

Evaluate the following integrations using residue theorem : 5+5

(a) $\displaystyle\int_0^{\infty} \frac{dx}{e^{x^2}}$

(b) $\displaystyle\int_0^{2\pi} \frac{d\theta}{5 + 12\sin(\theta)}$.