SCRA, 2012

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

T.B.C. : Q-TDSB-M-NDT

Test Booklet Series

A

TEST BOOKLET

MATHEMATICS

Paper – III

Time Allowed : Two Hours ***Maximum Marks : 200***

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET DOES *NOT* HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. ENCODE CLEARLY THE TEST BOOKLET SERIES **A, B, C,** OR **D** AS THE CASE MAY BE IN THE APPROPRIATE PLACE IN THE ANSWER SHEET.
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. ***DO NOT*** write ***anything else*** on the Test Booklet.
4. This Test Booklet contains **100** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ***ONLY ONE*** response for each item.
5. You have to mark all your responses ***ONLY*** on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. All items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator ***only the Answer Sheet.*** You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**
    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third (0·33)** of the marks assigned to that question will be deducted as penalty.
    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.
    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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1. What is the coefficient of x^n in the expansion of $\left(1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^n}{n!}\right)^2$?

(a) $\frac{2^n}{n!}$

(b) 2^n

(c) $n!$

(d) $\frac{1}{n!}$

2. Two real numbers x and y are selected from the closed interval [0, 4]. What is the probability that the selected numbers satisfy the inequation $y^2 \leq x$?

(a) 3/16

(b) 2/3

(c) 1/3

(d) 1/2

3. What is $\int_{10}^{100} (x - [x])\, dx$ equal to, where [x] denotes the greatest integer function ?

(a) 90

(b) 45

(c) 0

(d) –1

4. Let $f(x) = x^2 - 1$ for $0 < x < 2$ and $2x + 3$ for $2 \leq x < 3$. The quadratic equation whose roots are $\lim_{x \to 2-} f(x)$ and $\lim_{x \to 2+} f(x)$ is

(a) $x^2 - 4x + 21 = 0$

(b) $x^2 - 6x + 9 = 0$

(c) $x^2 - 10x + 21 = 0$

(d) $x^2 + 10x - 21 = 0$

5. Let f(x) be a continuous function defined for $1 \leq x \leq 3$. If f(x) takes rational values for all x and f(2) = 10, then what is f(1·5) equal to ?

(a) 0

(b) 1

(c) 10

(d) Cannot be determined as the data is insufficient

6. If $A = \{-2, -1, 0, 1, 2\}$ and $f : A \to Z$, $f(x) = x^2 - 2x - 3$, then what is the pre-image(s) of -3 ?

(a) 0 only

(b) 2 only

(c) 0, 2

(d) Φ

*For the next **two** (02) questions that follow :*

Consider the following determinant :

$$\begin{vmatrix} a & b & ax + by \\ b & c & bx + cy \\ ax + by & bx + cy & 0 \end{vmatrix}$$

7. What is the value of the determinant if $b^2 - ac < 0$ and $a > 0$?

(a) Positive

(b) Negative

(c) Zero

(d) Sign cannot be determined

8. What is the value of the determinant if $b^2 - ac = 0$ and $a > 0$?

(a) 0

(b) 1

(c) $b^2 + ac$

(d) abc

9. If f(x) is continuous for all real values of x, then what is $\sum_{r=1}^{n} \int_{0}^{1} f(r-1+x)\,dx$ equal to ?

(a) $\int_{0}^{n} f(x)\,dx$

(b) $\int_{0}^{1} f(x)\,dx$

(c) $n\int_{0}^{1} f(x)\,dx$

(d) $(n-1)\int_{0}^{1} f(x)\,dx$

10. What is the number of bijective functions from a set A to itself when $|A| = 106$?

(a) 106

(b) $(106)^2$

(c) 106 !

(d) 2^{106}

11. Ten coins are thrown simultaneously. What is the probability of getting at least seven heads ?

(a) 3/64

(b) 5/64

(c) 7/64

(d) 11/64

12. What is the locus of a complex number $z = x + iy$ where $i = \sqrt{-1}$ in the Argand plane satisfying the relation $\arg(z - a) = \pi/4$, where 'a' is a real number ?

(a) $x^2 - y^2 = a^2$

(b) $x^2 + y^2 = a^2$

(c) $x + y = a$

(d) $x - y = a$

13. If $|x^2 - 5x + 6| > x^2 - 5x + 6$, then which one of the following is correct ?

(a) $x > 3$

(b) $x < 2$

(c) $2 < x < 3$

(d) $-3 < x < -2$

14. If $3^{49}(x + iy) = \left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^{100}$ and $x = ky$, then what is the value of k ? $(i = \sqrt{-1})$

(a) $-\frac{1}{3}$

(b) $\sqrt{3}$

(c) $-\sqrt{3}$

(d) $-\frac{1}{\sqrt{3}}$

15. If A and B are two events such that $P(A) = \frac{3}{5}$, $P(B) = \frac{7}{10}$, then which one of the following is correct ?

(a) $P(A \cap B) \geq \frac{3}{10}$

(b) $P(A \cap B) \leq \frac{7}{10}$

(c) $\frac{3}{5} < P(A \cup B) < \frac{7}{10}$

(d) $\frac{3}{5} < P(A \cup B) \leq \frac{7}{10}$

*For the next **two** (02) questions that follow :*

Consider the function $f(x) = [x] \sin(\pi x)$ where [x] is the greatest integer not exceeding x.

16. What is the limit of f(x) as $x \to k$, where k is an integer ?

(a) -1

(b) 0

(c) $k - 1$

(d) None of the above

17. What is the left hand derivative of f(x) at $x = k$, where k is an integer ?

(a) $(-1)^k (k - 1)\pi$

(b) $(-1)^{k-1} (k - 1)\pi$

(c) $(-1)^k k\pi$

(d) $(-1)^{k-1} k\pi$

18. A determinant of the second order is made with the elements 0 and 1. What is the probability that the determinant made is non-negative ?

(a) 13/16

(b) 3/16

(c) 3/4

(d) 7/8

19. What is the sum of real roots of the equation $x^2 + 4|x| - 5 = 0$?

(a) 4

(b) 1

(c) 0

(d) -1

20. If $y^2 = P(x)$, a polynomial of degree $n \geq 3$, then what is $2\frac{d}{dx}\left(y^3 \frac{d^2y}{dx^2}\right)$ equal to ?

(a) $-P(x)\, P'''(x)$

(b) $P(x)\, P'''(x)$

(c) $P(x)\, P''(x)$

(d) $-P(x)\, P''(x)$

21. How many natural numbers less than a million can be formed using the digits 0, 7 and 8 ?

(a) 728

(b) 726

(c) 730

(d) 724

22. Three persons A, B, C are to speak at a function along with five others. If they all speak in random order, what is the probability that A speaks before B and B speaks before C ?

(a) 3/8

(b) 1/6

(c) 3/5

(d) 5/6

23. If A is a skew-symmetric matrix of order 3, then matrix A^3 is a/an

(a) Orthogonal matrix

(b) Diagonal matrix

(c) Symmetric matrix

(d) Skew-symmetric matrix

24. The solution of $\frac{dy}{dx} = \frac{ax + g}{by + f}$ represents a circle if

(a) $a = b$

(b) $a = -b$

(c) $a = -2b$

(d) $a = 2b$

25. Let ABCD be a parallelogram whose diagonals intersect at P. If O is the origin, then what is $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$ equal to ?

(a) $2\overrightarrow{OP}$

(b) $3\overrightarrow{OP}$

(c) $4\overrightarrow{OP}$

(d) $6\overrightarrow{OP}$

26. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function and $f(1) = 4$. What is the value of $\lim_{x \to 1} \int_4^{f(x)} \frac{2t\,dt}{x-1}$ equal to ?

(a) $8\,f'(1)$

(b) $4\,f'(1)$

(c) $2\,f'(1)$

(d) $f'(1)$

27. All curves in the xy-plane having the property that the tangents pass through the origin are

(a) $y = cx^2$

(b) $y = cx$

(c) $x = cy^2$

(d) $xy = c$

28. Consider the following statements :

1. If $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^3 + y^3 = 1\}$ and $B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x - y = 1\}$, then $A \cap B$ contains exactly one element.
2. If $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^3 + y^3 = 1\}$ and $B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x + y = 1\}$, then $A \cap B$ contains exactly two elements.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

29. For any $n \geq 2$, let $M_n(\mathbb{R})$ denote the set of all $n \times n$ matrices over the set of real numbers. Consider the following statements :

1. If $A \in M_n(\mathbb{R})$ is a non-zero matrix with $\det A = 0$, then $\det(\text{adj } A) = 0$.
2. For any $A \in M_n(\mathbb{R})$, $\det(\text{adj } A) = (\det A)^{n-1}$

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

30. If the set of integers with the operation defined by $a * b = a + b - 1$ forms a group, what is the inverse of a ?

(a) $-a$

(b) $2a$

(c) $2 - a$

(d) $1 - a$

31. Consider the following statements :

1. If for some positive integers p, q, r ($p < q < r$), 2^{2p+1}, 2^{2q+1}, 2^{2r+1} are in GP, then p^{-1}, q^{-1}, r^{-1} will be in HP.

2. If x, y, z are positive integers such that x^{-1}, y^{-1}, z^{-1} are in HP, then for any real number $u \neq 0$, $(xu)^{-1}$, $(yu)^{-1}$, $(zu)^{-1}$ will also be in HP.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

32. Let us define the length of a vector $a\hat{i} + b\hat{j} + c\hat{k} = a + b + c$. The definition coincides with the usual definition of length of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ iff

(a) $a = b = c = 0$

(b) any two of a, b, c are zero

(c) any one of a, b, c is zero

(d) $a + b + c = 0$

*For the next **three** (03) questions that follow :*

Consider the following integral :

$$I_n = \int_0^{\pi/4} \tan^n x \, dx \quad \text{where } n \in N, n > 1$$

33. What is $I_n + I_{n-2}$ equal to ?

(a) $\frac{1}{n-1}$

(b) $\frac{1}{n}$

(c) $\frac{1}{n+1}$

(d) $\frac{1}{n+2}$

34. What is $I_{n-1} + I_{n+1}$ equal to ?

(a) $\frac{1}{n-1}$

(b) $\frac{1}{n}$

(c) $\frac{1}{n+1}$

(d) $\frac{1}{n+2}$

35. Consider the following statements :

1. $\frac{1}{n+1} < 2I_n < \frac{1}{n-1}$

2. $\frac{1}{n} < 2I_{n-1} < \frac{1}{n-2}$

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

36. Out of 13 applicants for a job, there are 5 women and 8 men. It is desired to select 2 persons for this job. What is the probability that at least one of the selected persons will be a woman ?

(a) 5/13

(b) 10/13

(c) 14/39

(d) 25/39

37. If p is chosen at random in the closed interval [0, 5], the probability that the equation $x^2 + px + \frac{p+2}{4} = 0$ will have real roots is equal to

(a) 1/2

(b) 1/5

(c) 2/3

(d) 3/5

38. The set of matrices

$$S = \left\{ \begin{bmatrix} x & -x \\ -x & x \end{bmatrix} \text{ such that } 0 \neq x \in \mathbb{R} \right\}$$

forms a group under multiplication with identity element

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

39. If A, B, C are acute positive angles such that $A + B + C = \pi$ and $\cot A \cdot \cot B \cdot \cot C = k$, then

(a) $k \leq \frac{1}{3\sqrt{3}}$

(b) $k \geq \frac{1}{3\sqrt{3}}$

(c) $k < \frac{1}{9}$

(d) $k > \frac{1}{3}$

40. If $A = \sin^2\theta + \cos^4\theta$, then for all real θ, which one of the following is correct ?

(a) $1 \leq A \leq 2$

(b) $\frac{3}{4} \leq A \leq 1$

(c) $\frac{13}{16} \leq A \leq 1$

(d) $\frac{3}{4} \leq A \leq \frac{13}{16}$

41. If S is a finite set containing n elements, then what is the total number of binary operations on S ?

(a) n^n

(b) 2^{n^2}

(c) n^{n^2}

(d) n^2

42. Let A be the fixed point (0, 4) and B be a moving point (2t, 0). Let the mid-point of AB be M. Let the perpendicular bisector of AB meet the y-axis at N. What is the locus of mid-point P of MN ?

(a) $x^2 + (y - 2)^2 = 1/2$

(b) $-x^2 + (y - 2)^2 = 1/2$

(c) $x^2 + y = 2$

(d) $x + y^2 = 2$

43. If z_1, z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then what is $\arg(z_1) - \arg(z_2)$ equal to ?

(a) $-\pi$

(b) $-\pi/2$

(c) 0

(d) $\pi/2$

44. What is the equation to the line passing through $a\hat{i}$ and perpendicular to $\hat{j}$ and $\hat{k}$?

(a) x-axis

(b) y-axis

(c) z-axis

(d) None of the above

45. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-zero vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$, then consider the following statements :

1. $\vec{a}$, $\vec{b}$, $\vec{c}$ are orthogonal in pairs.

2. Each of $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

46. Consider the following statements :

1. The sum of two unit vectors can be a unit vector.

2. The magnitude of the difference between two unit vectors can be greater than the magnitude of a unit vector.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

47. If $\vec{a}$ and $\vec{b}$ are two non-zero non-collinear vectors and m, n are scalars such that $m\vec{a} + n\vec{b} = \vec{0}$, then

(a) $m \neq 0, n \neq 0$

(b) $m = 0, n \neq 0$

(c) $m \neq 0, n = 0$

(d) $m = 0, n = 0$

48. What is $\int e^x (2 + \sin 2x) \sec^2 x \, dx$ equal to ?

(a) $e^x (1 + \cos 2x) \sin x + c$

(b) $e^x (\sec x + \tan x) + c$

(c) $2e^x \tan x + c$

(d) $2e^x \sec x + c$

where c is the constant of integration.

*For the next **two** (02) questions that follow :*

The last term in the binomial expansion of $\left(2^{1/3} - \frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3\sqrt[3]{9}}\right)^{\log_3 8}$.

49. What is n equal to ?

(a) 8

(b) 9

(c) 10

(d) 11

50. What is the 5^{th} term in the expansion ?

(a) 840

(b) 720

(c) 360

(d) 210

*For the next **two** (02) questions that follow :*

Let f be a twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$.
Let $h(x) = \{f(x)\}^2 + \{g(x)\}^2$ where $h(5) = 11$.

51. What is $h'(x)$ equal to ?

(a) 0

(b) 1

(c) x

(d) x^2

52. What is h(10) equal to ?

(a) 0

(b) 11

(c) 22

(d) 44

53. If $B = B^2$ and $I - B = A$, then which one of the following is correct ?

(a) $A^2 = B$

(b) $A^2 = A$

(c) $A^2 = I$

(d) $A^2 = -A$

54. What is the equation of the curve through the point (1, 1) and whose slope is $\frac{2ay}{x(y-a)}$?

(a) $y^a x^{2a} = e^{y-1}$

(b) $y^a x^{2a} = e^y$

(c) $y^{2a} x^a = e^{y-1}$

(d) $y^{2a} x^a = e^y$

55. What is the degree of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0 \ ?$$

(a) 1

(b) 2

(c) 3

(d) 4

56. Consider the following statements in respect of the function $f(x) = 1 + |\sin x|$

1. f(x) is continuous everywhere.

2. f(x) is differentiable at x = 0.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

57. What do the lines $|x| + 2|y| = 1$ represent ?

(a) Sides of a triangle

(b) Sides of a rhombus

(c) Sides of a square

(d) None of the above

58. The function $\frac{a \sin x + b \cos x}{c \sin x + d \cos x}$, $x \in R$ attains neither maximum nor minimum if

(a) $\frac{a}{c} = \frac{b}{d}$

(b) $\frac{a}{d} = \frac{b}{c}$

(c) $\frac{a}{d} = \frac{c}{b}$

(d) None of the above

59. What is $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$ equal to ?

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{12}$

(c) $\frac{\pi^2}{12}$

(d) None of the above

60. If the three vertices of a parallelogram ABCD are A(1, 0), B(2, 3), C(3, 2), what are the coordinates of the fourth point ?

(a) (2, 1)

(b) (2, –1)

(c) (–1, 2)

(d) (–1, –2)

61. If one of the lines represented by $6x^2 + kxy + y^2 = 0$ is $2x + y = 0$, then what is the value of k ?

(a) 3

(b) 4

(c) 5

(d) – 5

62. If H is an orthogonal square matrix, then what is the determinant of H ?

(a) 0

(b) 1

(c) 2

(d) 4

63. A family of curves involves four parameters. Let the order of differential equation representing the family be m. What is 'm' equal to ?

(a) 1

(b) 2

(c) 3

(d) 4

64. The number of pairs of parallel tangents that an ellipse has, is

(a) Zero

(b) 1

(c) 2

(d) Infinite

65. Consider the function $f(x) = |x - 1|$ defined on an interval [–1, 2]. The point x_0 where $f'(x) = 0$ on that interval

(a) is 0

(b) is 1

(c) is –1

(d) does not exist

66. Let $x^m + y^m = 1$. If $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{1/3}$, then what is the value of m ?

(a) 1/3

(b) 2/3

(c) 1

(d) 3/2

67. Consider the functions $f(x) = e^x$, $g(x) = \log_e x$. Which one of the following statements is ***not*** correct ?

(a) f(x) is always positive

(b) $f(x) > g(x)$ for all values of x

(c) g(x) is always positive

(d) f(x) and g(x) curves never intersect

68. Let $f(x) = \frac{1}{|x|}$. In which one of the following intervals is the function discontinuous ?

(a) (0, 1)

(b) (–1, 0)

(c) [0, 1]

(d) (0, 1]

69. A line makes equal angles with the diagonals of a cube. What is the sine of the angle ?

(a) $\sqrt{\frac{2}{3}}$

(b) $\sqrt{\frac{1}{3}}$

(c) $\sqrt{\frac{1}{2}}$

(d) None of the above

70. In an arithmetic series of 16 terms with first term 16, the sum is equal to the square of the last term. What is the common difference ?

(a) 8/5

(b) $-4/5$

(c) $-8/5$

(d) None of the above

71. The solution of $|x - 2| < 5$ is all the real numbers satisfying

(a) $-2 < x < 5$

(b) $-3 < x < 7$

(c) $-5 < x < 7$

(d) $-3 < x < 5$

72. Let $\bar{z}$ be the complex conjugate of z. The curve represented by $\text{Re}(\bar{z})^2 = k^2$ is a

(a) parabola

(b) rectangular hyperbola

(c) hyperbola but not rectangular

(d) pair of straight lines

73. Which one of the following functions is inverse of itself ?

(a) $\frac{1-\sin x}{1+\sin x}$

(b) $\frac{1-x^2}{2+x^2}$

(c) $\frac{1}{2} \ln\left(\frac{e^x + e^{-x}}{e^x - e^{-x}}\right)$

(d) None of the above

74. Let N_1 be the total number of injective mappings from a set with m elements to a set with n elements when $m \leq n$. When $m > n$, let the number of injective mappings be N_2. Then

(a) $N_1 > N_2$

(b) $N_1 < N_2$

(c) $N_1 = N_2$

(d) $N_1 + N_2 = m + n$

75. Let A_1 be the area enclosed in between an ellipse and the circle drawn with the major axis as a diameter and A_2 be the area enclosed between the same ellipse and the circle drawn with the minor axis as diameter. Then which one of the following is correct ?

(a) $A_1 = A_2$

(b) $A_1 < A_2$

(c) $A_1 > A_2$

(d) $A_1 - A_2 = \pi (b^2 - a^2)$

76. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b}$ be a vector such that $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$. Then which one of the following is correct ?

(a) $\vec{b} = \lambda \vec{a}$ for some scalar λ

(b) $\vec{b} = \vec{0}$

(c) $\vec{b}$ is perpendicular to $\vec{a}$

(d) $\vec{b}$ is a non-zero vector

77. What is $\int_0^1 \frac{x^2\,dx}{\sqrt{x^6+1}}$ equal to ?

(a) $\frac{1}{3} ln (\sqrt{2} + 1)$

(b) $\frac{1}{2} ln (\sqrt{2} + 1)$

(c) $\frac{1}{3} ln (\sqrt{2} - 1)$

(d) $\frac{1}{2} ln (\sqrt{2} - 1)$

78. Let f(x) be a function defined in the closed interval I where I = [0, 1].

$$f(x) = \begin{cases} 0, & \text{when x is rational} \\ 1, & \text{when x is irrational} \end{cases}$$

Then which one of the following is correct ?

(a) f(x) is continuous on I

(b) f(x) is continuous on I except for a finite number of points

(c) f(x) is continuous nowhere on I

(d) None of the above

79. If A is a 3 × 3 matrix and det(3A) = k det(A), then what is the value of k ?

(a) 3

(b) 9

(c) 27

(d) 81

80. What is the area of the region in the first quadrant bounded by the y-axis and the curves y = sin x and y = cos x ?

(a) $\sqrt{2}$ square units

(b) $\sqrt{2} + 1$ square units

(c) $\sqrt{2} - 1$ square units

(d) $2\sqrt{2} - 1$ square units

81. What is $\int_{-1}^{1} \frac{|x|}{x}\,dx$ equal to ?

(a) 2

(b) 0

(c) 1

(d) 1/2

82. The straight line ax + by + c = 0 and the co-ordinate axes form an isosceles triangle when

(a) $|a| = |b|$

(b) $|a| = |c|$

(c) $|b| = |c|$

(d) None of the above

83. If the planes $\vec{r} \cdot (2\hat{i} - \lambda\hat{j} + \hat{k}) = 3$ and $\vec{r} \cdot (4\hat{i} + \hat{j} - \mu\hat{k}) = 5$ are parallel, then what is λ equal to ?

(a) 1/2

(b) 1

(c) –1/2

(d) –1

84. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-zero vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $m = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, then which one of the following is correct ?

(a) $m < 0$

(b) $m > 0$

(c) $m = 0$

(d) Cannot be determined

85. The least positive integer n for which $\left(\frac{1+i}{1-i}\right)^{2n}$, where $i = \sqrt{-1}$, has a negative value is

(a) 1

(b) 2

(c) 3

(d) 4

86. The factors of $\sin\theta + \sin\varphi - \cos\theta \sin(\theta + \varphi)$ are

(a) $\sin\theta$ and $1 + \sin(\theta + \varphi)$

(b) $\sin\theta$ and $1 - \cos(\theta + \varphi)$

(c) $\sin\varphi$ and $1 - \cos(\theta + \varphi)$

(d) None of the above

87. The inverse of a symmetric matrix, if it exists, is

(a) symmetric

(b) skew-symmetric

(c) always unit matrix

(d) None of the above

88. Which one of the following points does ***not*** lie on the circle with centre at (3, 4) and radius 5 ?

(a) (0, 0)

(b) (–1, 1)

(c) (2, 3)

(d) (3, –1)

89. The equation $Ax + By + C = 0$ involves only

(a) One arbitrary constant

(b) Two arbitrary constants

(c) Three arbitrary constants

(d) None of the above

90. Let X be the set of all persons living in a state. Elements x, y in X are said to be related if 'x < y', whenever y is 5 years older than x. Which one of the following is correct ?

(a) The relation is an equivalence relation

(b) The relation is transitive only

(c) The relation is transitive and symmetric, but not reflexive

(d) The relation is neither reflexive, nor symmetric, nor transitive

*For the next **three** (03) questions that follow :*

Consider $f(x) = \frac{1+x}{1-x}$, $(x \neq 0)$ and

$g(x) = fofofof(x)$

91. What is g(x) equal to ?

(a) $\frac{1}{1-x}$

(b) $\frac{1}{x}$

(c) x

(d) $\frac{1}{1+x}$

92. What is $g(x)\, g\left(\frac{1}{x}\right)$ equal to ?

(a) $\frac{1}{1-x}$

(b) $\frac{1}{x^2}$

(c) x^2

(d) 1

93. Consider the following statements :

1. g(x) is an identity function.
2. f o f(x) is an identity function.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

*For the next **three** (03) questions that follow :*

Consider the polynomial

$p(x) = (x - \alpha)(x - \beta)(x - \gamma)$ with

$\alpha = \cos 75°$, $\beta = \cos 45°$ and $\cos \gamma = \cos 165°$.

94. What is the constant term of p(x) equal to ?

(a) $\frac{1}{\sqrt{2}}$

(b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{4\sqrt{2}}$

(d) $-\frac{1}{4\sqrt{2}}$

95. What is the coefficient of x^2 in p(x) ?

(a) -1

(b) 1

(c) 0

(d) None of the above

96. What is the coefficient of x in p(x) ?

(a) $\frac{2\sqrt{2}+1}{4}$

(b) $-\frac{2\sqrt{2}+1}{4}$

(c) $\frac{\sqrt{2}+1}{4}$

(d) None of the above

*For the next **four** (04) questions that follow :*

The series of natural numbers is divided into groups (1), (2, 3, 4), (5, 6, 7, 8, 9) and so on.

97. How many numbers are there in the n^{th} group ?

(a) n

(b) $2n - 1$

(c) $2n$

(d) $2n + 1$

98. What is the first term in the n^{th} group ?

(a) $2n - 1$

(b) $n^2 - 2n - 1$

(c) $n^2 - 2n + 2$

(d) None of the above

99. What is the sum of the numbers in the n^{th} group ?

(a) $(2n + 1)(n^2 - n + 1)$

(b) $n^3 - 3n^2 + 3n - 1$

(c) $n^3 + (n + 1)^3$

(d) None of the above

100. What is the last term in the n^{th} group ?

(a) $4n + 1$

(b) $2n^2$

(c) n^2

(d) None of the above

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