

**DEPARTMENT OF MATHEMATICS & ASTRONOMY  
FACULTY OF SCIENCE / FACULTY OF ARTS  
LUCKNOW UNIVERSITY**

**Syllabus for M.A./M.Sc. Course under Semester System**

**FIRST SEMESTER**

(Effective from 2009-10 and onwards)

*There will be FIVE compulsory core courses each having 60 marks.*

**Paper I: Topology I**

**Unit I**

Countable and uncountable sets, Infinite sets and the axiom of choice, Cardinal numbers and its arithmetic, Schroeder-Bernstein theorem, Cantor's Theorem and Cantor's continuum hypothesis, Zorn's Lemma, Well ordering principle.

**Unit II**

Definition and examples of topological spaces, Closed sets, Closure, Dense subsets, Neighbourhoods, Interior, exterior and boundary, Accumulation points and derived sets, Bases and subbases, Subspaces and relative topology.

**Unit III**

Alternative methods of defining a topology in terms of Kuratowski closure operator, interior operator and neighbourhood systems, Continuous functions and homeomorphism, First & Second countable spaces, Lindeloff theorem and separable spaces and their relationships.

**Unit IV**

Separation axioms  $T_0$ ,  $T_1$ ,  $T_2$ , Nets and filters, Topology and convergence of nets. Hausdorffness and nets, Filters and their convergence, Ultra filters, Canonical way of converting nets to filters and vice-versa.

Books Recommended:

1. GF Simmons: Introduction to Topology and Modern Analysis, Mc Graw Hill, 1963.
2. JL Kelly: Topology, Von Nostrand Reinhold Co. New York, 1995.

**Paper II: Advanced Algebra**

**Unit I**

Group Theory- Series of groups, Schreier Theorem, Jordan Holder Theorem, Solvable groups, Nilpotent groups, Insolubility of  $S_n$  for  $n > 5$

**Unit II**

Field Theory- Field extensions, algebraic extensions, finite extensions, Splitting fields, algebraically closed fields, Normal extensions, Separable extension, Primitive element theorem,

**DEPARTMENT OF MATHEMATICS & ASTRONOMY  
FACULTY OF SCIENCE / FACULTY OF ARTS  
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**Unit III**

Galois Theory- Galois group, Galois extension, Fundamental Theorem of Galois Theory, Artin's Theorem, Fundamental Theorem of Algebra (Algebraic Proof),

**Unit IV**

Radical extensions, insolvability of a quintic, constructibility.

Books Recommended :

1. Serge Lang: Algebra, Addison Wesley
2. V.Sahai & V.Bist: Algebra, Second edition, Narosa.

**Paper III : Differential Geometry of Manifolds**

**Unit I**

Definition and examples of differentiable manifolds, Tangent Spaces, Vector fields, Jacobian map, Distributions, Hypersurface of  $\mathbb{R}^n$ ,

**Unit II**

Standard connection on  $\mathbb{R}^n$ , Covariant derivative, Sphere map, Weierstrass map, Gauss equation, the Gauss curvature equation and Codazzi-Mainardi equations.

**Unit III**

Invariant view point, Cartan view point, coordinate view point, Difference Tensor of two connections, Torsion and curvature tensors.

**Unit IV**

Riemannian Manifolds, Length and distance in Riemannian manifolds, Riemannian connection and curvature, Curves in Riemannian manifolds, Submanifolds.

Books Recommended:

1. NJ Hicks: Notes on Differential Geometry, D. Van Nostrand, 1965.
2. Y Matsushima: Differentiable Manifolds

**Paper IV: Integral Equations**

**Unit I**

Linear Integral Equations-Definition and Classification of conditions, Special kinds of Kernels, Eigen values and eigen functions, Convolution integral, Inner product Integral Equations with separable Kernels- Reduction to a system of algebraic equations,

**Unit II**

Fredholm alternative, Fredholm Theorem, Fredholm alternative theorem, Approximate method, Method of successive approximations- Iterative scheme, solution of Fredholm and Volterra integral equation. Results about resolvent Kernel.

**Unit III**

Classical Fredholm Theory- Method of solution of Fredholm equations. Fredholm first theorem (statement and Proof), Fredholm's second and third theorem (statement only).

**DEPARTMENT OF MATHEMATICS & ASTRONOMY  
FACULTY OF SCIENCE / FACULTY OF ARTS  
LUCKNOW UNIVERSITY**

Application to ODE- initial value problem, boundary value problem, Dirac Delta function.

**Unit IV**

Existence and uniqueness of solution of scalar differential equation Lipschitz's condition, Method of successive approximation, theorem of Existence theorem & unique solution to the initial value problem, Family of equicontinuous functions, G.A.Scoli's lemma (statement only), Peanos existence theorem,. Differential and integral inequalities.

Books Recommended :

1. R.P. Kanwal 1971, Linear Integral Equations Academic Press, New York.
2. Shair Ahmad & M.R.M. Rao, 1999, Theory of ordinary differential equations, Affiliated East-West Press Pvt. Ltd., New Delhi.

Reference Book-

1. P. Hartman, 1964, Ordinary Differential Equations, John Wiley.

**Paper V : Real Analysis**

**Unit I**

Algebra of sets, outer Measure, Measurable Sets and Lebesgue measure, non-measurable sets, measurable functions.

**Unit II**

The Lebesgue integration of bounded function over a set of finite measure, the integral of a non-negative functions, The general Lebesgue integral.

**Unit III**

The four derivatives, differentiation of monotone functions, functions of bounded variation, Lebesgue differentiation theorem, Differentiation of an integral. Absolute continuity.

**Unit IV**

Inequalities and the  $L_p$  Spaces, The  $L_p$  Spaces, convex functions, Jensen's inequality, the inequalities of Holder and Minkowski, completeness of  $L_p(\mu)$ . Convergence in Measure, almost uniform convergence.

Books Recommended:

1. Real Analysis by H.L. Royden
2. Measure Theory and Integration, by G.de Barra

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FACULTY OF SCIENCE / FACULTY OF ARTS  
LUCKNOW UNIVERSITY**

**SECOND SEMESTER**

**Paper I : Topology II**

**Unit I**

Separation axioms  $T_3$ ,  $T_{3\frac{1}{2}}$ ,  $T_4$  and their basic properties, Urysohn's lemma, Tietze extension theorem. Metric spaces, compactness and its basic properties, Local compactness and one point compactification.

**Unit II**

Compactness in metric spaces, Bolzano-Weierstrass property, Sequential compactness, countable compactness, equivalence of compactness, Countable compactness, Sequential compactness in metric space. Connected spaces, connectedness on the real line, Components, Locally connected spaces.

**Unit III**

Tychonoff product topology in terms of standard subbase and its characterization, Projection maps, separation axioms and product spaces, Connectedness and Compactness (Tychonoff theorem) with product spaces, Countability and product spaces. Embedding and metrization, Embedding lemma and Tychonoff embedding. The Urysohn's metrization theorem.

**Unit IV**

The fundamental group and covering spaces- Homotopy of paths. The fundamental group, covering spaces, The fundamental group of circle and the fundamental theorem of algebra.

Books Recommended:

1. James R Munkres: Topology, A first course, Prentice Hall, New Delhi, 2000
2. GF Simmons: Introduction to Topology and Modern Analysis, Mc Graw Hill, 1963.
3. JL Kelly: Topology, Van Nostrand Reinhold Co. New York, 1995.

**Paper II: Module Theory**

**Unit I**

Modules- Definition and examples, simple modules, submodules, Module Homomorphisms, Quotient modules, Direct sum of modules, Exact sequences, Short exact sequence, split exact sequences. Torsion free and torsion modules.

**Unit II**

Free modules- Definition and examples, modules over division rings are free modules, free modules over PID's, Invariant factor theorem for submodules,

**Unit III**

Finitely generated modules over PID, Chain of invariant ideals, Fundamental structure theorem for finitely generated module over a PID,

**Unit IV**

Projective and injective modules.

Books Recommended :

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FACULTY OF SCIENCE / FACULTY OF ARTS  
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1. Serge Lang: Algebra, Addison Wesley.
2. V.Sahai & V.Bist, Algebra, Second Edition, Narosa.

**Paper III: Riemannian manifolds, Lie Algebra and Bundle Theory**

**Unit I**

Sectional Curvature, Schur's Theorem, Geodesic in a Riemannian Manifold, Projective Curvature tensor, Conircular Curvature Tensor, Conformal curvature tensor, Conharmonic curvature tensor, Einstein Manifolds

**Unit II**

Tensor and forms, Exterior derivative, contraction, Lie derivative, general covariant derivative.

**Unit III**

Lie groups and Lie algebras with examples, homomorphism, isomorphism, one parameter subgroups and exponential map, The Lie transformations group.

**Unit IV**

Principal fibre bundle, Linear frame bundle, Associated bundles, tangent bundle.

Books Recommended:

1. NJ Hicks: Notes on Differential Geometry, D. Van Nostrand, 1965.
2. BB Sinha: An introduction to Modern Geometry

**Paper IV: Ordinary Differential Equations**

**Unit I**

Linear System- Introduction, properties of linear homogeneous systems, Periodic linear System, Floquet's theorem, Inhomogeneous linear system.

**Unit II**

System of first order equation: Linear system, Homogenous linear system with constant coefficient, Nonlinear system, Volterra's prey & predator equation, Non Linear equation: Autonomous system. The phase plane & its phenomena, types of critical points & stability.

**Unit III**

Critical points & stability for linear system, stability by Liapunov's direct method simple critical points of non linear system & non linear mechanics. Conservative system, Periodic solution, Poincare – Bendixson Theorem.

**Unit IV**

Second order differential equation Introduction, Preliminary results, Boundedness of solution, Oscillatory equation, number of zeroes, Pruffer's transformation, Strum theorem, Strurm's comparison theorem.

Books Recommended :

1. Shair Ahmad and M.R.M Rao, 1999, Theory of ordinary differential equations.

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FACULTY OF SCIENCE / FACULTY OF ARTS  
LUCKNOW UNIVERSITY**

Affiliated East-West Press Private Ltd. New Delhi.

2. Differential Equation – G.F. Simmons, TMH

Reference Book:

1. P.Hartman, 1964, Ordinary differential equations, John Wiley

**Paper V: Complex Analysis**

**Unit I**

Complex Riemann-Stieltjes Integral, piecewise smooth paths, complex line integral, Cauchy's theorem for a starshaped domains, Cauchy's integral formula. Taylor's theorem, Liouville's theorem, fundamental theorem of algebra, maximum modulus theorem, minimum modulus theorem, Schwarz Lemma, Hadamard's three circle theorem, inverse function theorem

**Unit II**

Singularities, Laurent's series, Meromorphic Functions, Argument Principle, Rouches Theorem, residue theorem, Evaluation of real integrals.

**Unit III**

Functions spaces: Hurwitz's Theorem, infinite products, Weirstrass factorization theorem, Mittag-Leffler's Theorem, Gamma functions and its properties, Riemann's Zeta Function.

**Unit IV**

Analytic Continuation, Uniqueness of direct Analytic Continuation, Power series Method of Analytic Continuation, Harmonic Functions on a disk, Harnack's inequality and theorem, Canonical products Poisson Formula, Jensen's Formula, Poisson Jensen's Formula, Hadamard's three circle Theorem as convexity theorem, Hadamard's factorization theorem, order of an entire function.

Books Recommended:

1. Complex Analysis by J.V. Deshpande
2. Theory of Functions by E.C. Titchmarsh
3. Functions of one complex variable by John B. Conway

**DEPARTMENT OF MATHEMATICS & ASTRONOMY  
FACULTY OF SCIENCE / FACULTY OF ARTS  
LUCKNOW UNIVERSITY**

**Syllabus for M.A./M.Sc. Course under Semester System  
( 2010-11 and onwards)**

*There will be THREE compulsory core courses and TWO OPTIONAL PAPERS each having 60 marks.*

**SEMESTER THIRD**

**Compulsory Papers**

**Paper I: Functional Analysis**

**Unit I**

Banach Spaces- the definition and some examples, continuous linear transformations, The Hahn Banach theorem,

**Unit II**

The natural imbedding of  $N$  in  $N^{**}$ , the open mapping theorem, the conjugate of an operator.

**Unit III**

Hilbert spaces- the definition and some simple properties, Orthogonal complements, orthogonal sets, the Conjugate space  $H^*$ ,

**Unit IV**

The adjoint of an operator, Self adjoint operators, normal and unitary operators, Projections. Finite dimensional spectral theory – Spectrum of an operator, the spectral theorem.

Books :Recommended :

G F Simmons: Introduction to Topology & Modern Analysis (Mc Graw Hill).

**Paper II: Structures on even dimensional differentiable manifolds**

**Unit I**

Almost complex manifolds, Nijenhuis tensor, contravariant and covariant analytic vector. Almost Hermite manifold, almost analytic vector fields curvature tensors, Linear connections.

**Unit II**

Kahler manifolds, affine connections, curvature tensors, contravariant almost analytic vectors.

**Unit III**

Nearly Kahler manifold, curvature identities, Curvature tensors, almost analytic vectors.

**Unit III**

Almost Kahler manifolds, analytic vectors conformal transformations, curvature identities.

**DEPARTMENT OF MATHEMATICS & ASTRONOMY  
FACULTY OF SCIENCE / FACULTY OF ARTS  
LUCKNOW UNIVERSITY**

**Books Recommended :**

R.S. Mishra, Structures on a differentiable manifold and their applications, Chandrama Prakashan, Allahabad.

**Paper III: Fluid Mechanics I**

**Unit I**

Types of fluids, Lagrangian and Eulerian Method of describing fluid motion, Motion of Fluid element: Translation, rotation and deformation. Stream lines, Path lines and streak lines. Material derivative. Acceleration Components of a fluid particle in Cartesian, Cylindrical Polar and Spherical Polar Coordinates (without proof). Vorticity vector, Vortex lines, Rotational and irrotational motion, Velocity, Potential Boundary surface, Boundary condition.

**Unit II**

Reynold transport theorem. Principle of conservation of mass-Equation of continuity (By Lagrangian and Eulerian method. Equation of Continuity in different coordinate systems. Body force and Surface force. Euler's equation of motion-conservation of momentum, Energy Equation. Bernoulli's Equation and function.

**Unit III**

Irrotational motion in two dimensions: Stream function, Physical significance of stream function, Complex Velocity Potential. Sources, Sinks, Doublets and their images in two dimension. Milne-Thompson circle theorem. Simple problems. Vortex motion. Complex Potential due to Vortex circulation, Kelvin's theorem on Vortex motion, Blasius Theorem and Kutta-Joukowski Theorem.

**Unit IV**

Two dimensional Irrotational motion produced by motion of circular and Co-axial cylinders in an infinite mass of liquid, Liquid Streaming past Circular cylinder, Kinetic energy of liquid, Irrotational motion in three-dimension: Motion of sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere, Equation of motion of a sphere, Axis-Symmetric flow, Stoke's function.

**Books Recommended :**

1. Frank Chorlton, Text Book of Fluid Dynamics, C.B.S. Publishers, Delhi, 1985
2. Z.U.A. Warsi, Fluid Dynamics, Theoretical and Computational Approaches, C.R.C. Press
3. S.W. Yuan, Foundation of Fluid Mechanics, Prentice Hall of India Pvt. Ltd. New Delhi, 1969



**DEPARTMENT OF MATHEMATICS & ASTRONOMY  
FACULTY OF SCIENCE / FACULTY OF ARTS  
LUCKNOW UNIVERSITY**

**Optional Paper**

**Paper IV: Analytic Number Theory I**

**Unit I**

**Arithmetical Functions:** The Mobius function, the Euler totient functions and relation connecting them. Properties of Euler's totient. The Dirichlet product of Arithmetical functions. Dirichlet inverses and the Mobius inversion formula. The Mangoldt function and Multiplicative functions. Multiplicative functions and Dirichlet multiplication. The inverse of a completely multiplicative function. Liouville's function. The divisor functions  $d(n)$  and Generalized convolutions.

**Unit II**

The Bell series of arithmetical functions. Bell series and Dirichlet multiplication. Derivative of arithmetical functions. Euler's summation formula, Estimates for sums of Divisors, Estimate for the Number of Divisors, Highly composite Numbers. The average order of  $d(n)$  The average order of the divisor functions  $d(n)$ . the average order of Euler's totient. The partial sums of a Dirichlet product. Application of Mobius and Mangoldt functions. Multiplicatively perfect number numbers and super Perfect numbers.

**Unit III**

Introduction to Modular forms : Congruences Residue classes and complete residue system. Linear congruences. Reduced residue system and the Euler-Fermat theorem. Polynomials congruences modulo  $p$ , Lagrange's theorem. Simultaneous linear congruences, The Chinese remainder theorem, Application of Chinese remainder theorem. Polynomial congruences with prime power moduli. The principle of cross classification, A decomposition property of reduced residue system.

**Unit IV**

Quadratic residues, Legendre's Symbol and its properties Gauss Law, the quadratic reciprocity law, Applications of reciprocity law. The Jacobi symbol and reciprocity law for Jacobi symbols. Applications of reciprocity law to Diophantine equations, Primitive roots, Primitive roots and reduced residue systems. The existence of primitive roots mod  $p$  for odd prime's  $p$ . Primitive roots and quadratic residue.

**Books Recommended:** 1. T.M. Apostol, Introduction to Analytic Number Theory, Norosa Publication House, 1980.

1. J.P. Serre – A course in Arithmetic, Springer Verlag, 1973.
2. Jeffrey Stopple – A Primer of Analytic Number Theory, Cambridge University, Press, 2003.

**Paper V: Special Functions I**

**Unit I**

The Gamma Functions: Analytic Character, Tannery's theorem, Euler's limit formula, Duplication formula, Eulerian integral of the first kind, Euler's Constant, Canonical product, Asymptotic expansions, Watson's lemma, Asymptotic expansion of  $\Gamma(z)$  and its range of validity, Asymptotic behaviour of  $\Gamma(x+iy)$ , Hankel's contour integral.

**Unit II**

**DEPARTMENT OF MATHEMATICS & ASTRONOMY**  
**FACULTY OF SCIENCE / FACULTY OF ARTS**  
**LUCKNOW UNIVERSITY**

The Hypergeometric Functions: Solution of homogeneous linear differential equation of order two near an ordinary point and near a regular singularity, Convergence of the series solution near a regular singularity, solutions valid for large value of  $|z|$ , solution when the exponent difference is an integer or zero, second-order differential equation with three regular singularity, Hypergeometric equation and its solution, generalized hypergeometric equation.

**Unit III**

Integral representation of  $F(a,b,c,z)$ , value of  $F(a,b,c;i)$  when  $\text{Re}(c-a-b) > 0$ , Analytical continuation of  $f(a,b,c;za)$ , Barnes's contour integral for  $F(a,b,c;z)$ , behaviour between contiguous hypergeometric functions, hypergeometric function, Confluent hypergeometric function,  $F_1(\ ; ; z)$ , Asymptotic expansion, Asymptotic expansion of  ${}_1F_1(\ ; ; z)$ .

**Unit IV**

Bessel Functions: Bessel's differential equation and its series solutions, recurrence formulae for  $J(z)$ , Schlaflis contour integral for  $J(z)$ , generating functions for  $j_n(z)$  solution of Bessel's equation by Complex integrals, Hankel functions, Connexion between the Bessel and Hankel functions, complete solution of Bessel's equation, Bessel function of the second kind, series for  $Y_n(z)$ , Asymptotic expansion of the Bessel's functions, Neumann polynomials, Neumann's expansion theorem.

**Paper VI : Advanced Discrete Mathematics I**

**Unit I**

Formal logic statements, symbolic representations, propositional logic, statement calculus, tautology, logic operators, truth table, validity of arguments of statement calculus, inference theory of statement calculus, predicate calculus, quantifiers, inference theory of predicate calculus, deduction system for predicate calculus, validity of arguments of predicate calculus.

**Unit II**

Lattice theory, partially ordered set and their properties, lattice as algebraic system, sub lattice, direct product, homomorphism, special lattices: complete, distributive and complemented, Boolean algebra as lattice, Boolean identities, switching algebra, direct product, homomorphism, join irreducible elements.

**Unit III**

Digital logic, logic gates (AND, OR, NOT, NAND, NOR, EXOR etc.), minterms, maxterms, sum of products and product of sum forms, canonical forms (disjunctive and conjunctive), expression minimization using deduction system, Karnaugh map, expression minimization using Karnaugh map, Applications of Boolean algebra: simple logic circuits using gates, realization of circuits using universal gates.

**Unit IV**

Algebraic systems : semi groups and monoid definitions and examples (including those pertaining to concatenation operation), homomorphism of semi groups and monoids, Congruence relation and quotient semi groups, sub semi groups, sub manifold, direct product, homomorphism theorem.

**DEPARTMENT OF MATHEMATICS & ASTRONOMY  
FACULTY OF SCIENCE / FACULTY OF ARTS  
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References :

1. J.P. Trembley and R Manohar : Discrete Mathematical Structures with Applications to Computer Science; MGH
2. Discrete Mathematics; Seymour Lipschutz, Mark Lipson; T M H
3. C.L. Lieu: Elements of Discrete Mathematics; MGH

**Paper VIII: Mathematical Biology I**

**Unit 1**

Continuous population Models for single species : Continuous Growth Models, Insect Out break Model: Spruce Budworm, Delay models, Linear Analysis of Delay Population Models : Periodic solutions

**Unit 2**

Delay Models in Physiology; I Dynamic Diseases, Harvesting a single Natural Population, Population Model with Age Distribution, Simple Discrete Models.

**Unit 3**

Continuous Models for Interacting Population : Interaction between species: two species models, definition of stability, community matrix approach, Qualitative behavior of the community matrix, Competition: Lotka-Volterra models, Extension to Lotka\_Volterra models, Competition in field experiments, Competition for space, Models for Mutualism.

**Unit 4**

Predator: Prey interaction: Lotka-Volterra Models, dynamic of the simple Lotka\_Volterra models, Role of density dependent in the Prey, Classic laboratory experiment on predator, predation in natural system. Some predator- prey models.

**Books recommended :**

1. Mathematical Biology : J.D. Murray.
2. Population Biology : Alan Hastings Concepts and Models , Springer

**Paper X Ordinary and Partial Differential Equations**

**Unit I**

Two point Boundary value Problem: Introduction, The homogeneous boundary problem, The adjoint boundary problem, The nonhomogeneous boundary problem, Self-Adjoint boundary problem.

**Unit II**

Introduction, basic concepts and definitions, Second order linear equation and methods of characteristics, the methods of separation of variables.

**Unit III**

Fourier transform and Initial boundary value problems Green's functions and boundary value problems.

**DEPARTMENT OF MATHEMATICS & ASTRONOMY  
FACULTY OF SCIENCE / FACULTY OF ARTS  
LUCKNOW UNIVERSITY**

**Unit IV**

Sturm-Liouville system, eigen function, Bessel function, singular Sturm-Liouville System, Legendre Function, boundary value problem for ordinary differential equations and Green's functions.

**Recommended Books :**

1. Non- linear Partial Differential Equations IInd Edition, Loknath Debnath, Birkhauser.
2. Partial Differential Equations of Mathematical Physics by Tyn Myint-U, Elseveir Publication

**Optional Paper XI : Approximation Theory & Wavelets I**

**Unit I**

Different types of Approximations, Least squares polynomial approximation Weierstrass Approximation Theorem, Monotone operators, Markoff inequality, Bernstein inequality, Fejers theorem for HF interpolation.

**Unit II**

Erdoes- Turan Theorem, Jackson's Theorems (I to V), Dini-Lipschitz theorem, Inverse of Jackson's Theorem, Bernstein Theorems (I,II, III), Zygmund theorem.

**Unit III**

Lobetto and Radau Quadrature, Hermite and HF interpolation, (0,2)-interpolation on the nodes of  $\pi(x)$ , existence, uniqueness, explicit representation and convergence.

**Unit IV**

Spline interpolation, existence, uniqueness, explicit representation of cubic spline, certain external properties and uniform approximation.

**Book Recommended:**

1. T.J. Rivlin ; An Introduction to the Approximation of functions, Dover Publications
2. E.W. Cheney: Introduction to Approximation Theory, McGraw-Hill Book Company
3. A. Ralston, A First Course in Numerical Analysis, MacGraw –Hill Book Company

**DEPARTMENT OF MATHEMATICS & ASTRONOMY  
FACULTY OF SCIENCE / FACULTY OF ARTS  
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**SEMESTER IV**

**Compulsory Paper**

**Paper I: Lie Algebras**

**Unit I**

Basic Concepts – definition and construction of Lie and associative algebras, algebras of linear transformations, derivations, inner derivations of associative and lie algebras, determinations of Lie algebras of low dimensionalities,

**Unit II**

Representations and modules, some basic module operations, Ideals, solvability, nilpotency, extension of the base field.

**Unit III**

Solvable and Nilpotent Lie algebras- Weakly closed subsets of an associative algebra, nil weakly closed sets, Engel's theorem, Primary components, weight spaces.

**Unit IV**

Lie algebras with semi simple enveloping associative algebras, Lie's theorems, Applications to abstract Lie algebras, some counter examples, Universal enveloping algebras- definition and basic properties, The Poincare Birkhoff Witt theorem.

Recommended Books :

1.Lie Algebras – N. Jacobson, John Wiley

**Paper II: Structures on odd dimensional differentiable manifolds, F-structure manifolds, Submanifolds**

**Unit I**

Almost contact manifold, Lie derivative, affinely almost Co-Symplectic manifold.

**Unit II**

Almost Grayan manifold, almost Sasakian manifold, K-contact Riemannian manifold, Properties of curvature on these manifolds.

**Unit III**

Co-symplectic structure, F- structure manifold.

**Unit IV**

Submanifolds of almost Hermite manifolds and Kahler manifolds, Almost Grayan submanifolds.

**Books recommended :**

R.S. Mishra, Structures on a differentiable manifold and their applications. Chandrama Prakashan, Allahabad.

**DEPARTMENT OF MATHEMATICS & ASTRONOMY  
FACULTY OF SCIENCE / FACULTY OF ARTS  
LUCKNOW UNIVERSITY**

**Paper III : Fluid Mechanics II**

**Unit I**

Newtons' Law of viscosity, Nature of stress, Stress component in real fluid, Symmetry of stress tensor. Transformation of stress components. Stress invariants, Principal Stresses, Nature of Strain, Rates of strain components, transformation of rate of strain components, Rate of Strain Quadric. Relation between Stress and rate of Strain. Stokesian fluids, Boundary conditions for viscous fluid.

**Unit II**

Navier-Stoke's equation of motion-Conservation of momentum. Energy Equation-Conservation of Energy. Energy dissipation function. Energy dissipation due to viscosity. Diffusion of vorticity.

**Unit III**

Exact Solutions:

Plane Poiseuille and Couette flows between two parallel plates, Steady viscous flow through tubes of uniform cross-section in form of Circle, ellipse and equilateral triangle under constant pressure gradient. Flow between two co-axial cylinders and concentric spheres, unsteady viscous flow over a flat plate.

**Unit IV**

Reynolds number, slow viscous flow, flow past a sphere, Stoke's flow. Prandtl's Boundary layer concept, Boundary layer thickness-displacement, momentum of energy. Momentum and energy integrals, condition for separation, boundary layer flow along a semi-infinite plate at zero incidence in a uniform stream, Blasius solution. Wave motion in a gas, solution of one dimensional wave equation, Speed of Sound, Sub-sonic sound, Supersonic flows of a gas, Isentropic gas flows (elementary ideas)

Books Recommended :

1. Frank Chorlton, Text Book of Fluid Dynamics, C.B.S. Publishers, New Delhi, 1985
2. Z.U.A. Warsi, Fluid Dynamics, Theoretical and Computational approaches, C.R.C. Press
3. S.W. Yuan Foundation of Fluid Mechanics, Prentice Hall of India Pvt. Ltd., New Delhi, 1964
4. L. Rosenhead, Laminar Boundary layer, Oxford Press

**Optional Papers**

**Paper IV: Analytic Number Theory II**

**Unit I**

Dirichlet series, the half Plane of absolute convergence of Dirichlet series, the function defined by a Dirichlet series, Uniqueness theorem, Multiplication of Dirichlet series, The identities related to Riemann zeta function and various Arithmetical functions.

**DEPARTMENT OF MATHEMATICS & ASTRONOMY  
FACULTY OF SCIENCE / FACULTY OF ARTS  
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The Analytic version of fundamental theorem of Airthematic due to Euler and its appliation. The half plane of convergence of a Dirichlet series.

**Unit II**

Integral representation of the Hurwitz zeta function. A contour integral representation for the Hurwitz zeta function. The analytic continuation of the Hurwitz zeta function. Hurwitz's formula, the functional equation for the Riemann zeta function. A functional equation for the Hurwitz zeta function.

**Unit III**

Primes : A probability Argument, Mersenne primer sophie Germain Primes and Fermat Numbers. Elementary properties of  $\zeta(x)$ . Amicable pairs, Chebyshev's functions  $\psi(x)$  and  $\theta(x)$ . Relations connecting  $\psi(x)$  and  $\theta(x)$ . Some equivalent forms of the prime numbers theorem. The relation of the prime number theorem to the asymptotic value of the  $n$ th prime. Inequalities for  $\psi(n)$  and  $\theta(n)$ . Shapiro's Tauberian theorem, Application of Shapiro's theorem, Applications of Shapior's theorem. The partial sums of Mobius function. Selberg's asymptotic formula.

**Unit IV**

Analytic Proof of the Prime Number Theorem. A Contour integral representation for  $\zeta(x)$ . Upper bounds for Riemann zeta functions and their derivative. Inequalities related to Riemann zeta function. Zero- free region for Riemann zeta function. The Riemann Hypothesis classally abundant numbers. Application of prime number theorem to the divisor function and Euler's totient.

**Books Recommended :** 1. T.M. Apostal – Introduction to Analytic Number Theory, Narosa Publication House, 1980.

2. Jeffrey Stopple – A primer of Analytic Number Theory, Cambridge University Press, 2003.

**Paper V :Special Function II**

**Unit I**

**Legendre Functions:** Legendre's differential equation, Legendre polynomials, Laplace's integral for the Legendre Polynomials, Generating function, recurrence formulae, integral of a product of Legendre polynomials complete solution of Legendre's equation when  $n$  is an integer,

**Unit II**

Behaviour of  $Q_n(z)$  at infinity, Integral formula for  $Q_n(z)$  Heine's integrals for  $Q_n(z)$ , Neumann's integral for  $Q_n(z)$ , Heine's expansion of  $(z-u)^{-1}$  as a series of Legendre polynomials, Neumann's expansion theorem, Associated Legendre functions, Jacobi's Lemma, Integral representations of  $P_{mn}(z)$  and  $Q_{mn}(z)$ , addition-theorem for the Legendre polynomials.

**DEPARTMENT OF MATHEMATICS & ASTRONOMY  
FACULTY OF SCIENCE / FACULTY OF ARTS  
LUCKNOW UNIVERSITY**

**Unit III**

**Elementary theory of Orthogonal polynomials:** Introduction, Moment functional and orthogonality, Existence of OPS, Fundamental recurrence formula, Zeros, Gauss quadrature, Kernel Polynomials, Symmetric Moment functionals, related recurrence relations.

**Unit IV**

**The Representation Theorem and Distribution Functionals:** Introduction, Preliminary theorems (omitting proof of Helley's theorems), Representation theorem, Spectral points and zeros of orthogonal polynomials, determinacy of  $L$  in the bounded case.

**Books Recommended:**

- 1.E.T. Copson- Introduction to the theory of the functions of a complex variable.
- 2.T.S. Chihara- An introduction to orthogonal polynomials, Gordan & Breach.

**Optional Paper VI: Advanced Discrete Mathematics II**

**Unit I**

Graph Theory, Definitions and terminology related with directed and undirected graphs, sub graph, connected and disconnected graph, planar graph and its properties, Euler's formula for connected graph, complete graph, bipartite graph, Kuratowski's theorem (statement only) and its application, tree, forest, spanning tree, minimum spanning tree, cut sets, Kruskal's algorithm, Prim's algorithm, Adjacency matrix, Incidence matrix, linked list representation, coloring of graph, some theorems.

**Unit II**

Koingsberg bridge problem, Euler theorem of existence of Eulerian path and circuit, algorithm for finding Eulerian Circuit, In degree and out degree of a vertex, weighted undirected and directed graph, Hamiltonian path, weak and strong connectivity, path matrix, Warshall's algorithm, shortest path, Dijkstra's algorithm, directed tree, tree traversal: preorder, in order and post order traversal, expression evaluation by Polish and RPN expression.

**Unit III**

Introductory computability theory, non deterministic and deterministic finite automata and their equivalence, transition table and transition diagram,  $\epsilon$ - transition, conversion, partial recursive functions and Turing machines.

**Unit IV**

Grammars and languages, regular expression, Finite automata and regular language, context free and context sensitive grammar, rewriting rules, derivations and derivation trees, pumping lemma, sentential forms, syntax analysis, decision algorithms for CFL, LR(k) : LR(o) grammar and LR(1) grammar.



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References :

1. JP Trembley & R Manohar: Discrete Mathematical Structure with Applications to Computer Science ; MGH
2. Discrete Mathematics ; Seymour Lipschutz, Mark Lipson; TMH
3. N Deo; Graph Theory; Prentice Hall
4. Hope Craft and Ullman; Introduction to Automata Theory, Languages and Computations; Narosa Pub. House.

**Optional Paper: Non-Commutative Rings II**

**Unit I**

Modules, Irreducible Modules, Schur's Lemma, The Jacobson Radical, Semisimple Rings .

**Unit II**

Artinian Rings, Structure of Semisimple Artinian Rings, Group Algebra, Maschke's Theorem.

**Unit III**

Primitive Rings, Density Theorem, Prime Rings, Wedderburn-Artin Theorem.

**Unit IV**

Direct Product, Subdirect Sums, Applications of Wedderburn's Theorem.

Books Recommended :

1. Non-Commutative Rings, I.N. Herstein, Monograph of the Mathematical Association of America, 1968.
2. Basic Algebra II, N. Jacobson, W.H. Freeman, 1980.

**Optional Paper VIII : Mathematical Biology II**

**Unit I**

Historical asides of Epidemics, Simple Epidemic models and practical application, modeling venereal diseases

AIDS: Modeling the transmission dynamics of the HIV.

**Unit II**

HIV: Modeling combination drug therapy, delay models for HIV infection with drug therapy, modeling the population dynamics of Acquired Immunity to parasite infection.

**Unit III**

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Age dependent epidemic models and threshold criteria, Simple drug use epidemic model and threshold analysis, tuberculosis infection in Badgers and Cattle, derivation of diffusion equation, models of animal dispersion.

**Unit IV**

Background and the travelling waveform, Fisher-Kolmogoroff equation and propagating wave solution, asymptotic solution and stability of wavefront solution of the Fisher-Kolmogoroff equation, density dependent diffusion equation-reaction diffusion models some exact solution.

**Paper X Ordinary Differential Equations**

**Unit I**

Two point Boundary value Problem: Introduction, The homogeneous boundary problem, The adjoint boundary problem, The nonhomogeneous boundary problem, Self-Adjoint boundary problem.

**Unit II**

Introduction, basic concepts and definitions, Second order linear equation and methods of characteristics, the methods of separation of variables.

**Unit III**

Fourier transform and Initial boundary value problems Green's functions and boundary value problems.

**Unit IV**

Sturm-Liouville system, eigen function, Bessel function, singular Sturm-Liouville System, Legendre Function, boundary value problem for ordinary differential equations and Green's functions.

Recommended Books :

3. Non- linear Partial Differential Equations IInd Edition, Loknath Debnath, Birkhausen.
4. Partial Differential Equations of Mathematical Physics by Tyn Myint-U, Elseveir Publication

**Optional Paper XI : Approximation Theory & Wavelets II**

**Unit I**

Continuous Wavelets Transform: The Heisenberg uncertainty principle, the Shannon sampling theorem, Definition and examples of continuous wavelet transforms. A Plancherel formula, Inversion formulas, the Kernel functions, Decay of Wavelet transform.

**Unit II**

Frames : Geometrical considerations, Notion of frames, Discrete wavelet transforms signal decomposition (analysis), relation with filter banks, signal reconstruction.

**Unit III**

Multiresolution analysis, axiomatic, description, the scaling function, construction of Fourier domain.

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**Unit IV**

Orthonormal wavelets with compact support: the basic idea, Algebraic constructions, binary interpolation, spline wavelets.

Reommended Books:

1. Christian Blatter, Wavelets: A Premier, AK Peters, 2002
2. C.K. Chui, An Introduction to Wavelets, Academic Press
3. Daubechies, Ten Lectures on Wavelets, SIAM Publication, Philadepphia
4. G. Kaiser, A friendly Guide to Wavelets, Birkhauser Boston, 1994