QUANTITATIVE METHODS IN BUSINESS
PROBLEM #1: DECISION MAKING WITH PROBABILITIES

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Introduction

Managers are often faced with making decisions in an uncertain or risky environment. The decision maker must choose the best decision alternative without having any control of the events that affect the profits resulting from the decision. When the probabilities of various events happening are known, we refer to these decision-making situations as decision making under risk or decision making with probabilities.

Mathematics:

PROBABILITY

Classical Probability : Suppose that the sample space *S*, of an experiment contains *n* equally likely outcomes. The probability of any event is defined as the ratio *s/n*, where *s* is the number of favorable outcomes (outcomes contained in the event).

Example 1: Suppose that you roll one fair die. S = {1, 2, 3, 4, 5, 6}. Let *E* be the event that you roll a number less than or equal to 4 , so that *E* = {1, 2, 3, 4}. The probability of the event is *E* is *p(E)* = 4/6 = 2/3 since there are 4 favorable outcomes out of the 6 equally likely outcomes in the sample space.

The probability just computed is a theoretical one, which represents the way we would expect our experiment to behave over time. That is, if we repeated the experiment again and again over time, approximately 2/3 of the time, we would get a roll less than or equal to 4 . That is, in 3,000 rolls, for example, approximately 2,000 would be less than or equal to 4 . In reality, it is unlikely that the number would be exactly 2,000 . The larger the number of repetitions of the experiment, the closer the ratio would be to

There is another approach to computing probabilities using *relative frequency of occurrence.*

Frequency Interpretation of Probability : The probability of an event is the proportion of time that events of the same kind will occur in the long run.

Example 2: The weather service predicts a 40% chance for rain (the probability of rain is .40). This means that under the same weather conditions, it will rain 40% of the time.

We then *estimate* the probability of an event by *observing* what fraction of the time similar events have occurred in the past.

Example 3: Data shows that over a period of time, 524 of 700 jets arrived from N.Y. on time at a particular airport. We estimate that the probability of any one flight from N.Y. to this airport arriving on time is 524/700 = .749 (rounded off).

When probabilities are estimated this way, it is reasonable to question how good these estimates are.

Law of Large Numbers : If a situation is repeated again and again , the proportion of successful outcomes will tend to approach the constant probability that any one of the outcomes will be a success.

Random Variable

An experiment, game, business strategy, or other situation may lead to different outcomes with various probabilities of getting those outcomes. A random variable, X, is a mathematical description of these situations. A random variable takes on numeric values corresponding to the outcomes. Each possible outcome has an assigned probability.

Notation: If x_i represents a particular value of the random variable X, then $p(x_i)$ is the probability of getting that value. (Note: In practice, we will often not write in the subscript, even though it should really be there.)

A random variable is described by a probability distribution.

Probability Distributions

A probability distribution gives the possible outcomes of an experiment, along with the probabilities of getting the various outcomes.

In a discrete probability distribution.: the outcomes and their probabilities can be listed in a table.

x	p(x)
outcomes	probability

Example: You may have two games (or business strategies) that lead to different random variables. These two random variables can be listed in tables as below.

Game 1

x	p
$100	1/2
0	1/2

Game 2

x	p
$75	5/6
- $25	1/6

Random Variables as a Description of Reality

The same random variable can describe different situations.

Example:

Situation 1: The random variable for game 1 can describe a game in which a coin is tossed. You win $100 if heads comes up, otherwise you win nothing.

Situation 2: This random variable can also describe a business situation. If the item is sold, you get $100 profit. There is a 50% chance that the item will be sold. If the item is not sold, it may be returned to the supplier, at no cost.

Exercise:

Describe a game and a business situation that can lead to the random variable for game 2.

Making Decisions

Suppose you have the opportunity to play either game. Which one do you choose?

The answer depends on your objective, or <u>decision criteria</u>.. There are many possible criteria.

a) If you want to maximize the change of winning something, you choose game 2.
b) If you want to minimize the chance of losing, choose game 1.
c) If you want the game with the largest potential winning, choose game 1.

Decision criteria **b)** is sometimes called the "**risk aversion**" approach.

A more sophisticated way to choose is based on the <u>expected value criterion</u>. This forms the basis of decision theory. The expected value criterion is especially useful if the same situation will be occurring over and over again.

<u>Expected Value</u> of a Random Variable

If an experiment is performed many times, and the outcome of the experiment is a number, we can compute the mean of those numbers, $\bar{x}$.

If the experiment is performed many, many times, these means will approach a theoretical value, μ (mu), also called the <u>expected value</u> of the random variable, and denoted E(x).

Formula:

$$\mu = \mathbf{E(x)} = \sum_{i=1}^{N} x_i p(x_i)$$

Using E(x):

In a game, if $E(x) > 0$ play the game, for you should win in the long run.
If $E(x) < 0$ do not play. If $E(x) = 0$, we call this a "fair" game.

Expected Value Criterion for Making a Decision

Choose the game that leads to the higher expected value.

Example:

The expected value of game 1 is: $100 * .5 + 0 * .5 = 50$
The expected value of game 2 is $75 * (5/6) + (-25) * 1/6 = 58.33$.

Using the expected value criteria, choose game 2.

Exercise:

Game 1

x	p(x)
0	1/8
-1	3/8
2	3/8
7	1/8

Game 2

x	p(x)
0	2/52
1	2/52
2	40/52
3	8/52

Which game do you choose?

a) If you want to maximize the change of winning something: _____.
b) If you want to minimize the chance of losing:: _______
c) If you want the game with the largest potential winning: :_____.
d) If you want the highest expected value: ___

Answer: a) Game 2
b) Game 2
c) Game 1
d) Game 2

Excel

=SUMPRODUCT
For a discussion of the Excel =SUMPRODUCT function, see Introduction to Computers problem set #2, Salaries. The concept of vector is also discussed in the mathematics section of this problem set.

Absolute Address
For a discussion of absolute address ($B$9:$D$9), see MIS problem set #1, ABC Company. The copy command is also discussed in that problem set.

=Max and =Min
To find the maximum or minimum component of a vector, we use the function =max(vector) or =min(vector). A vector is denoted in Excel by the cell address of its first and last component, separated by a colon. Thus, if the vector [.2, .5, .3] is in cells B9 through D9, we indicate this vector by **B9:D9**.

=max(B9:D9) = .5
=min(B9:D9) = .2

Business Application

Software Development Corporation (SDC) has developed a new encryption software to facilitate secure commercial transactions over the Internet. The feasibility of the product has been proven, but each sale will require significant customer support. SDC must make a decision regarding the level of sales and development resources that must be allocated to this product next year.

The least expensive decision alternative (d_1) is to start selling the new product through existing sales channels and provide customer support as needed. The next alternative (d_2) is to assign one full-time sales person and one software specialist to focus on this product. The third alternative (d_3) is to have a team of six people dedicated to this product. Finally, a complete division (d_4) consisting of about twelve people may be created to fully automate the product and engage in an extensive marketing campaign.

The potential profit from each decision alternative depends on the market acceptance or demand for this product which may be high, moderate or low. If market acceptance is high, each of the four decision alternatives, d_1 through d_4 will yield a profit of -200, 0, 300, and 900 thousand dollars respectively. If there is a moderate demand, the profits are likely to be 100, 100, 200, and -200 thousand dollars respectively. If the demand turns out to be low, then the profits will be 200, 150, -200, and -500 thousand dollars respectively.

The industry experience with such products provides a probability estimate of demand to be high, moderate and low as .3, .5, and .2 respectively. Which of the four decision alternatives should be selected by SDC? What will be the expected profit from this decision? If a market research firm can provide perfect information about demand to SDC (i.e. whether it will be high, moderate, or low) before a product launching decision is made, how much is that information worth to SDC?

Solution to the Business Problem

To structure this decision making problem, we begin by constructing a *payoff table*. The rows of a payoff table represent the *decision alternatives* which in our example consist of four different levels of resources (d_1, d_2, d_3, d_4,) assigned to the new product by SDC. The columns of a payoff table represent *events* or *states of nature* which in our example indicate market acceptance (low, moderate, high) or demand. Our payoff table will, therefore, have 4 rows and 3 columns.

The numbers inside the payoff table will represent the profit we will make for each combination of demand and decision alternative. The payoff table (in thousands of dollars) for the SDC decision making problem is shown below along with probabilities of each event or demand level.

		Demand or Events		
		Low	Moderate	High
	d_1	200	100	-200
Decision	d_2	150	100	0
Alternatives	d_3	-200	200	300
	d_4	-500	-200	900
Probability of each event		.2	.5	.3

The decision maker can choose the appropriate decision (row) of the table to use, but the demand (or column) is beyond his or her control.

The most common approach to solve such decision making problems with known probabilities is to use the *expected value* approach. The decision alternative with the highest *expected profit* is selected.

Using the expected value formula we obtain:

E (profit from d_1) = .2 x 200 + .5 x 100 + .3 x (-200) = 30
E (profit from d_2) = .2 x 150 + .5 x 100 + .3 x (0) = 80
E (profit from d_3) = .2 x (-200) + .5 x 200 + .3 x (300) = 150
E (profit from d_4) = .2 x (-500) + .5 x (-200) + .3 x (900) = 70

Solution: The best decision is d_3 and E (profit) = \$150,000. The highest expected profit results from decision alternative d_3. This implies that the average profit from d_3 is \$150,000. Even though the actual profit from d_3 will be either -\$200,000, \$200,000, or \$300,000 depending on the demand, the average profit will be \$150,000 if this decision situation or experiment is repeated a large number of times. SDC is going to make this decision only once, so it is, therefore, possible that the company may lose \$200,000 by selecting d_3.

If the management of SDC is risk adverse, however, they may instead choose decision d_2, since there is no possibility of loss with that decision.

Finding Expected Value of Perfect Information (EVPI)

Now suppose that the market research gave us perfect information about the demand before we made the decision. We will then be able to pick the highest profit decision alternative for each demand level (i.e. the highest number in each column of the payoff table). The expected profit from perfect information (EPPI) is as follows:

$$EPPI = .2 \times 200 + .5 \times 200 + .3 \times 900 = 410$$

We can, therefore, make an average of $410,000 if we have perfect information about the demand.

$$\begin{aligned} EVPI &= EPPI - E\text{ (profit)} \\ &= \$410{,}000 - \$150{,}000 \\ &= \$260{,}000 \end{aligned}$$

The perfect information increases our average profit from $150,000 to $410,000. Therefore, the value of this information, or EVPI, is $260,000. SDC should consider getting market research information if the cost of getting such information is reasonable (i.e., significantly less than $260,000).

Excel Spreadsheet Construction

	A	B	C	D	E
1	Decision	Low	Moderate	High	Expected Profit
2	Alternatives	Demand	Demand	Demand	from each decision
3					
4	d1	200	100	-200	=SUMPRODUCT(B4:D4,$B$9:$D$9)
5	d2	150	100	0	=SUMPRODUCT(B5:D5,$B$9:$D$9)
6	d3	-200	200	300	=SUMPRODUCT(B6:D6,$B$9:$D$9)
7	d4	-500	-200	900	=SUMPRODUCT(B7:D7,$B$9:$D$9)
8					
9	Probabilities	0.2	0.5	0.3	
10					
11	Max Profit	=MAX(B4:B7)	=MAX(C4:C7)	=MAX(D4:D7)	
12				E (Profit)	=MAX(E4:E7
13				EPPI	=SUMPRODUCT (B11:D11, $B$9:$D$9)
14				EVPI	=E13-E12

Enter heading, data, and formulas as shown above. Your spreadsheet will show resulting numbers instead of formulas as shown below:

	A	B	C	D	E
1	Decision	Low	Moderate	High	Expected Profit
2	Alternatives	Demand	Demand	Demand	from each decision
3					
4	d1	200	100	-200	30
5	d2	150	100	0	80
6	d3	-200	200	300	150
7	d4	-500	-200	900	70
8					
9	Probabilities	0.2	0.5	0.3	
10					
11	Max Profit	200	200	900	
12				E (Profit)	150
13				EPPI	410
14				EVPI	260

Additional Problems

1. For the decision situation represented by the following payoff table, find the best decision, E (profit) and EVPI.

		Events			
		s_1	s_2	s_3	s_4
Decisions	d_1	10	7	9	5
	d_2	3	10	12	8
	d_3	-4	8	14	9
	d_4	-7	12	10	14
Probability		.1	.4	.3	.2

2. Suppose the numbers in the payoff table represent costs rather than profits, what will be the best decision, E (cost), and EVPI?

<u>Research Problem</u>:

Find a real life situation that routinely occurs in any business (or industry) where the expected value approach could be used for making decisions.