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UNIVERSITY OF MUMBAI

Program: M.Sc.
Course: Mathematics
Syllabus for Semester I and Semester II

(Credit Based Semester and Grading System with effect from the academic year 2012–2013)

M.Sc. Mathematics Syllabus Semester I and Semester II

Credit Based and Grading System

To be implemented from the Academic year 2012-2013

SEMESTER I

Course Code	UNIT	TOPIC HEADINGS	Credits	L / Week

PSMT101	**I**	Linear Transformations	**5**	**1**
	II	Determinants		**1**
	III	Characteristic polynomial		**1**
	IV	Inner product spaces, Bilinear forms		**1**

PSMT102	**I**	Metric Spaces, Euclidean space IR^n	**5**	**1**
	II	Continuous functions		**1**
	III	Differentiable functions		**1**
	IV	Inverse function theorem, Implicit function theorem.		**1**

PSMT103	**I**	Power series	**5**	**1**
	II	Analytic functions		**1**
	III	Complex differentiability		**1**
	IV	Cauchy's theorem		**1**

PSMT104	**I**	Number theory	**5**	**1**
	II	Advanced counting		**1**
	III	Pigeon-hole principle		**1**
	IV	Boolean algebra, Graph theory		**1**

PSMT105	**I**	Introduction to logic	**4**	**1**
	II	Sets and functions		**1**
	III	Partial order		**1**
	IV	Permutations		**1**

SEMESTER II

Course Code	UNIT	TOPIC HEADINGS	Credits	L / Week
PSMT201	**I**	Groups, group Homomorphisms	**5**	**1**
	II	Groups acting on sets		**1**
	III	Rings		**1**

	IV	Divisibility in integral domains		**1**

PSMT202	**I**	Topological spaces	**5**	**1**
	II	Connected topological spaces		**1**
	III	Compact topological spaces		**1**
	IV	Compact metric spaces, Complete metric spaces		**1**

PSMT203	**I**	Cauchy's theorem	**5**	**1**
	II	Maximum Modulus and Open Mapping Theorem		**1**
	III	Singularities		**1**
	IV	Residue calculus		**1**

PSMT204	**I**	Ordinary Differential Equations	**5**	**1**
	II	Picard's Theorem		**1**
	III	First Order Partial Differential Equation (PDE)		**1**
	IV	First Order Nonlinear Partial Differential Equation		**1**

PSMT205	**I**	Basics of Probability theory	**4**	**1**
	II	Central Limit Theorem		**1**
	III	Random Walks		**1**
	IV	Markov Chains		**1**

In addition, there will be tutorials, seminars, as necessary, for each of the five courses.

M. Sc. Mathematics Syllabus
Credit and Grading System
To be implemented from the Academic year 2012-2013

Semester I

Course Code: PSMT101

Unit I. Linear Transformations (15 Lectures)

Linear equations, Vector spaces, Review of Linear Equations, Homogeneous and non-homogeneous system of linear equations, Gaussian Elimination, Vector spaces, linear independence, basis, dimension, dual spaces, dual basis, the double dual, Linear transformations, kernel and image, rank and nullity of a linear transformation, relationship with linear equations, the algebra of linear trans- formations, relationship with matrices, rank of a matrix, transpose of a linear transformation, invertible operators.

Unit II. Determinants (15 Lectures)

Determinants as alternating n-forms, existence and uniqueness, Laplace expansion of determinant, determinants of products and transpose, determinants and invertible operators, determinant of a linear operator.

Unit III. Characteristic polynomial (15 Lectures)

Eigen values and Eigen vectors of a linear transformation, Characteristic polynomial, Minimal polynomial, Cayley Hamilton Theorem, Triangulable and diagonalizable linear operators, invariant subspaces, nilpotent operators, Jordan Canonical Form (Statement only).

Unit IV. Inner product spaces, Bilinear forms (15 Lectures)

Inner product spaces, orthonormal basis, Adjoint of an operator, Unitary operators, Self adjoint and normal operators, Spectral theorems, Bilinear forms, Symmetric bilinear forms, Classification.

Recommended Books

1. Hoffman K, Kunze R : Linear Algebra, Prentice-Hall India.
2. Serge Lang : Linear Algebra, Springer-Verlag Undergraduate Text in Mathematics.
3. Michael Artin: Algebra, Prentice-Hall India.

Course Code: PSMT102

Unit I. Metric Spaces, Euclidean space R^n (15 Lectures)

Metric spaces, examples, examples of function spaces, open and closed sets, equivalent metrics, sequences and convergence in metric spaces, metrics on R^n, equivalent metrics on R^n, Standard topology on R^n, Compact subsets of R^n, Connected subsets of R^n, Bolzano-Weirstrass Theorem, Heine Bore Theorem, Lebesgue Covering Lemma.

Unit II. Continuous functions (15 Lectures)

Continuous functions on R^n, sequential continuity, continuity and compactness, Uniform continuity, continuity and connectedness.

Unit III. Differentiable functions (15 Lectures)

Differentiable functions on R^n, Total derivative, Partial derivatives, directional derivatives, Chain rule, derivatives of higher order, C^k- functions, $C^\infty-$ functions.

Unit IV. Inverse function theorem, Implicit function theorem (15 Lectures)

Mean value theorem, maxima, minima, Taylor expansion, Inverse function theorem, Implicit function theorem.

Recommended Books

1. Walter Rudin: Principles of Mathematical Analysis, Mcgraw-Hill India.
2. Tom Aposol: Mathematical Analysis, Narosa.
3. Richard R. Goldberg: Methods of Real Analysis, Oxford and IBH Publishing Company.
4. Michael Spivak: Calculus on Manifolds, Harper-Collins Publishers.
5. Steven Krantz: Real analysis and foundations, Chapman and Hall.
6. Charles Chapman Pugh: Real mathematical analysis, Springer UTM

Course Code : PSMT103

Unit I. Power series **(15 Lectures)**

Complex Numbers, Geometry of the complex plane, Riemann sphere, Complex sequences and series, Complex power series, Sequences and series of functions in C, Uniform convergence, Power series, radius of convergence of a power series.

Unit II. Analytic functions **(15 Lectures)**

Analytic functions, Trigonometric functions, Exponential function, Branches of logarithm.

Unit III. Complex differentiability **(15 Lectures)**

Complex differentiable functions, Cauchy Riemann equations, Mobius transformations.

Unit IV. Cauchy's theorem **(15 Lectures)**

Complex functions and Line integrals, primitive, Cauchy's Theorem for a disc.

Recommended Books

1. J. B .Conway: Functions of one complex variable, Narosa.
2. Serge Lang: Complex Analysis, Springer.
3. James W. Brown & Ruel V. Churchill: Complex variables and applications, Mcgraw-Hill Asia.
4. Anant R. Shastri: An introduction to complex analysis, Macmillan.
5. R.Remmert: Theory of complex functions, Springer.

Course Code : PSMT104

Unit I. Number theory (15 Lectures)

Divisibility, Linear Diophantine equations, Cardano's Method, Congruences, Quadratic residues, Arithmetic functions, Elementary Set theory: The sum rule and the product rule, two-way counting, permutations and combinations, Binomial and multinomial coefficients, Pascal identity, Binomial and multinomial theorems.

Unit II. Advanced counting (15 Lectures)

Advanced counting : Types of occupancy problems, distribution of distinguish- able and indistinguishable objects into distinguishable and indistinguishable boxes (with condition on distribution) Stirling numbers of second and first kind, Inclu- sion Exclusion Principle and its application to derangement, Mobius inversion formula.

Unit III. Pigeon-hole principle (15 Lectures)

Pigeon-hole principle, generalized pigeon-hole principle and its applications, Erdos-Szekers theorem on monotone subsequences

Unit IV. Boolean algebra, Graph theory (15 Lectures)

Partially ordered set, Lattices, Distributive and Modular Lattices, complements, Boolean Algebra, Boolean expressions, Applications, Elementary Graph theory, Eulerian and Hamiltonian graphs, Planar graphs, Directed graphs, Trees.

Recommended Books

1. V. Krishnamurthy: Combinatorics: Theory and applications, Affiliated East-West Press.

2. Richard A. Brualdi: Introductory Combinatorics, Pearson.

3. A. Tucker: Applied Combinatorics, John Wiley & Sons.

4. Norman L. Biggs: Discrete Mathematics, Oxford University Press.

5. Kenneth Rosen: Discrete Mathematics and its applications, Tata McGraw Hills.

Course Code : PSMT105

Unit I. Introduction to logic **(15 Lectures)**

Statements, Propositions and Theorems, Truth value, Logical connectives and Truth tables, Conditional statements, Logical inferences, Methods of proof, examples.

Unit II. Sets and functions **(15 Lectures)**

Basic Set theory: Union , intersection and complement, indexed sets, the algebra of sets, power set, Cartesian product, relations, equivalence relations, partitions, discussion of the example congruence modulo-m relation on the set of integers, Functions, composition of functions, surjections, injections, bijections, inverse functions, Cardinality Finite and infinite sets, Comparing sets, Cardinality , $|A| < |P(A)|$, Schroeder-Bernstein theorem , Countable sets, Uncountable sets, Cardinalities of N , $N \times N$, Q, R, $R \times R$.

Unit III. Partial order **(15 Lectures)**

Order relations, order types, partial order, Total order, Well ordered sets, Principle of Mathematical Induction, Russel's paradox, introduction to axiomatic set theory, Statements of the Axiom of Choice, the Well Ordering Theorem, Zorn's lemma, applications of Zorn's lemma to maximal ideals and to bases of vector spaces.

Unit IV. Permutations **(15 Lectures)**

Permutations, decomposition into cycles, product of permutations, permutations and geometric symmetry, computing the order of a permutation, even and odd permutations.

Recommended Books

1. Larry J. Gerstein: Introduction to mathematical structures and proofs, Springer.
2. Joel L. Mott, Abraham Kandel, Theodore P. Baker: Discrete mathematics for computer scientists and mathematicians, Prentice-Hall India.
3. Robert R. Stoll: Set theory and logic, Freeman & Co.
4. Robert Wolf: Proof , logic and conjecture, the mathematician's toolbox, W.H.Freemon.
5. James Munkres: Topology, Prentice-Hall India.

Semester II

Course Code : PSMT201

Unit I. Groups, group Homomorphisms

Groups ,examples, subgroups, cyclic groups, Lagrange's theorem , normal sub- groups, Examples such as Permutation groups, Dihedral groups, Matrix groups, Groups of units in Z_n , Group homomorphisms, Quotient groups, direct product of groups, homomorphisms, isomorphism theorems, automorphisms, inner automorphisms, Structure theorem of Abelian groups(statement only) and applications

Unit II. Groups acting on sets

Center of a group, centralizer, normalizer, Groups acting on sets, Class equation, p-groups, Syllow's theorem, Classification of groups upto order 15.

Unit III. Rings

Rings , ideals , integral domains and fields, ring homomorphisms, quotient rings, Isomorphism theorems, polynomial rings, Quotient field, Characteristic of a ring.

Unit IV. Divisibility in integral domains

Prime elements, irreducible elements, Unique Factorization Domains, Principle Ideal Domains, Gauss's lemma, Z[X] is a UFD, Irreducibility criterions, Eisen- stein's criterion, Euclidean domains.

Recommended Books

1. Michael Artin: Algebra, Prentice-Hall India.
2. Joseph A. Gallian: Contemporary Abstract Algebra, Narosa.
3. I. N. Herstein: Topics in Algebra, Wiley-India.
4. David Dummit, Richard Foot: Abstract Algebra, Wiley-India.

Course Code : PSMT202

Unit I. Topological spaces (15 Lectures)

Topological spaces, basis, subbasis, product topology, subspace topology, closure, interior, continuous functions, Countability Axioms, Separation Axioms, Hausdorff topological spaces, Regular topological spaces, Normal topological spaces.

Unit II. Connected topological spaces (15 Lectures)

Connected topological spaces, Path-connected topological spaces, continuity and connectedness, local connectedness, Connected components of a topological space, Path components of a topological space.

Unit III. Compact topological spaces (15 Lectures)

Compact spaces, limit point compact spaces, continuity and compactness, Tube lemma, compactness and product topology, local compactness, one point compactification.

Unit IV: Compact metric spaces, Complete metric spaces (15 Lectures)

Complete metric spaces, Completion of a metric space, Total boundedness, compactness in Metric spaces, sequentially compact metric spaces, uniform continuity, Lebesgue covering lemma.

Recommended Books

1. James Munkres: Topology, Pearson.
2. George Simmons: Topology and Modern Analysis, Tata Mcgraw-Hill.
3. M.A.Armstrong: Basic Topology, Springer UTM.

Course Code : PSMT203

Unit I. Cauchy’s theorem (15 Lectures)

Cauchy’s theorem and applications, Cauchy’s estimate, Cauchy integral formula, Holomorphic functions holomorphic functions, entire functions, Louville’s theorem, Morera’s theorem.

Unit II Maximum Modulus and Open Mapping Theorem (15 Lectures)

Holomorphic functions and their properties, Maximum modulus theorem, Open mapping theorem, zeros of analytic functions, analytic continuation.

Unit III. Singularities (15 Lectures)

Isolated singularities, poles and essential singularities, Laurent Series, removable singularities, Riemann’s theorem, Casorati-Weirstrass theorem, Argument principle, Rouche’s theorem.

Unit IV. Residue calculus (15 Lectures)

Residue Theorem and its applications, evaluation of standard types of integrals by the residue calculus method.

Recommended Books

1. J. B. Conway: Functions of one complex variable, Narosa.
2. Serge Lang: Complex Analysis, Springer.
3. James W. Brown & Ruel V. Churchill: Complex variables and applications, Mcgraw-Hill Asia.
4. Anant R. Shastri: An introduction to complex analysis, Macmillan.
5. R.Remmert: Theory of complex functions, Springer.

Course Code : PSMT204

Unit I. Ordinary Differential Equations (15 Lectures)

Linear ODE with constant coefficients, linear dependence and independence of solutions, Wronskian, Solutions in the form of power series for second order linear equations, Chebyshev equation, Legendre equation, Hermite equation, Hypergeometric equation, Regular Singular points; Method of Frobenius, indicial equation, Bessel functions, Sturm- Liouville Theory Qualitative properties of solutions of Boundary value problems of linear equation of second order, Oscillation proper- ties of solutions, Sturm Liouville Separation and comparison Theorems, Eigen values and eigen functions and the vibrating string

Unit II. Picard's Theorem (15 Lectures)

Existence and Uniqueness of solutions to Initial value problem of first order ODE; Approximate solutions, Theorems of Picard and Cauchy, First order Systems of linear and nonlinear ODEs, vector notation and existence and uniqueness theorems, Application to n-th order equations

Unit III. First Order Partial Differential Equation (PDE)

(15 Lectures)

Geometric interpretation of First order Partial Differential Equations in three variables, solution as curves and surfaces, Necessary and sufficient condition for equation of the type $P\,dx + Qdy + Rdz = 0$ to be integrable, General and special solutions of the simultaneous equation of the form: $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$,

Orthogonal trajectories of a system of curves on a surface, Cauchy problem for First order Partial Differential Equations, General solution of First order Linear PDE and its connection to solutions of the simultaneous equation of the form: $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$,

Integral surfaces passing through a given curve, Surfaces orthogonal to a given system of surfaces.

Unit IV. First Order Nonlinear Partial Differential Equation
(15 Lectures)

Nonlinear PDE of First Order, Cauchys method of characteristics, Compatible systems, Charpits method, Special types of first order PDEs, Solutions satis- fying the given conditions, Jacobis method, Application of First order Partial Differential Equations.

Recommended Books

1. G.F. Simmons: Differential equations with applications and historical notes, McGraw-Hill international edition.
2. E.A. Codington: Introduction to ordinary differential Equations, Prentice- Hall India
3. Fritz John: Partial Differential Equations, Springer.
4. Hurewicz W.: Lectures on ordinary differential equations, M.I.T. Press.
5. N.Sneddon: Elements of partial differential equations, Mcgraw-Hill Newyork.

Course Code : PSMT205

Unit I. Basics of Probability theory (15 Lectures)

Review of probability basics, random variable and distributions (Bernoulli, Binomial, Poisson and normal distributions), sigma algebra, probability as a measure, probability triple (brief treatment), Simple random variable, random variable with density, general random variable and expectation for all cases, basic properties of expectation, variance and covariance, Conditional probability and independence, Conditional probability and independence, basic formulas, Borel-Cantelli theorem, Kolmogorov zero one law. Polya's urn model, genetic models

Unit II. Central Limit Theorem (15 Lectures)

Central limit theorem , Law of large numbers, Normal distributions, central limit theorem, Law of large numbers, Sterling formula, De Moivre-Laplace theorem.

Unit III. Random Walks (15 Lectures)

Random walks in one, two and three dimensions, Gambler's ruin, limiting schemes, transitional probabilities

Unit IV. Markov Chains (15 Lectures)

Markov chains, Martingales, Basic structure of Markov chain, steady state, re-currence and transience, Kolmogorov- Chapman theorem, brief treatment of a Martingale, application to pricing model.

Recommended Books

1. Kai Lai Chung and Fareed Aitsahlia: Elementary probability theory, Springer.
2. Marec Capinsky and Zastawniak T.: Probability through problems, Springer.
3. Yuan Shih Chow and Henry Teicher: Probability theory, Springer interna- tional edition.

The scheme of examination for the revised course in the subject of Mathematics at the M.A./M.Sc. Part I Programme (semesters I & II) will be as follows.

Scheme of Examination

In each semester, the performance of the learners shall be evaluated into two parts. The learners performance in each course shall be assessed by a mid- semester test of 40 marks in the first part, by conducting the Semester End Examinations with 60 marks in the second part.

External Theory examination of 60 marks:

(i) Duration:- Examination shall be of 2 Hours duration. (ii) Theory Question Paper Pattern:-

1. There shall be five questions each of 12 marks.
2. On each unit there will be one question and the fifth one will be based on entire syllabus.
3. All questions shall be compulsory with internal choice within each question.
4. Each question may be subdivided into sub-questions a, b, c, ·· and the allocation of marks depend on the weightage of the topic.
5. Each question will be of 18 marks when marks of all the subquestions are added (including the options) in that question.

Questions		Marks
Q1	Based on Unit I	12
Q2	Based on Unit II	12
Q3	Based on Unit III	12
Q4	Based on Unit IV	12
Q5	Based on Units I,II,III& IV	12
Total Marks		60