This booklet contains 24 printed pages

PAPER - 1: PHYSICS, MATHEMATICS & CHEMISTRY

Do not open this Test Booklet until you are asked to do so.

Read carefully the Instructions on the Back Cover of this Test Booklet.

Important Instructions:

1. Immediately fill the particulars on this page of the Test Booklet with *Blue / Black Ball Point Pen. Use of pencil is strictly prohibited.*
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of 3 hours duration.
4. The Test Booklet consists of **90** questions. The maximum marks are **360**.
5. There are ***three*** parts in the question paper A, B, C consisting of **Physics, Mathematics and Chemistry** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for each correct response.
6. *Candidates will be awarded marks as stated above in Instructions No. 5 for correct response of each question. ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 6 above.
8. ***Use Blue/Black Ball Point Pen only*** for writing particulars/marking responses on ***Side-1*** and ***Side-2*** of the Answer Sheet. ***Use of pencil is strictly prohibited.***
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, paper, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **3** pages at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. *However, the candidates are allowed to take away this Test Booklet with them.*
12. The CODE for this Booklet is Q. Make sure that the CODE printed on **Side-2** of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. **Do not fold or make any stray marks on the Answer Sheet.**

Name of the Candidate (in Capital letters) : ______________________

Roll Number : in figures : | | | | | | | | | in words : ______________________

Examination Centre Number : | | | | | | |

Name of Examination Centre (in Capital letters) : ______________________

Candidate's Signature : ______________ Invigilator's Signature : ______________

PART - A (PHYSICS)

1. The transverse displacement y(x, t) of a wave on a string is given by

$y(x,\ t) = e^{-\left(ax^2 + bt^2 + 2\sqrt{ab}xt\right)}$ This represents a :

1. wave moving in $-x$ direction with speed $\sqrt{\frac{b}{a}}$
2. standing wave of frequency $\sqrt{b}$
3. standing wave of frequency $\frac{1}{\sqrt{b}}$
4. wave moving in $+x$ direction with speed $\sqrt{\frac{a}{b}}$

Ans. **1**

Sol. $y(x,\ t) = e^{-[\sqrt{ax} + \sqrt{bt}]^2}$

It is transverse type $y(x,\ t) = e^{-(ax + bt)^2}$

Speed $v = \frac{\sqrt{b}}{\sqrt{a}}$

and wave is moving along –x direction.

2. A screw gauge gives the following reading when used to measure the diameter of a wire
Main scale reading : 0 mm
Circular scale reading : 52 divisions
Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.
The diameter of wire from the above data is :

1. 0.052 cm 2. 0.026 cm 3. 0.005 cm 4. 0.52 cm

Ans. **1**

Sol. Least count of screw gauge = $\frac{1}{100}$ mm = 0.01 mn

Diameter - Divisions on circular scale × least count + main scale reading

$= 52 \times \frac{1}{100} + 0$

= 0.52 mm

diameter = 0.052 cm

3. A mass m hangs with the help of a string wrapped around a pulley of a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect unifrom circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is:

1. g 2. $\frac{2}{3}g$ 3. $\frac{g}{3}$ 4. $\frac{3}{2}g$

Ans. **2**

Sol. mg – T = ma

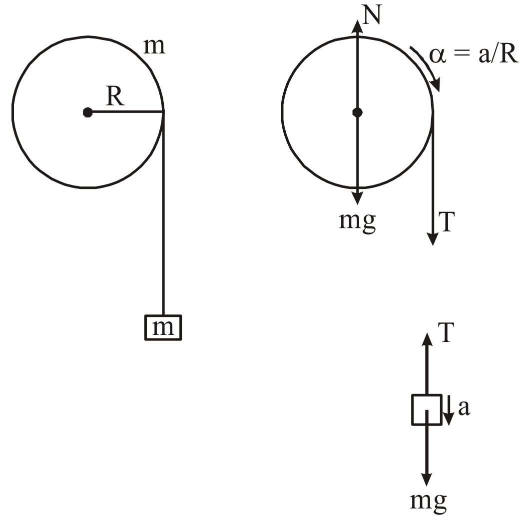
$$TR = \frac{mR^2\alpha}{2}$$

$$T = \frac{mR\alpha}{2} = \frac{ma}{2}$$

$$mg - \frac{ma}{2} = ma$$

$$\frac{3ma}{2} = mg$$

$$a = \frac{2g}{3}$$



4. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = 0.03 Nm^{-1}) :

1. 0.2π mJ 2. 2π mJ 3. 0.4π mJ 4. 4π mJ

Ans. **3**

Sol. $W = T\Delta A$

$= 0.03\ (2 \times 4\pi \times (5^2 - 3^2)\ 10^{-4}$

$= 24\pi\,(16) \times 10^{-6}$

$= 0.384\,\pi \times 10^{-3}$ Joule

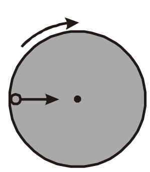
$\cong 0.4\,\pi$ mJ

5. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc:

1. continuously decreases 2. continuously increases
3. first increases and then decreases 4. remains unchanged

Ans. **3**

Sol.



From angular momentum conservation about vertical axis passing through centre. When insect is coming from circumference to center. Moment of inertia first decrease then increase. So angular velocity increase then decrease.

6. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x – axis. Their mean position is separated by distance $X_0(X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is :

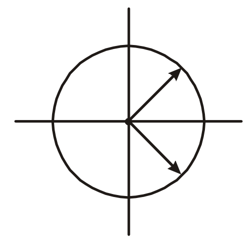
1. $\frac{\pi}{3}$ 2. $\frac{\pi}{4}$ 3. $\frac{\pi}{6}$ 4. $\frac{\pi}{2}$

Ans. **1**

Sol. $x_1 = A\sin(\omega t + \phi_1)$

$x_2 = A\sin(\omega t + \phi_2)$

$x_1 - x_2 = A\left[2\sin\left[\omega t + \frac{\phi_1 + \phi_2}{2}\right]\sin\left[\frac{\phi_1 - \phi_2}{2}\right]\right]$



$A = 2A\sin\left(\frac{\phi_1 - \phi_2}{2}\right)$

$\frac{\phi_1 - \phi_2}{2} = \frac{\pi}{6}$

$\phi_1 = \frac{\pi}{3}$

7. Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is:

1. $-\frac{4Gm}{r}$ 2. $-\frac{6Gm}{r}$ 3. $-\frac{9Gm}{r}$ 4. zero

Ans. **3**

Sol. $\frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$

$\frac{1}{x} = \frac{2}{r - x}$

$r - x = 2x$

$3x = \frac{r}{3}$

$x = \frac{r}{3}$

$-\frac{Gm}{r/3} - \frac{G(4m)}{2r/3}$

$-\frac{3Gm}{r} - \frac{6Gm}{r} = \frac{9Gm}{r}$

8. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance d(d << l) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between, them

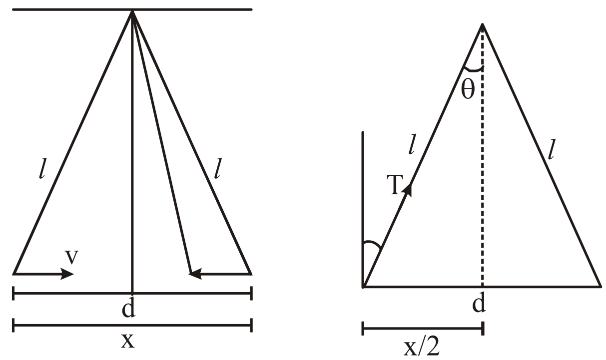
1. $v \propto x^{-1}$ 2. $v \propto x^{1/2}$ 3. $v \propto x$ 4. $v \propto x^{-1/2}$

Ans. **4**

Sol. $\sin\theta = \frac{kq^2}{d^2}$

$\cos\theta = mg$

$\tan\theta = \frac{k}{mg}\cdot\frac{q^2}{x^2}$



$\frac{x}{2l}\cdot\frac{k}{mg}\cdot\frac{q^2}{x^2}$

$x^3 = \frac{2kl}{mg}q^2$

$q^2 \propto x^3$

$q \propto x^{3/2}$

$\frac{dq}{dt} \propto \frac{3}{2}x^{1/2}\frac{dx}{dt}$ (dq/dt is constant)

$c \propto x^{1/2}\, v$

$v \propto x^{-1/2}$

9. A boat is moving due east in a region where the earth's magnetic field is $5.0 \times 10^{-5}\ NA^{-1}m^{-1}$ due north and horizontal. The boat carries a vertical aerial 2m long. If the speed of the boat is $1.50\ ms^{-1}$, the magnitude of the induced emf in the wire of aerial is:

1. 0.75 mV 2. 0.50 mV 3. 0.15 mV 4. 1 mV

Ans. **3**

Sol. $E_{ind} = B \times v \times l$

$= 5.0 \times 10^{-5} \times 1.50 \times 2$

$= 10.0 \times 10^{-5} \times 1.5$

$= 15 \times 10^{-5}$ vot.

= 0.15 mv

10. An object moving with the speed of 6.25 m/s, is decelerated at a rate given by :

$\frac{dv}{dt} = -2.5\sqrt{v}$

where v is the instantaneous speed. The time taken by the object, to come to rest would be:

1. 2s 2. 4s 3. 8s 4. 1s

Ans. **1**

Sol. $\int_{6.25}^{0}\frac{dv}{\sqrt{v}} = -2.5\int_{0}^{t} dt$

$\left|2\sqrt{\upsilon}\right|_{6.25}^{0} = -2.5t$

$2.\sqrt{6.25} = 2.5t$

t = 2 sec.

11. A fully charged capacitor C with initial change q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic field is :

1. $\frac{\pi}{4}\sqrt{LC}$ 2. $2\pi\sqrt{LC}$ 3. $\sqrt{LC}$ 4. $\pi\sqrt{LC}$

Ans. **1**

Sol. In LC oscillation energy is transfered C to L

or L to C maximum energy in L is $= \frac{1}{2}LI^2_{max}$

Maximum energy in C is = $\frac{q^2_{max}}{2C}$

Equal energy will be when

$$\frac{1}{2}LI^2 = \frac{1}{2}\frac{1}{2} LI^2_{max}$$

$$I = \frac{1}{\sqrt{2}}I_{max}$$

$$I = I_{max} \sin\omega t = \frac{1}{\sqrt{2}}I_{max}$$

$$\omega t = \frac{\pi}{4}$$

or $\frac{2\pi}{T}t = \frac{\pi}{4}$ or $t = \frac{T}{8}$

$$t = \frac{1}{8}2\pi\sqrt{LC} = \frac{\pi}{4}\sqrt{LC}$$

12. Let the x – z plane be the boundary between two transparent media. Medium 1 in z ≥ 0 has a refractive index of $\sqrt{2}$ and medium 2 with z < 0 has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\,\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is :

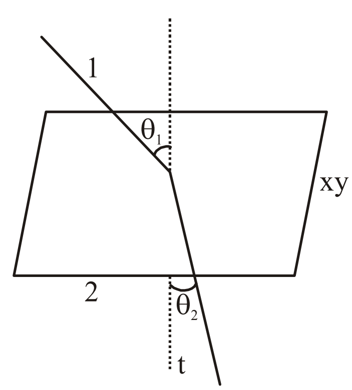
1. 45º 2. 60º 3. 75º 4. 30º

Ans. **1**

Sol. X – Y Plane

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$$

$$\cos\theta_1 = \frac{10}{\sqrt{\left(6\sqrt{3}\right)^2 + \left(8\sqrt{3}\right)^2 + 100}} = \frac{10}{\sqrt{400}} = \frac{10}{20}$$



$$\cos\theta_1 = \frac{1}{2}$$

$$\theta_1 = 60º$$

$$\sqrt{2}\sin 60º = \sqrt{3}\sin\theta_2$$

$$\sqrt{2}\times\frac{\sqrt{3}}{2} = \sqrt{3}\sin\theta_2$$

$$\sin\theta_2 = \frac{1}{\sqrt{2}}$$

$$\theta_2 = 45º$$

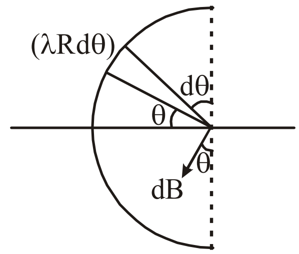
13. A current I flows in an infinitely long wire with cross section in the form of a semicircular ring of radius R. The magnitude of the magnetic induction along its axis is:

1. $\frac{\mu_0 I}{2\pi^2 R}$ 2. $\frac{\mu_0 I}{2\pi R}$ 3. $\frac{\mu_0 I}{4\pi R}$ 4. $\frac{\mu_0 I}{\pi^2 R}$

Ans. **4**

Sol. $v = \frac{1}{\pi R}$

$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{2I}{R}$ $\quad I = \lambda R\, d\theta$



$\therefore \quad B = \int_{-\pi/2}^{\pi/2} dB\cos\theta$

$= \frac{\mu_0\lambda}{2\pi}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta$

$= \frac{\mu_0\lambda}{\pi} = \frac{\mu_0 I}{\pi^2 R}$

14. A thermally insulated vessel contains an ideal gas of molecular mass M and radio of a specific heats γ. It is moving with speed v and its suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by:

1. $\frac{(\gamma - 1)}{2\gamma R} Mv^2 k$ 2. $\frac{\gamma Mv^2}{2R} K$

3. $\frac{(\gamma - 1)}{2\gamma R} Mv^2 k$ 4. $\frac{(\gamma - 1)}{2(\gamma + 1)R} Mv^2 k$

Ans. **3**

Sol. $\frac{1}{2}Mv^2 = C_v . \Delta T$

$\frac{1}{2}Mv^2 = \frac{R}{\gamma - 1} . \Delta T$

$\Delta T = \frac{M . v^2 (\gamma - 1)}{2R} = \frac{(\gamma - 1)Mv^2}{2R}$

15. A mass M, attached to a horizontal spring executing S.H.M with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is :

1. $\frac{M + m}{M}$ 2. $\left(\frac{M}{M + m}\right)^{1/2}$ 3. $\left(\frac{M + m}{M}\right)^{1/2}$ 4. $\frac{M}{M + m}$

Ans. **3**

Sol. C.O.L.M. $MV_{max} = (m + M)V_{new}$, $V_{max} = A_1\omega_1$

$V_{new} = \frac{MV_{max}}{(m + M)}$

Now, $V_{new} = A_2 . \omega_2$

$$\frac{M.A_1}{(m+M)}\sqrt{\frac{K}{M}} = A^2\sqrt{\frac{K}{(m+M)}}$$

$$A_2 = A_2\sqrt{\frac{M}{(m+M)}} \qquad \frac{A_1}{A_2} = \left(\frac{m+M}{M}\right)^{1/2}$$

16. Water is flowing continuously from a tap having an internal diameter 8×10^{-3}m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1}m below the tap is close to :

1. 7.5×10^{-3}m
2. 9.6×10^{-3}m
3. 3.6×10^{-3}m
4. 5.0×10^{-3}m

Ans. **3**

Sol. Diameter = 8×10^{-3} m

v = 0.4 m/s

$$v = \sqrt{u^2 + 2gh}$$

$$= \sqrt{(0.4)^2 + 2 \times 10 \times 0.2}$$

= 2m/s

$A_1v_1 = A_2v_2$

$$\pi\left(\frac{8 \times 10^{-3}}{4}\right)^2 \times 0.4 = \pi \times \frac{d^2}{4} \times 2$$

$$d \approx 3.6 \times 10^{-3} \text{ m.}$$

17. This question has statement – 1 and statement – 2. Of the four choices given after the statements, choose the one that best describes the two statements

Statement – 1 : Sky wave signals are used for long distance radio communication. These signals are in general, less stable then ground wave signals.

Statement – 2 : The state of ionosphere varies from hour to hour, day to day and season to season

1. Statement – 1 is true, Statement – 2 is true, Statement – 2 is the correct explanation of Statement – 1
2. Statement – 1 is true, Statement – 2 is true, Statement – 2 is not correct explanation of Statement – 1.
3. Statement – 1 is false, Statement – 2 is true
4. Statement – 1 is true, Statement – 2 is false.

Ans. **3**

18. Three perfect gases at absolute temperature T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is :

1. $\dfrac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$
2. $\dfrac{n_1T_1^2 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$
3. $\dfrac{n_1^2T_1^2 + n_2^2T_2^2 + n_3^2T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$
4. $\dfrac{(T_1 + T_2 + T_3)}{3}$

Ans. **1**

Sol. $T = \dfrac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

19. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion if reversed, is

1. more than 3 but less than 6
2. more than 6 but less than 9
3. more than 9
4. less than 3.

Ans. **1**

Sol. To reverse the direction $\int \tau d\theta = 0$ (work done is zero)

$\tau = (20\,t - 5t^2)\ 2 = 40t - 10t^2$

$\alpha = \dfrac{\tau}{I} = \dfrac{40t - 10t^2}{10} = 4t - t^2$

$\alpha = \int_0^t \alpha dt = 2t^2 - \dfrac{t^3}{3}$

ω is zero at

$2t^2 - \dfrac{t^3}{3} = 0$

$t^3 = 6t^3$

t = 6 sec.

$\theta = \int \omega dt$

$= \int_0^6 \left(2t^2 - \dfrac{t^2}{2}\right) dt$

$\left[\dfrac{2t^3}{3} - \dfrac{t^4}{12}\right]_0^6 = 216\left[\dfrac{2}{3} - \dfrac{1}{2}\right] = 36$ rad.

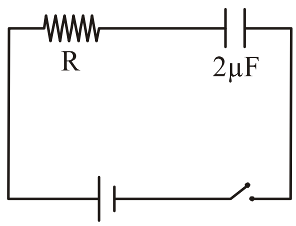
No of revolution $\dfrac{36}{2\pi}$ Less than 6

20. A resistor 'R' and 2μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. ($\log_{10}2.5 = 0.4$)

1. $1.7 \times 10^5 \Omega$
2. $2.7 \times 10^6 \Omega$
3. $3.3 \times 10^7 \Omega$
4. $1.3 \times 10^4 \Omega$

Ans. 2

Sol.



$v = 200(1 - e^{-t/t})$

$120 = 200(1 - e^{-t/t})$

$$e^{-t/t} = \frac{200 - 120}{200} = \frac{80}{200}$$

$t/\tau = \log(2.5) = 0.4$

$5 = (0.4) \times R \times 2 \times 10^{-6}$

$$\Rightarrow \quad R = \frac{5}{(0.4) \times 2 \times 10^{-6}}$$

$\Rightarrow \quad R = 2.7 \times 10^6$

21. A Carnot engine operating between temperature T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are respectively:

1. 372 K and 330 K
2. 330 K and 268 K
3. 310 K and 248 K
4. 372 K and 310 K

Ans. 4

Sol. $$\eta = 1 - \frac{T_2}{T_1} = \frac{1}{6} \quad \Rightarrow \quad \frac{T_2}{T_1} = 1 - \frac{1}{6} = \frac{5}{6}$$

$$\frac{1}{3} = 1 - \frac{(T_2 - 62)}{T_1} \quad \Rightarrow \quad \frac{T_2 - 62}{T_1} = \frac{2}{3}$$

$$\frac{5(T_2 - 62)}{T_2} = \frac{2}{3}$$

$5T_2 - 310 = 4T_2$

$T_2 = 310$ and $T_1 = \frac{6 \times 310}{5}$

$T_1 = 372$ K

22. If a wire is stretched to make it 0.1% longer, its resistance will :

1. increase by 0.2 %
2. decrease by 0.2%
3. decrease by 0.05%
4. increase by 0.05%

Ans. **1**

Sol. $R = \frac{\rho l}{A}$ $\quad (\because V = Al \text{ const.})$

$V = Al$

By differentiation $\quad O = l dA + A dl \quad$ ...(1)

By differentiation $\quad dR = \frac{\rho(Adl - ldA)}{A^2} \quad$ ...(2)

$dR = \rho \frac{2Adl}{A^2}$

$dR = \frac{2\rho dl}{A}$ $\quad$ or $\quad \frac{dR}{R} = 2.\frac{dl}{l}$

So, $\frac{dR}{R}\% = 2.\frac{dl}{l}\% \quad = 2 \times 0.1\%$

$\frac{dR}{R}\% = 0.2\%$

23. Direction:

The question has a paragraph followed by two statements, Statement – 1 and Statement – 2. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane–convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement – 1 :

When light reflects from the air–glass plate interface, the reflected wave suffers a phase change of π.

Statement – 2 :

The centre of the interference pattern is dark.

1. Statement – 1 is true, Statement – 2 is true and Statement – 2 is the correct explanation of statement – 1.
2. Statement – 1 is true, Statement – 2 is true and Statement – 2 is not the correct explanation of Statement – 1.
3. Statement – 1 is false, Statement – 2 is true.
4. Statement – 1 is true, Statement – 2 is false

Ans. **1**

Sol. When light reflects from deuser med. (Glass) a phase dift of π is generated.

Central fringe the path difference is zero therefore the centre fringe should have been bright, but due to the phase change of π from air to glass is interface the centre of fringe is dark. Therefore the statement 2 is correct and confirms statement 1.

24. A car is fitted with a convex side–view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is:

1. $\frac{1}{15}\text{m/s}$ 2. 10 m/s 3. 15 m/s 4. $\frac{1}{10}\text{m/s}$

Ans. **1**

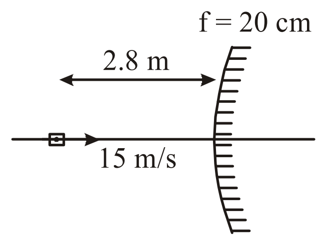
Sol. Mirrorformula:

$$\frac{1}{v} + \frac{1}{-280} = \frac{1}{20}$$

$$\frac{1}{v} + \frac{1}{20} = \frac{1}{280}$$

$$\frac{1}{v} + \frac{14 + 1}{280}$$

$$v = \frac{280}{15}$$



$$v_1 = -\left(\frac{v}{u}\right)^2 . v_{om}$$

$$\therefore \ V_1 = -\left(\frac{280}{15 \times 280}\right)^2 .15$$

$$\therefore \ V_1 = \frac{-15}{15 \times 15}$$

$$V_1 = -\frac{1}{15} \ \text{m/s}$$

25. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is :

1. 36.3 eV 2. 108.8 eV 3. 122.4 eV 4. 12.1 eV

Ans. **2**

Sol. $E_1 = -\frac{13.6(3)^2}{(1)^2}$

$$E_3 = -\frac{13.6(3)^2}{(3)^2}$$

$$\therefore \quad \Delta E = E_3 - E_1$$

$$= 13.6(3)^2 \left[1 - \frac{1}{9}\right]$$

$$= \frac{13.6 \times 9 \times 8}{9}$$

$$\Delta E = 108.8 \ \text{eV.}$$

26. The electrostatic potential indise a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is:

1. $-6a\varepsilon_0 r$ 2. $-24\pi a\varepsilon_0$ 3. $-6a\varepsilon_0$ 4. $-24\pi a\varepsilon_0 r$

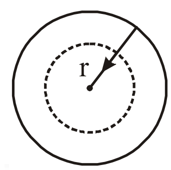
Ans. **3**

Sol. $\phi = ar^2 + b$

$$E = -\frac{d\phi}{dt} = -2ar$$

$$\oint \vec{E}\,.\,\vec{dS} = \frac{q}{\varepsilon_0}$$

$$-2ar\,.\,4\pi r^2 = \frac{q}{\varepsilon_0}$$



$$q = -8\,\varepsilon_0 a\pi r^3$$

$$\rho = \frac{q}{\frac{4}{3}\pi r^3}$$

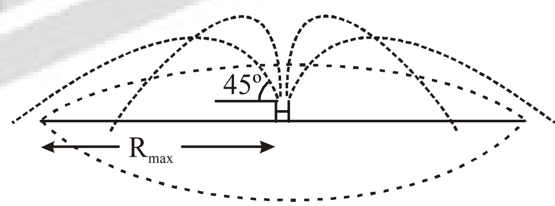
$$\rho = -6a\varepsilon_0$$

27. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is:

1. $\pi\frac{v^4}{g^2}$ 2. $\frac{\pi}{2}\frac{v^4}{g^2}$ 3. $\pi\frac{v^2}{g^2}$ 4. $\pi\frac{v^2}{g}$

Ans. **1**

Sol. $$R_{max} = \frac{v^2}{g}\sin 2\theta = \frac{v^2}{g}$$



$$\text{area} = \pi R^2$$

$$= \pi\frac{v^4}{g^2}$$

28. 100g of water is heated from 30ºC to 50ºC. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184J/kg/K) :

1. 8.4 kJ 2. 84 kJ 3. 2.1 kJ 4. 4.2 kJ

Ans. **1**

Sol. $\Delta Q = M, S, \Delta T$

$= 100 \times 10^{-3} \times 4.184 \times 20 = 8.4 \times 10^3$

$\Delta Q = 84\,kJ,$ $\Delta W = 0$

$\Delta Q = \Delta V + \Delta W$

$\therefore$ $\Delta V = 8.4\,kJ.$

29. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is :

1. 14 min 2. 20 min 3. 28 min 4. 7 min

Ans. 2

Sol. $\frac{2}{3}N_0 = N_0 e^{-\lambda t_1}$

$\frac{1}{3}N_0 = N_0 e^{-\lambda t_2}$

$2 = e^{\lambda(t_2 - t_1)}$

$\lambda(t_2 - t_1) = ln\,2$

$(t_2 - t_1) = \frac{ln\,2}{\lambda} = 20$ min.

30. This question has statement – 1 and statement – 2. Of the four choices given after the statements, choose the one that best describes the two statements

Statement – 1:

A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement – 2 :

The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

1. Statement – 1 is true, Statement – 2 is true and Statement – 2 is the correct explanation of statement – 1.
2. Statement – 1 is true, Statement – 2 is true and Statement – 2 is not the correct explanation of Statement – 1.
3. Statement – 1 is false, Statement – 2 is true.
4. Statement – 1 is true, Statement – 2 is false

Ans. 3

Sol. $hv = hv_0 + k_{max}$

$k_{max} = hv - hv_0$

PART B – MATHEMATICS

31. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersects the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R

Statement-1:

The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

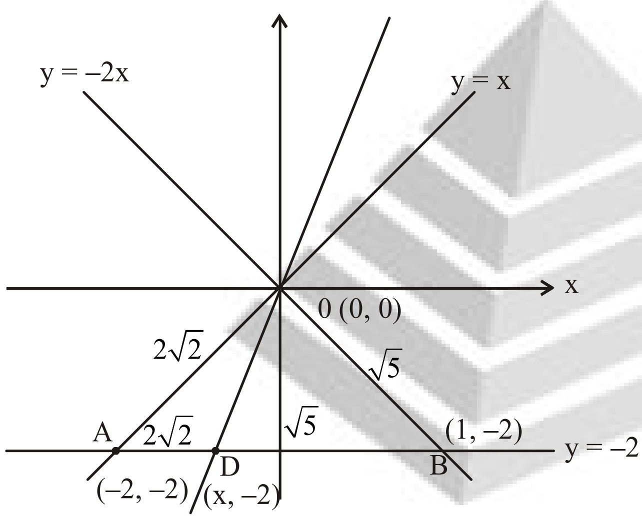
Statement-2:

In any triangle, bisector of an angle divides the triangle into two similar triangles.

1. Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
2. Statement-1 is true, Statement-2 is false
3. Statement-1 is false, Statement-2 is true
4. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Ans. **2**

Sol. $\therefore$ AD : DB = $2\sqrt{2} : \sqrt{5}$



$\because$ OD is angle bisector of angle AOB

$\therefore$ St : 1 true

St. 2 false (obvious)

32. If $A = \sin^2 x + \cos^4 x$, then for all real x:

1. $\frac{13}{16} \le A \le 1$
2. $1 \le A \le 2$
3. $\frac{3}{4} \le A \le \frac{13}{16}$
4. $\frac{3}{4} \le A \le 1$

Ans. **4**

Sol. $A = \sin^2 x + \cos^4 x$

$= \sin^2 x + (1 - \sin^2 x)^2$

$= \sin^4 x - \sin^2 x + 1$

$= \left(\sin^2 \times - \frac{1}{2}\right)^2 + \frac{3}{4} \quad = \frac{3}{4} \le A \le 1$

33. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is:

1. –132 2. –144 3. 132 4. 144

Ans. **2**

Sol. $(1 - x - x^2 + x3)^6$

$(1 - x)^6 (1 - x^2)^6$

$({}^6C_0 - {}^6C_1 x^1 + {}^6C_2 x^2 - {}^6C_3x^3 + {}^6C_4x^4 - {}^6C_5x^5 + {}^6C_6x^6) ({}^6C_0 - {}^6C_1x^2 + {}^6C_2x^4 - {}^6C_3x^6 + {}^6C_4x^8 + ... + {}^6C_6x^{12})$

Now coefficient of $x^7 = {}^6C_1{}^6C_3 - {}^6C_3{}^6C_2 + {}^6C_5{}^6C_1$

$= 6 \times 20 - 20 \times 15 + 36$

$= 120 - 300 + 36$

$= 156 - 300$

$= - 144$

34. $\lim_{x \to 2} \left(\frac{\sqrt{1 - \cos\{2(x - 2)\}}}{x - 2} \right)$

1. equals $\sqrt{2}$ 2. equals $-\sqrt{2}$

3. equals $\frac{1}{\sqrt{2}}$ 4. does not exist

Ans. **4**

Sol. $\lim_{x \to 2} \sqrt{2} \frac{|\sin(x - 2)|}{(x - 2)}$

$\therefore$ does not exist

35. **Statement-1:**

The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Statement-2:

The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

1. Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
2. Statement-1 is true, Statement-2 is false
3. Statement-1 is false, Statement-2 is true
4. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Ans. **1**

Sol. **Statement - 1 :**

$B_1 + B_2 + B_3 + B_4 = 10$

= coefficient of x^{10} in $(x^1 + x^2 + ...+ x^7)^4$

= coefficient of x^6 in $(1 - x^7)^4 (1 - x)^{-4}$

$= {}^{4+6-1}C_6 = {}^9C_3$

Statement - 2 :

Obviously $^9C_3$

36. $\frac{d^2x}{dy^2}$ equals:

1. $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

2. $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

3. $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

4. $\left(\frac{d^2y}{dx^2}\right)^{-1}$

Ans. **3**

Sol. $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{dx/dy}\right) = \frac{d}{dy}\left(\frac{1}{dx/dy}\right).\frac{dy}{dx} = -\frac{1}{\left(\frac{dx}{dy}\right)^2}.\frac{\frac{d^2x}{dy^2}}{\frac{dx}{dy}} = \frac{-\frac{d^2x}{dy^2}}{\left(\frac{dx}{dy}\right)^3}$$

37. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to:

1. 5 2. 13 3. –2 4. 7

Ans. **4**

Sol. $\frac{dy}{dx} = y + 3$

$\frac{dy}{y+3} = dx$

$ln(y + 3) = x + c$

given at $x = 0, y = 2$

$ln5 = c$

$\therefore \quad ln(y + 3) = x + ln5$

$ln\left(\frac{y+3}{5}\right) = x$

$y + 3 = 5e^x$

$y = 5e^x - 3$

$\therefore \quad y(ln2) = 5e^{ln2} - 3 = 7$

38. Let R be the set of real numbers.

Statement-1:
$A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R.

Statement-2:
$B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R.

1. Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
2. Statement-1 is true, Statement-2 is false
3. Statement-1 is false, Statement-2 is true
4. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Ans. **4**

Sol. **Statement - 1 :**

(i) $x - x$ is an integer $\forall\ x \in R$ so A is reflexive relation.

(ii) $y - x \in I \Rightarrow x - y \in I$ so A is symmetric relation.

(iii) $y - x \in I$ and $z - y \in I \Rightarrow y - x + z - y \in I$

$\Rightarrow z - x \in I$ so A is transitive relation.

Therefore A is equivalence relation.

Statement - 2 :

(i) $x = \alpha x$ when $\alpha = 1 \Rightarrow$ B is reflexive relation

(ii) for $x = 0$ and $y = 2$, we have $0 = \alpha (2)$ for $\alpha = 0$

But $2 = \alpha (0)$ for no α

so B is not symmetric so not equivalence.

39. The value of $\int_0^1 \frac{8 \log (1 + x)}{1 + x^2} dx$ is:

1. $\frac{\pi}{8} \log 2$ 2. $\frac{\pi}{2} \log 2$ 3. $\log 2$ 4. $\pi \log 2$

Ans. **4**

Sol. $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$$I = \int_0^{\pi/4} \frac{8 \ln(1 + \tan\theta)}{\sec^2\theta} \sec^2 d\theta = 8 \int_0^{\pi/4} \ln(1 + \tan\theta)\, d\theta$$

$$\Rightarrow \quad I = 8 \int_0^{\pi/4} \ln (1 + \tan (\pi/4 - \theta))\, d\theta$$

$$= 8 \int_0^{\pi/4} \ln\left(\frac{2}{1 + \tan\theta}\right) d\theta$$

$$= 8 \int_0^{\pi/4} l\,n(2)\, d\theta$$

$$= 8 \frac{\pi}{4} l\,n\,2 = 2\pi l\,n\,2$$

$\Rightarrow \quad I = \pi l\,n\,2$

40. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line $\text{Re}\, z = 1$, then it is necessary that:

1. $\beta \in (-1, 0)$ 2. $|\beta| = 1$ 3. $\beta \in (1, \infty)$ 4. $\beta \in (0, 1)$

Ans. **3**

Sol. Let roots be p + iq and p – iq $p, q \in R$

root lie on line Re(z) = 1

$\Rightarrow \quad p = 1$

product of roots = $p^2 + q^2 = \beta = 1 + q^2$

$\Rightarrow \quad \beta \in (1, \infty)$, $(q \neq 0, \because$ roots are distinct)

41. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval:

1. $\left(\frac{3}{4}, \frac{11}{12}\right]$ 2. $\left[0, \frac{1}{2}\right]$ 3. $\left(\frac{11}{12}, 1\right]$ 4. $\left(\frac{1}{2}, \frac{3}{4}\right]$

Ans. **2**

Sol. $1 - P^6 \geq \frac{31}{32}$

$P^5 \leq \frac{1}{32}$

$P \leq \frac{1}{2}$

$P \in \left[0, \frac{1}{2}\right]$

42. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 0 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after:

1. 19 months 2. 20 months 3. 21 months 4. 18 months

Ans. **3**

Sol. a = Rs. 200

d = Rs. 40

savings in first two months = Rs. 400

remained savings = 200 + 240 + 280 + ... upto n terms

$= \frac{n}{2}\left[400 + (n - 1)40\right] = 11040 - 400$

$200n + 20n^2 - 20n = 10640$

$20n^2 + 180\,n - 10640 = 0$

$n^2 + 9n - 532 = 0$

$(n + 28)(n - 19) = 0$

$n = 19$

$\therefore$ no. of months = 19 + 2 = **21**

43. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is:

1. $(0, \infty)$
2. $(-\infty, 0)$
3. $(-\infty, \infty) - \{0\}$
4. $(-\infty, \infty)$

Ans. **2**

Sol. $f(x) = \frac{1}{\sqrt{|x| - x}}$

$|x| - x > 0$

$|x| > x$

$\Rightarrow \quad x < 0$

$\therefore \quad x \in (-\infty, 0)$

44. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals:

1. $\frac{3}{2}$
2. $\frac{2}{5}$
3. $\frac{5}{3}$
4. $\frac{2}{3}$

Ans. **4**

Sol. $\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-3}{\lambda}$ ...(1)

$x + 2y + 3z = 4$ ...(2)

Angle between the line and plane is

$$\cos(90 - \theta) = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

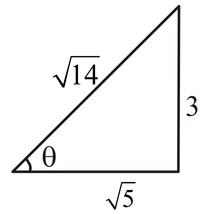
$$\Rightarrow \quad \sin\theta = \frac{1 + 4 + 3\lambda}{\sqrt{14} \times \sqrt{5 + \lambda^2}} = \frac{5 + 3\lambda}{\sqrt{14} \times \sqrt{5 + \lambda^2}} \quad ...(3)$$

But given that angle between line and plane is

$$\theta = \cos^{-1}\left(\sqrt{\frac{5}{14}}\right) = \sin^{-1}\left(\frac{3}{\sqrt{14}}\right)$$

$\Rightarrow \quad \sin\theta = \dfrac{3}{\sqrt{14}}$

$\therefore \quad$ from (3)



$$\frac{3}{\sqrt{14}} = \frac{5 + 3\lambda}{\sqrt{14} \times \sqrt{5 + \lambda^2}}$$

$\Rightarrow \quad 9(5 + \lambda^2) = 25 + 9\lambda^2 + 30\lambda$

$\Rightarrow \quad 30\lambda = 20$

$\lambda = \dfrac{2}{3}$

45. If $\vec{a} = \dfrac{1}{\sqrt{10}}(3\hat{i} + \hat{k})$ and $\vec{b} = \dfrac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$, then the value of $\left(2\vec{a} - \vec{b}\right).\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} + 2\vec{b}\right)\right]$ is:

1. –3 2. 5 3. 3 4. –5

Ans. **4**

Sol. $(2\vec{a} - \vec{b}).[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})]$

$= -(2\vec{a} - \vec{b}).[(\vec{a} + 2\vec{b}) \times (\vec{a} \times \vec{b})]$

$= -(2\vec{a} - \vec{b}).[((\vec{a} + 2\vec{b}).\vec{b})\vec{a} - ((\vec{a} + 2\vec{b}).\vec{a})\vec{b}]$

$= -(2\vec{a} - \vec{b}).[(\vec{a}.2\vec{b}) + 2\vec{b}.\vec{b})\vec{a} - (\vec{a}.\vec{a} + 2\vec{b}.\vec{a})\vec{b}]$

$= -(2\vec{a} - \vec{b}).[0 + 2\vec{a} - (0 + \vec{b})]$

$= -(2\vec{a} - \vec{b}).(2\vec{a} - \vec{b})$

$= -(2\vec{a} - \vec{b})^2 = -4\vec{a}^2 + 4\vec{a}.\vec{b} - \vec{b}^2$

$= -4 + 0 - 1 = -5$

46. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\dfrac{2}{5}}$ is:

1. $5x^2 + 3y^2 - 48 = 0$
2. $3x^2 + 5y^2 - 15 = 0$
3. $5x^2 + 3y^2 - 32 = 0$
4. $3x^2 + 5y^2 - 32 = 0$

Ans. **1, 4**

Sol. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{9}{a^2} + \frac{1}{b^2} = 1$ ...(1)

case - 1 when a > b

$b^2 = a^2 (1 - e^2)$

$b^2 = a^2 (1 - 2/5)$

$5b^2 = 3a^2$ ...(2)

from (1) & (2)

$\frac{9 \times 3}{5b^2} + \frac{1}{b^2} = 1 \quad \Rightarrow \quad b^2 = \frac{32}{5}$

$\therefore \quad a^2 = \frac{32}{3}$

$\therefore \quad \frac{3x^2}{32} + \frac{5y^2}{32} = 1 \quad \Rightarrow \quad 3x^2 + 5y^2 . 32 = 0$

case - 2 when b > a

$a^2 = b^2 (1 . e^2)$

$= \frac{3}{5} b^2$ ...(3)

from (1) & (3)

$a^2 = \frac{48}{5}, \ b^2 = 16$

$\therefore \quad \frac{5x^2}{48} + \frac{y^2}{16} = 1$

$\Rightarrow \quad 5x^2 + 3y^2 - 48 = 0$

47. Let I be the purchase value of an equipment and V(t) be the value after it has been useed for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T - t)$, where k > 0 is a constant and T is the total like in years of the equipment. Then the scrap value V(T) of the equipment is:

1. $I - \frac{kT^2}{2}$
2. $I - \frac{k(T - t)^2}{2}$
3. e^{-kT}
4. $T^2 - \frac{I}{k}$

Ans. **1**

Sol. $\frac{dv(t)}{dt} = k(T - t)$

$$\int dv(t) = \int (-KT)dt + \int ktdt$$

$$V(t) = -kTt + k\frac{t^2}{2} + c$$

at t = 0 C = I

$$V(T) = -kTt + \frac{kt^2}{2} + I$$

Now at t = T

$$V(T) = -kT^2 + k\frac{T^2}{2} + I$$

$$V(T) = I - \frac{1}{2}kT^2$$

48. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying: $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a} \cdot \vec{d} = 0$. Then the vector $\vec{d}$ is equal to:

1. $\vec{c} + \left(\frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right)\vec{b}$
2. $\vec{b} + \left(\frac{\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right)\vec{c}$
3. $\vec{c} - \left(\frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right)\vec{b}$
4. $\vec{b} - \left(\frac{\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right)\vec{c}$

Ans. **3**

Sol. $\vec{a}.\vec{b} \neq 0,\ \vec{b}\times\vec{c} = \vec{b}\times\vec{d},\ \vec{a}.\vec{d} = 0$

$$(\vec{b}\times\vec{c}) \times \vec{a} = (\vec{b} \times \vec{d}) \times \vec{a}$$

$$(\vec{b}.\vec{a})\vec{c} - (\vec{c}.\vec{a})\vec{b} = (\vec{b}.\vec{a})\vec{d} - (\vec{d}.\vec{a})\vec{b}$$

$$\vec{d} = \vec{c} - \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$$

49. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2\ (c > 0)$ touch each other if:

1. $|a| = c$ 2. $a = 2c$ 3. $|a| = 2c$ 4. $2|a| = c$

Ans. **1**

Sol. $x^2 + y^2 = ax$ ...(1)

$\Rightarrow$ centre $c_1\left(-\frac{a}{2}, 0\right)$ and radius $r_1 = \left|\frac{a}{2}\right|$

$x^2 + y^2 = c^2$ ...(2)

$\Rightarrow$ centre c_2 (0, 0) and radius $r_2 = c$

both touch each other iff

$|c_1c_2| = r_1 \pm r_2$

$$\frac{a^2}{4} = \left(\pm\frac{a}{2} \pm c\right)^2 \quad \Rightarrow \frac{a^2}{4} = \frac{a^2}{4} \pm |a|c + c^2 \quad \Rightarrow |a| = c$$

50. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is:

1. $P(C|D) \geq P(C)$
2. $P(C|D) < P(C)$
3. $P(C|D) = \frac{P(D)}{P(C)}$
4. $P(C|D) = P(C)$

Ans. **1**

Sol. $$P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)}$$

$$\frac{1}{P(D)} \geq 1$$

$$\frac{P(C)}{P(D)} \geq P(C)$$

$$P(C) \leq P\left(\frac{C}{D}\right)$$

51. The number of values of k for which the linear equations

$4x + ky + 2z = 0$

$kx + 4y + z = 0$

$2x + 2y + z = 0$

possess a non-zero solution is:

1. 2
2. 1
3. zero
4. 3

Ans. **1**

Sol. $$\Delta = \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad 8 - k(k - 2) - 2(2k - 8) = 0$

$\Rightarrow \quad 8 - k^2 + 2k - 4k + 16 = 0$

$\Rightarrow \quad -k^2 - 2k + 24 = 0$

$\Rightarrow \quad k^2 + 2k - 24 = 0$

$\Rightarrow \quad (k + 6)(k - 4) = 0$

$\Rightarrow \quad k = -6, 4$

Number of values of k is 2

52. Consider the following statements

P: Suman is brillant
Q: Suman is rich
R: Suman is honest

The negation of the statement "Suman is brillant and dishonest if and only if Suman is rich" can be expressed as:

1. $\sim (Q \leftrightarrow (P \wedge \sim R))$
2. $\sim Q \leftrightarrow \sim P \wedge R$
3. $\sim (P \wedge \sim R) \leftrightarrow Q$
4. $\sim P \wedge (Q \leftrightarrow \sim R)$

Ans. **1**

Sol. Negation of $(P \wedge \sim R) \leftrightarrow Q$ is $\sim ((P \wedge \sim R) \leftrightarrow Q)$

It may also be written as $\sim (Q \leftrightarrow (P \wedge \sim R))$

53. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is:

1. $\frac{3\sqrt{2}}{8}$
2. $\frac{8}{3\sqrt{2}}$
3. $\frac{4}{\sqrt{3}}$
4. $\frac{\sqrt{3}}{4}$

Ans. **1**

Sol. $y - x = 1$

$y^2 = x$

$$2y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2y} = 1$$

$$y = \frac{1}{2}$$

$$x = \frac{1}{4}$$

tangent at $\left(\frac{1}{4}, \frac{1}{2}\right)$

$$\frac{1}{2}y = \frac{1}{2}\left(x + \frac{1}{4}\right)$$

$$y = x + \frac{1}{4}$$

$$y - x = \frac{1}{4}$$

$$\text{distance} = \left|\frac{1 - \frac{1}{4}}{\sqrt{2}}\right| = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$$

54. If the mean deviation about the median of the numbers a, 2a, ..., 50a is 50, then |a| equals:

1. 3 2. 4 3. 5 4. 2

Ans. **2**

Sol. Median = 25.5 a

Mean deviation about median = 50

$$\Rightarrow \quad \frac{\Sigma\left|x_1 - 25.5a\right|}{50} = 50$$

$\Rightarrow$ 24.5 a + 23.5a + ... + 0.5a + 0.5a + ... + 24.5a = 2500

$\Rightarrow$ a + 3a + 5a + ... + 49a = 2500

$$\Rightarrow \quad \frac{25}{2}(50a) = 2500 \quad \Rightarrow \quad a = 4$$

55. **Statement-1:**

The point A(1, 0, 7) is the mirror image of the point B(1, 6, 3) in the line: $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

Statement-2:

The line: $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining A(1, 0, 7) and B(1, 6, 3)

1. Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
2. Statement-1 is true, Statement-2 is false
3. Statement-1 is false, Statement-2 is true
4. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Ans. **1**

Sol. Mid- point of AB ≡ M(1, 3, 5)

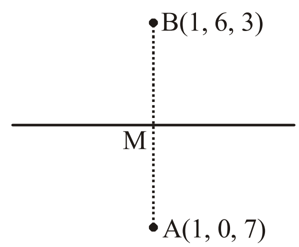
M lies on line

Direction ratios of AB is < 0, 6, – 4 >

Direction ratios of given line is < 1, 2, 3 >

As AB is perpendicular to line

$\therefore$ 0.1 + 6.2 – 4.3 = 0



56. Let A and B be two symmetric matrices of order 3.

Statement-1:

A(BA) and (AB)A are symmetric matrices.

Statement-2:

AB is symmetric matrix if matrix multiplication of A with B is commutative.

1. Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
2. Statement-1 is true, Statement-2 is false
3. Statement-1 is false, Statement-2 is true
4. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for

Statement-1

Ans. **1**

Sol. $A' = A,\ B' = A$

$P = A(BA)$

$P' = (A(BA))'$

$= (BA)'A'$

$= (A'B')A'$

$= A(BA)$

$\therefore$ A(BA) is symmetric

similarly (AB) A is symmetric

Statement (2) is correct but not correct explanation of statement (1).

57. If $\omega (\neq 1)$ is a cube root of unity, and $(1 + \omega)^7 = A + B\omega$. Then (A, B) equals:

1. (1, 1) 2. (1, 0) 3. (–1, 1) 4. (0, 1)

Ans. **1**

Sol. $(1 + \omega)^7 = A + B\omega$

$(-\omega^2)^7 = A + B\omega$

$-\omega^{14} = A + B\omega$

$-\omega^2 = A + B\omega$

$1 + \omega = A + B\omega$

$\therefore$ (A, B) = (1, 1)

58. The values of p and q for which the function $f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x} & , x < 0 \\ q & , x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}} & , x > 0 \end{cases}$ is continuous for all x in R, are:

1. $p = \frac{5}{2}, q = \frac{1}{2}$ 2. $p = -\frac{3}{2}, q = \frac{1}{2}$

3. $p = \frac{1}{2}, q = \frac{3}{2}$ 4. $p = \frac{1}{2}, q = -\frac{3}{2}$

Ans. **2**

Sol. $f(0) = q$

$$f(0^+) = \lim_{x\to 0^+} \frac{(1+x)^{1/2} - 1}{x} = \lim_{x\to 0^+} \frac{1 + \frac{1}{2} \times + ... - 1}{x} = \frac{1}{2}$$

$$f(0^-) = \lim_{x\to 0^-} \frac{\sin(p+1)x + \sin x}{x}$$

$$f(0^-) = \lim_{x \to 0^-} \frac{(\cos(p+1)x)\,(p+1) + (\cos x)}{1}$$

$$= (p + 1) + 1 = p + 2$$

$$\therefore \qquad p + 2 = q = \frac{1}{2} \qquad \Rightarrow \qquad p = -\frac{3}{2},\ q = \frac{1}{2}$$

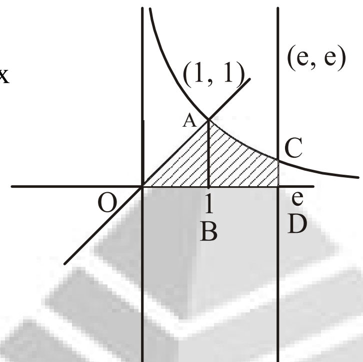
59. The area of the region enclosed by the curves $y = x$, $x = e$, $y = 1/x$ and the positive x-axis is:

1. 1 square units
2. 3/2 square units
3. 5/2 square units
4. 1/2 square units

Ans. 2

Sol. Required area

= OAB + ACDB



$$= \frac{1}{2} \times 1 \times 1 + \int_1^e \frac{1}{x}\,dx$$

$$= \frac{1}{2} + (l\,n\,x)_1^e$$

$$= \frac{3}{2} \text{ square unit}$$

60. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t}\,\sin t\,dt$. Then f has:

1. local minimum at π and 2π
2. local minimum at π and local maximum at 2π
3. local maximum at π and local minimum at 2π
4. local maximum at π and 2π

Ans. **3**

Sol. $f(x) = \int_0^x \sqrt{t}\ \sin\ tdt$

$f'(x) = \sqrt{x}\ \sin\ x$

$+ \quad - \quad +$

$0 \quad \pi \quad 2\pi \quad 5\pi/2$

local maximum at π

and local minimum at 2π

PART C – CHEMISTRY

61. Among the following the maximum covalent character is shown by the compound:

1. $SnCl_2$ 2. $AlCl_3$ 3. $MgCl_2$ 4. $FeCl_2$

Ans. **2**

Sol. Covalent character in ionic compounds is governed by Fazan's Rule. $AlCl_3$ will show Maximum covalent character on account of higher polarising power of Al^{3+} because of its having higher positive charge and smaller size.

62. The presence or absence of hydroxy group on which carbon atom of sugar differentiates RNA and DNA

1. 2^{nd} 2. 3^{rd} 3. 4^{th} 4. 1^{st}

Ans. **1**

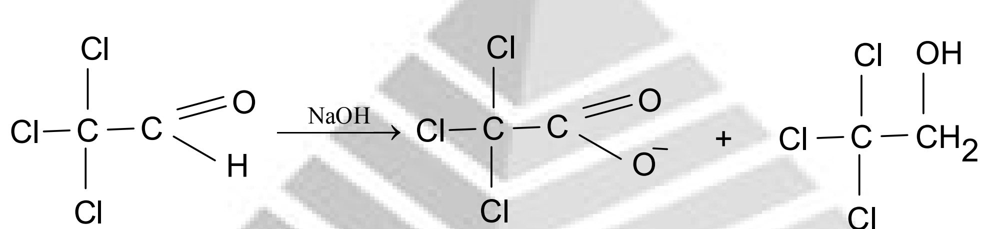
Sol. RNA and DNA has ribose and deoxyribose sugars, which differs in absence of hydroxy group at 2^{nd} carbon.

63. Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate and another compound. The other compound is.

1. Trichloromethanol 2. 2, 2, 2 - Trichloropropanol
3. Chloroform 4. 2, 2, 2 - Trichloroethanol

Ans. **4**

Sol. The cannizzaro product of given reaction yields 2, 2, 2-trichloroethanol.

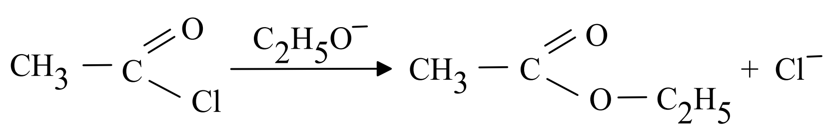


64. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in the above reaction is.

1. 2- Butanone 2. Ethyl chloride
3. Ethyl ethanoate 4. Diethyl ether

Ans. **3**

Sol.



Ethylethanoate

65. The reduction potential of hydrogen half- cell will be negative if

1. $p(H_2) = 1$ atm and $[H^+] = 1.0M$ 2. $p(H_2) = 2$ atm and $[H^+] = 1.0M$
3. $p(H_2) = 2$ atm and $[H^+] = 2.0M$ 4. $p(H_2) = 1$ atm and $[H^+] = 2.0M$

Ans. **2**

Sol. $2H^+ (aq) + 2e^- \rightarrow H_2 (g)$

$$E_{red} = E^o_{red} - \frac{0.0591}{2} \log \frac{P_{H_2}}{(H^+)^2}$$

$$E_{red} = 0 - \frac{0.0591}{2} \log \frac{2}{(1)^2}$$

$$E_{red} = -\frac{0.0591}{2} \log^2$$ $\therefore$ E_{red} is found to be negative for (3) option.

66. The strongest acid amongst the following compounds is

1. HCOOH
2. $CH_3CH_2CH(Cl)CO_2H$
2. $ClCH_2CH_2CH_2COOH$
4. CH_3COOH

Ans. **2**

Sol. α -chlorobutyric acid is more stronger acid than others due to – I effect of Cl.

67. The degree of dissociation (α) of a weak electrolyte, A_xB_y is related to van't Hoff factor (i) by the expression.

1. $\alpha = \frac{i-1}{x+y+1}$
2. $\alpha = \frac{x+y-1}{i-1}$
3. $\alpha = \frac{x+y+1}{i-1}$
4. $\alpha = \frac{i-1}{(x+y-1)}$

Ans. **4**

Sol. $A_xB_y \longrightarrow xA^{y+} + yB^{x-}$

$1-\alpha \quad\quad x\alpha \quad\quad y\alpha$

$i = 1 - \alpha + x\alpha + y\alpha$

$i = 1 + \alpha(x + y - 1)$

$$\alpha = \frac{i - 1}{(x + y - 1)}$$

68. **'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because**

1. a and b for Cl_2 < a and b for C_2H_6.

2. a for Cl_2 < a for C_2H_6 but b for Cl_2 > b for C_2H_6

3. a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6

4. a and b for Cl_2 > a and b for C_2H_6

Ans. *3*

Sol.

	a	**b**
Cl_2	**6.579 L^2 bar mol^{-2}**	**0.05622 L mol^{-1}**
C_2H_5	**5.562 L^2 bar mol^{-2}**	**0.06380 L mol^{-1}**

69. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm, the value of K is.

1. 3 atm
2. 0.3 atm
3. 0.18 atm
4. 1.8 atm

Ans. **4**

Sol. $CO_2(g) + C(S) \rightleftharpoons 2CO(g)$

0.5 atm

0.5–p 2p

Total pressure = 0.5 – P + 2P = 0.8

P = 0.3

$$K_p = \frac{P_{CO}^2}{P_{CO_2}} = \frac{(2P)^2}{(0.5 - P)} = \frac{(0.6)^2}{(0.5 - 0.3)}$$

$K_p = 1.8$

70. Boron cannot form which one of the following anions

1. BH_4^- 2. $B(OH)_4^-$ 3. BO_2^- 4. BF_6^{3-}

Ans. **4**

Sol. Due to non-availability of d-orbitals, boron is unable to expand its octet. Therefore, the maximum covalence of boron cannot exceed 4.

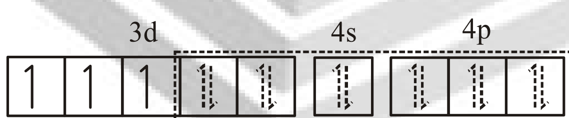
71. Which of the following facts about the complex $[Cr[NH_3]_6Cl_3$ is **wrong**

1. The complex is paramagnetic
2. The complex is an outer orbital complex.
3. The complex is gives white precipitate with silver nitrate solution.
4. The complex involves d^2sp^3 hybridisation and is octahedral in shape

Ans. **2**

Sol. In case of d^3 configuration, the number of unpaired electrons remains 3 whether the ligand is strong field or weak field. The hybridisation scheme can be shown as follow :

$[Cr(NH_3)_3]^3 =$



d^2sp^3 hybridisation

Hence the complex is inner orbital complex as it involves (n – 1) d orbitals for hybridisation, $3.93 = \sqrt{n(n + 2)}$; so n = 3 (here n is number of unpaired electron(s)).

72. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it from freezing at –6°C will be: (K_f for water = 1.86 K kg mol^{-1}, and molar mass of ethylene glycol= 62gmol–1)

1. 204.30 g 2. 400.00 g 3. 304.60g 4. 804.32 g

Ans. **4**

Sol. $\Delta T_f = iK_f m$

$\Delta T_f = 6^oC$

i = 1

$$6 = 1 \times 1.86 \times \frac{w}{62 \times 4}$$

w = 804.32 g.

73. Which one of the following orders presents the correct sequence of the increasing basic nature of
the given oxides

1. $MgO < K_2O < Al_2O_3 < Na_2O$
2. $Na_2O < K_2O < MgO < Al_2O_3$
3. $K_2O < Na_2O < Al_2O_3 < MgO$
4. $Al_2O_3 < MgO < Na_2O < K_2O$

Ans. **4**

Sol. As metallic character of element attached to oxygen atom increases, the difference between the electronegativity values of element and oxygen increases and thus basic character of oxides increases and vice-versa. Hence the increasing correct order of basic nature is $Al_2O_3 < MgO < Na_2O < K_2O$.

74. The rate of a chemical reaction doubles for every 10°C rise of temperature. If the temperature is raised by 50°C, the rate of the reaction increases by about.

1. 24 times
2. 32 times
3. 64 times
4. 10 times

Ans. **2**

Sol. $$\frac{\text{Rate at } 50^\circ C}{\text{Rate at } T_1^\circ C} = (2)^{\frac{\Delta T}{T_1}} = (2)^{\frac{50}{10}} = 2^5$$

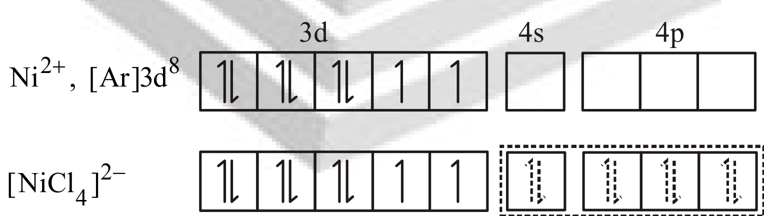
$= 32$ times

75. The magnetic moment (spin only) of $[NiCl_4]^{2-}$ is.

1. 5.46 BM
2. 2.82 BM
3. 1.41 BM
4. 1.82 BM

Ans. **2**

Sol. In the paramagnetic and tetrahedral complex $[NiCl_4]^{2-}$, the nickel is in +2 oxidation state and the ion has the electronic configuration $3d^8$. The hybridisation scheme is as shown in figure



sp^3 hybrid orbitals

$$\mu_{B.M.} = \sqrt{n(n+2)} = \sqrt{2(2+2)} = \sqrt{8} = 2.82 \text{ BM}$$

76. The hybridisation of orbitals of N atom in NO_3^-, NO_2^+ and NH_4^+ are respectively.

1. *sp^2, sp, sp^3*
2. *sp, sp^3, sp^2,*
3. *sp^2, sp^3, sp*
4. *sp, sp^2, sp^3*

Ans. **1**

Sol. NO_2^+ Number of electron pairs = 2

Number of bond pairs = 2

Number of lone pair = 0

So, the species is linear with sp hybridisation.

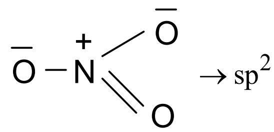
$$O = \underset{sp}{\overset{+}{N}}O$$

NO_3^- Number of electron pairs = 3

Number of bond pairs = 3

Number of lone pair = 0

So, the species is trigonal planar with sp^2 hybridisation.

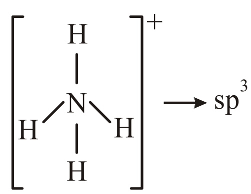


NH_4^+ Number of electron pairs = 4

Number of bond pairs = 4

Number of lone pair = 0

So, the species is tetrahedral with sp^3 hybridisation



77. In context of the lanthanoids, which of the following statements is not correct

1. All the members exhibit +3 oxidation state
2. Because of similar properties the separation of lanthanoids is not easy
3. Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series.
4. There is a gradual decrease in the radii of the members with increasing atomic number in the series.

Ans. **3**

Sol. Availability of 4f electrons donot results in the formation of compounds in +4 state for all the members of the series.

78. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH, is supplied. What is the mole fraction of methyl alcohol in the solution.

1. 0.190 2. 0.086 3. 0.050 4. 0.100

Ans. **2**

Sol. $$X_{\text{ethyl alcohol}} = \frac{5.2}{5.2 + \frac{1000}{18}} = 0.086$$

79. Which of the following statement is **wrong**.

1. nitrogen cannot form π –pπ bond.
2. Single N- N bond is weaker than the single P-P bond.
3. N_2O_4 has two resonance structures
4. The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table

Ans. **4**

Sol. The stability of hydrides decreases from NH_3 to BiH_3 which can be observed from their bond dissociation enthalpy. The correct order is $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$.

Property	NH_3	PH_3	AsH_3	SbH_3	BiH_3

$\Delta_{diss} H^{\Theta}$ (E–H) / kJ mol^{-1}	389	322	297	255	–

80. The outer electron configuration of Gd (Atomic No.: 64) is:

1. $4f^8\ 5d^0\ 6s^2$ 2. $4f^4\ 5d^4\ 6s^2$ 3. $4f^7\ 5d^1\ 6s^2$ 4. $4f^3\ 5d^5\ 6s^2$

Ans. **3**

Sol. Gadolinium ($_{64}$Gd) = $[Xe]^{54}4f^7 5d^1 6s^2$

81. Which of the following statements regarding sulphur is **incorrect**

1. The vapour at 200°C consists mostly of S_8 rings.
2. At 600°C the gas mainly consists of S_2 molecules
3. The oxidation state of sulphur is never less than +4 in its compounds.
4. S_2 molecule is paramagnetic

Ans. **3**

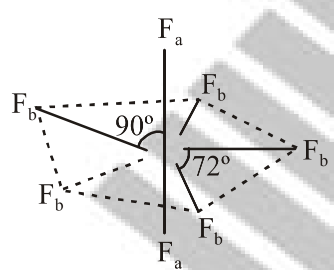
Sol. Sulphur exhibit + 2, + 4, + 6 oxidation states but + 4 and + 6 are more common.

82. The structure of IF_7 is

1. trigonal bipyramid 2. octahedral
3. pentagonal bipyramid 4. square pyramid

Ans. **3**

Sol. The structure is pentagonal bipyramid having $sp^3\ d^3$ hybridisation as given below :



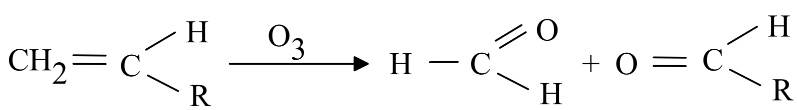
$F_b - I - F_b = 72°$ (5 number) ; $F_b - I - F_a = 90°$ (10 number)
$F_b - I$ bond length = 1.858 ± 0.004 Å ; $F_a - I$ bond length = 1.786 ± 0.007 Å.

83. **Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of:**

1 vinyl group **2. an isopropyl group**
3. an acetylenic triple bond **4. two ethylenic double bonds**

Ans. **1**

Sol.



Presence of one vinyl group gives formaldehyde as one of the product in ozonolysis.

84. A gas absorbs a photon of 355 nm and emits at two wavelengths. if one of the emissions is at 680 nm, the other is at.

1. 325 nm 2. 743 nm 3. 518 nm 4. 1035 nm

Ans. **2**

Sol. $E = E_1 + E_2$

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

$$\frac{1}{355} = \frac{1}{680} + \frac{1}{\lambda_2}$$

$$\lambda_2 = 742.76 \text{ nm.}$$

85. Silver Mirror test is given by which one of the following compounds

1. Acetone
2. Formaldehyde
3. Benzophenone
4. Acetaldehyde

Ans. **2, 4**

Sol. $CH_3-C(=O)-H \xrightarrow{[Ag(NH_3)_2]^+} CH_3-C(=O)-O^- + Ag\downarrow$

$H-C(=O)-H \xrightarrow{[Ag(NH_3)_2]^+} H-C(=O)-O^- + Ag\downarrow$

86. Which of the following reagents may be used to distinguish between phenol and benzoic acid

1. Tollen's reagent
2. Molisch reagent
3. Neutral $FeCl_3$
4. Aqueous NaOH

Ans. **3**

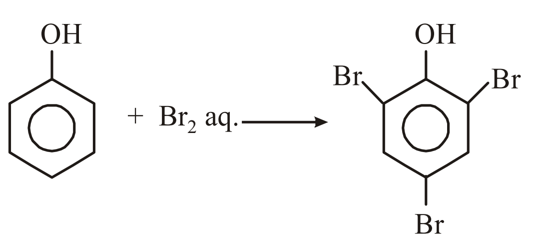
Sol. Neutral $FeCl_3$ reacts with phenol and give violet coloured complex.

87. Phenol is heated with a solution of mixture of KBr and $KBrO_3$. The major product obtained in the above reaction is

1. 3 – Bromophenol
2. 4 – Bromophenol
2. 2, 4, 6 – Tribromophenol
4. 2– Bromophenol

Ans. **3**

Sol. KBr (aq.) + $KBrO_3$ (aq.) $\rightarrow$ Br_2 (aq.)



2, 4, 6-tribromophenol

88. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. if one atom of B is missing from one of the face centred points, the formula of the compound is

2. AB_2 2. A_2B_3 3. A_2B_5 4. A_2B

Ans. 3

A B

$8 \times \frac{1}{8}$ $5 \times \frac{1}{2}$

Formula of compound A_2B_5.

89. The entropy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of 10dm3 to a volume of 100 dm^3 at 27°C is

1. $35.8\ J\ mol^{-1}K^{-1}$ 2. $32.8\ J\ mol^{-1}K^{-1}$
3. $32.3\ J\ mol^{-1}K^{-1}$ 4. $38.3\ J\ mol^{-1}K^{-1}$

Ans. 4

Sol. $\Delta S = nR \ln \frac{V_2}{V_1}$

$= 2.303\ nR \log \frac{V_2}{V_1}$

$= 2.303 \times 2 \times 8.314 \times \log \frac{100}{10}$

$= 38.3\ J\ mol^{-1}\ K^{-1}$

90. Identify the compound that exhibits tautomerism

1. Lactic acid 2. 2- Pentanone 3. Phenol 4. 2-Butene

Ans. 2

Sol. $CH_3-\overset{\overset{O}{||}}{C}-CH_2-CH_2-CH_3 \rightleftharpoons CH_3-\overset{\overset{OH}{|}}{C}=CH-CH_2-CH_3$

Read the following instructions carefully:

1. The candidates should fill in the required particulars on the Test Booklet and Answer Sheet (Side-1) with Blue/Black Ball Point Pen.

2. For writing/marking particulars on Side-2 of the Answer Sheet, use Blue/ Black Ball - Point Pen only.

3. The candidates should not write their Roll Numbers anywhere else (except in the specified space) on the Test Booklet/ Answer Sheet.

4. Out of the four options given for each question, only one option is the correct answer.

5. For each incorrect response, one-fourth (1/4) of the total marks allotted to the question would be deducted from the total score. No deduction from the total score, however, will be made if no response is indicated for an item in the Answer Sheet.

6. Handle the Test Booklet and Answer Sheet with care, as under no circumstance (except for discrepancy in Test Booklet Code and Answer Sheet Code), will another set be provided.

7. The candidates are not allowed to do any rough work or writing work on the Answer l Sheet. All calculations / writing work are to be done in the space provided for this purpose in the Test Booklet itself, marked 'Space for Rough Work'. This space is given at the bottom of each page and in 3 pages (Pages 21 — 23) at the end of the booklet.

8. On completion of the test, the candidates must hand over the Answer Sheet to the l lnvigilator on duty in the Room/ Hall. However, thecandidates are allowed to take away this Test Booklet with them.

9. Each candidate must show on demand his / her Admit Card to the lnvigilator.

10. No candidate, without special permission of the Superintendent or lnvigilator, should leave his / her seat.

11. The candidates should not leave the Examination Hall without handing over their Answer Sheet to the lnvigilator on duty and sign the Attendance Sheet again. Cases where a candidate has not signed the Attendance Sheet a second time will be deemed not to have handed over the Answer Sheet and dealt with as an unfair means case. The candidates are also required to put their left hand THUMB impression in the space provided in the Attendance Sheet.

12. Use of Electronic/ Manual Calculator and any Electronic ltem like mobile phone, pager etc. is prohibited.

13. The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination Hall. All cases of unfair means will be deal.t with as per Rules and Regulations of the Board.

14. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.

15. Candidates are not allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, electronic device or any other material except the Admit Card inside the examination hall/ room.