# **CIRCULAR MOTION**

#### Introduction-

- **Motion:** The change in position of a body.
- Linear Motion: It is simplest type of motion of a body. Also known as tranlational motion.
- **<u>Circular motion</u>**: Motion of an object along the circumference of a circle is called <u>circular</u> motion.
- **Rigid body:**
- It is defined as a body which dose not changes its shape and size under the action of force.
- with external unbalanced torque or moment of force body perform rotational motion.
- Due to the distance between partical of body remains unchanged even under the action of force. shape and size of rigid body remains constant.
- Velocity:
- The rate of change of displacement with respect to time.
- Average velocity -
- It is the ratio of total displacement to the time taken.

Average velocity =  $\frac{\text{Total displacement}}{\text{Total time taken}}$ 

#### A **Uniform velocity -**

≻ A body is said to be moving with uniform velocity, if its speed & its direction of motion, both are constant with respect to time.

#### A Variable velocity -

A body is said to be moving with variable velocity if its speed or its direction of motion or both  $\succ$ are changed with respect to time.

#### A Instanteneous velocity -

It is defined as the velocity of body at particular instant of time or at a particular point of it's  $\geq$ path.

Instantaneous velocity =  $\frac{\lim_{\delta t \to 0} \vec{\delta s}}{\delta t \to 0} = \frac{\vec{ds}}{\delta t}$ 

#### A Acceleration -

It is the rate of change of velocity with respect to time.

#### A Average acceleration -

It is the ratio of the total change in velocity during motion to the total time taken.

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Z	Angular displacement -
-	The rate of change of angular displacement with timeis called the <b>angular velocity.</b>
-	The rate of change of angular velocity with time is called the <b>angular acceleration.</b>
-	Uniform circular motion is the motion of particle along the circumference of a circle with
	constant linear speed. It can also be defined as the motion of particle along the circumference
	of a circle <u>with constant angular velocity.</u>
Z	Linear velocity = radius x angular velocity
Z	Linear acceleration = radius x angular acceleration
Z	Retardation -
	It is the negative acceleration
X	Speed -
	It is the distance covered by a body in unit time.

### 1.1 CIRCULAR MOTION

Circular motion -Defination :- Motion of particle along the circumference of circle is called circular motion.

#### OR

Motion of an object along curved path is called circular motion.

### > Examples :

- When car takes turn, is circular path.
- Electron moves around nucleus in an atom.
- Particles of spining top.

### > It is also called roatational motion -

- **UCM** The motion of the particle along the circum ference of circle with constant linear speed is uniform circular motion.
- Periodic phenomenon : Any motion which goes on repeating in equal intervals of time is called periodic motion.
- > **Period** Time taken for a particle performing uniform circular motion to complete one revolution is called it's period.  $\therefore T = 2\pi/\omega$
- > Frequency (n) The number of revolution

performed per unit time by the particle (1s).

Axis of rotation - The line passing through the centre of the circular path and perpendicular to the plane of rotation is called Axis of rotation.

Radius Vector :- A vector drawn from the centre of circle to position of particle in circular motion is called radius vector. OR

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 $\therefore n = \frac{1}{T} = \frac{\omega}{2\pi}$ 

The line joining the centre of the circle and the particle performing circular motion indicates radius vector. The vector is directed towards the point of position of particle.

1.2 ANGULAR DISPLACEMENT.

### Angular displacement ( $\theta$ ) :

Angle traced by radius vector at the center in given time is called angular displacement

### It is along the Axis of rotation.

Angular displacement of all points of rigid body is same.

- It's SI unit is radian.
- Practicle unit of it is degree.
  i.e. 1 rad = 57.3°
  - $\therefore$  2  $\pi$  rad = 360<sup>°</sup> = 1 revolution

#### 🕝 Note -

- Direction of infinitesimal angular displacement is given by right hand rule.
   Finite angular displacement does not obey commutative and associative laws of
- vector addition hence it is not true vector.
   While infinitesimal angular displacement obeys these laws hence it is true vector.
- Angular displacement possesses magnitude as well as direction, hence it is vector quantity.
- 🖎 <u>Right hand Rule :</u>
- Imagine the axis of rotation to be held in right hand with fingers curled round the axis and thumb stretched along the axis. If the curled fingers denotes sense of rotation, then thumb denotes direction of vector.
- 1.3 ANGULAR VELOCITY ( $\omega$ ):
- The rate of change of angular displacement with time is called angular velocity.

#### OR

Angle traced per unit time by the radius vector is angular velocity.

**i.e.** i.e. 
$$\omega = \frac{d\theta}{dt}$$

- SI unit is radian / second (rad/sec.)
- ➢ It dimensions are [M<sup>0</sup>L<sup>0</sup>T<sup>-1</sup>]
- It is a vector quantity and represented in magnitude and direction by using right hand rule.

$$\vec{\omega} = \frac{\delta \vec{\theta}}{\delta t}$$
 In vector notation.

> Angular speed of every point on earth is

$$\omega = \frac{2\pi}{24}$$
 rad / hr =  $\omega = \frac{2\pi}{24 \times 60 \times 60}$  rad/s.

- 1.4 ANGULAR ACCELERATION :
- Angular acceleration is defined as the rate of change of angular velocity.
- > If in a time  $\delta_t$  the angular velocity changes from  $\vec{\omega}_1$  to  $\vec{\omega}_2$  then angular acceleration,

$$\vec{\alpha} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{\delta t} = \frac{\delta \vec{\omega}}{\delta t}$$

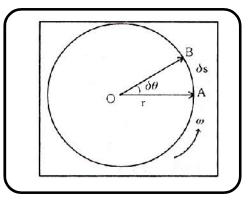
- > SI unit = radian per square second.  $(rad/s^2)$ 
  - **Dimension** =  $[M^0L^0T^{-2}]$

 $\geq$ 

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- It is a vector quantity having direction similar to angular velocity.
  - Angular acceleration is same for all points of rigid body.
- For UCM angular velocity is constant so
   angular acceleration is zero.
- 1.4 RELATION BETWEEN LINEAR VELOCITY & ANGULAR VEL-OCITY



Angular displacement  $\delta \theta = \frac{\delta s}{r}$  :  $\delta s = r \, \delta \theta$ 

$$\lim_{\delta t \to 0} \frac{\delta s}{\delta t} = r \lim_{\delta t \to 0} \frac{\delta \theta}{\delta t}$$

$$\frac{ds}{dt} = r \frac{d\theta}{dt} : v = r, w$$

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$$n = \frac{1}{T}$$
 ..... (3)

 Hence frequency is reciprocal of period put value of T in (3)

$$n = \frac{v}{2\pi r}$$

$$n = \frac{\omega}{2\pi} \quad -----(4)$$

$$\frac{v}{2\pi r} = \frac{\omega}{2\pi}$$

$$v = r \omega$$

$$\therefore \quad \omega = 2\pi n.$$

or 
$$v = 2\pi r n$$

Frequency of revolution or rotation:

- The number of circular rotations completed by the partical in one second is called rotational frequency of circular motion
- > It is express as revolution per second rotation per second.
- > often it is abbreviated as **rps**.
- 1.7 RADIAL ACCELERATION :

The acceleration of particle performing UCM is along radius and directed towards centre called radial (centripetal) acceleration.

- Acceration in UCM is perpendicular to tangential velocity.
- Accelaration in UCM is along the radius & towards center so it is radial accelaration.
- > Magnitude of acceration in UCM is given by

$$a = \frac{v^2}{r} = r\omega^2 = v\omega$$

- 1.8 CENTRIPETAL FORCES :-
- > Centripetal force can be produced in

#### inertial frame of reference.

- Centripetal force is along the radius and towards the centre of circle.
- Centripetal force is real force because known interaction such as gravitational, electrical or nuclear produces such force.
- Radialy directed inward force which provides necessary centripetal acceleration for circular motion is called centripetal force.
- > Centripetal force is given by

$$F = \frac{mv^2}{r} = mvw^2 = mvw$$

- In vector form,  $\vec{F} = -\frac{mv^2}{r}\vec{ro} = -mw^2\vec{r}$ 
  - where  $\vec{ro}$  is the unit vector in the direction of radius vector  $\vec{r}$ .
- In circular motion of a sattellite round the planet required centripetal force is provided
   by gravitational force of attration between

earth and satellite i.e.  $\frac{mv^2}{r} = \frac{GMm}{r^2}$ 

For circular motion of stone tied to a string, required centripetal force is provided by the tension in the string.

### • Examples :

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- 1) If object is tied to string and whirled in circle (horizontal) the necessary centripetal force for circular motion is provided by tension in the string.
- 2) Circular Motion of the moon round the earth is, due to gravitational force between the earth and the moon.
- 3) In an atom electron revolve in circular orbit round nucleus, necessary C.P.F. is provided by electrostatic force of attraction.

#### CENTRIFUGAL FORCE :-

- Centrifulgal force is equal in magnitude to centripetal force but opposite in direction.
- Centrifugal force is canbe produced in non inertial frame of reference.
- Radially directed outward force as experienced by body in circular motion is called C.F.
- Centrifugal force is not due to any known interactions such as gravitational, electrical or nuclear.
- > Centrifugal force is given by

$$F = \frac{mv^2}{r}mrw^2 = mvw$$

In vector form,

 $\vec{F} = \frac{mv^2}{ro} \vec{ro}$  OR  $\vec{F} = mw^2$ 

Where  $\vec{ro}$  is unit vector in direction of radius vector.

- In non-inertial frame of reference in order to apply Newton's laws of motion we have to consider an imaginary force called **Pseudo force.**
- Flattening of earth at the Poles, drier, cream separator are examples of C.F.
- 1.9 MAXIMUM SPEED OF VEHICLE ALONG A HORIZONTAL CURVED ROAD.
- If vehicle is moving on horizontal circular path, the weight of vehicle is balanced by normal reaction N and frictional force between tyers and road surface provides

necessary centripetal force  $\therefore \frac{mv^2}{r} = \mu mg$ 

This is max<sup>m</sup> speed of a vehicle of mass in which can take a circular turn of radius r without skidding.

- When vehicle takes a turn on a road, it has tendency to skid away from centre of curvature of the road due to inertia.
- At high speed and sharp turn, friction is not able to provide the required centripetal force.
- In order to take safe turn on curved road surface is banked since friction causes unneccessary wear and tears of the tyers.

### Banking of road -

Raising of outer edge of curved road slightly than inner edge to avoids, the danger of accidents is called banking of

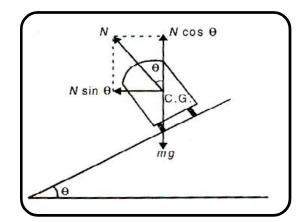
#### road. OR

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The surface of the road, along the curve is kept inclined to the horizontal at some suitable angle, so that outer edge of the road is at higher elevation than inner edge this arrangement is called banking of roads.

### 1.10 BANKING OF ROADS :

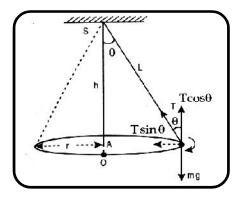


In banking of roads, road surface in inclined to the horizontal in such a way that outer edge of road is raised above the inner edge.

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help of string to revloved in horizontal circle.

• Period Of Conical Pendulum :-



- Consider a body of mass m is attached with string of length l and string is attached to rigid support at upper end revolving in horizontal circle of radius r.
- If string makes on angle Q with vertical then forces on mass m are
  - i) Weight (mg) ii) Tension in string (T)
- Tension (T) is resolved into two mutually perpendicular components
  - a)  $\mathbf{T} \cos \theta$  which balances weight  $\therefore \mathbf{T} \cos \theta = \mathbf{mg}$

b) T sin  $\theta$  which provided necessary centripetal force

$$\therefore \mathbf{T} \cos \theta = \mathbf{m} \mathbf{r} \omega^2.$$

$$\therefore \tan \theta \frac{r\omega^2}{g} \quad \text{i.e.} \quad \therefore w^2 = \frac{g \tan \theta}{r}$$

i.e. 
$$\therefore \omega = \frac{\sqrt{g \tan \theta}}{r}$$

$$\therefore \frac{2\pi}{T} = \frac{\sqrt{g \tan \theta}}{r}$$

ie. 
$$T = 2\pi \sqrt{\frac{l\sin\theta}{g\tan\theta}} = 2\pi \sqrt{\frac{l\sin\theta}{g\sin\theta/\cos\theta}}$$

Where  $r = l \sin \theta$ 

$$T = 2\pi \sqrt{\frac{l\cos\theta}{g}} = 2\pi \sqrt{\frac{h}{g}}$$
 where,

 $h = l \cos \theta$ 

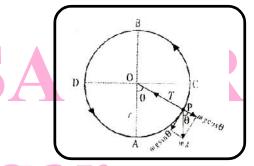
> **Period of conical pendulum** is given by

$$T = 2\pi \sqrt{\frac{l\cos\theta}{g}} = 2\pi \sqrt{\frac{h}{g}}$$

- > Period of Pendulum depends upon.
  - i) length
  - ii) angle of inclination
  - iii) height
- > Angular speed of the pendulum is,

$$\omega = \sqrt{\frac{g}{l\cos\theta}} = \sqrt{\frac{g}{h}}$$

1.12 VERTICAL CIRCULAR MOTION:



- The speed of the body goes on decresing because gravity oppses motion when body move from lowest position of vertical circle to the highest position.
- while the speed of body goes on incresing because gravity helps the motion when body moves from highest position to lowest position
- Hence speed of body is minimum at the highest position and it is maximum at the lowest position.
- consider the body of mass m tied to a string
   & whirled in vertical circle. At any point p
   with angular displacement θ various
   forces acting on the body are

weight mg & 2) tention in string
 Weight mg is resolved into two components:

a) mg  $\cos \theta$  along string

 $\sum_{n=1}^{\infty}$ 

b) mg sin θ perpendicular to string .  
For circular motion of body required centripetal force
$$\frac{mv^2}{r} = T - mgcos0$$

$$\therefore T = \frac{mv^2}{r} + mgcos0$$
ii) At highest Position Q = 180°.  

$$\therefore T_{\pi} = \frac{mv^2}{r} + mg (\because cos θ = -1)$$
At the highest position of a circle tension is minimum and velocity is minimum.
In circular motion, tension in the string T should be greater than or equal to zero.  

$$\therefore \frac{mv^2}{r} = mg \times V_n = \sqrt{rg}$$

$$\therefore T_{\epsilon} = \frac{mv_{\epsilon}^2}{r} + mg (\because cos θ = -1)$$
At the howest position of a circle tension is minimum velocity at the highest position to complete circular path is  $\sqrt{rg}$ .
ii) At the lowest position  $\theta = 0^{0}$ 

$$\therefore T_{\epsilon} = \frac{mv_{\epsilon}^2}{r} + mg (\because cos θ = 1)$$
Potential energy is minimum and tension is maximum at the lowest position increase in potential energy.  

$$K \cdot E_{\epsilon} + P \cdot E_{\epsilon} = K \cdot E_{\mu} + P \cdot E_{\mu}$$

$$(mg \times 2r)$$
By law of conservation of energy.  

$$\therefore \frac{1}{2} mv_{\epsilon}^2 + 0 = \frac{1}{2} mv_{\mu}^2 + mg \times 2r$$

$$\therefore V_{L}^2 = V_{H}^2 + 4gr$$
Minimum velocity at the highest position must be  $V_{\mu} = \sqrt{rg}$ 

$$\therefore V_{L}^2 = V_{\mu}^2 + 4gr$$
Minimum velocity at the highest position must be  $V_{\mu} = \sqrt{rg}$ 

$$\therefore V_{L}^2 = rg + 4rg$$
i.e.  $V_{L} = \sqrt{srg}$ 

$$\frac{V_{L}}{V_{L}} = \sqrt{srg}$$

$$\frac{V_{L}}{V_{L}} = \frac{V_{L}}{V_{L}} = \sqrt{srg}$$

$$\frac{V_{L}}{V_{L}} = \frac{V_{L}}{V_{L}} = \sqrt{srg}$$

$$\frac{V_{L}}{V_{L}} = \frac{V_{L}}{V_{L}} = \frac{V_{L}}{V$$

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CIRCULAR MOTION

#### 1.14 KINEMATIC EQUATION:-

Linear motion	<b>Rotational motion</b>
V = u + at	$\omega_2 = \omega_1 + \alpha t$
$S = ut + 1/2 at^2$	$\theta = \omega_1 t + 1/2 \alpha t^2$ .
$V^2=u^2+2as$	$W_{2}^{2} = \omega_{1}^{2} + 2 \alpha t$
Sn=u+a(n-1/2)	$\theta_n th = \omega_1 + \alpha (n-1/2)$
$\mathbf{S} = \left(\frac{V+u}{2}\right)\mathbf{t}$	$S = \frac{\omega_1 + \omega_2}{2}$

A EXAMPLES OFMOTION :-

CIRCULAR.

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- Bucket revolving in vertical cirtcle:-1)
- When bucket full of water is revolved in  $\succ$ vertical circle with sufficient speed then water in it does not fall out at highest point in inverted position because at this position, weight of water is balanced by radially outward cenrifugal force.

$$\frac{mv^2}{r} = mg$$

 $\therefore V^2 = rg$  OR V =  $\sqrt{rg}$ 

- Motion of aeroplane along a curve :-2)
- When aeroplane takes a curved filght it has  $\triangleright$ to bend inward because in this case reaction is produced by air.
- This reaction has two component  $\triangleright$
- i) Vertical component of reaction lifs the aeroplane  $\therefore R \cos \theta = mg$
- Horizontal component of reaction provdes ii) necessary centripetal force

 $\therefore S \sin \theta = mv^2 / r$ 

$$\therefore \tan \theta = \frac{v^2}{rg}$$

- **Bending of cyclist** 3)
- Motion of the blades of fan **4**)
- Motion of the tips of the hands in a clock. 5)

D **PSEUDO FORCE** 

A frame of referance which is fixed or ≻

moving with costant linear velocity relative to fix frame is called inertial frame of reference.

- A frame of referance which moves with  $\triangleright$ acceleration relative to fixed frame of refrance is called **non-inertial or** accelerated frame of referance.
- Newton's law's of motion are not obeyed  $\triangleright$ in non-inertial frame of referance.
- Here to apply Newton's laws existance of  $\triangleright$ one or more fictitious forces acting on object is assumed.
- $\triangleright$ Such forces are not real forces as they arises due to accelerated frame of reference called pseudo force.
- $\triangleright$ A real force is that force which is produced due to gravitational or electrical or nuclear intraction.
  - Centrifugal force does not belongs to any of these intractions but it arises due to accelerated frame of reference, hence C.F.F. called pseudo force.
- A The expression for safety speed of a vehicle moving along a plain curved road. Consider a vehicle of mass 'm' and moving with velocity 'V' along curved plain road.
- ≻ Let r = radius of curved road.
- $\succ$ The necessary centripetal force is supplied by friction between wheels and surface of road.
- $\succ$ The maximum frictional force is µmg. where  $\mu$  = coefficient of friction.

For equilibrium of car.

**Centripetal force = frictional force -- -----(1)** 

$$\frac{mv^2}{r} = \mu mg$$
$$v^2 = \mu rg$$

 $v = \sqrt{\mu r g}$  -----(2)

equation (2) gives maximum speed with which vehicle can move safely along curved plain road.

## VERTICLE CIRCULAR MOTION DUE TO EARTH'S GRAVITATION:

- When a body moves along circular path in verticle plain, it's motion is called 'verticle circular motion'.
- Consider a body of mass m tied to one end of string and whirl in a verticle circle of radius r as shown in figure, then the speed of the body at different position is different.
- As body moves from lowest point A of verticle circle to the highest point B, the speed of the body goes on decreasing because gravity opposes the motion.
- Tension in the string at point 'A & B': At point 'A' as weight & tension (T<sub>1</sub>) are in opposite direction .The centripetal force proved by (T<sub>1</sub>-mg)

$$\therefore T_1 - mg = \frac{mv_1^2}{r}$$
$$\therefore T_1 = \frac{mv_1^2}{r} + mg \rightarrow (1)$$

(maximum value of tension)

At point B  $T_2$  & weight both are in downward direction. The centripetal force is provided by ( $T_2$ +mg)

$$\therefore T_2 + mg = \frac{mv_2^2}{r}$$
$$\therefore T_2 = \frac{mv_2^2}{r} + mg \rightarrow (2)$$

(minimum value of tension) At any intermediate point the tension in the string is given by general equation,

$$T = \frac{m}{r} [v_1^2 + rg - 2gh] \rightarrow (3)$$

Where h is height of position of particle from lower point A

🖎 Net forces in string :

The componant of mg opposite to T is mg  $\cos \theta$ 

:. Net forces in string towards the centre of circular path.  $F = T - mg \cos \theta$ At lowest point -

 $A, \quad \theta = 0 \quad \therefore \cos \theta = 1 \& T = T_1$ 

 $\therefore$  F<sub>1</sub>=T<sub>1</sub>-mg **minimum** value of net force.

At highest point B -

 $\succ$ 

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 $\theta = 180^{\circ}$  ::  $\cos 180^{\circ} = -1$  &  $T = T_2$ 

 $\therefore$  F<sub>2</sub>=T<sub>2</sub>-(-mg)=t<sub>2</sub>+mg **maximum** value of net force.

Kinematical equation for circular motion.

> If in a circular motion speed changes continuously with constant angular acceleration  $\alpha$  then using anology between uniformly accelerated linear motion & circular mo-

tion. If the angular velocity of a body in circular motion changes from  $\omega_0$  to  $\omega$  in time t, with uniform angular acceleration  $\alpha$  then,

 $\omega = \omega_0 + \alpha t$  analogus to V = u + at

 $\theta = \omega_0 t + \frac{1}{2} at^2 \text{ analogus to } S = ut + \frac{1}{2} at^2$  $\omega^2 = \omega_0^2 + 2\alpha \ \theta \text{ analogus to } V^2 = u^2 + 2as$  $\bigstar \bigstar \bigstar \bigstar \bigstar$