## 1. CIRCULAR MOTION

## Introduction-

X Motion: The change in position of a body.
X Lincar Motion:It is simplest type of motion of a body.. Uso known as tranational motion.

Circular motion: $\backslash$ ation of an object along the circumbernce of a circk is called circular motion.
— Rigid body

- It is delined as a body which dose not chanees its shape and sion under the action of foree.

- Due to the distance between patical of body remains anchangederen under the action of foree. shapeandsico ofrigid boderemainsconstant.
Velocit!:
- The rate of change of displacement with respertotime.

X Werage velocity -

- It is the ration of total displatement on the time laken.

Werage velocity $=\frac{\text { Total displacement }}{\text { Total time taken }}$
2. Uniform velocity -
$>$ A body is said to be moving with uniform velocity, if its speed \& its directionof motion, both are constant with respect to time.
29 Variable velocity -
$>A$ body is said to be moving with variable velocity if its speed or its direction of motion or both are changed with respect to time.

## Instanteneous velocity -

$>\quad$ It is defined as the velocity of body at particular instant of time or at a particular point of it's path.
Instantaneous velocity $=\lim _{\delta t \rightarrow 0} \frac{\vec{\delta} s}{\delta t}=\frac{\vec{d} s}{d t}$
Acceleration -
It is the rate of change of velocity with respect to time.

## Average acceleration -

It is the ratio of the total change in velocity during motion to the total time taken.

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Angular displacement -
- The rate of change of:angular displacement with timets called the angular velocity.
- The rate of change of angular velocity with time is called the angular acceleration.
- Laform circular motion is the motion of particle along the circumference of a circle with
constan linear sped. It can also be delined as the motion of particle along the circumbence
of a circle with constant amgular velocity.
Lincar velocity = radius x angular velocity
Lincar acceleration \(=\) radius x angular acceleration
Retardation -
It is the ne gatiocacceleration
Speed -
It is the distance cosered by a body in unit time.
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### 1.1 CIRCULAR MOTION

## 2 Circular motion -

Defination :- Motion of particle along the circumference of circle is called circular motion.

OR
Motion of an object along curved path is called circular motion.
> Examples :

- When car takes turn, is circular path.
- Electron moves around nucleus in an atom.
- Particles of spining top.
$>\quad$ It is also called roatational motion UCM - The motion of the particle along the circum ference of circle with constant linear speed is uniform circular motion.
$>\quad$ Periodic phenomenon :Any motion which goes on repeating in equal intervals of time is called periodic motion.
$>\quad$ Period - Time taken for a particle performing uniform circular motion to complete one revolution is called it's period. $\therefore \mathrm{T}=2 \pi / \omega$
$>\quad$ Frequency $(\mathbf{n})$ - The number of revolution
performed per unit time by the particle (1s).

$$
\therefore n=\frac{1}{\mathrm{~T}}=\frac{\omega}{2 \pi}
$$

Axis of rotation-The line passing through the centre of the circular path and perpendicular to the plane of rotation is called Axis of rotation.
Radius Vector :- A vector drawn from the centre of circle to position of particle in circular motion is called radius vector. OR

The line joining the centre of the circle and the particle performing circular motion indicates radius vector. The vector is directed towards the point of position of particle.

## 1.2 $\mathcal{A N G U L A R}$ DISPLACEMESNT.

## 2. Angular displacement ( $\theta$ ):

Angle traced by radius vector at the center in given time is called angular displacement It is along the Axis of rotation.
Angular displacement of all points of rigid body is same.
$>\quad$ It's SI unit is radian.
$>$ Practicle unit of it is degree.
i.e. $\mathbf{1} \mathbf{r a d}=57 . \mathbf{3}^{0}$
$\because 2 \pi \mathrm{rad}=360^{\circ}=1$ revolution

## Note -

Direction of infinitesimal angular displacement is given by right hand rule. Finite angular displacement does not obey commutative and associative laws of vector addition hence it is not true vector. While infinitesimal angular displacement obeys these laws hence it is true vector.
Angular displacement possesses magnitude as well as direction, hence it is vector quantity.

## 2. Right hand Rule:

$>\quad$ Imagine the axis of rotation to be held in right hand with fingers curled round the axis and thumb stretched along the axis. If the curled fingers denotes sense of rotation, then thumb denotes direction of vector.
1.3 ANGULAR VELOCITY $(\omega):$
$>\quad$ The rate of change of angular displacement with time is called angular velocity.
OR
Angle traced per unit time by the radius vector is angular velocity.
i.e. i.e. $\omega=\frac{\mathrm{d} \theta}{\mathrm{dt}}$
$>\quad$ SI unit is radian / second (rad/sec.)
$>\quad$ It dimensions are $\left[\mathbf{M}^{0} \mathbf{L}^{0} \mathbf{T}^{-1}\right]$
$>\quad$ It is a vector quantity and represented in magnitude and direction by using right hand rule.
$\vec{\omega}=\frac{\vec{\theta}}{\delta t} \quad$ In vector notation.
$>\quad$ Angular speed of every point on earth is $\omega=\frac{2 \pi}{24} \mathrm{rad} / \mathrm{hr}=\omega=\frac{2 \pi}{24 \times 60 \times 60} \mathrm{rad} / \mathrm{s}$.

### 1.4 ANGULAR ACCELERATION:

$>\quad$ Angular acceleration is defined as the rate of change of angular velocity.
$>\quad$ If in a time $\delta$ the angular velocity changes from $\vec{\omega}_{1}$ to $\vec{\omega}_{2}$ then angular acceleration, $\vec{\alpha}=\frac{\vec{\omega}_{2}-\vec{\omega}_{1}}{\delta \mathrm{t}}=\frac{\delta \vec{\omega}}{\delta \mathrm{t}}$
SI unit $=$ radian per square second. $\left(\mathrm{rad} / \mathrm{s}^{2}\right)$
Dimension $=\left[\mathrm{M}^{0} \mathrm{~L}^{0} \mathrm{~T}^{-2}\right]$
It is a vector quantity having direction similar to angular velocity.
Angular acceleration is same for all points of rigid body.
For UCM angular velocity is constant so angular acceleration is zero.
1.4 RELATION BETWEEN LISNEAR VELOCITT \& ANGULAR VELOCITY


Angular displacement $\delta \theta=\frac{\delta s}{r} \therefore \delta s=r \delta \theta$
$\lim _{\delta t \rightarrow 0} \frac{\delta s}{\delta t}=r \lim _{\delta t \rightarrow 0} \frac{\delta \theta}{\delta t}$
$\frac{d s}{d t}=\mathrm{r} \frac{d \theta}{d t} \quad \therefore v=r . w$
liner velocity $=$ radius x angular velocity
$>$ In magnitude and direction relation $\vec{v}=\vec{\omega} \times \vec{r}$

## Note :

$\vec{v}$ and $\vec{r}$ are in plane of circle and perpendicular to each other, hence $\omega$ is perpendicular to plane of circle.
1.6 RELATION BETWEEN LINEAR ACCELERATION ASND ASGULAR ACCELERATION.
Consider a particle performing circuler motion having linear acceleration 'a' in tangential direction called tangential acceleration.
> Let linear velocity ' $v$ ' changes with time.

$$
\begin{array}{rlrl}
a & =\frac{d v}{d t} & & \text { but } \mathrm{v}=\mathrm{r} \omega \\
& =\frac{d}{d t}(r \omega) & & \\
& =r \frac{d \omega}{d t} & \mathrm{r}=\mathrm{constant} \\
& \therefore a=r \cdot \alpha & \{\alpha=d \omega / d t\} \\
\hline
\end{array}
$$

linear acceleration $=$ radius $x$ angular accleration

## (2) UNIFORM CIRCULAR MOTION -

## (U.C.M.)

U.C.M. is defined as the motion of a
particle along the circumference of a circle with constant angular velocity. Then period and frequency.

NOTE:
If time interval $\delta$ t is continuously reduced the point B will approach the point $\mathrm{A} \&$ in the limit when $\delta t \rightarrow 0, \delta \vec{V}$ will be parpendicular to tangent i.e. $\delta$ r will be along the radius and directed toward the center. Thus the acceleration in uniform circular motion is always directed along the radius toward the centre of the circle and it's magnitude is $\mathrm{v}^{2} / \mathrm{r}$ or $\mathrm{rw}^{2}$. Therefore it is called radial acceleration or centripetal acceleration.

2 Period :Time taken by particle performing UCM to complete one revolution is called period of revolution ( $T$ ).
During one period particle covers distance equal to perimeter of circle.

Period $=\frac{\text { Perimeter }}{\text { Linear velocity }}$
$\mathrm{T}=\frac{2 \pi \mathrm{r}}{v}$
where $r=$ radius of circle
$\rightarrow$ Also in one period angular displacement $=$ $2 \pi$ radian.
$\therefore$ Period $=\frac{\text { Perimeter }}{\text { angular vlocity }}$
$T=\frac{2 \pi}{\omega}$
2. Frequency:- The number of revolutions performed per unit time by particle is called frequency of revolution ( $n$ ).
In time T, particle completes one revolution

$$
\begin{equation*}
n=\frac{1}{T} \tag{3}
\end{equation*}
$$

$>\quad$ Hence frequency is reciprocal of period put value of T in (3)

$$
\begin{align*}
& n=\frac{v}{2 \pi r} \\
& n=\frac{\omega}{2 \pi} \tag{4}
\end{align*}
$$

$\frac{v}{2 \pi r}=\frac{\omega}{2 \pi}$

or $\quad v=2 \pi \mathrm{rn}$
Frequency of revolution or rotation:
$>$ The number of circular rotations completed by the partical in one second is called rotational frequency of circular motion
$>\quad$ It is express as revolution per second rotation per second .
$>\quad$ often it is abbreviated as rps.

### 1.7 RADIAL ACCELERATION:

The acceleration of particle performing UCM is along radius and directed towards centre called radial (centripetal) acceleration.
$>$ Acceration in UCM is perpendicular to tangential velocity.
$>$ Accelaration in UCM is along the radius \& towards center so it is radial accelaration.
> Magnitude of acceration in UCM is given by $a=\frac{v^{2}}{r}=r \omega^{2}=v \omega$

### 1.8 CENTRIPETAL FORCES :-

$>\quad$ Centripetal force can be produced in
inertial frame of reference.
Centripetal force is along the radius and towards the centre of circle.
Centripetal force is real force because known interaction such as gravitational, electrical or nuclear produces such force.
> Radialy directed inward force which provides necessary centripetal acceleration for circular motion is called centripetal force.
> Centripetal force is given by

$$
F=\frac{m v^{2}}{r}=m v w^{2}=m v w
$$

$>$ In vector form, $\vec{F}=-\frac{m v^{2}}{r} \overrightarrow{r o}=-m w^{2} \vec{r}$ where $\overrightarrow{r_{0}}$ is the unit vector in the direction of radius vector $\vec{r}$.
In circular motion of a sattellite round the planet required centripetal force is provided by gravitational force of attration between earth and satellite i.e. $\frac{m v^{2}}{r}=\frac{G M m}{r^{2}}$
For circular motionof stone tied to a string, required centripetal force is provided by the tension in the string.

- Examples :

1) If object is tied to string and whirled in circle (horizontal) the necessary centripetal force for circular motion is provided by tension in the string.
2) Circular Motion of the moon round the earth is, due to gravitational force between the earth and the moon.
3) In an atom electron revolve in circular orbit round nucleus, necessary C.P.F. is provided by electrostatic force of attraction.

## CENTRIFUGAL FORCE :-

Centrifulgal force is equal in magnitude to centripetal force but opposite in direction.
> Centrifugal force is canbe produced in non inertial frame of reference.
$>\quad$ Radially directed outward force as experienced by body in circular motion is called C.F.
> Centrifugal force is not due to any known interactions such as gravitational, electrical or nuclear.
$>\quad$ Centrifugal force is given by

$$
F=\frac{m v^{2}}{r} m r w^{2}=m v w
$$

$>$ In vector form,

$$
\begin{equation*}
\vec{F}=\frac{m v^{2}}{r} r \vec{O} \quad \text { OR } \quad \vec{F}=m w^{2} \vec{r} \tag{1}
\end{equation*}
$$

Where $\underset{r o}{ }$ is unit vector in direction of radius vector.
$>$ In non-inertial frame of reference in order to apply Newton's laws of motion we have to consider an imaginary force called Pseudo force.
$>\quad$ Flattening of earth at the Poles, drier, cream separator are examples of C.F.

### 1.9 MAXIMUM SPEED OF VEHICLE

 ALONG A HORIZONTAL CURVED ROAD.$>\quad$ If vehicle is moving on horizontal circular path, the weight of vehicle is balanced by normal reaction N and frictional force between tyers and road surface provides necessary centripetal force $\therefore \frac{m v^{2}}{r}=. \mu m g$
i.e. $V_{\max }=\sqrt{\mu r g}$

This is $\max ^{\mathrm{m}}$ speed of a vehicle of mass in which can take a circular turn of radius $r$ without skidding.
$>\quad$ When vehicle takes a turn on a road, it has tendency to skid away from centre of curvature of the road due to inertia.
At high speed and sharp turn, friction is not able to provide the required centripetal force.
$>\quad$ In order to take safe turn on curved road surface is banked since friction causes unneccessary wear and tears of the tyers.

## Banking of road -

Raising of outer edge of curved road slightly than inner edge to avoids, the danger of accidents is called banking of road.
OR
The surface of the road, along the curve is kept inclined to the horizontal at some suitable angle, so that outer edge of the road is at higher elevation than inner edge this arrangement is called banking of roads.
1.10
$\mathfrak{B A} \mathcal{N K} I \mathcal{N} G$ OF ROADS :

$>\quad$ In banking of roads, road surface in inclined to the horizontal in such a way that outer edge of road is raised above the inner edge.
$>\quad$ In order to make turing of vehicle safe on curved road, without depeding upon friction, road surface is inclined to horizontal with suitable angle.
$>\quad$ In banking of roads, normal reaction N has two components.
i) $\quad \mathbf{N} \cos \theta$ which balances weight of the vehicle $\therefore \mathrm{N} \cos \theta=\mathrm{mg}$.
ii) $\quad \mathbf{N} \boldsymbol{\operatorname { s i n }} \theta$ which provides necessary centripetal force
$\therefore \mathrm{N} \sin \theta=\frac{\mathrm{mv}^{2}}{r} \quad \therefore \tan \theta=\frac{\mathrm{v}^{2}}{r g}$
a) Thus, maximum safety speed limit on banked road is given by
$V=\sqrt{r g \tan \theta}$
b) Angle of banking is given by
$\theta=\tan ^{-1}\left(\frac{v^{2}}{r g}\right)$

## 2 Angle of Banking:

Angle made by curved banked road with the horizontal is called as angle of banking.
Necessicity :
When vehicle move along the curved road, it performs circular motion.
$>\quad$ The necessary centripetal force is provided by force of friction between road and tyres.
> However on sharp curves frictional force may not be sufficient, to provide required centripetal force.
> In such cases force of friction is not reliable hence there is possiblity of over turn of vehicle.
$>$ Hence to avoid the danger of accidents curved raods are banked.
2. An expression for safety speed of vehicle, when moving along banked road.

Consider a car of mass ' $m$ ' moving along a curved road of radius 'r' and with speed 'v' $\theta^{\prime}$

Forces are acting on car setting
i) Its weight ' mg ' acting vertically downward.
ii) Normal reaction which is perpendicular to road surface and can be resolved into two components.
a) $\quad$ Vertical component $=\mathbf{N} \boldsymbol{\operatorname { c o s }} \theta$
b) $\quad$ Horizontal component $=\mathbf{N} \sin \theta$
> Weight of car balanced by
$\mathrm{N} \cos \theta=\mathrm{mg}$
The horizontal component $\mathrm{N} \sin \theta$ is directed towards the centre of curve and acts as centripetal force.
$N \sin \theta=\frac{m \nu^{2}}{r}$
divide (2) by (1)
$\frac{N \sin \theta}{N \cos \theta}=\frac{m v^{2}}{r} \times \frac{1}{m g}$
$\tan \theta=\frac{V^{2}}{r g}$
$\theta=\tan ^{-1}\left(\mathrm{~V}^{2} / \mathrm{rg}\right)$
$\mathrm{V}^{2}=\mathrm{rg} \tan \theta$.
$V=\sqrt{r g \tan \theta}$
Equation (3) gives angle of banking and equation (4) gives the maximum speed with which vehicle can be safely driven.
$>\quad$ As there is no mass term in equation (4) hence velocity does not depend on mass of vehicle.
1.11 CONICAL PEDULUM :-

Conical pendium sweeps over the surface of the cone.
Conical pedulum consist of a heavy point mass suspended from rigid support with the
help of string to revloved in horizontal circle.

## - Period Of Conical Pendulum :-



Consider a body of mass $m$ is attached with string of length 1 and string is attached to rigid support at upper end revolving in horizontal circle of radius $r$.
$>\quad$ If string makes on angle $Q$ with vertical then forces on mass $m$ are
i) Weight (mg) ii) Tension in string (T) Tension (T) is resolved into two mutually perpendicular components
a) $\mathbf{T} \boldsymbol{\operatorname { c o s }} \theta$ which balances weight

$$
\therefore \mathbf{T} \cos \theta=\mathbf{m g}
$$

b) $\mathrm{T} \sin \theta$ which provided necessary centripetal force
$\therefore \mathbf{T} \boldsymbol{\operatorname { c o s }} \theta=\mathbf{m r} \omega^{\mathbf{2}}$.
$\therefore \tan \theta \frac{r \omega^{2}}{g}$ i.e. $\therefore w^{2}=\frac{g \tan \theta}{r}$
i.e. $\therefore \omega=\frac{\sqrt{g \tan \theta}}{r}$
$\therefore \frac{2 \pi}{T}=\frac{\sqrt{g \tan \theta}}{r}$
ie. $T=2 \pi \sqrt{\frac{l \sin \theta}{g \tan \theta}}=2 \pi \sqrt{\frac{l \sin \theta}{g \sin \theta / \cos \theta}}$
Where $\mathrm{r}=l \sin \theta$
$T=2 \pi \sqrt{\frac{l \cos \theta}{g}}=2 \pi \sqrt{\frac{h}{g}}$ where,
$\mathrm{h}=l \cos \theta$
$>\quad$ Period of conical pendulum is given by
$T=2 \pi \sqrt{\frac{l \cos \theta}{g}}=2 \pi \sqrt{\frac{h}{g}}$
$>$ Period of Pendulum depends upon.
i) length
ii) angle of inclination
iii) height
$>$ Angular speed of the pendulum is,
$\omega=\sqrt{\frac{g}{l \cos \theta}}=\sqrt{\frac{g}{h}}$
1.12 VERTICAL CIRCULAR MOTION:


The speed of the body goes on decresing because gravity oppses motion when body move from lowest position of vertical circle to the highest position .
$>\quad$ while the speed of body goes on incresing because gravity helps the motion when body moves from highest position to lowest position
Hence speed of body is minimum at the highest position and it is maximum at the lowest position.
$>\quad$ consider the body of mass $m$ tied to a string \& whirled in vertical circle. At any point p with angular displacement $\theta$ various forces acting on the body are

1) weight mg \&
2) tention in string
$>$ Weight mg is resolved into two components:
a) $m g \cos \theta$ along string
b) $\mathrm{mg} \sin \theta$ perpendicular to string.
$>\quad$ For circular motion of body required centripetal force

$$
\begin{aligned}
& \frac{\mathbf{m v}^{2}}{\mathbf{r}}=\mathrm{T}-\mathbf{m g} \cos \theta \\
& \therefore \mathrm{T}=\frac{\mathbf{m v ^ { 2 }}}{\mathbf{r}}+\mathbf{m g c o s} \theta
\end{aligned}
$$

ii) At highest Position $Q=\mathbf{1 8 0}^{\boldsymbol{0}}$.

$$
\therefore \mathbf{T}_{H}=\frac{\mathbf{m} \mathbf{v}^{2}}{\mathbf{r}}+\mathbf{m g}(\because \cos \theta=-1)
$$

$>\quad$ At the highest position of a circle tension is minimum and velocity is minimum .
$>\quad$ In circular motion, tension in the string T should be greater than or equal to zero.

$$
\therefore \frac{\mathbf{m v}_{H}}{\mathbf{r}}=\mathbf{m g} \quad \therefore \mathrm{V}_{\mathrm{H}}=\sqrt{r g}
$$

$\therefore$ minimum velocity at the highest position to complete circular path is $\sqrt{r g}$.
ii) At the lowest position $\theta=0^{0}$

$$
\therefore T_{L}=\frac{m v_{L}^{2}}{r}+m g \quad(\because \cos \theta=1)
$$

$>\quad$ Potential energy is minimum and tension is maximum at the lowest position increase in potential energy
$K . E_{L}+P \cdot E_{L}=K . E_{H}+P \cdot E_{H}$
$(m g \times 2 r)$
$>$ By law of conservation of energy
$\therefore \frac{1}{2} m v_{L}{ }^{2}+0=\frac{1}{2} m v_{H}{ }^{2}+m g \times 2 r$
$\therefore \quad \mathbf{V}_{\mathbf{L}}{ }^{2}=\mathrm{V}_{\mathbf{H}}{ }^{2}+\mathbf{4 g r}$
> Minimum velocity at the highest position must be $\mathrm{V}_{\mathrm{H}}=\sqrt{\mathbf{r g}}$
$\therefore \mathrm{V}_{\mathrm{L}}{ }^{2}=\mathbf{r g}+\mathbf{4 r g}$
i.e. $\mathrm{V}_{\mathrm{L}}=\sqrt{\mathbf{5 r g}}$
$\therefore$ Velocity of the particle in vertical circular motion is maximum at the lowest position and should be greater than $\sqrt{5 \mathrm{rg}}$
> Maximum tension at the lowest position is

$$
T_{L}=\frac{m v^{2}{ }_{L}}{r}+m g=\frac{m}{r} \times(5 r g)+m g=6 m g
$$

$>$ Tension on horizontal diameter is 3 mg Minimum speed at the end of horizontal diameter when particle completes circle is $\sqrt{3 g r}$

## 2 ACCELERATION IN VERTICAL

 CIRCULAR MOTION :-$>\quad$ In order to continue circular motion velocity at lowest position must be greater than $\sqrt{5 \mathrm{rg}}$
Velocity at highest position must be greater than $\sqrt{r g}$
$>\quad$ Velocity at point N must be greater than $\sqrt{3 g r}$
Therefore acceleration at the lowest position is $a=\frac{V^{2}{ }_{L}}{r}=\frac{5 r g}{r}=5 g$
> Acceleration at the highest position is
$a=\frac{V^{2}{ }_{H}}{r}=\frac{r g}{r}=g$

### 1.13 EQUATIOSS FOR ENERGY AT $\operatorname{DIFFERENT}$ POSITION OF VERTICAL CIRCULAR MOTION.

| NO. | Physical <br> Quantity | Lowest <br> Position | Highest <br> Position | On <br> Horizontal |
| :---: | :--- | :--- | :--- | :---: |
| 1 | Velocity | $\sqrt{5 r g}$ | $\sqrt{r g}$ | $\sqrt{3 g r}$ |
| 2 | K.E. | $5 / 2 \mathrm{mgr}$ | $1 / 2 \mathrm{mgr}$ | $3 / 2 \mathrm{mgr}$ |
| 3 | P.E. | 0 | 2 mgr | mgr |
| 4 | Totalenergy | $5 / 2 \mathrm{mgr}$ | $5 / 2 \mathrm{mgr}$ | $5 / 2 \mathrm{mgr}$ |

### 1.14

KISEMATIC EQUATION:-

Linear motion
$\mathbf{V}=\mathbf{u}+\mathbf{a t}$
$S=u t+1 / 2 a t^{2}$
$V^{2}=\mathbf{u}^{2}+\mathbf{2 a s}$
$\mathbf{S n}=\mathbf{u}+\mathbf{a}(\mathbf{n}-1 / 2)$
$\mathrm{S}=\left(\frac{V+u}{2}\right) \mathrm{t}$

EXAMPLES
OF
$\omega_{2}=\omega_{1}+\alpha t$
$\theta=\omega_{1} \mathrm{t}+1 / 2 \alpha \mathbf{t}^{2}$.
$\mathbf{W}_{2}{ }^{2}=\omega_{1}{ }^{2}+2 \alpha \mathbf{t}$
$\theta_{\mathrm{n}} \mathrm{th}=\omega_{1}+\alpha(\mathrm{n}-1 / 2)$

MOTION :-

1) Bucket revolving in vertical cirtcle:-
$>$ When bucket full of water is revolved in vertical circle with sufficient speed then water in it does not fall out at highest point in inverted position because at this position, weight of water is balanced by radially outward cenrifugal force.

$$
\begin{aligned}
\frac{m v^{2}}{r} & =m g \\
\therefore V^{2} & =r g \quad \text { OR } \quad \mathrm{V}=\sqrt{\mathrm{rg}}
\end{aligned}
$$

2) Motion of aeroplane along a curve :-
> When aeroplane takes a curved filght it has to bend inward because in this case reaction is produced by air.
> This reaction has two component
i) Vertical component of reaction lifs the aeroplane $\therefore R \cos \theta=m g$
ii) Horizontal component of reaction provdes necessary centripetal force
$\therefore S \sin \theta=m v^{2} / r$
$\therefore \tan \theta=\frac{v^{2}}{r g}$
3) Bending of cyclist
4) Motion of the blades of fan
5) Motion of the tips of the hands in a clock.

PSEUDO FORCE
A frame of referance which is fixed or
moving with costant linear velocity relative to fix frame is called inertial frame of reference.
$>\quad$ A frame of referance which moves with acceleration relative to fixed frame of refrance is called non-inertial or accelerated frame of referance.
Newton's law's of motion are not obeyed in non-inertial frame of referance.
Here to apply Newton's laws existance of one or more fictitious forces acting on object is assumed.
Such forces are not real forces as they arises due to accelerated frame of reference called pseudo force.
A real force is that force which is produced due to gravitational or electrical or nuclear intraction.
Centrifugal force does not belongs to any of these intractions but it arises due to accelerated frame of reference, hence C.F.F. called pseudo force.
2. The expression for safety speed of a vehicle moving along a plain curved road. Consider a vehicle of mass ' m ' and moving with velocity 'V' along curved plain road.
Let $\mathrm{r}=$ radius of curved road.
The necessary centripetal force is supplied by friction between wheels and surface of road.
$>\quad$ The maximum frictional force is $\mu \mathrm{mg}$. where $\mu=$ coefficient of friction.
$>\quad$ For equilibrium of car.
Centripetal force $=$ frictional force
$\frac{m v^{2}}{r}=\mu m g$
$v^{2}=\mu r g$
$v=\sqrt{\mu r g}$
equation (2) gives maximum speed with which vehicle can move safely along curved plain road.
2. VERTICLE CIRCULARMOTIONDUE TOEARTH'S GRAVITATION:
$>\quad$ When a body moves along circular path in verticle plain, it's motion is called 'verticle circular motion'.
$>\quad$ Consider a body of mass $m$ tied to one end of string and whirl in a verticle circle of radius $r$ as shown in figure, then the speed of the body at different position is different.
$>\quad$ As body moves from lowest point A of verticle circle to the highest point $B$, the speed of the body goes on decreasing because gravity opposes the motion .
$>\quad$ Tension in the string at point 'A \& B' : At point ' $A$ ' as weight \& tension $\left(T_{1}\right)$ are in opposite direction .The centripetal force proved by ( $\mathrm{T}_{1}-\mathrm{mg}$ )
$\therefore T_{1}-m g=\frac{m v_{1}{ }^{2}}{r}$
$\therefore T_{1}=\frac{m v_{1}{ }^{2}}{r}+m g \rightarrow$ (1)
(maximum value of tension)
At point $\mathrm{B}_{2} \&$ weight both are in downward direction. The centripetal force is provided by $\left(\mathrm{T}_{2}+\mathrm{mg}\right)$
$\therefore T_{2}+m g=\frac{m v_{2}{ }^{2}}{r}$
$\therefore T_{2}=\frac{m v_{2}{ }^{2}}{r}+m g \rightarrow(2)$
(minimum value of tension)
At any intermediate point the tension in the string is given by general equation,

$$
T=\frac{m}{r}\left[v_{1}^{2}+r g-2 g h\right] \rightarrow(3)
$$

Where $h$ is height of position of particle from lower point A

Xet forces in string:
The componant of omporite of T is $\cos \theta$
$\therefore \quad$ Net forces in string towards the centre of circular path. $F=T-m g \cos \theta$
At lowest point -
$A, \quad \theta=0 \quad \therefore \cos \theta=1 \& T=T_{1}$
$\therefore \quad \mathrm{F}_{1}=\mathrm{T}_{1-}-\mathrm{mg}$ minimum value of net force.
At highest point B-
$\theta=180^{\circ} \quad \therefore \cos 180^{\circ}=-1 \& T=T_{2}$
$\therefore \quad \mathrm{F}_{2}=\mathrm{T}_{2}-(-\mathrm{mg})=\mathrm{t}_{2}+\mathrm{mg}$ maximum value of net force.
2. Kinematical equation for circular motion.
If in a circular motion speed changes continuously with constant angular acceleration $\alpha$ then using anology between uniformly accelerated linear motion \& circular motion.
If the angular velocity of a body in circular motion changes from $\omega_{0}$ to $\omega$ in time $t$, with uniform angular acceleration $\alpha$ then, $\omega=\omega_{0}+\alpha \mathrm{t}$ analogus to $\mathrm{V}=\mathrm{u}+\mathrm{at}$
$\theta=\omega_{0} \mathrm{t}+\frac{1}{2} \mathrm{at}^{2}$ analogus to $\mathrm{S}=\mathrm{ut}+\frac{1}{2} \mathrm{at}^{2}$
$\omega^{2}=\omega_{0}^{2}+2 \alpha \theta$ analogus to $\mathrm{V}^{2}=\mathrm{u}^{2}+2$ as


