BE-5765

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Seat No.________

B. Sc. (Sem. V) Examination

May/June - 2014

Mathematics : CC-MAT-503

(Differential Equations)

Time : **3** Hours] [Total Marks : **70**

Instructions : (i) All questions are compulsory.

(ii) Figures to the right indicate marks of the question.

1 (a) Prove that : **6**

(i) $$\frac{1}{D^2 + a^2}\cos ax = \frac{x}{2a}\sin ax$$

(ii) $$\frac{1}{D^2 + a^2}\sin ax = \frac{-x}{2a}\cos ax$$

(b) $\left(D^2 - 3D + 2\right)y = \sin\left(e^{-x}\right)$, solve it. **6**

(c) Solve : $\left(D^3+1\right)y=\cos 2x$. **6**

OR

1 (a) If $f(D)=(D-a)^r\cdot\phi(D)$ then **6**

prove that $\frac{1}{f(D)}e^{ax}=\frac{x^r\cdot e^{ax}}{f^{(r)}(a)},\ \phi(a)\neq 0$.

(b) Solve : $\left(D^2+n^2\right)y=\sec nx$. **6**

(c) Solve : $\left(D^2+2\right)y=x^2\cdot e^{3x}$. **6**

2 (a) Obtain the first integral of **6**

$x^2y_3+xy_2+(2xy-1)y_1+y^2+2x=0$.

(b) Solve : $y(1-\log y)y^{(2)}+(1+\log y)\left\{y^{(1)}\right\}^2=0$. **6**

(c) Solve : $y^{(2)}+\operatorname{cosec}^2 y\cdot\cot y=0$, given that **6**

$y=\frac{\pi}{2},\ y^{(1)}=1$ when $x=0$.

OR

2 (a) Solve : $xy^{(2)} - y^{(1)} = x^3 \cdot e^{x^2/2}$. **6**

(b) Solve : $y^{(2)} = x^2 \cdot \sin x$. **6**

(c) Solve : $x^2 yy_2 + \left(xy_1 - y\right)^2 - 3y^2 = 0$. **6**

3 (a) Solve : $xy^{(2)} - (2x-1)\,y^{(1)} + (x-1)\,y = 0$. **6**

(b) Solve : $y^{(2)} - 2\tan x \cdot y^{(1)} + 5y = e^x \cdot \sec x$. **6**

[by normal form]

(c) Solve : $y^{(2)} + y = \operatorname{cosec} x$. **6**

[by variation of parameter method]

OR

3 (a) Solve : $\left(1-x^2\right)y^{(2)} + xy^{(1)} - y = \left(1-x^2\right)^{3/2}$. **6**

(b) Solve : $y^{(2)} - 4xy^{(1)} + \left(4x^2 - 1\right)y = -3e^{x^2} \cdot \sin x$. **6**

[by normal form]

(c) Solve : $y^{(2)} + (3\sin x - \cot x)\, y^{(1)} + 2y\sin^2 x$

$$= e^{-\cos x} \cdot \sin^2 x$$

[by changing independent variable]

4 Solve any four : **16**

(1) $(D-a)^n y = e^{ax}$, $D = \frac{d}{dx}$

(2) $\left(D^4 - 4D^3 + 8D^2 - 8D + 4\right) y = 0$, $D = \frac{d}{dx}$

(3) $\sqrt{\left(a^2 - x^2\right)^3} \cdot y^{(2)} = x$.

(4) $y^{(2)} + 2y^{(1)} + 2y = 1 + x^2$.

[by method of undetermine cofficient]

(5) $y^{(2)} + 4y = \sec^2 2x$

[by variation of parameter method]

(6) $\left(y^{(2)}\right)^2 = 1 + \left\{y^{(1)}\right\}^2$.