Note: ***For the benefit of the students, specially the aspiring ones, the question of JEE(advanced), 2019 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test. For this test the time allocated in Physics, Chemistry & Mathematics are 30 minutes, 21 minutes and 25 minutes respectively.***

FIITJEE

SOLUTIONS TO JEE (ADVANCED) – 2019

PART I: PHYSICS

Section 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question have **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct option is chosen.
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

*Q.1 Consider a spherical gaseous cloud of mass density $\rho(r)$ in free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common center with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time, the particle number density $n(r) = \rho(r)/m$ is
[G is universal gravitational constant]

A. $\frac{3K}{\pi r^2 m^2 G}$

B. $\frac{K}{2\pi r^2 m^2 G}$

C. $\frac{K}{6\pi r^2 m^2 G}$

D. $\frac{K}{\pi r^2 m^2 G}$

Sol. **B**

$$\rho = \frac{1}{4\pi G r^2}\frac{d(gr^2)}{dr} \quad \ldots (i)$$

Because $\frac{mv^2}{r} = mg$

so $\frac{1}{2}mv^2 = K = \frac{mgr}{2}$

so from equation (i)

$$\rho = \frac{1}{4\pi G r^2}\frac{d}{dr}\left(\frac{2k}{m}r\right) = \frac{K}{2\pi G m r^2}$$

so, $\frac{\rho}{m} = \frac{K}{2\pi G m^2 r^2}$

Q.2 A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with a small area $\alpha 4\pi R^2 (\alpha << 1)$ is made on the shell without affecting the rest of the shell. Which one of the following statements is correct?

A. The magnitude of electric field at a point, located on a line passing through the hole and shell's center, on a distance 2R from the center of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$

B. The magnitude of electric field at the center of the shell is reduced by $\frac{\alpha V_0}{2R}$

C. The ratio of the potential at the center of the shell to that of the point at $\frac{1}{2}R$ from center towards the hole will be $\frac{1-\alpha}{1-2\alpha}$

D. The potential at the center of the shell is reduced by $2\alpha V_0$

Sol. **C**

$V_0 = \frac{\sigma 4\pi R^2}{4\pi\varepsilon_0 R} \quad \Rightarrow \sigma = \frac{V_0\varepsilon_0}{R}$

so V at R/2 $= v_0 - \frac{1}{4\pi\varepsilon_0}\frac{2}{R}\frac{V_0\varepsilon_0}{R}\alpha 4\pi R^2 = V_0(1-2\alpha)$

and V at centre $= V_0 - \frac{1}{4\pi\varepsilon_0}\frac{1}{R}\frac{V_0\varepsilon_0\alpha 4\pi R^2}{R} = V_0(1-\alpha)$

*Q.3 A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as

$T(t) = T_0(1 + \beta t^{1/4})$

where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of the metal is

A. $\frac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$

B. $\frac{4P(T(t)-T_0)}{\beta^4 T_0^2}$

C. $\frac{4P(T(t)-T_0)^2}{\beta^4 T_0^2}$

D. $\frac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$

Sol. **D**

At equilibrium, $C\frac{dT}{dt} = P$

$\frac{dT}{dt} = \frac{T_0\beta}{4}t^{-\frac{3}{4}}$

So heat capacity $C = \frac{4P}{\beta T_0}t^{\frac{3}{4}}$

From the given equation $\frac{T(t)-T_0}{\beta T_0} = t^{\frac{1}{4}}$

So $t^{\frac{3}{4}} = \frac{(T(t)-T_0)^3}{\beta^3 T_0^3}$

So $C = \frac{4P}{\beta^4 T_0^4}(T(t)-T_0)^3$

Q.4 In a radioactive sample ${}^{40}_{19}K$ nuclei either decay into stable ${}^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{-10} per year or into stable ${}^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the

stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei are produced by the ${}^{40}_{19}K$ nuclei only. In time t × 10^9 years, if the ratio of the sum of stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei to the radioactive ${}^{40}_{19}K$ nuclei is 99, the value of t will be
[Given : ln10 = 2.3]

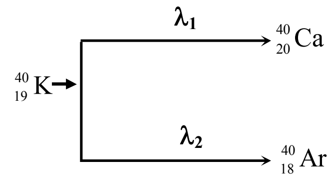
A. 1.15
B. 4.6
C. 9.2
D. 2.3

Sol. C

So equivalent decay constant = $\lambda_1 + \lambda_2 = 5 \times 10^{-10}$ per year

$\frac{N}{N_0} = e^{-\lambda_{eq} t}$ and given that $\frac{N_0 - N}{N} = 99$

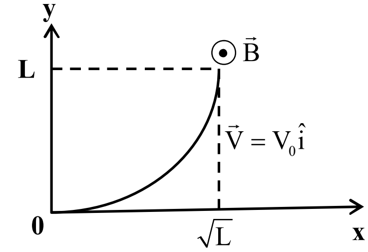
So $t = 9.2 \times 10^9$ year



Section 2 (maximum marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme.
  Full Marks : +4 If only (all) the correct option(s) is (are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial marks : +2 if three or more options are correct but ONLY two options are chosen and both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –1 mark.

Q.1 A conducting wire of parabolic shape, initially $y = x^2$, is moving with velocity $\vec{V} = V_0\hat{i}$ in a non uniform magnetic field $\vec{B} = B_0\left(1 + \left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V_0, B_0, L and β are positive constants and Δϕ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are:



A. |Δϕ| is proportional to the length of the wire projected on the y-axis.

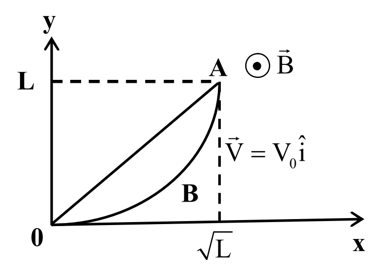
B. $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, y = x initially, of length $\sqrt{2}\,L$

C. $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for $\beta = 0$

D. $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for $\beta = 2$

Sol. **A, B, D**

These is no change in flux through the loop OABO due to the movement of loop. So potential difference developed in curved wire and the straight wire OA is same.

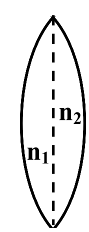


For $\beta = 0$, $|\Delta\phi| = 2B_0V_0L$

For $\beta = 2$, $|\Delta\phi| = \int_0^L B_0\left(1+\frac{y^2}{L^2}\right)V_0\,dy$

$= \frac{4}{3}B_0V_0L$

Q.2 A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surfaces are equal. *f* is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n - 1)$ and $1 < n < 2$. The correct statement(s) is/are.



A. $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

B. If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

C. For n = 1.5, $\Delta n = 10^{-3}$ and f = 20 cm, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place).

D. The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surfaces of the same radius of curvature.

Sol. **B, C, D**

When $n_1 = n_2 = n$

$$\frac{1}{f} = (n-1)\left(\frac{2}{R}\right) \qquad \ldots(i)$$

When, $n_1 = n$ and $n_2 = n + \Delta n$

$$\frac{1}{f+\Delta f} = (n-1)\left(\frac{1}{R}\right) + (n+\Delta n-1)\left(\frac{1}{R}\right) \qquad \ldots(ii)$$

So from equation (i) and (ii)

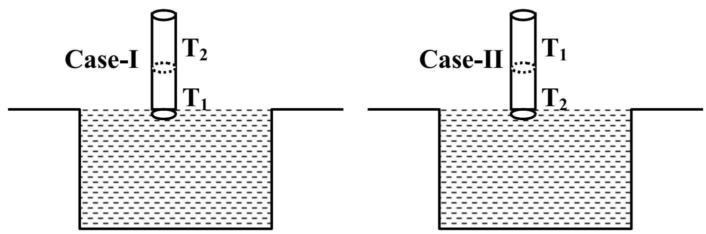
$$\frac{1}{f} - \frac{1}{f+\Delta f} = -(\Delta n)\left(\frac{1}{R}\right)$$

$$\Rightarrow \frac{\Delta f}{f^2} = -(\Delta n)\left(\frac{1}{R}\right)$$

So $\frac{\Delta f}{f} = -\frac{\Delta n}{2(n-1)} \approx -\frac{\Delta n}{2n}$

*Q.3 A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T_1 and T_2 of different materials having water contact angles of 0^0 and 60^0, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is (are) correct?

[Surface tension of a water = 0.075 N/m, density of water = 1000 kg/m^3, take g = 10 m/s^2]



A. For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)

B. For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)

C. For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)

D. The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.

Sol. **A, C, D**

When T_1 is in contact with water

then $h = \frac{2T\cos\theta_1}{r\rho g} = 7.5\,\text{cm} < 8\text{cm}$.

But in option (B) height is insufficient.

When T_2 is in contact with water

then $h = \frac{2T\cos\theta_2}{r\rho g} = 3.75\text{ cm} < 5\,\text{cm}$

Volume of water in the meniscus depends upon the angle of contact.

Q.4 A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct?

[ε_0 is permittivity of free space]

A. If $h < 8R/5$ and $r = 3R/5$ then $\phi = 0$

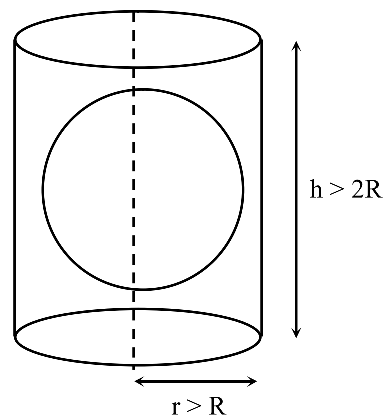
B. If $h > 2R$ and $r > R$ then $\phi = Q/\varepsilon_0$

C. If $h > 2R$ and $r = 4R/5$ then $\phi = Q/5\varepsilon_0$

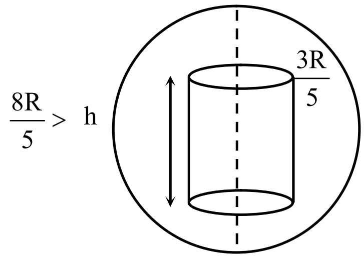
D. If $h > 2R$ and $r = 3R/5$ then $\phi = Q/5\varepsilon_0$

Sol. **A, B, D**

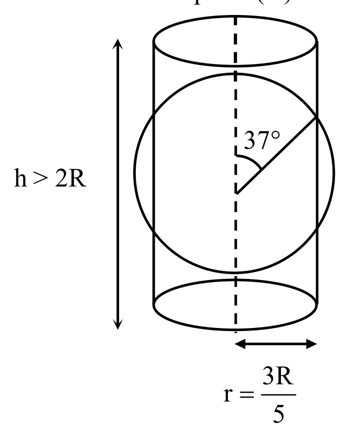
Option (B)



Option (A)



Option (D)



$$\phi = \frac{Q}{4\pi\varepsilon_0}(2)2\pi(1-\cos 37°)$$

*Q.5 Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement(s) is/are correct?

A. The dimension of energy is L^{-2}

B. The dimension of force is L^{-3}

C. The dimension of power is L^{-5}

D. The dimension of linear momentum is L^{-1}

Sol. **A, B, D**

$$\left[M^0L^0T^0\right] = \left[ML^2T^{-1}\right]$$

$$\Rightarrow \left[L^2\right] = [T]$$

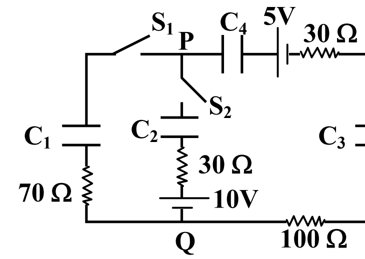
$$\text{Energy} = \left[MLT^{-2}L\right] = L^{-2}$$

$$\text{Force} = \left[MLT^{-2}\right] = L^{-3}$$

$$\text{Power} = \left[MLT^{-2}LT^{-1}\right] = L^{-4}$$

$$\text{linear momentum} = MLT^{-1} = L^{-1}$$

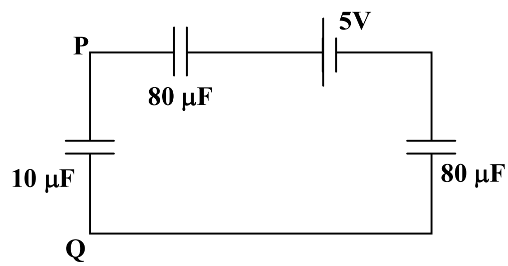
Q.6 In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are $C_1 = 10\ \mu F$, $C_2 = 30\ \mu F$ and $C_3 = C_4 = 80\ \mu F$. Which of the statement(s) is/are correct?



A. The key S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time, the instantaneous current across 30Ω resistor (between points P and Q) will be 0.2 A (round off to 1st decimal place).
B. If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitor C_1 will be 4V.
C. At time t = 0, the key S_1 is closed, the instantaneous current in the closed circuit will be 25 mA
D. If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10 V.

Sol. **B, C**

S_1 closed for long time

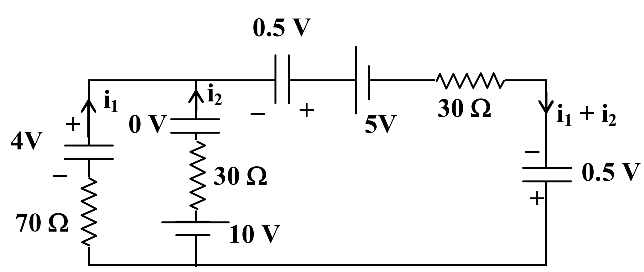


$$V_{10\mu F} = \left(\frac{40}{40+10}\right)5 = 4V$$

$$V_P - V_Q = 4V$$

$\Rightarrow t = 0$, key S is closed

$$i = \frac{5}{70+100+30} = 25 \text{ mA}$$

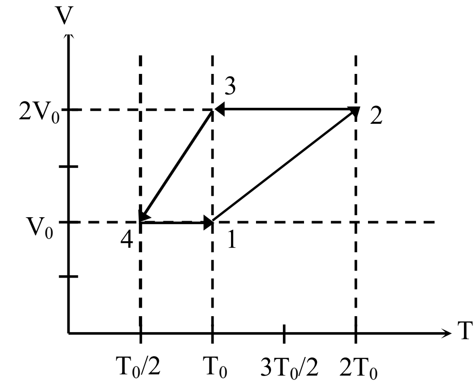


$$-70i_1 + 30\ i_2 - 6 = 0$$

$$-30i_1 - 60\ i_2 + 6 = 0$$

$\Rightarrow i_2 = 0.11$ A

*Q.7 One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V – T) diagram. The correct statement(s) is/are:
[R is the gas constant]



A. Work done in this thermodynamic cycle ($1 \to 2 \to 3 \to 4 \to 1$) is $|W| = \frac{1}{2}RT_0$

B. The ratio of heat transfer during processes $1 \to 2$ and $2 \to 3$ is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{5}{3}$

C. The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

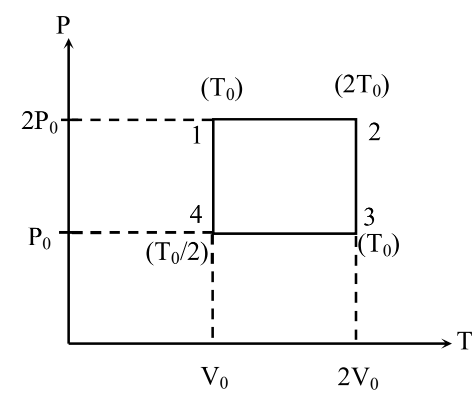
D. The ratio of heat transfer during processes $1 \to 2$ and $3 \to 4$ is $\left|\frac{Q_{1\to2}}{Q_{3\to4}}\right| = \frac{1}{2}$

Sol. **A, B**

$$W_{cycle} = P_0V_0 = \frac{RT_0}{2}$$

$$\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \left|\frac{nC_p(T_2 - T_1)}{nC_V(T_3 - T_2)}\right| = \left|-\frac{5}{3}\right| = \frac{5}{3}$$

$$\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \left|\frac{nC_p(T_2 - T_1)}{nC_V(T_4 - T_3)}\right| = 2$$



Q.8 Two identical moving coil galvanometers have 10 Ω resistance and full scale deflection at 2 μA current. One of them is converted into a voltmeter of 100 mV full scale reading and the other into an Ammeter of 1mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with R = 1000 Ω resistor by using an ideal cell. Which of the following statement(s) is/are correct?

A. The measured value of R will be 978 Ω < R < 982 Ω

B. The resistance of the Voltmeter will be 100 kΩ

C. If the ideal cell is replaced by a cell having internal resistance of 5Ω then the measured value of R will be more than 1000 Ω

D. The resistance of the Ammeter will be 0.02Ω (round off to 2nd decimal place)

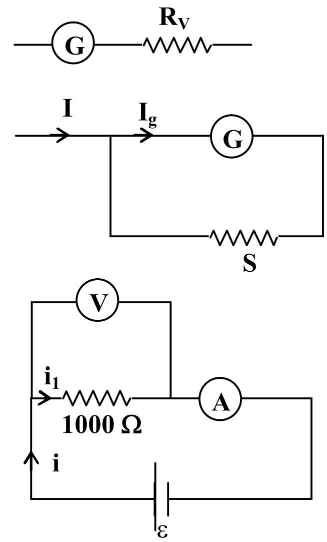
Sol. **A, D**

$V = I_g\left(G + R_V\right)$

$G + R_V \simeq R_V = 5\times10^4\,\Omega$

$I_g G = \left(I - I_g\right)S$

$S = 20\ \text{m}\,\Omega$



$$R_{ammeter} = \frac{GS}{G+S} = 20\times10^{-3}\,\Omega$$

$$i = \frac{\varepsilon}{\left(\dfrac{1000R_V}{1000+R_V}\right)} = \frac{51\varepsilon}{5\times10^4}\left(R_A \text{ is small}\right)$$

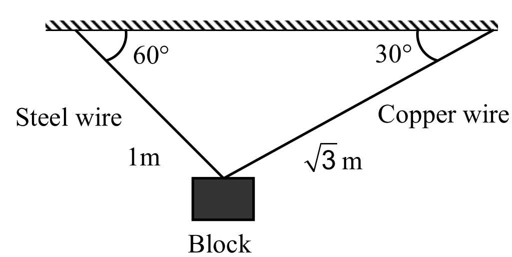
$$i_1 = i\left(\frac{1000+R_V}{1000}\right) = \frac{\varepsilon}{1000}$$

$$R_{measured} = \frac{i_1(1000)}{i} = 980.4\,\Omega$$

SECTION 3 (Maximum Marks: 18)

- This section contains SIX (06) questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered;
  Zero Marks : 0 In all other cases.

*Q.1 A block of weight 100 N is suspended by copper and steel wires of same cross sectional area 0.5 cm^2 and, length $\sqrt{3}$ m and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30° and 60°, respectively. If elongation in copper wire is $(\Delta\ell_C)$ and elongation in steel wire is $(\Delta\ell_S)$, then the ratio $\dfrac{\Delta\ell_C}{\Delta\ell_S}$ is ______

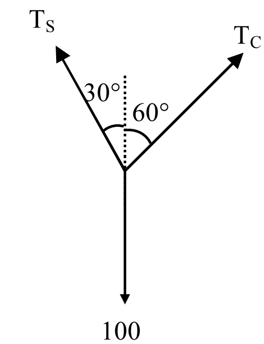


[Young's modulus for copper and steel are 1×10^{11} N/m^2 and 2×10^{11} N/m^2, respectively.]

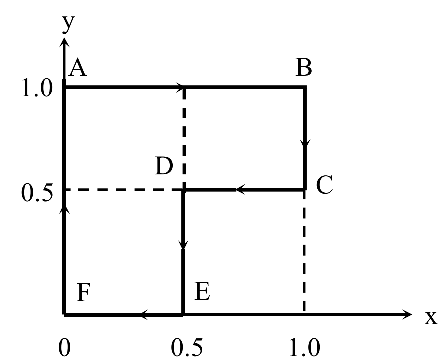
Sol. **2.00**

$T_S \sin 30° = T_C \sin 60°$

$\frac{\Delta \ell_C}{\Delta \ell_S} = \frac{T_C \ell_C}{A_C Y_C}\left(\frac{A_S Y_S}{T_S \ell_S}\right) = 2.00$



*Q.2 A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})$ N, where x and y are in meter and $\alpha = -1\ Nm^{-1}$. The work done on the particle by this force $\vec{F}$ will be _______ Joule.



Sol. **0.75**

$w_{AB} = \int_0^1 \alpha y dx = -1$

$w_{BC} = \int_1^{0.5} 2\alpha x dy = +1$

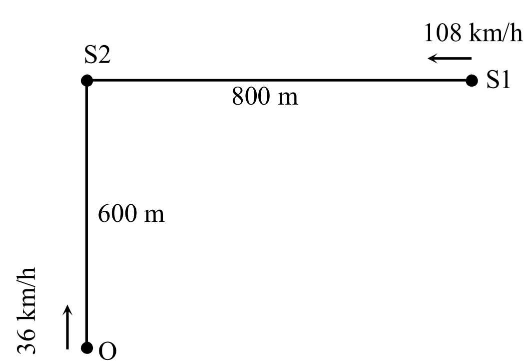
$w_{CD} = \int_1^{0.5} \alpha y dx = +0.25$

$w_{DE} = \int_{0.5}^{0} 2\alpha x dy = +0.5$

$w_{EF} = w_{FA} = 0$

$W_{net} = 0.75$

* Q.3 A train S1, moving with a uniform velocity of 108 km/h, approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S2, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from S2 and distance between S1 and S2 is 800 m, the number of beats heard by O is _____________.
[Speed of the sound = 330 m/s]

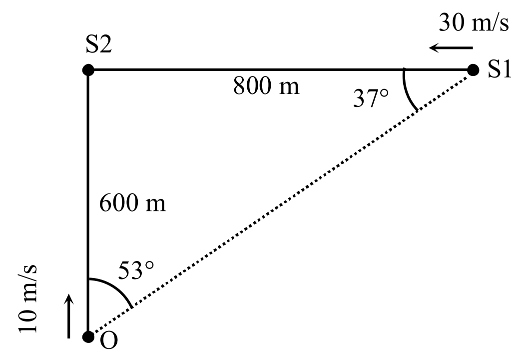


Sol. **8.13**

$$f_1 = 120\left(\frac{330+10\cos 53}{330-30\cos 37^\circ}\right)$$

$$f_2 = 120\left(\frac{330+10}{330}\right)$$

$$f_b = \left|f_1 - f_2\right| = 8.128\text{Hz} = 8.13\text{ Hz}$$



*Q.4 A liquid at 30°C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be ____________°C.
[Neglect the heat exchange with surrounding]

Sol. **270.00**

$5(s)(50)+5L = C(30)$ …(i)

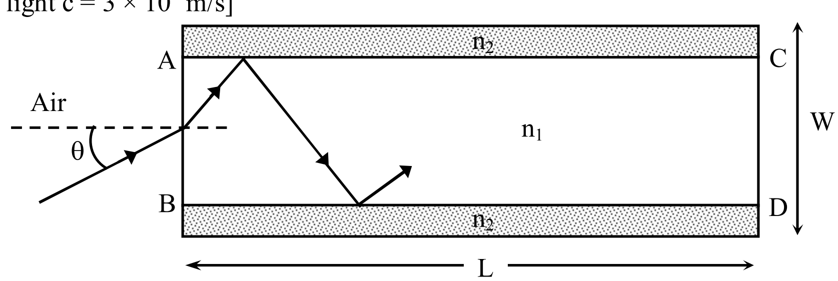
$80(s)(20) = C(30)$ …(ii)

$\therefore$ from (i) and (ii)

$\frac{L}{S} = 270°C$

$\therefore$ 270.00

Q.5 A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If L >> W, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For L = 9.6 m, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}$ s, where t is ________.
[Speed of light $c = 3 \times 10^8$ m/s]



Sol. **50.00**

$1.5\sin\theta_C = 1.44\sin 90°$

$\sin\theta_C = \frac{24}{25}$

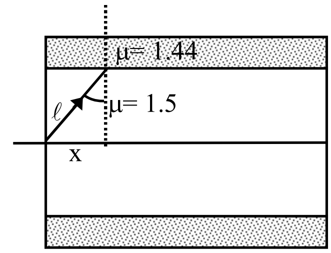
$\ell = \frac{x}{\sin\theta_C} = \frac{25}{4}x$

total length for light to travel

$\ell' = \frac{25}{4}\times 9.6 = 10\text{m}$

$\therefore \text{time} = \frac{\ell'}{c/1.5} = 5\times10^{-8}\text{s} \Rightarrow 50\times10^{-9}\text{s}$

t = 50.00



Q.6 A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the m^{th} layer is $K_m = K\left(1+\frac{m}{N}\right)$. For a very large $N\left(>10^3\right)$, the capacitance C is $\alpha\left(\frac{K\varepsilon_0 A}{d\ln 2}\right)$. The value of α will be ___________.

[$\in_0$ is the permittivity of free space]

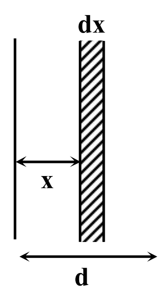
Sol. **1.00**

$$\frac{1}{dC} = \frac{dx}{k\left(1+\frac{m}{N}\right)\varepsilon_0 A}$$

$$\int\frac{1}{dC} = \int_0^d \frac{dx}{k\varepsilon_0\left(1+\frac{x}{d}\right)}$$

$$\Rightarrow C_{eq} = \frac{K\varepsilon_0 A}{d\ln 2}$$

$\alpha = 1.00$



$x = \frac{d}{N}(m)$

PART II: CHEMISTRY

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+3** If ONLY the correct option is chosen.
  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : **−1** In all other cases.

*Q.1 The green colour produced in the borax bead test of a chromium(III) salt is due to

(A) CrB (B) Cr_2O_3

(C) $Cr_2(B_4O_7)_3$ (D) $Cr(BO_2)_3$

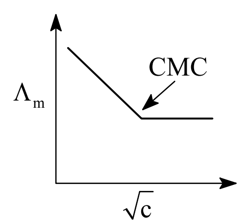
Sol. **(D)**

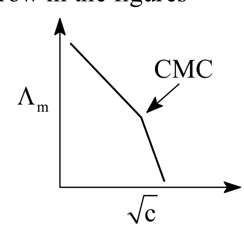
$Cr(BO_2)_3$

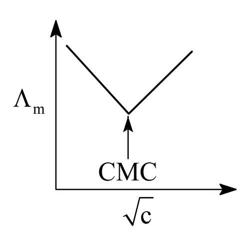
$$Na_2B_4O_7 \cdot 10H_2O \xrightarrow{\Delta} Na_2B_4O_7 \xrightarrow{\Delta} NaBO_2 + B_2O_3$$

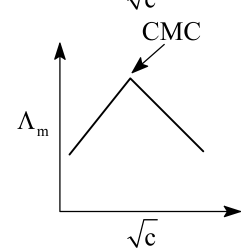
$$Cr_2O_3 + B_2O_3 \longrightarrow \underset{\text{(green colour bead)}}{Cr(BO_2)_3}$$

Q.2 Molar conductivity (Λ_m) of aqueous solution of sodium stearate, which behaves as a strong electrolyte, is recorded at varying concentrations (c) of sodium stearate. Which one of the following plots provides the correct representation of micelle formation in the solution?
(critical micelle concentration (CMC) is marked with an arrow in the figures

(A)


(B)


(C)


(D)


Sol. **(B)**

Q.3 Calamine, malachite, magnetite and cryolite, respectively, are

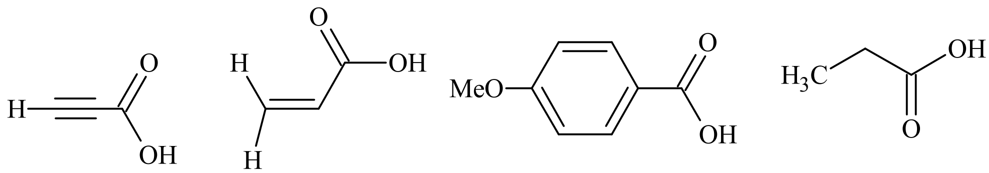
(A) $ZnSO_4$, $Cu(OH)_2$, Fe_3O_4, Na_3AlF_6 (B) $ZnCO_3$, $CuCO_3.Cu(OH)_2$, Fe_3O_4, Na_3AlF_6

(C) $ZnSO_4$, $CuCO_3$, Fe_2O_3, AlF_3 (D) $ZnCO_3$, $CuCO_3$, Fe_2O_3, Na_3AlF_6

Sol. **(B)**

Calamine – $ZnCO_3$
Malachite – $CuCO_3.Cu(OH)_2$
Magnetite – Fe_3O_4
Cryolite – Na_3AlF_6

*Q.4 The correct order of acid strength of the following carboxylic acids is



I II III IV

(A) I > III > II > IV (B) III > II > I > IV
(C) I > II > III > IV (D) II > I > IV > III

Sol. **(C)**
I > II > III > IV

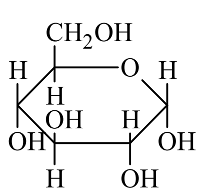
SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.
  Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.
  Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.
  Partial Marks **:** **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : **−1** In all other cases.
- **For Example:** If (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  Choosing ONLY (A), (B) and (D) will get +4 marks,
  Choosing ONLY (A) and (B) will get +2 marks,
  Choosing ONLY (A) and (D) will get +2 marks,
  Choosing ONLY (B) and (D) will get +2 marks,
  Choosing ONLY (A) will get +1 mark,
  Choosing ONLY (B) will get +1 mark,
  Choosing ONLY (D) will get +1 mark,
  Choosing no option (i.e. the question is unanswered) will get 0 marks, and
  Choosing any other combination of options will get –1 marks.

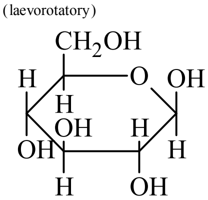
Q.1 Which of the following statements(s) is (are) true ?
(A) Oxidation of glucose with bromine water gives glutamic acid
(B) The two six-membered cyclic hemiacetal forms of D-(+)-glucose are called anomers
(C) Monosaccharides cannot be hydrolysed to give polyhydroxy aldehydes and ketones
(D) Hydrolysis of sucrose gives dextrorotatory glucose and laevorotatory fructose

Sol. **B, C, D**

$$\text{sucrose} \xrightarrow[\text{Hydrolysis}]{H^+,\, H_2O} \underset{(\text{Dextrorotatory})}{D(+)\ \text{Glucose}} + \underset{(\text{laevorotatory})}{D(-)\ \text{Fructose}}$$



α - D - glucopyranose



β - D - glucopyranose

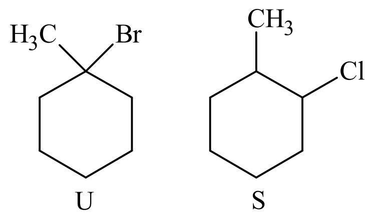
α-D-glucopyranose and β-D-glucopyranose are anomers of each other.

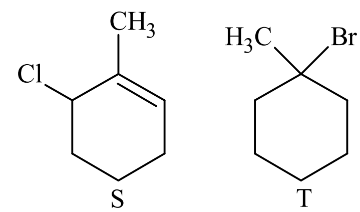
Q.2 Choose the correct option(s) for the following set of reactions

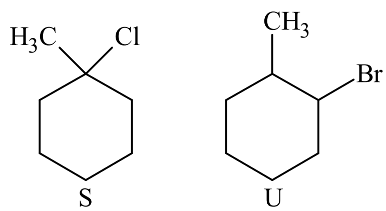
$$C_6H_{10}O \xrightarrow[\text{ii) } H_2O]{\text{i) MeMgBr}} Q \xrightarrow{\text{conc. HCl}} \underset{\text{(major)}}{S}$$

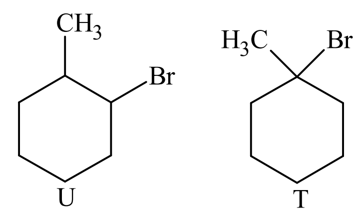
$$Q \xrightarrow{20\%\ H_3PO_4,\ 360K} R$$

$$\underset{\text{(major)}}{T} \xrightarrow[\text{ii) } Br_2,\ h\nu]{\text{i) } H_2,\ Ni} \underset{\text{(major)}}{R} \xrightarrow[\Delta]{\text{HBr, benzolyl peroxide}} \underset{\text{(major)}}{U}$$

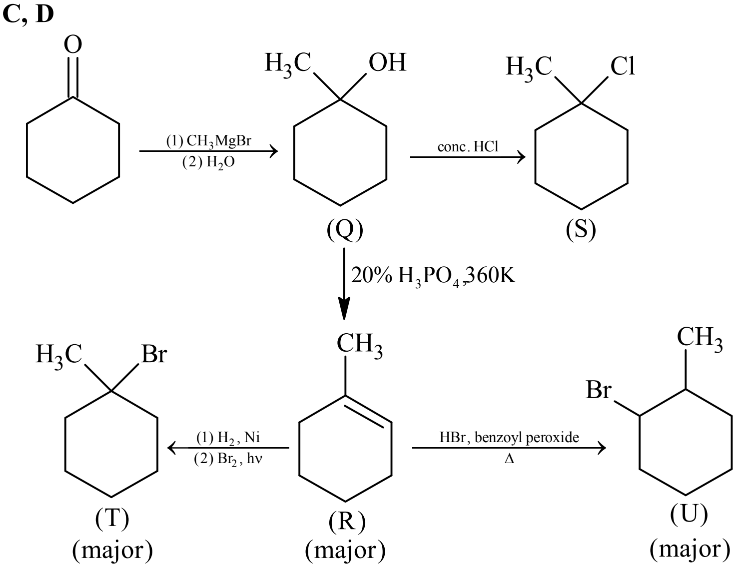
(A) 

(B) 

(C) 

(D) 

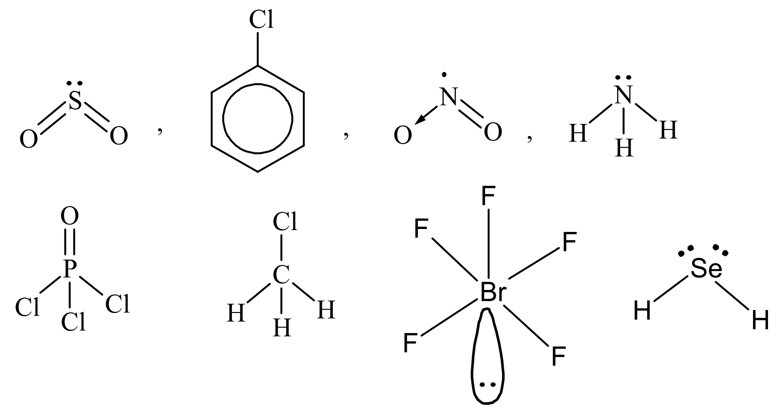
Sol. **C, D**



*Q.3 Each of the following options contains a set of four molecules, Identify the option(s) where all four molecules possess permanent dipole moment at room temperature.

(A) SO_2, C_6H_5Cl, H_2Se, BrF_5
(B) $BeCl_2$, CO_2, BCl_3, $CHCl_3$
(C) BF_3, O_3, SF_6, XeF_6
(D) NO_2, NH_3, $POCl_3$, CH_3Cl

Sol. **A, D**



*Q.4 Choose the reaction(s) from the following options, for which the standard enthalpy of reaction is equal to the standard enthalpy of formation.

(A) $2C(g)+3H_2(g)\longrightarrow C_2H_6(g)$

(B) $\frac{3}{2}O_2(g)\longrightarrow O_3(g)$

(C) $2H_2(g)+O_2(g)\longrightarrow 2H_2O(\ell)$

(D) $\frac{1}{8}S_8(s)+O_2(g)\longrightarrow SO_2(g)$

Sol. **B, D**

Standard enthalpy of formation of a compound is the standard enthalpy when one mole of a compound is formed from the elements in their stable state of aggregation.

Q.5 Fusion of MnO_2 with KOH in presence of O_2 produces a salt W. Alkaline solution of W upon electrolytic oxidation yields another salt X. The manganese containing ions present in W and X, respectively, are Y and Z. Correct statement(s) is(are)

(A) In aqueous acidic solution, Y undergoes disproportionation reaction to give Z and MnO_2

(B) In both Y and Z, π-bonding occurs between p-orbitals of oxygen and d-orbitals of manganese

(C) Y is diamagnetic in nature while Z is paramagnetic

(D) Both Y and Z are coloured and have tetrahedral shape

Sol. **A, B, D**

$$MnO_2 \xrightarrow{KOH/O_2} \underset{w}{MnO_4^{2-}} \xrightarrow[\text{Oxidation}]{\text{Electrolytic}} \underset{x}{MnO_4^-}$$

$$\downarrow \text{acidic medium}$$

$$MnO_2 + MnO_4^-$$

$y = Mn^{+6}$ and $z = Mn^{+7}$

*Q.6 Which of the following statement(s) is(are) correct regarding the root mean square speed (U_{rms}) and average translational kinetic energy (ε_{av}) of a molecule in a gas at equilibrium ?

(A) ε_{av} at a given temperature **does not** depend on its molecular mass

(B) U_{rms} is doubled when its temperature is increased four times

(C) ε_{av} is doubled when its temperature is increased four times

(D) U_{rms} is inversely proportional to the square root of its molecular mass

Sol. **A, B, D**

$$E_{av} = \frac{3}{2}RT \qquad U_{rms} = \sqrt{\frac{3RT}{M}}$$

E_{av} does not depend on its molecular mass but depends upon absolute temperature.

*Q.7 A tin chloride Q undergoes the following reactions (not balanced)

$$Q + Cl^- \longrightarrow X$$

$$Q + Me_3N \longrightarrow Y$$

$$Q + CuCl_2 \longrightarrow Z + CuCl$$

X is a monoanion having pyramidal geometry. Both Y and Z are neutral compounds. Choose the correct option(s)

(A) The central atom in Z has one lone pair of electrons
(B) The central atom in X is sp^3 hybridized
(C) There is a coordinate bond in Y
(D) The oxidation state of the central atom in Z is +2

Sol. **B, C**

$$\underset{(Q)}{SnCl_2} + Cl^- \longrightarrow \underset{(X)}{SnCl_3^-}$$

$$\underset{(Q)}{SnCl_2} + Me_3N \longrightarrow \underset{(Y)}{SnCl_2\left[N\left(CH_3\right)_2\right]}$$

$$\underset{(Q)}{SnCl_2} + 2CuCl_2 \longrightarrow \underset{(Z)}{SnCl_4} + CuCl$$

Q.8 In the decay sequence,

$$^{238}_{92}U \xrightarrow{-x_1} {}^{234}_{90}Th \xrightarrow{-x_2} {}^{234}_{91}Pa \xrightarrow{-x_3} {}^{234}Z \xrightarrow{-x_4} {}^{230}_{90}Th$$

x_1, x_2, x_3 and x_4 are particles /radiation emitted by the respective isotopes. The correct option(s) is(are)

(A) x_3 is γ-ray
(B) Z is an isotope of uranium
(C) x_2 is β^-
(D) x_1 will deflect towards negatively charged plate

Sol. **B, C, D**

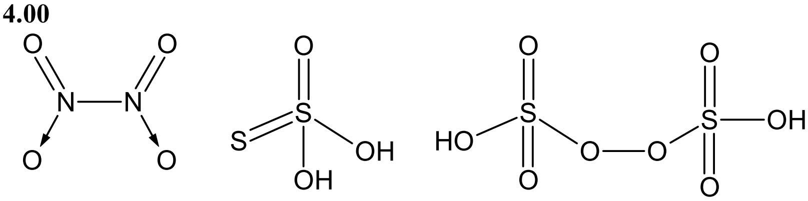
$X_1 = \alpha$
$X_2 = \beta$
$X_3 = \beta$
$X_4 = \alpha$

SECTION 3 (Maximum Marks: 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places. Truncate/round-off the value to TWO decimal places
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+3** If ONLY the correct numerical value is entered as answer.
  Zero Marks : **0** In all other cases.

*Q.1 Among B_2H_6, $B_3N_3H_6$, N_2O, N_2O_4, $H_2S_2O_3$ and $H_2S_2O_8$, the total number of molecules containing covalent bond between two atoms of the same kind is

Sol. **4.00**



Q.2 On dissolving 0.5 g of a non-volatile non-ionic solute to 39 g of benzene, its vapour pressure decreases from 650 mm Hg to 640 mm Hg. The depression of freezing point of benzene (in K) upon addition of the solute is

(Given data: Molar mass and the molal freezing point depression constant of benzene are 78 g mol^{-1} and 5.12 K kg mol^{-1}, respectively)

Sol. **1.02**

$$\frac{p_0 - p_s}{p_s} = i\left(\frac{n_{solute}}{n_{solvent}}\right)$$

$$\frac{650-640}{640} = 1\times\frac{0.5\times 78}{M\times 39}$$

$\Rightarrow M_{solute} = 64$ g

$\Delta T_f = K_f \times \text{molality} = 5.12\times\frac{0.5\times 1000}{64\times 39}$

$\Delta T_f = 1.02$

*Q.3 For the following reaction, the equilibrium constant K_c at 298 K is 1.6×10^{17}

$$Fe^{2+}(aq) + S^{2-}(aq) \rightleftharpoons FeS(s)$$

When equal volumes of 0.06 M Fe^{2+}(aq) and 0.2 M S^{2-}(aq) solutions are mixed, the equilibrium concentration of Fe^{2+}(aq) is found to be $Y\times 10^{-17}$ M. The value of Y is ……… .

Sol. **8.93**

	$Fe^{2+}(aq)$ +	$S^{2-}(aq) \rightleftharpoons FeS(s)$	$K_c = 1.6\times 10^{17}$
Initial	0.06 M	0.2 M	
After mixing	0.03 M	0.1 M	
	?	0.07 M	

$$1.6\times 10^{17} = \frac{1}{\left[Fe^{2+}\right]\times 0.07} \quad \text{or}$$

$$\left[Fe^{2+}\right] = \frac{10^{-17}}{1.6\times 0.07} = \frac{10^{-15}}{11.2} = 8.928\times 10^{-17}$$

or $8.93\times 10^{-17} = Y\times 10^{-17}$

Q.4

Experiment No.	[A] (mol dm^{-3})	[B] (mol dm^{-3})	[C] (mol dm^{-3})	Rate of reaction (mol dm^{-3}s^{-1})
1	0.2	0.1	0.1	6.0×10^{-5}
2	0.2	0.2	0.1	6.0×10^{-5}
3	0.2	0.1	0.2	1.2×10^{-4}
4	0.3	0.1	0.1	9.0×10^{-5}

The rate of the reaction for [A] = 0.15 mol dm^{-3}, [B] = 0.25 mol dm^{-3} and [C] = 0.15 mol dm^{-3} is found to be $Y\times 10^{-5}$ mol dm^{-3}s^{-1}. The value of Y is ……… .

Sol. **6.75**

Rate $k[A]^x[B]^y[C]^z$

By exp. No. 1 & 2 $y = 0$

By exp. No. 1 & 3 $z = 1$

By exp. No. 1 & 4 $x = 1$

Rate = $k[A]^1[B]^0[C]^1$

From Exp. No.1 $6\times 10^{-5} = k(0.2)(0.1)$

$\Rightarrow k = 3\times 10^{-3}$

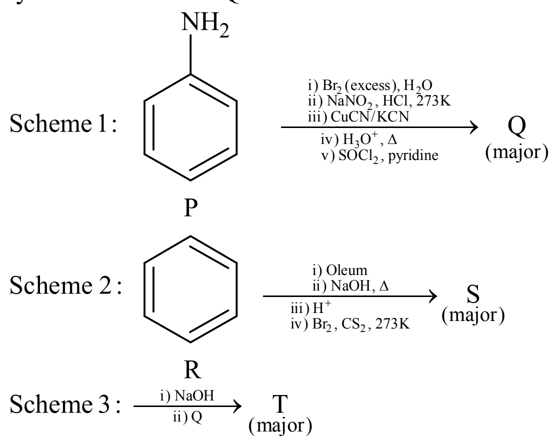
Now for [A] = 0.15 [B] = 0.25 [C] = 0.15

Rate = $k[A]^1[B]^0[C]^1$

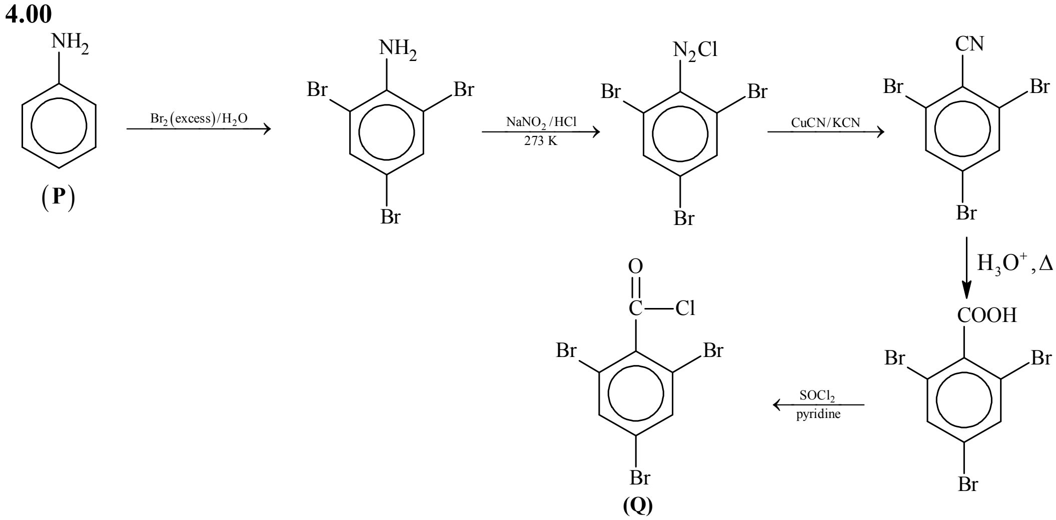
$= 3\times 10^{-3}\times 0.15\times 1\times 0.15$

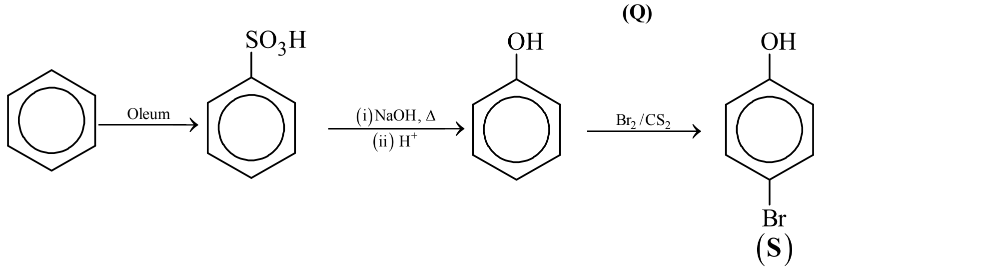
$$= 3\times 0.0225\times 10^{-3} = \underbrace{6.75}_{Y}\times 10^{-5}\ \text{mol L}^{-1}\ \text{sec}^{-1}$$

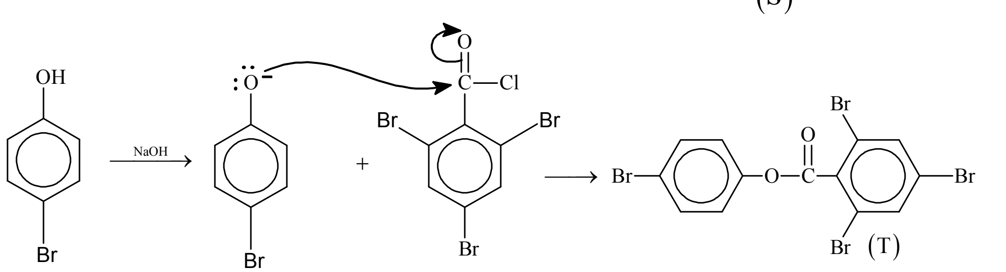
Q.5 Schemes 1 and 2 describe the conversion of P to Q and R to S, respectively. Scheme 3 describes the synthesis of T from Q and S. The total number of Br atoms in a molecule of T is



Sol. **4.00**



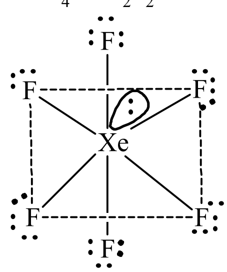




Q.6 At 143 K, the reaction of XeF_4 with O_2F_2 produces xenon compound Y. The total number of lone Pair(s) of electrons present on the whole molecule of Y is ……… .

Sol. **19.00**

$$XeF_4 + O_2F_2 \longrightarrow XeF_6 + O_2$$



PART III: MATHEMATICS

Section 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question have **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct option is chosen.
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

Q. 1 Let $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$, where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers, and I is the 2×2 identity matrix. If α^* is the minimum of the set $\{\alpha(\theta) : \theta \in [0, 2\pi)\}$ and β^* is the minimum of the set $\{\beta(\theta) : \theta \in [0, 2\pi)\}$, then the value of $\alpha^* + \beta^*$ is

A. $-\dfrac{37}{16}$ B. $-\dfrac{31}{16}$

C. $-\dfrac{17}{16}$ D. $-\dfrac{29}{16}$

Sol. **D**

$$M = \alpha\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \beta\begin{bmatrix} \cos^4\theta & 1+\sin^2\theta \\ -1-\cos^2\theta & \sin^4\theta \end{bmatrix}$$

on comparing we have

$$\sin^4\theta = \alpha + \frac{\beta\cos^4\theta}{|M|}$$

$$-1-\sin^2\theta = \alpha + \frac{\beta\left(1+\sin^2\theta\right)}{|M|}$$

$\alpha = \sin^4\theta + \cos^4\theta$

Now $\alpha = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$$= 1 - \frac{\sin^2 2\theta}{2}$$

$$\Rightarrow \alpha^* = \frac{1}{2}$$

We have, $|M| = \sin^4\theta\cos^4\theta + (1+\sin^2\theta)(1+\cos^2\theta)$

$= 2 + \sin^2\theta\cos^2\theta + \sin^4\theta\cos^4\theta$

$= (\sin^2\theta\cos^2\theta + 1/2)^2 + 7/4$

$$\Rightarrow \beta = -|M| = -\frac{7}{4} - \left(\sin^2\theta\cos^2\theta + \frac{1}{2}\right)^2$$

$$\Rightarrow \beta^* = -\frac{7}{4} - \left(\frac{1}{4} + \frac{1}{2}\right)^2 = -\frac{7}{4} - \frac{9}{16} = -\frac{37}{16}$$

$$\alpha^* + \beta^* = -\frac{29}{16}.$$

Q. 2 The area of the region $\{(x, y) : xy \le 8, 1 \le y \le x^2\}$ is

A. $16\log_e 2 - 6$

B. $8\log_e 2 - \frac{14}{3}$

C. $16\log_e 2 - \frac{14}{3}$

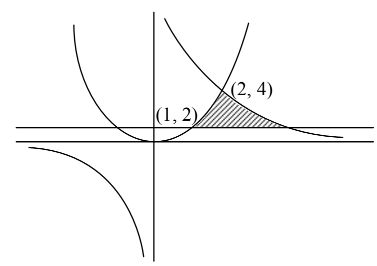
D. $8\log_e 2 - \frac{7}{3}$

Sol. **C**

Required area $= \int_1^4 \left(\frac{8}{y} - \sqrt{y}\right)dy$

$= \left(8\ln y - \frac{2}{3}y^{3/2}\right)_1^4$

$= 16\ln 2 - \frac{14}{3}$



*Q. 3 Let S be the set of all complex numbers z satisfying $|z - 2 + i| \ge \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0 - 1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|} : z \in S\right\}$, then the principal argument of $\frac{4 - z_0 - \overline{z_0}}{z_0 - \overline{z_0} + 2i}$ is

A. $\frac{3\pi}{4}$

B. $\frac{\pi}{4}$

C. $-\frac{\pi}{2}$

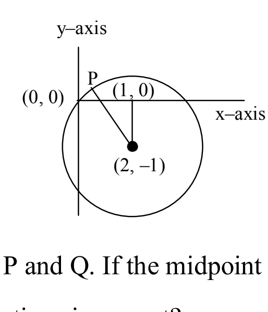
D. $\frac{\pi}{2}$

Sol. **C**

Clearly location of required point z_0 is at P with abscissa < 1 & ordinate > 0

Now $\arg\left[\frac{4 - z_0 - \overline{z}_0}{z_0 - \overline{z}_0 + zi}\right] = \text{Arg}\left(\frac{x-2}{y+1}\right)i = \text{Arg } ki$ & $k < 0$

$\Rightarrow$ Required argument $= -\pi/2$



*Q. 4 A line $y = mx + 1$ intersects the circle $(x - 3)^2 + (y + 2)^2 = 25$ at the points P and Q. If the midpoint of the line segment PQ has x–coordinate $-\frac{3}{5}$, then which one of the following options is correct?

A. $4 \le m < 6$

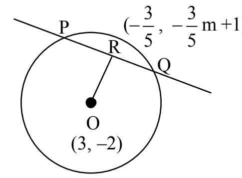
B. $-3 \le m < -1$

C. $2 \le m < 4$

D. $6 \le m < 8$

Sol. **C**

PQ $\perp$ OR $\Rightarrow$ Stope OR = $-\frac{1}{m} = \frac{-\frac{3}{5}m+1+2}{-\frac{3}{5}-3}$



$\Rightarrow \quad m^2 - 5m + 6 = 0$
$\Rightarrow \quad m = 2, 3$

Section 2 (maximum marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial marks	:	+2	if three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –1 mark.

*Q. 1 Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n, define $a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$, $n \geq 1$, $b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}$, $n \geq 2$. Then which of the following options is/are correct?

A. $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

B. $b_n = \alpha^n + \beta^n$ for all $n \geq 1$

C. $a_1 + a_2 + a_3 + \ldots.. + a_n = a_{n+2} - 1$ for all $n \geq 1$

D. $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

Sol. **B, C, D**

Clearly we have $\alpha + \beta = 1$ & $\alpha\beta = -1$

$\alpha, \beta = \frac{1 \pm \sqrt{5}}{2}$

As $b_n = a_{n-1} + a_{n+1}$

$= \frac{\alpha^{n-1} - \beta^{n-1}}{\alpha - \beta} + \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta}$

$$= \frac{\alpha^{n-1}\left(1+\alpha^2\right)-\beta^{n-1}\left(1+\beta^2\right)}{\alpha-\beta}$$

$$= \frac{\alpha^{n-1}(\alpha+2)-\beta^{n-1}(\beta+2)}{\alpha-\beta}$$

$$= \frac{\alpha^{n-1}\left(\frac{5+\sqrt{5}}{2}\right)-\beta^{n-1}\left(\frac{5-\sqrt{5}}{2}\right)}{\alpha-\beta}$$

$$= \frac{\sqrt{5}\left(\alpha^n+\beta^n\right)}{\alpha-\beta} = \alpha^n+\beta^n \quad : \text{As } \alpha-\beta=\sqrt{5}$$

(D) $\displaystyle\sum_{n=1}^{\infty}\frac{a_n}{10^n} = \sum_{n=1}^{\infty}\frac{\alpha^n-\beta^n}{(\alpha-\beta)10^n}$

$$= \frac{\dfrac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} - \dfrac{\frac{\beta}{10}}{1-\frac{\beta}{10}}}{(\alpha-\beta)}$$

$$= \frac{\alpha(10-\beta)-\beta(10-\alpha)}{(10-\alpha)(10-\beta)(\alpha-\beta)} = \frac{10}{89}$$

(A) $\displaystyle\sum_{n=1}^{\infty}\frac{b_n}{10^n} = \sum_{n=1}^{\infty}\frac{\alpha^n+\beta^n}{10^n} = \frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}}$

$$= \frac{\alpha(10-\beta)+\beta(10-\alpha)}{(10-\alpha)(10-\beta)}$$

$$= \frac{10(\alpha+\beta)-2\alpha\beta}{100-(\alpha+\beta)10+\alpha\beta} = \frac{12}{89}$$

(C) $a_1 + a_2 + a_3 + ... + a_n = \displaystyle\sum_{r=1}^{n} a_r = \sum_{r=1}^{n}\frac{\alpha^r-\beta^r}{\alpha-\beta}$

$$= \frac{\dfrac{\alpha\left(1-\alpha^n\right)}{1-\alpha} - \dfrac{\beta\left(1-\beta^n\right)}{1-\beta}}{\alpha-\beta}$$

$$= \frac{(\alpha-\alpha\beta)\left(1-\alpha^n\right)-\beta(1-\alpha)\left(1-\beta^n\right)}{(\alpha-\beta)(1-\alpha)(1-\beta)}$$

$$= \frac{(\alpha-\beta)-\alpha^n(1+\alpha)+\beta^n(1+\beta)}{-(\alpha-\beta)}$$

$$= \frac{(\alpha-\beta)-\alpha^{n+2}+\beta^{n+2}}{-(\alpha-\beta)}; \text{ As } 1+x=x^2$$

$$= -1+\frac{\alpha^{n+2}-\beta^{n+2}}{\alpha-\beta} = -1+a_{n+2}$$

*Q. 2 In a non–right–angled triangle ΔPQR, let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. If $p = \sqrt{3}$, $q = 1$, and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct?

A. length of $OE = \frac{1}{6}$

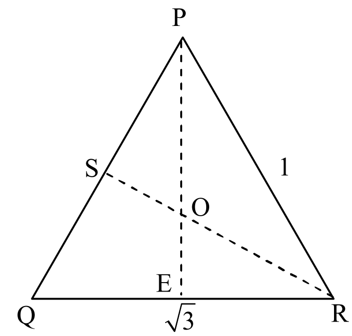
B. Radius of incircle of $\Delta PQR = \frac{\sqrt{3}}{2}\left(2-\sqrt{3}\right)$

C. Length of $RS = \frac{\sqrt{7}}{2}$

D. Are of $\Delta SOE = \frac{\sqrt{3}}{12}$

Sol. **A, B, C**

By sine rule



Circum radius = 1

$$\frac{\sin P}{P} = \frac{1}{2R} \Rightarrow \sin P = \frac{\sqrt{3}}{2} \Rightarrow P = 60^o \text{ or } 120^o$$

$$\frac{\sin\theta}{2} = \frac{1}{2R} \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^o,\ 150^o$$

so only possible combination, $P = 120^o$ & $Q = 30^o \Rightarrow \angle R = 30^o$

(C) Length of $RS \Rightarrow (PR)^2 + (QR)^2 = 2\,(RS^2 + PS^2) \Rightarrow RS = \frac{\sqrt{7}}{2}$

(A) One $\Delta PQR = ½\ PQ\ .\ PR \sin 120^o = \frac{\sqrt{3}}{4} \Rightarrow PE = ½$

$\Rightarrow OE = 1/3\ PE = 1/6$

(B) $r = \frac{\Delta}{s} = \frac{2\Delta}{p+q+r} = \frac{2\cdot\frac{\sqrt{3}}{4}}{\sqrt{3}+2} = \frac{\sqrt{3}}{2\left(2+\sqrt{3}\right)} = \frac{\sqrt{3}}{2}\left(2-\sqrt{3}\right)$

(D) are of $\Delta OSE = ½\ SE\ .\ OE \sin\angle SEO$

$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{6}\sin 60^o = \frac{\sqrt{3}}{48}$ unit^2

*Q. 3 Let Γ denote a curve $y = y(x)$ which is in the first quadrant and let the point (1, 0) lie on it. Let the tangent to Γ at a point P intersect the y–axis at Y_P. If PY_P has length 1 for each point P on Γ, then which of the following option is/are correct?

A. $y = \log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$

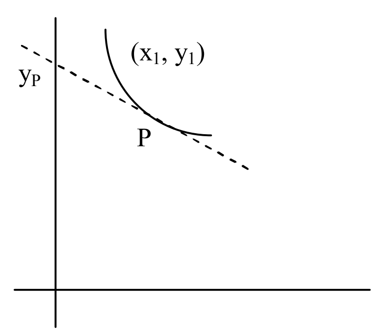
B. $xy' + \sqrt{1-x^2} = 0$

C. $y = -\log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$

D. $xy' - \sqrt{1-x^2} = 0$

Sol. **A, B**

Equation tangent



$$y - y_1 = \frac{dy_1}{dx_1}(x - x_1)$$

$$y_p\left(0,\ y_1 - x_1\frac{dy_1}{dx_1}\right)$$

$$Py_p = \sqrt{x_1^2 + \left(-x_1\frac{dy_1}{dx_1}\right)^2} = 1$$

$$\Rightarrow \frac{dy}{dx} = \pm\frac{\sqrt{1-x^2}}{x} \Rightarrow dy = \pm\int\frac{\sqrt{1-x^2}}{x}dx,\ 1 - x^2\ t^2,\ -x\,dx = t\,dt$$

$$dy = \pm\int\frac{-t^2}{1-t^2}dt$$

$$y = \pm\left(\int 1\,dt + \int \frac{1}{1-t^2}\,dt\right)$$

$$y = \pm\left(t - \frac{1}{2}\ell n\left|\frac{1+t}{1-t}\right|\right) + c$$

$$y = \pm\left(\sqrt{1-x^2} - \frac{1}{2}\left|\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\right|\right) + c$$

y is passing through (1, 0) so c = 0

$$y = \pm\left(\sqrt{1-x^2} - \frac{1}{2}\ell n\left(\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\right)\right)$$

$$y = \pm\left(\sqrt{1-x^2} - \ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right)\right)$$

As curve y = y(x) lies in the first quadrant so option A and B will only satisfy.
so AB are correct.

Q. 4 Let $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and adj $M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$ where a and b area real numbers. Which of the following options is/are correct?

A. $(\text{adj } M)^{-1} + \text{adj } M^{-1} = -M$

B. If $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

C. $\det(\text{adj } M^2) = 81$

D. $a + b = 3$

Sol. **A, B, D**

M adj M = |M| I $\Rightarrow$ a = 2, b = 1

$$\Rightarrow \quad M = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \Rightarrow |M| = -2$$

(A) $(\text{adj } M)^{-1} + (\text{adj } M^{-1}) = (|M|\, M^{-1})^{-1} + |M^{-1}|\, M = \frac{M}{|M|} + \frac{M}{|M|} = \frac{2M}{|M|} = -M$

(B) $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$\alpha = 1, \beta = -1, \gamma = 1 \Rightarrow \alpha - \beta + \gamma = 3$

(C) $|\text{adj } M^2| = |M^2|^2 = |M|^4 = 16$

(D) a = 2, b = 1, a + b = 3

*Q.5 Define the collections $\{E_1, E_2, E_3, \ldots\ldots.\}$ of ellipses and $\{R_1, R_2, R_3, \ldots\}$ of rectangles as follows:

$E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1;$

R_1 : rectangle of largest area, with sides parallel to the axes, inscribed in E_1;

E_n : ellipse $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ of largest area inscribed in R_{n-1}, $n > 1$;

R_n: rectangle of largest area, with sides parallel to the axes, inscribed in E_n, $n > 1$.

Then which of the following options is/are correct?

A. $\sum_{n=1}^{N}(\text{area of } R_n) < 24$, for each positive integer N

B. The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

C. The eccentricities of E_{18} and E_{19} are NOT equal

D. The length of latus rectum of E_9 is $\frac{1}{6}$

Sol. **A, D**

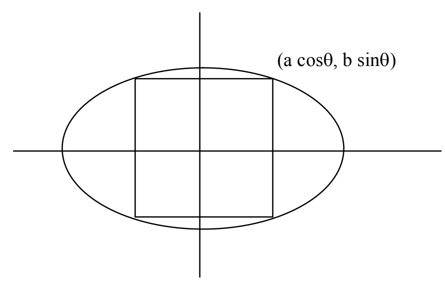
Area of $R_n = 2a\cos\theta \times 2b\sin\theta$

$= 2ab\sin 2\theta$

which will be maximum when $\theta = 45^\circ$

$\therefore R_n|_{max} = 2ab$

	a	b
E_1	3	2
E_2	$\frac{3}{\sqrt{2}}$	$\frac{2}{\sqrt{2}}$
E_3	$\frac{3}{(\sqrt{2})^2}$	$\frac{2}{(\sqrt{2})^2}$
$\vdots$		
E_n	$\frac{3}{(\sqrt{2})^{n-1}}$	$\frac{2}{(\sqrt{2})^{n-1}}$



(A) Area of R_1 + Area of R_2 ... Area of R_n

< Area of R_1 + Area of R_2 .... ∞

$< 2\left(6 + \frac{6}{2} + \frac{6}{4} + ...\right)$

$< 12\left(1 + \frac{1}{2} + ...\right)$

$< 12 \times \frac{1}{1-\frac{1}{2}} < 24$

(B) $b^2 = a^2(1 - e^2)$

for equation

$$\left(\frac{2}{(\sqrt{2})^8}\right)^2 = \left(\frac{3}{(\sqrt{2})^8}\right)^2\left(1 - e_9^2\right)$$

$e_9^2 = 1 - \frac{4}{9} = \frac{5}{9}$

$e_9 = \frac{\sqrt{5}}{3}$

Distance between centre and focus = ae

$= \frac{3}{(\sqrt{2})^8} \times \frac{\sqrt{5}}{3} = \frac{\sqrt{5}}{16}$

(C) $e_{18} = e_{19}$

(D) Latus rectum (length) $= \frac{2b^2}{a} = \frac{2\times\left(\frac{2}{(\sqrt{2})^8}\right)^2}{\frac{3}{(\sqrt{2})^8}} = \frac{1}{6}$

Q. 6 Let $f : R \to R$ by given by $f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0; \\ x^2 - x + 1, & 0 \le x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \le x < 3; \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \ge 3. \end{cases}$

Then which of the following options is/are correct?

A. f′ has a local maximum at x = 1
B. f′ is NOT differentiable at x = 1
C. f is onto
D. f is increasing on $(-\infty, 0)$

Sol. **A, B, C**

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1 & x < 0 & \to & (-\infty, 1) \\ x^2 - x + 1 & 0 \le x < 1 & \to & \left[\frac{3}{4}, 1\right] \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3} & 1 \le x < 3 & \to & \left[\frac{1}{3}, 1\right) \\ (x-2)\ell n(x-2) - x + \frac{10}{3} & x \ge 3 & \to & \left[\frac{1}{3}, \infty\right) \end{cases}$$

(Range will contain set)

$$f'(x) = \begin{cases} 5(x^4 + 4x^3 + 6x^2 + 4x + 1) - 2 & x < 0 \\ 2x - 1 & 0 \le x < 1 \\ 2x^2 - 8x + 7 & 1 \le x < 3 \\ \ell n(x-2) & x \ge 3 \end{cases}$$

(A) $f'(1) > f'(1^+)$ & $f'(1) > f(1^-)$ so $f'(x)$ has local max. at x = 1

(B) L.H.D. = 2 are R.H.D. = – 2, f′ is not differentiable at x = 1

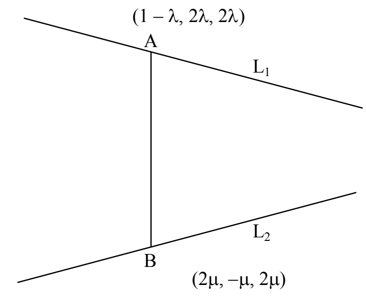
(C) f is containing $(-\infty, \infty)$, so f is onto

(D) $f'(x) = 5(x + 1)^4 - 2$ is changing sign in $(-\infty, 0)$, so if is not increasing

Q. 7 Let L_1 and L_2 denote the lines $\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in R$ and $\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in R$ respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe (s) L_3?

A. $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$
B. $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$
C. $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$
D. $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

Sol. **A, B, C**



Equation of L_1 $\vec{r} = \hat{i} + \lambda\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right)$

Equation of L_2 $\vec{r} = \mu\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$

L_1 & L_2 are skew lines

The direction ratios of line AB which is perpendicular to L_1 and L_2 will be

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{vmatrix} = 6\hat{i} + 6\hat{j} - 3\hat{k}$$

Hence direction ratios of AB will be (2, 2, –1)

direction ratios of AB proportional to (2, 2, –1)

$1 - \lambda - 2\mu = 2\,k$ ... (i)

$2\lambda + \mu = 2k$ ... (ii)

$2\lambda - 2\mu = -\,k$ ... (iii)

solving (i) (ii) & (iii)

we get $\lambda = 1/9$

$\mu = 2/9$

$$A\left(\frac{8}{9}, \frac{2}{9}, \frac{2}{9}\right),\ B\left(\frac{4}{9}, -\frac{2}{9}, \frac{4}{9}\right)$$

Equation of line L_3 (A, B) passing through A

$$= \left(\frac{8}{9}\hat{i} + \frac{2}{9}\hat{j} + \frac{2}{9}\hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right),\ t \in R$$

option (A) correct

Equation of line L_3 passing through B

$$= \left(\frac{4}{9}\hat{i} - \frac{2}{9}\hat{j} + \frac{4}{9}\hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$$

Option (C) is correct, option (B) also satisfy

Q.8 There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}, \frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct?

A. Probability that the chosen ball is green equals $\frac{39}{80}$

B. Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

C. Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

D. Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{5}{13}$

Sol. **A, B**

$$P(B_1) = \frac{3}{10},\ P(B_2) = \frac{3}{10},\ P(B_3) = \frac{4}{10}$$

(A) $P(G) = P(B_1) \times P\left(\frac{G}{B_1}\right) + P(B_2) \times P\left(\frac{G}{B_2}\right) + P(B_3) \times P\left(\frac{G}{B_3}\right)$

$$= \frac{3}{10} \times \frac{5}{10} + \frac{3}{10} \times \frac{5}{8} + \frac{4}{10} \times \frac{3}{8}$$

$$= \frac{60+75+60}{400} = \frac{195}{400} = \frac{39}{80}$$

(B) $P\left(\frac{G}{B_3}\right) = \frac{3}{8}$

(C) $P\left(\frac{B_3}{G}\right) = \frac{P(B_3) \times P\left(\frac{G}{B_3}\right)}{P(G)} = \frac{\frac{4}{10} \times \frac{3}{8}}{\frac{39}{80}} = \frac{4}{13}$

SECTION 3 (Maximum Marks: 18)

- This section contains SIX (06) questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered;
  Zero Marks : 0 In all other cases.

*Q.1 Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set $\{|a + b\omega + c\omega^2|^2 : a, b, c$ distinct non–zero integers$\}$ equals _____

Sol. **3.00**

$|a + b\omega + c\omega^2|^2$

$= (a + b\omega + c\omega^2)(a + b\bar{\omega} + c\bar{\omega}^2)$

$= a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

Let $a > b > c \Rightarrow |a - b| \geq 1, |b - c| \geq 1, |a - c| \geq 2$

$\geq \frac{1}{2}(1+1+4) \geq 3$

Q.2 If $I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$ then $27\ I^2$ equals _____

Sol. **4.00**

$I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$ ... (i)

$x = -t$

$I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{e^{\sin t}}{(1+e^{\sin t})(2-\cos 2t)}dt$ ... (ii)

add (i) and (ii)

$I = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \frac{1}{2-x\cos 2t}dt = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \frac{\sec^2 t}{1+3\tan^2 t}dt = \frac{1}{\pi}\cdot\frac{2\pi}{3\sqrt{3}}$

$3\sqrt{3}I = 2 \Rightarrow 27I^2 = 4$

Q.3 Let S be the sample space of all 3 × 3 matrices with entries from the set {0, 1}. Let the events E_1 and E_2 be given by

$E_1 = \{A \in S : \det A = 0\}$ and

$E_2 = \{A \in S : \text{sum of entries of A is } 7\}$

If a matrix is chosen at random from S, then the conditional probability $P(E_1/E_2)$ equals _____

Sol. **0.50**

$n(E_2)$ = arrangement of 7, 1 and 2 or = $\frac{9!}{7!\,2!} = 36$

$n(E_1 \cap E_2)$ = both zero should be in a row or a column

$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ (number of ways of arranging of (1, 0, 0) = 3 and arrangement of row = 3

total = 9

in same way for (1, 0, 0) for columns number of ways will be = 9

total ways = 18

$$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{18}{36} = \frac{1}{2} = 0.50$$

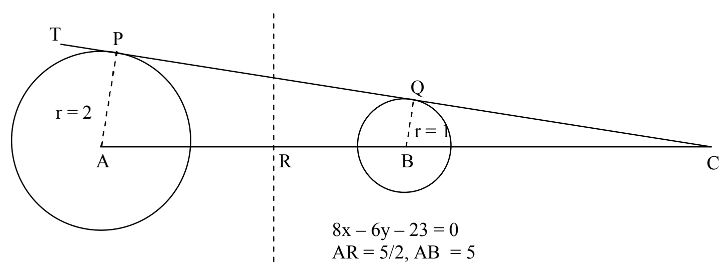
*Q.4 Let the point B be the reflection of the point A(2, 3) with respect to line $8x - 6y - 23 = 0$. Let Γ_A and Γ_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is _____

Sol. **10.00**

now ΔAPC and BQC are similarly

$$\frac{BC}{AC} = \frac{1}{2} \Rightarrow 2(AC - AB) = AC$$

AC = 2AB = 10



Q.5 Let AP(a; d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference d > 0. If $AP(1; 3) \cap AP(2; 5) \cap AP(3; 7) = AP(a; d)$ then a + d equals_____

Sol. **157.00**

$AP(1, 3) \equiv \{1, 4, 7, 10, ...\} = \{n / n = 3k + 1, k \in W\}$

$AP(2, 5) \equiv \{2, 7, 12, ... \} = \{n / n = 5k + 2, k \in W\}$

$AP(3, 7) \equiv \{3, 10, 17, ...\} = \{n / n = 7k + 3, k \in W\}$

Let common term is M

$M \equiv 1 \pmod 3$, $M \equiv 2 \pmod 5$, $M \equiv 3 \pmod 7$

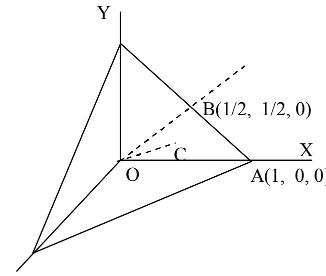
$\Rightarrow$ $M \equiv 52 \pmod{105}$

so a = 52, d = 105 and a + d = 157

Q. 6 Three lines are given by $\vec{r} = \lambda\hat{i}, \lambda \in R$, $\vec{r} = \mu(\hat{i} + \hat{j}), \mu \in R$ and $\vec{r} = v(\hat{i} + \hat{j} + \hat{k}), v \in R$. Let the lines cut the plane x + y + z = 1 at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals

Sol. **0.75**

O is origin point C will be foot of perpendicular from O to plane



so $C\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

so, $\overrightarrow{AB} = -\frac{1}{2}\hat{i} + \frac{1}{2}\hat{j}$

$\overrightarrow{AC} = -\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{1}{3}\hat{k}$

$$\Delta = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}\left|\frac{\hat{i}}{6} + \frac{\hat{j}}{6} + \frac{\hat{k}}{6}\right| = \frac{\sqrt{3}}{12}$$

$$(6\Delta)^2 = \frac{3}{4} = 0.75$$