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SEAT No.________

[41/A-15]

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SARDAR PATEL UNIVERSITY

M. Sc. (Semester III) Examination (NC)

Date: 21-4-2018 **Time: 2.00 To 5.00 p.m.**

Subject: MATHEMATICS **Paper No. PS03EMTH25 – (Graph Theory – I)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) If $K_{1,n} = K_{n+1}$, then

(a) $n = 1$ (b) $n = 2$ (c) $n > 2$ (d) none of these

(2) The complete graph K_8 is

(a) bipartite (b) Hamiltonian (c) Eulerian (d) disconnected

(3) Let T be a spanning in-tree with root R. Then

(a) $d^+(R) > 0, d^-(R) > 0$ (c) $d^+(R) = 0, d^-(R) > 0$

(b) $d^-(R) = 0, d^+(R) > 0$ (d) $d^+(R) = 0, d^-(R) = 0$

(4) If G is a simple digraph with vertices $\{v_1, v_2, \ldots, v_n\}$ & e edges, then $\sum_{i=1}^{n} d^+(v_i) =$

(a) ne (b) e^2 (c) 2e (d) e

(5) Which of the following graphs is not uniquely colourable?

(a) K_5 (b) P_6 (c) C_5 (d) C_6

(6) The coefficient c_3 in chromatic polynomial of K_4 is

(a) 0 (b) 1 (c) 3 (d) 3!

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is

(a) maximum $\Rightarrow$ perfect (c) maximal $\Rightarrow$ maximum

(b) maximal $\Rightarrow$ perfect (d) maximum $\Rightarrow$ maximal

(8) If $G = K_{2,n}$, then $\alpha(G) =$ ____.

(a) 2 (b) n (c) $\max\{2, n\}$ (d) $\min\{2, n\}$

2. Attempt any SEVEN: [14]

(a) Find the radius of $K_{2,3}$.

(b) Find $|E(G)|$, if G is a complete, symmetric digraph with n vertices.

(c) Define fundamental circuit matrix in a digraph.

(d) Define Hamiltonian cycle and Hamiltonian graph.

(e) Prove or disprove: For any planer graph G, $\chi(G) \leq 3$.

(f) Prove: The chromatic polynomial of P_4 and $K_{1,3}$ are same.

(g) Define isomorphic graphs and give one example of it.

(h) Prove or disprove: For any graph G, $\alpha(G) = \beta'(G)$.

(i) Define maximum matching and perfect matching in a graph.

(P. T. O.)

3. (a) Prove that if G is connected Euler digraph, then it is balanced. [6]

(b) Define symmetric and complete symmetric digraph and give one example of each. Also, discuss the relation between them. [6]

OR

(b) Prove that for each $n \geq 1$, there is a simple digraph with n vertices $v_1, v_2, \ldots, v_n$ such that $d^+(v_i) = i - 1$ and $d^-(v_i) = n - i$ for each $i = 1, 2, \ldots n$. [6]

4. (a) Define arborescence and show that an arborescence is a tree in which every vertex other than the root has an in – degree exactly one. [6]

(b) Show that the determinant of every square sub matrix of incidence matrix A(G) of a digraph G is 1, −1 or 0. [6]

OR

(b) Let G be a connected digraph with n vertices. Prove that rank of $A(G) = n - 1$. [6]

5. (a) Prove: For a connected graph G, $\chi(G) = 2$ if and only if G has no odd cycle. [6]

(b) Prove: If G is Hamiltonian, then, for each $S \subset V(G)$, $c(G - S) \leq |S|$. [6]

OR

(b) Find the Chromatic polynomial of graph C_4. [6]

6. (a) Prove: If G is a bipartite graph, then $\alpha'(G) = \beta(G)$. [6]

(b) State Hall's theorem and show that a k-regular bipartite graph has a perfect matching. [6]

OR

(b) Define $\alpha(G)$ and $\beta(G)$ and find it with corresponding sets, for $G = K_{3,5}$. [6]

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