P-445(H/E) HIGHER MATHEMATICS 2015

Time : 3 Hours] Class : 12th [M. M. : 100

Instructions- (i) All questions are compulsory. (ii) Read instructions carefully of the question paper and then answers of the questions. (iii) Question paper has two sections – Section – 'A' and Section – 'B'. (iv) In the Section – 'A' Question Nos. 1 to 5 are objective type, which contain the – choose the correct option, answer in one word/sentence, fill in the blanks, True/False and match the columns. Each question carries 5 marks. (v) In the Section – 'B' question Nos. 6 to 24 has Internal option. (vi) Q.Nos. 6 to 10 carry 2 marks each. (vii) Q.Nos. 11 to 17 carry 4 marks each. (viii) Q.Nos. 18 to 22 carry 5 marks each. (ix) Q.Nos. 23 and 24 carry 6 marks each.

Section 'A'

Q.1. Choose the correct options- $(5 \times 1 = 5)$

(a) Fraction form of $\frac{1}{(x+3)(x+4)}$ is-

(i) $\frac{1}{(x+3)} + \frac{1}{(x+4)}$

(ii) $\frac{1}{(x+3)} - \frac{1}{(x+4)}$

(iii) $\frac{1}{(x+4)} - \frac{1}{(x+3)}$

(iv) $\frac{1}{2}\left[\frac{1}{x+3} + \frac{1}{x+4}\right]$

(b) The perpendicular distance of the plane $3x - 6y + 5z = 12$ from origin is be-

(i) $\frac{-\sqrt{70}}{12}$ (ii) $\frac{-12}{\sqrt{70}}$

(iii) $\frac{12}{\sqrt{70}}$ (iv) $\frac{\sqrt{70}}{12}$

(c) The unit vector in the direction of "$\hat{i}+\hat{j}+\hat{k}$" is be-

(i) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$

(ii) $\sqrt{3}(\hat{i}+\hat{j}+\hat{k})$

(iii) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}+\hat{k})$

(iv) $\sqrt{2}(\hat{i}+\hat{j}+\hat{k})$

(d) Differential coefficient of "$\log(\sin x)$" with respect to 'x' is-

(i) $\cot x$ (ii) $\operatorname{cosec} x$

(iii) $\tan x$ (iv) $\sec x$

(e) By Newton-Raphson's method the formula for finding the square root of any number "y" is-

(i) $x_{n+1} = \frac{1}{2}\left[x_n + \frac{y}{x_n}\right]$

(ii) $x_{n+1} = \frac{1}{2}\left[x_0 + \frac{y}{x_0}\right]$

(iii) $x_{n+1} = \frac{1}{3}\left[2x_n + \frac{y}{x_n^2}\right]$

(iv) $x_{n+1} = \frac{1}{3}\left[2x_0 + \frac{y}{x_0^2}\right]$

Q.2. Answers in one word/sentences- $(5 \times 1 = 5)$

(i) Write the equation of a straight line which passes through the point (2, 1, 3) and has direction-rations (1, 3, 2)

(ii) If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertises of the triangle ABC, then write the formula of the area of ΔABC.

(iii) Write the value of $\int \frac{dx}{ax+1}$.

(iv) Define the positive co-relation.

(v) What is the value of $\sqrt{12}$ by Newton-Raphson's method after first iteration?

Q3. Fill in the blanks- $(5 \times 1 = 5)$

(i) Is be $\sin^{-1} x + \cos^{-1} x =$ ....

(ii) Sphere $3x^2 + 3y^2 + 3z^2 - 6x - 12y + 6z + 2 = 0$ has centre ....

(iii) If $\vec{a}, \vec{b}, \vec{c}$ are coplanar then $\left[\vec{a}, \vec{b}, \vec{c}\right]$ will be ....

(iv) The arithmetic mean of regression coefficients always .... the corelation.

(v) Related to the numerical method, the formula by the trapezoidal rule is ....

Q4. Write the True/False- $(5 \times 1 = 5)$

(i) Distance the point P (x, y, z) from the plane- X-Y is be $\sqrt{x^2 + y^2 + z^2}$.

(ii) Differential coefficient of e^x with respect to $\sqrt{x}$ is $\sqrt{x}.e^x$.

(iii) $f(x) = 2x^3 - 21x^2 + 36x - 30$ is maximum at $x = 1$.

(iv) According to the Newton-Raphson's method the approximate root of the equation $f(x) = 0$ is x_a then be $x_n = x_{n+1} - \dfrac{f(x)}{f'(x_n)}$.

(v) By the method of Newton-Raphson, the cube root of 10, after first iteration is 2.167.

Q5. Match the correct pair- $(5 \times 1 = 5)$

'A'	'B'
(a) $\int \frac{dx}{x^2 + a^2}$	(i) $\log\left[x - \sqrt{x^2 - a^2}\right]$
(b) $\int \frac{dx}{\sqrt{a^2 - x^2}}$	(ii) $\frac{1}{a} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1} \frac{x}{a}$

(c) $\int \sqrt{a^2 - x^2}\, dx$ (iii) $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

(d) $\int \frac{dx}{\sqrt{x^2 - a^2}}$ (iv) a. $\tan^{-1}$

(e) $\int \sqrt{a^2 + x^2}\, dx$ (v) $\sin^{-1}\left(\frac{x}{a}\right)$

(vi) $\frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log\left[x + \sqrt{x^2 + a^2}\right]$

(vii) $\log\left[x + \sqrt{x^2 - a^2}\right]$

Section 'B'

Q.6. Prove that- 2

$$\vec{AB} + \vec{BC} + \vec{CA} = 0$$

(Or) If $\vec{OP} = \hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{OQ} = 2\hat{i} - 2\hat{j} - k$ then find the modulus of $\vec{PQ}$.

Q.7. Prove that vectors $2\hat{i} - 3\hat{j} + 5\hat{k}$ and $-2\hat{i} + 2\hat{j} + 2\hat{k}$ are mutually perpendicular.

(Or) If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j}$, then find $\vec{a} \times \vec{b}$.

Q.8. Find the vector equation of sphere whose centre is (2, -3, 4) and radius is 5. 2

(Or) Find the distance of point (2, -1, 3) from the plane

$$\vec{r}.(3\hat{i} + 2\hat{j} - 6\hat{k}) + 15 = 0.$$

Q.9. Evaluate- $\int \frac{dx}{1+\cos 2x}$ 2

(Or) Evaluate- $\int \frac{1}{1-4x} dx$

Q.10. Evaluate- $\int_{0}^{\pi/4} \sin 2x \, dx$. 2

(Or) Evaluate- $\int \frac{\sec x}{(\sec x - \tan x)} dx$.

Q.11. Resolve the following fraction into partial fractions- $\frac{16}{(x+2)(x^2-4)}$ 4

(Or) Resolve the following fraction into partial fractions- $\frac{2x+1}{(x-1)(x^2+1)}$

Q.12. Prove that- 4

$$\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right].$$

(Or) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, then prove that, $xy + yz + zx = 1$.

Q.13. Find the differential coefficient of sinx by first principle. 4

(Or) If $y = \log(\log \sin x)$, then evaluate $\frac{dy}{dx}$.

Q.14. Differentiate, $\tan^{-1}\left[\frac{\cos x + \sin x}{\cos x - \sin x}\right]$ with respect to x. 4

(Or) Find the differential coefficient with respect to x of $\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$.

Q.15. If the edge of a cube is increasing at the rate of 5cm/sec., find the rate of increasing of its volume when its edge is 8cm long? 4

(Or) Prove that, $f(x) = x^3 - 3x^2 + 3x - 100$ is an increasing function in R.

Q.16. If "r" is a coefficient of conetation of two variables x and y, then prove that-

$$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x . \sigma_y}$$ Where σ_x^2, σ_y^2 and σ_{x-y}^2 are the variance of x, y and x - y respectively. 4

(Or) If $n = 10$, $\sum x = 50$, $\sum y = 30$, $\sum x^2 = 290$, $\sum y^2 = 300$,

$\sum xy = -115$, then find the coefficient of correlation.

Q.17. If "θ" be the angle between two regression lines and regression coefficients are $b_{yx} = 1.6$ and $b_{xy} = .4$, then find the value of $\tan\theta$. 4

(Or) Prove that coefficient of correlation is the Geometric mean of regression co-efficients.

Q.18. If $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are direction-cosines of any straight line, then prove that- $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ 5

(Or) Equation of the sphere is,

$2x^2 + 2y^2 + 2z^2 - 8x + 12y - 16z + 8 = 0$ find its centre and radius.

Q.19. Prove that- $\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)=1$ 5

(Or) Evaluate- $\lim_{x\to 0}\left(\frac{1-\cos 2x}{x}\right)$.

Q.20. Find the area of circle, $x^2+y^2=a^2$. 5

(Or) Prove that- $\int_0^1 \tan^{-1} x\,dx=\frac{\pi}{4}-\frac{1}{2}\log 2$.

Q.21. Solve the Differential Equation, 5

$(1+x)\,y\,dx+(1-y)\,x\,dy=0$.

(Or) Solve the differential equation, $(x^2+xy)\,dy=(x^2+y^2)dx$.

Q.22. Write theorem of total probaility and prove it. 5

(Or) A bag contains 8 black and 5 white balls. 2 balls are drawn. Find the probability that both the balls are white.

Q23. Prove that the lines. 6

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and } \frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$$ are coplanar. Find also the point of intersection.

(Or) Find the equation of the sphere which passes through the points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ and whose centre lies on the plane $3x - y + z = 2$.

Q24. Prove by vector method. 6

$\cos(A - B) = \cos A . \cos B + \sin A . \sin B$.

(Or) Find the shortest distance between the lines.

$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + t(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = 2\hat{i} + 4\hat{j} + 5\hat{k} + s(3\hat{i} + 4\hat{j} + 5\hat{k})$.