CAREER POINT

JEE Main Online Exam 2019

Questions & Solutions

8th April 2019 | Shift - I

MATHEMATICS

Q.1 $\int \dfrac{\sin\frac{5x}{2}}{\sin\frac{x}{2}} dx$ is equal to – (where c is a constant of integration)

(1) 2x + sin x + 2 sin 2x + c (2) 2x + sin x + sin 2x + c

(3) x + 2 sin x + 2 sin 2x + c (4) x + 2 sin x + sin 2x + c

Ans. **[4]**

Sol. $I = \int \dfrac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{x}{2}\right)} dx$

$I = \int \dfrac{2\sin\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right)}{\sin x} dx$

$I = \int \dfrac{\sin(3x) + \sin(2x)}{\sin x} dx$

$I = \int \dfrac{(3\sin x - 4\sin^3 x) + 2\sin x \cos x}{(\sin x)} dx$

$I = \int (3 - 4\sin^2 x + 2\cos x) dx$

$I = \int (3 - 2(1 - \cos 2x) + 2\cos x) dx$

$I = \int (3 - 2 + 2\cos 2x + 2\cos x) dx$

$I = \int (1 + 2\cos 2x + 2\cos x) dx$

I = x + sin 2x + 2sin x + c

Q.2 The equation of a plane containing the line of intersection of the planes 2x – y – 4 = 0 and y + 2z – 4 = 0 and passing through the point (1, 1, 0) is -

(1) x – 3y – 2z = – 2 (2) x – y – z = 0 (3) x + 3y + z = 4 (4) 2x – z = 2

Ans. **[2]**

Sol. Required equation of plane
$(2x - y - 4) + \lambda (y + 2z - 4) = 0$
Required plane passes through (1, 1, 0)
$(2 - 1 - 4) + \lambda (1 + 0 - 4) = 0$
$\lambda = -1$
$(2x - y - 4) - (y + 2z - 4) = 0$
$2x - 2y - 2z = 0$
$x - y - z = 0$

Q.3 Let y = y(x) be the solution of the differential equation, $(x^2 + 1)^2 \frac{dy}{dx} + 2x(x^2 + 1)y = 1$ such that y(0) = 0. If $\sqrt{a}\, y(1) = \frac{\pi}{32}$, then the value of 'a' is -

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{16}$ (4) 1

Ans. **[3]**

Sol. $(x^2 + 1)^2 \frac{dy}{dz} + 2x (x^2 + 1) y = 1$

$\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right)y = \frac{1}{(x^2+1)^2}$

I.F. = $e^{\int \frac{2x}{1+x^2}dx} = (1 + x^2)$
Solution of diff. eq.

$y (1 + x^2) \times \int \frac{1}{(1+x^2)^2} \times (1 + x^2)\, dx$

$y (1 + x^2) = \tan^{-1}(x) + c$
Given that y(0) = 0
c = 0
$y(1 + x^2) = \tan^{-1}(x)$
Put x = 1

$y \cdot (2) = \frac{\pi}{4}$

$y = \frac{\pi}{8}$

$\frac{1}{4} \cdot y = \frac{\pi}{32}$

i.e. $\sqrt{a} = \frac{1}{4}$

$a = \frac{1}{16}$

Q.4 If $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$, |x| < 1, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to -

(1) $2f(x)$ (2) $2f(x^2)$ (3) $(f(x))^2$ (4) $-2f(x)$

Ans. **[1]**

Sol. $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$ |x| + 1

$$f\left(\frac{2x}{1+x^2}\right) = \log_e\left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+x^2}}\right)$$

$$= \log_e\left(\frac{(1-x)^2}{(1+x)^2}\right)$$

$$= 2\log_e\left(\frac{1-x}{1+x}\right) = 2f(x)$$

Q.5 If $f(x) = \frac{2-x\cos x}{2+x\cos x}$ and $g(x) = \log_e x$, $(x > 0)$ then the value of the integral $\int_{-\pi/4}^{\pi/4} g(f(x))dx$ is -

(1) $\log_e 1$ (2) $\log_e 3$ (3) $\log_e e$ (4) $\log_e 2$

Ans. **[1]**

Sol. $f(x) = \frac{2-x\cos x}{2+x\cos x}$ $g(x) = \log_e x$

$$I = \int_{-\pi/4}^{\pi/4} g(f(x))dx$$

$$I = \int_{-\pi/4}^{\pi/4} \log_e\left(\frac{2-x\cos x}{2+x\cos x}\right)dx \quad \ldots.(i)$$

Applying

$$I = \int_{-\pi/4}^{\pi/4} \log_e\left(\frac{2+x\cos x}{2-x\cos x}\right)dx \quad \ldots.(ii)$$

Equation (i) + (ii)

$$2I = \int_{-\pi/4}^{\pi/4} \log_e(1)\,dx$$

$2I = 0$ $I = 0$

Q.6 The sum of the squares of the length of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is -

(1) 160 (2) 320 (3) 210 (4) 105

Ans. **[3]**

Sol. Equation of circle $x^2 + y^2 = 16$

Equation of chord $x + y = N$ $n \in N$

Length of chord = $2\sqrt{16-\frac{n^2}{2}}$

$(\text{Length of chord})^2 = 2(32 - n^2)$

Possible value of n = 1, 2, 3, 4, 5

Sum of squares of the length of chords

= 2(31) + 2(28) + 2(23) + 2(16) + 2(7)

= 210

Q.7 The sum of the solutions of the equation $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$, $(x > 0)$ is equal to -

(1) 4 (2) 10 (3) 9 (4) 12

Ans. **[2]**

Sol. $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$ $(x > 0)$

let $\sqrt{x} = t$

$|t - 2| + t(t - 4) + 2 = 0$

$t \geq 2$	$t < 2$
$t - 2 + t^2 - 4t + 2 = 0$	$2 - t + t^2 - 4t + 2 = 0$
$t^2 - 3t = 0$	$t^2 - 5t + 4 = 0$
$t = 0$ (Reject)	$t = 4$ (Reject)
$t = 3$ $\sqrt{x} = 3$	$t = 1$ $\sqrt{x} = 1$
$x = 9$	$x = 1$

Sum of all colution = 9 + 1 = 10

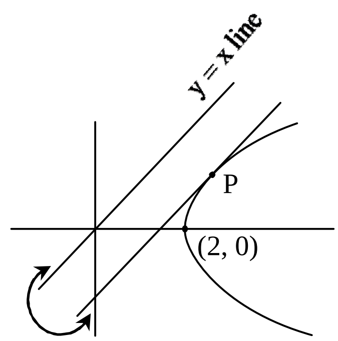
Q.8 The shortest distance between the line $y = x$ and the curve $y^2 = x - 2$ is -

(1) $\frac{7}{4\sqrt{2}}$ (2) 2 (3) $\frac{7}{8}$ (4) $\frac{11}{4\sqrt{2}}$

Ans. **[1]**

Sol. $y = x$ line

$y^2 = x - 2$ parabola



for point P $\left(\frac{dy}{dx}\right)_P = 1$

$\left(\frac{1}{2y}\right)_P = 1$

$(y_{costiate}) = \frac{1}{2}$

$(x_{coordinate}) = \frac{9}{4}$

Point P $\left(\frac{9}{4}, \frac{1}{2}\right)$

Shortest distance = length of perpendicular from P on line $x - y = 0$

$$= \frac{\left|\frac{9}{4} - \frac{1}{2}\right|}{\sqrt{2}} = \frac{7}{4\sqrt{2}}$$

Q.9 The sum of the co-efficients of all even degree terms in x in the expansion of $(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6$, $(x > 1)$ is equal to -

(1) 24 (2) 26 (3) 29 (4) 32

Ans. **[1]**

Sol. $(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6$

$= 2\,[{}^6C_0\, x^6\, (x^3 - 1)^0 + {}^6C_2\, x^4\, (x^3 - 1) + {}^6C_4\, x^2\, (x^3 - 1)^2 + {}^6C_6\, x^0\, (x^3 - 1)^3]$

Sum of coefficient at all even power

$= 2\,[{}^6C_0 + {}^6C_2\,\{-1\} + {}^6C_4\, x^2\,\{1 + 1\} + {}^6C_6\,\{{}^{-3}C_1 - {}^3C_3\}]$

$= 2\,[1 + 15\,(-1) + 15\,(2) + (-4)]$

$= 2\,[1 - 15 + 30 - 4]$

$= 2 \times 12 = 24$

Q.10 Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct ?

(1) $P(A|B) = P(B) - P(A)$ (2) $P(A|B) \geq P(A)$ (3) $P(A|B) = 1$ (4) $P(A|B) \leq P(A)$

Ans. **[2]**

Sol. $A < B$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$$

P(B) lies between (0, 1]

So that

$P(A/B) \geq P(A)$

Q.11 The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is -

(1) 3221 (2) 3303 (3) 3203 (4) 3121

Ans. **[4]**

Sol. H.C.F. $(91, n) > 1$

where $100 < n < 200$

sum of all positive values of 'n'

$= 7\,[15 + 16 + \ldots.. + 28] + 13\,[8 + \ldots.. + 15] - (13 \times 14)$

$$= 7 \times \frac{14}{2}[15 + 28] + 13 \times \frac{8}{2}[8 + 15] - (13 \times 14)$$

$= (49 \times 43) + (52 \times 23) - 182$

$= 2107 + 1196 - 182$

$= 3121$

Q.12 A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in -

(1) 1^{st}, 2^{nd} and 4^{th} quadrants (2) 1^{st} and 2^{nd} quadrants (3) 4^{th} quadrants (4) 1^{st} quadrants

Ans. **[2]**

Sol. 3x + 5y = 15 equation of straight line

$x = y$	$x = -y$
$8x = 15$	$-2x = 15$
$x = 15/8$	$x = -15/2$
$y = 15/8$	$y = 15/2$
$P\left(\frac{15}{8}, \frac{15}{8}\right)$	$Q\left(-\frac{15}{2}, \frac{15}{2}\right)$
I Quadrant	II Quadrant

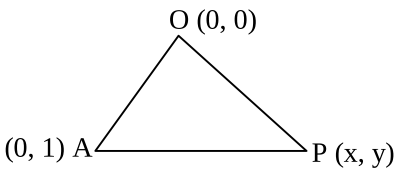
Q.13 Let O(0, 0) and A(0, 1) be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is -

(1) $8x^2 + 9y^2 - 9y = 18$ (2) $9x^2 + 8y^2 - 8y = 16$

(3) $9x^2 - 8y^2 + 8y = 16$ (4) $8x^2 - 8y^2 + 9y = 18$

Ans. **[2]**

Sol.



Given that

$OA + AP + OP = 4$

$1 + \sqrt{x^2 + (y-1)^2} + \sqrt{x^2 + y^2} = 4$

$(\sqrt{x^2 + y^2})^2 = (3 - \sqrt{x^2 + (y-1)^2})^2$

$x^2 + y^2 = 9 + x^2 + y^2 - 2y + 1 - 6\sqrt{x^2 + (y-1)^2}$

$3\sqrt{x^2 + (y-1)^2} = (5 - y)$

$9(x^2 + (y-1)^2) = (5 - y)^2$

$9x^2 + 8y^2 - 8y = 16$

Q.14 If $2y = \left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2$, $x \in \left(0, \frac{\pi}{2}\right)$ then $\frac{dy}{dx}$ is equal to -

(1) $x - \frac{\pi}{6}$ (2) $\frac{\pi}{3} - x$ (3) $\frac{\pi}{6} - x$ (4) $2x - \frac{\pi}{3}$

Ans. **[1]**

Sol. $2y = \left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2 \quad x \in \left(0, \frac{\pi}{2}\right)$

$2y = \left(\cot^{-1}\left(\frac{\sqrt{3} + \tan x}{1 - \sqrt{3}\tan x}\right)\right)^2$

$$2y = \left(\frac{\pi}{2} - \tan^{-1}\left(\tan\left(\frac{\pi}{3} + x\right)\right)\right)^2$$

Case 1: $0 < x < \frac{\pi}{6}$

$\left(\frac{\pi}{3} < x + \frac{\pi}{3} < \frac{\pi}{2}\right)$

$2y = \left(\frac{\pi}{2} - \frac{\pi}{3} - x\right)^2$

$2y = \left(\frac{\pi}{6} - x\right)^2$

Differentiation

$2\cdot\frac{dy}{dx} = 2\left(\frac{\pi}{6} - x\right)(-1)$

$\frac{dy}{dx} = x - \frac{\pi}{6}$

Case 2: at $x = \frac{\pi}{6}$

f×u is undefined
(non-diff)

Case 3: $\frac{\pi}{6} < x < \frac{\pi}{2}$

$\frac{\pi}{2} < x + \frac{\pi}{3} < \frac{5\pi}{6}$

$2y = \left(\frac{\pi}{2} - \left(\frac{\pi}{3} + x - \pi\right)\right)^2$

$2y = \left(\frac{2\pi}{6} - x\right)^2$

Differentiation

$2\cdot\frac{dy}{dx} = 2\left(\frac{7\pi}{6} - x\right)(-1)$

$\frac{dy}{dx} = x - \frac{7\pi}{6}$

Q.15 All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is -

(1) 180 (2) 175 (3) 160 (4) 162

Ans. **[1]**

Sol. Total number of possible numbers

$= \left({}^4C_3 \times \frac{3!}{2!}\right)\left(\frac{6!}{4!\,2!}\right)$

$= 180$

Q.16 If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points (1, 2) and (a, b) are perpendicular to each other, then a^2 is equal to -

(1) $\frac{2}{17}$ (2) $\frac{64}{17}$ (3) $\frac{128}{17}$ (4) $\frac{4}{17}$

Ans. **[1]**

Sol. Equation of ellipse $4x^2 + y^2 = 8$

$\frac{dy}{dx} = -\frac{4x}{y}$

tangent at (1, 2) and (a, b) are perpendicular

$\left(-\frac{y}{2}\right)\left(-\frac{4a}{b}\right) = -1$

$b = -8a$ ....(i)

(a, b) lies on ellipse

$4a^2 + b^2 = 8$ (from eq. (i))

$4a^2 + 64a^2 = 8$

$a^2 = \frac{8}{68} = \frac{2}{17}$

Q.17 $\lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals -

(1) 4 (2) $2\sqrt{2}$ (3) $4\sqrt{2}$ (4) $\sqrt{2}$

Ans. **[3]**

Sol. $\lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$

$$\lim_{x\to 0}\left(\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}\right)\left(\frac{\sqrt{2}+\sqrt{1+\cos x}}{\sqrt{2}-\sqrt{1+\cos x}}\right)$$

$$\lim_{x\to 0}\left(\frac{\sin^2 x}{1-\cos x}\right)(\sqrt{2}+\sqrt{1+\cos x})$$

$$\lim_{x\to 0}\frac{\sin^2 x}{2\sin^2\left(\frac{x}{2}\right)}(\sqrt{2}+\sqrt{1+\cos x})$$

$$\lim_{x\to 0}\frac{1}{2}\left(\frac{\sin x}{x}\right)^2 \times x^2 \times \left(\frac{x/2}{\sin x/2}\right)^2 \times \frac{1}{\left(\frac{x^2}{4}\right)} \times (\sqrt{2}+\sqrt{1+\cos x})$$

$$= \frac{1}{2}\times 4 \times 2\sqrt{2} = 4\sqrt{2}$$

Q.18 If $\cos(\alpha+\beta) = \frac{3}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to -

(1) $\frac{63}{52}$ (2) $\frac{63}{16}$ (3) $\frac{33}{52}$ (4) $\frac{21}{16}$

Ans. **[2]**

Sol. $\cos(\alpha+\beta) = \frac{3}{5}$ $\quad\tan(\alpha+\beta) = \frac{4}{3}$

$\sin(\alpha-\beta) = \frac{5}{13}$ $\quad\tan(\alpha-\beta) = \frac{5}{12}$

$\tan(2\alpha) = \tan((\alpha+\beta)+(\alpha-\beta))$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}} = \frac{48+15}{36-20} = \frac{63}{16}$$

Q.19 The sum of the series $2\cdot{}^{20}C_0 + 5\cdot{}^{20}C_1 + 8\cdot{}^{20}C_2 + 11\cdot{}^{20}C_3 + \ldots + 62\cdot{}^{20}C_{20}$ is equal to -

(1) 2^{23} (2) 2^{25} (3) 2^{26} (4) 2^{24}

Ans. **[2]**

Sol. $S = 2\cdot{}^{20}C_0 + 5\cdot{}^{20}C_1 + \ldots\ldots\ 59\cdot{}^{20}C_{19} + 62\cdot{}^{20}C_{20}$ .... (i)

$S = 62\cdot{}^{20}C_{20} + 59\cdot{}^{20}C_{19} + \ldots\ldots + 5\cdot{}^{20}C_1 + 2\cdot{}^{20}C_0$ .... (ii)

Add equation (i) & (ii)

$2S = 64\ [{}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \ldots\ldots + {}^{20}C_{20}]$

$2S = 64 \cdot 2^{20}$

$S = 64 \cdot 2^{19}$

$S = 2^6 \cdot 2^{19} = 2^{25}$

Q.20 The length of the perpendicular from the point (2, –1, 4) on the straight line, $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is -

(1) greater than 3 but less then 4

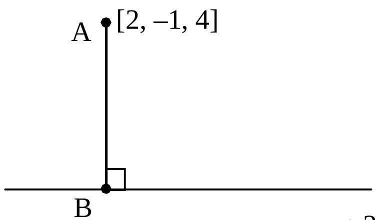
(2) greater than 2 but less than 3

(3) greater than 4

(4) less than 2

Ans. **[1]**

Sol.



$[10\lambda - 3, -7\lambda + 2, \lambda]$ $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{3}{1} = \lambda$

Drs of AB = $(10\lambda - 5, -7\lambda + 3, \lambda - 4)$

Drs of given line = (10, –7, 1)

$10(10\lambda - 5) - 7(-7\lambda + 3) + (\lambda - 4) = 0$

$100\lambda - 50 + 49\lambda - 21 + \lambda - 4 = 0$

$150\lambda = 75$

$\lambda = \frac{1}{2}$

Point B $\left(2, -\frac{3}{2}, \frac{1}{2}\right)$

Distance length AB = $\frac{5}{\sqrt{2}} = 3.53$

Q.21 Let $f : [0, 2] \to R$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2 - x)$, then ϕ is -

(1) increasing on (0, 2)

(2) increasing on (0, 1) and decreasing on (1, 2)

(3) decreasing on (0, 2)

(4) decreasing on (0, 1) and increasing on (1, 2)

Ans. **[4]**

Sol. Given that $f''(x) > 0$ $\quad x \in (0, 2)$

i.e. $f'(x)$ is increasing function

$\phi(x) = f(x) + f(2 - x)$

$\phi'(x) = f'(x) - f'(2 - x)$

$\phi(x)$ is increasing	$\phi(x)$ is decreasing
i.e. $\phi'(x) > 0$	i.e. $\phi'(x) < 0$
$f'(x) - f'(2 - x) > 0$	$f'(x) - f'(2 - x) < 0$
$f'(x) > f'(2 - x)$	$f'(x) < f'(2 - x)$
$x > 2 - x$	$x < 2 - x$
$2x > 2$	$x > 1$
$x > 1$	i.e. $x \in (0, 1)$
i.e. $x \in (1, 2)$	

Q.22 The greatest value of $c \in R$ for which the system of linear equations

$x - cy - cz = 0$

$cx - y + cz = 0$

$cx + cy - z = 0$

has a non-trivial solution, is -

(1) $\frac{1}{2}$ (2) 0 (3) 2 (4) –1

Ans. **[1]**

Sol. $\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0$

$1\,(1 - c^2) + c\,(-c - c^2) - c\,(c^2 + c) = 0$

$1 - c^2 - c^2 - c^3 - c^3 - c^2 = 0$

$2c^3 + 3c^2 - 1 = 0$

$c = -1$

$c = \frac{1}{2}$

Maximum value of $c = \frac{1}{2}$

Q.23 The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, is -

(1) $\frac{\sqrt{3}}{2}$ (2) $\sqrt{\frac{3}{2}}$ (3) $3\sqrt{6}$ (4) $\sqrt{6}$

Ans. **[2]**

Sol. Vector $\perp$ to given to vectors is

$$\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$$

projection of $\vec{b}$ in direction of $\vec{a}$ $= \dfrac{|\vec{a}\cdot\vec{b}|}{|\vec{a}|}$

$$= \frac{|(\hat{i} - 2\hat{j} + \hat{k})\cdot(2\hat{i} + 3\hat{j} + \hat{k})|}{\sqrt{6}}$$

$$= \frac{|2-6+1|}{\sqrt{6}} = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

Q.24 The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is -

(1) 45 (2) 48 (3) 49 (4) 40

Ans. **[2]**

Sol. Let unknown observation are x_1 & x_2

$$\frac{2+4+10+12+14+x_1+x_2}{7} = 8$$

$x_1 + x_2 = 14$ ....(i)

$$\frac{1}{7}\Sigma x_1^2 - (\bar{x})^2 = 16 \text{ (given)}$$

$$\frac{1}{7}(2^2 + 4^2 + 10^2 + 12^2 + 14^2 + x_1^2 + x_2^2) - (8) = 16$$

$x_1^2 + x_2^2 = 100$

$(x_1 + x_2)^2 - 2x_1x_2 = 100$

$2x_1x_2 = 96$

$x_1x_2 = 48$

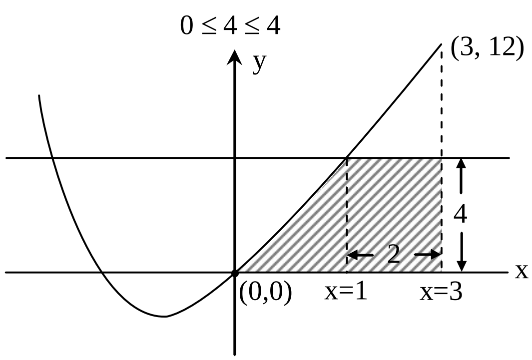
Q.25 The area (in sq. units) of the region $A = \{(x, y) \in R \times R | 0 \le x \le 3, 0 \le y \le 4, y \le x^2 + 3\}$ is -

(1) 8 (2) $\frac{53}{6}$ (3) $\frac{26}{3}$ (4) $\frac{59}{6}$

Ans. **[4]**

Sol. $y \le x^2 + 3x$

$0 \le x \le 3$



Required region

$$= \int_0^1 [x^2 + 3x]dx + [2 \times 4]$$

$$= \left[\frac{x^3}{3} + \frac{3}{2}x^2\right]_0^1 + 8$$

$$= \frac{2+9+48}{6} = \frac{59}{6}$$

Q.26 If $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$, $\beta = \tan^{-1}$, where $0 < \alpha, \beta < \frac{\pi}{2}$, then $\alpha - \beta$ is equal to -

(1) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (2) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (3) $\tan^{-1}\left(\frac{9}{14}\right)$ (4) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

Ans. **[2]**

Sol. $\alpha = \cos^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{4}{3}\right)$

$\beta = \tan^{-1}\left(\frac{1}{3}\right)$

$\alpha - \beta = \tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\frac{1}{3}\right)$

$$= \tan^{-1}\left(\frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3} \times \frac{1}{3}}\right) = \tan^{-1}\left(\frac{1}{1 + \frac{4}{9}}\right) = \tan^{-1}\left(\frac{9}{13}\right)$$

$$= \sin^{-1}\left(\frac{9}{\sqrt{(13)^2 + 9^2}}\right) = \sin^{-1}\left(\frac{9}{\sqrt{250}}\right)$$

$$= \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$$

Q.27 If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$, $x \in R$, then -

(1) $S_1 = \{-2, 0\}$; $S_2 = \{1\}$ (2) $S_1 = \{-1\}$; $S_2 = \{0, 2\}$

(3) $S_1 = \{-2, 1\}$; $S_2 = \{0\}$ (4) $S_1 = \{-2\}$; $S_2 = \{0, 1\}$

Ans. **[3]**

Sol. $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$

$f'(x) = 36x^3 + 36x^2 - 72x$

$f'(x) = 36x\,(x^2 + x - 2)$

$f'(x) = 36x\,(x + 2)\,(x - 1)$

Sign of $f'(x)$

⊖ ⊕ ⊖ ⊕

–2 0 1

↑ ↑ ↑

Minina Maxima Minima

$S_1 = \{-2, 1\}$

$S_2 = \{0\}$

Q.28 Let A = $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, ($a \in R$) such that $A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then a value of α is -

(1) $\frac{\pi}{64}$ (2) 0 (3) $\frac{\pi}{32}$ (4) $\frac{\pi}{16}$

Ans. **[1]**

Sol. $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$

Similarly

$A^{32} = \begin{bmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$32\alpha = \frac{\pi}{2}$

$\alpha = \frac{\pi}{64}$

Q.29 If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is -

(1) 3 (2) 5 (3) 4 (4) 2

Ans. **[3]**

Sol. $x^2 - 2x + 2 = 0$

$(x - 1)^2 = -1$

$x = 1 + i$ & $1 - i$

Let $\alpha = 1 + i$

$\beta = 1 - i$

$\frac{\alpha}{\beta} = \frac{1+i}{1-i} = \frac{(1+i)^2}{2} = \frac{1+i^2+2i}{2}$

$\frac{\alpha}{\beta} = i$

$\left(\frac{\alpha}{\beta}\right)^n = (i)^n = 1$

least value of n = 4

Q.30 The contrapositive of the statement ''If you are born in India, then you are a citizen of India'', is -

(1) If you are a citizen of India, then you are born in India

(2) If you are born in India, then you are not a citizen of India

(3) if you are not a citizen of India, then you are not born in India

(4) If you are not born in India, then you are not a citizen of India

Ans. **[3]**

Sol. Contrapositive of given statement is. If you are not a citizen of India, then you are not born in India.