Section 1
Mathematics

I. Linear Equations

Linear Algebra refers to equations in the form $y=mx+b$, where

y = y-value at a certain point in the graph
x = x-value at a certain point in the graph
b = a constant
m = the slope of the line

    This is useful for a variety of problems and is often represented by a graph that has a simple, straight line (Which is why it is called *'linear.'*). You can solve for specific characteristics of a linear equation by plugging in known values and using a technique called isolation, which involves isolating the unknown variable. The unknowns that many questions ask students to find would include the following:

The y-intercept, or the value of y when it intercepts the y-axis
The x-intercept, or the value of x when it intercepts the x-axis
Slope
The constant

Isolation is fairly straightforward to use, as in the following example.

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Example 1: What is the slope of the equation $y=mx+3$ if it intersects the x-axis at the point (4,0)?

So what do we know?

We know the equation, and that it passes through the point (4,0) somewhere along its graph, so we can simply plug in 4 for x and 0 for y to isolate m.
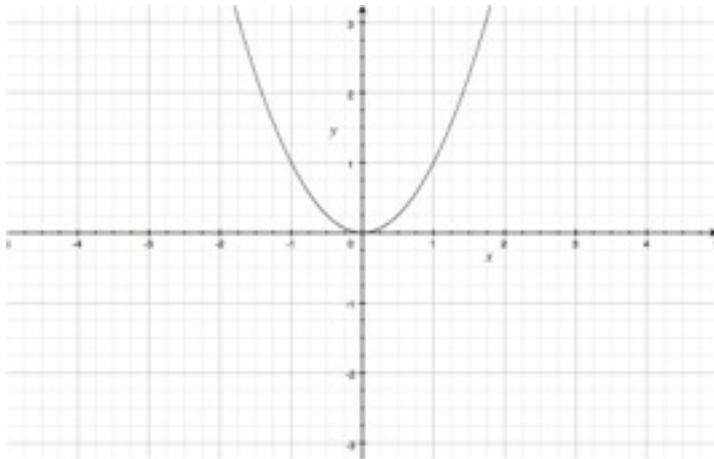
$y=mx+3$
$0=m(4)+3$
$-3=m(4)$
$-3/4=m$

We've now isolated m, or the slope, so we have our answer: **-3/4 or -0.75.**
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II. Quadratic Equations

Quadratic equations are, in simple terms, equations that have a variable in the second power. An equally simple example would be this equation, with its accompanying graph:

$y=x^2$

You should be familiar with this. This is a graph of a parabola, and interestingly enough, the graphs of quadratic equations are also termed 'conics' or conic sections. This is because they look like sections of three-dimensional cones. Included here would be your parabolas, circles and hyperbolas.

Hyperbolas and circles come out very rarely, if at all, in standardized tests, so we will focus on what is necessary, which would be the parabolas.

They come in the standard form of $y=a(x-h)^2+k$, with the vertex, or what you could describe as the 'tip' of the parabola at the point (h,k).

The problem is they don't usually give you such an easy time by giving you a quadratic equation in standard form. If it's in non-standard form, for the quadratic function (something like an equation) $f(x)$, its vertex is:

$(-b/2a , f(-b/2a))$, in $f(x)=ax^2+bx+c$

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Example 1: What is the vertex of the function $f(x)=2x^2+8x+3$?

You can't use the standard form, right? So use the vertex formula.

$a=2$, $b=8$. We have all we need, but we'll solve it one at a time. Let's start with $-b/2a$.

$-b/2a$ is the same as $-8/2(2)$. When we simplify this, we get *-2*, which is now our x-value for the vertex.

The y value is $f(-8/2(2))$. We solved for $-8/2(2)$ before already, and it's *-2,* so we plug in the value *-2* into the function $f(x)$. We'll get $2(2^2)+8(2)+3=y$, which result in -9, or our vertex's y-value.

Now we have both the *x*-value and *y*-value of our vertex, so our final answer is **(-2, -9)**, which you can confirm by inputting the equation in a graphing calculator.

III. Geometry

Geometric problems in the UPCAT are expected to involve some variation of solving for the area or volume of a shape or a solid. This is usually very doable, especially if you have the formulas at hand, so the key here would be knowing your geometric formulas and the different quirks of every shape or solid.

A. Triangles

Angles of a triangle: A sum of 180 degrees
Area of a triangle: {(base x height)/2}=area
Pythagorean Theorem: $a^2+b^2=c^2$, with c as hypotenuse, the side opposite the 90 degree angle of a triangle.
Trigonometry: Just remember SOH CAH TOA

SOH: sin=opposite/hypotenuse
CAH: cos=adjacent/hypotenuse
TOA: tan=opposite/adjacent

B. Circles

Area of a Circle= (pi)(radius)^2=area
Circumference of a Circle= 2(pi)(radius)=circumference
Length of an Arc of a Circle= {angle/360}{2(pi)(radius)}=length of arc

A triangle inside a circle with one side spanning the diameter is always a right triangle.

C. Quadrilaterals

Sum of Angles= 360 degrees
Parallelogram: Opposite angles are equal
Area of a Parallelogram: (base)(height)=area

D. Angles of regular polygons

The angle of any regular polygon, or a polygon whose side lengths are all equal is solvable with the equation *180(n-2)=angle sum*, with n as the number of sides a polygon has.

We can test this with the three sided polygon, the triangle, which we know has an angle sum of 180 degrees.

{180(3-2)}=x
180=x, which we know is correct.

To find the specific angle if a polygon is regular, simply divide that by *n*. Continuing from the previous example, this means that in a regular triangle, each angle is 60 degrees.

E. Three-Dimensional Shapes

Volume of a Cube = (side)^3
Volume of a Cuboid = (length)(width)(height)
Volume of a Sphere = 4/3(pi)(radius)^3
Volume of a Cylinder = (height)(pi)(radius)^2
Volume of a Cone = (pi)(radius)^2(height/3)

IV. Functions

Functions are usually written as something like an equation, except that instead of using the variable y, you use the term $f(x)$ instead, where you 'input' an x value at $f(x)$ to get a result.

For example, we have $f(x)=x+1$. If $x=2$, we can 'evaluate' the function and 'input' the given x value at $f(x)$. In the process, we will replace each x in the function with its given value here, which is two, so it becomes:
f(x)=2+1
f(x)=3

Another defining characteristic of a function is that each x-value is only assigned one corresponding $f(x)$ or y-value. For example, in the graph of a function, the following two points cannot co-exist: (2,3), (2,4). In such a situation, you call the relationship between the y-value and the x-value a 'relation' not a 'function.' Knowing this, figures such as circles and hyperbolas are clearly not functions.

You can test if something is a function graphically by making use of the vertical line test, wherein if at any x-value, a vertical line intersects with more than one point of a figure, it is not a function. See the following graph, with a vertical line at $x=0.4$, and a circle.
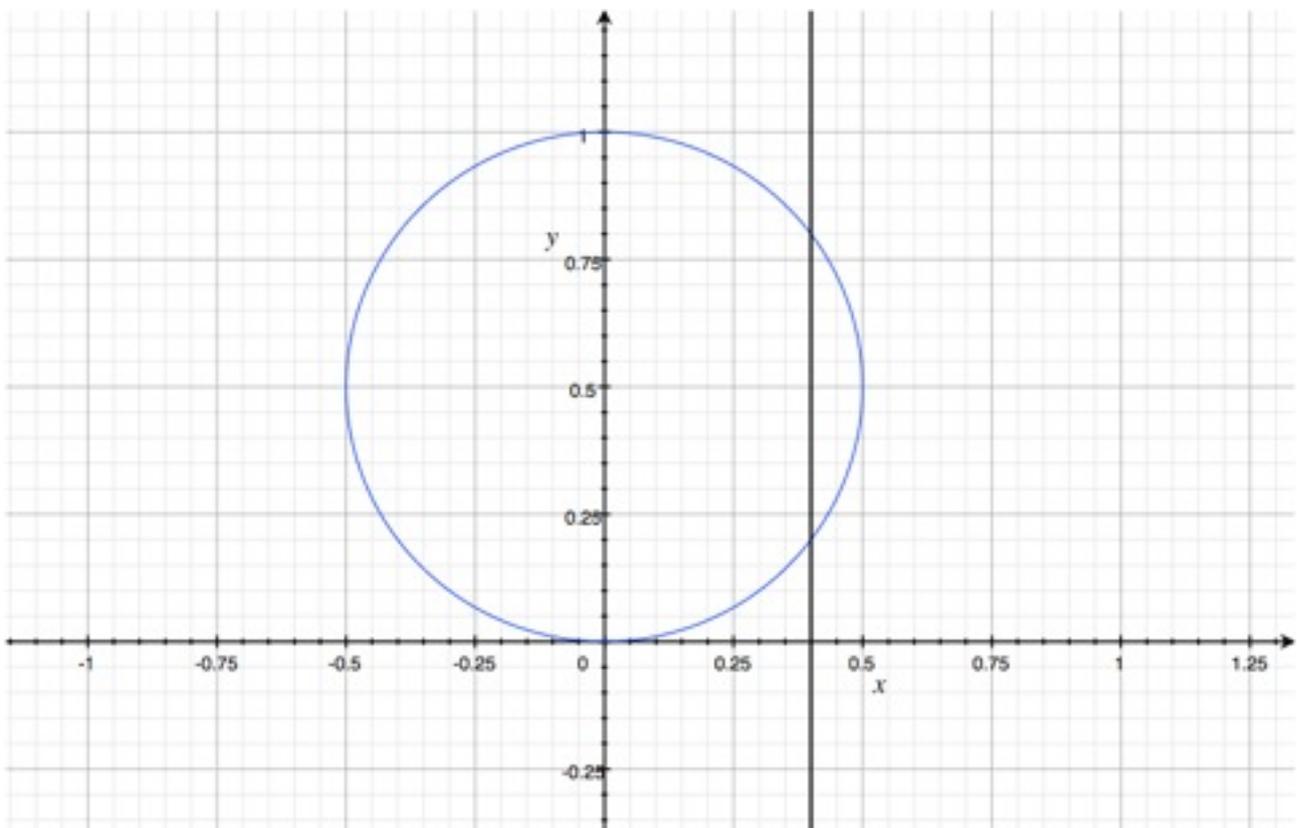
The usual Math 12 coverage of functions and relations often involves the different types of functions, such as rational functions and quadratic functions, which you should be familiar with by now. We will instead discuss some new types of functions introduced in Math 12 and transformation.

A. Transformation of Functions

Simply put, the transformation of functions involves moving the graph across the Cartesian plane without changing its shape or form. It is usually achieved by adding or subtracting a constant to the original function, which results in a new 'transformed' function.

Here we will use $f(x)$ to denote the original function, $g(x)$ to denote the new, transformed function, and k as an arbitrary constant.

No shift: $g(x) = f(x)$
Shit Upwards: $g(x) = f(x) + k$
Shift Downwards: $g(x) = f(x) - k$
Shift Left: $g(x) = f(x+k)$
Shift Right: $g(x) = f(x-k)$

Additionally, you can stretch and shrink a function by multiplying a constant a with the function. The graph of the function will change in shape and form this time, and can be generalized as follows:

$g(x) = af(x)$
$g(x) = f(ax)$, wherein points on the y-axis stay put.

B. Absolute Values

All you need to know about absolute values for the UPCAT is that any value x that comes in between two vertical lines in this manner, $|x|$, will 'turn into' its positive equivalent in the number line.

For example, if we have $x = -4$, then $f(x) = |x|$ in this case will be $f(x) = |-4|$ and $|-4|$ will become positive (because it's an absolute value), so the complete answer is $f(x) = 4$, when $x = -4$.

What if x=4? Will it become -4? No. The absolute value only turns negatives into positives, and not the other way around.

C. Logarithms

Logarithms are quite confusing to handle, because initially, they don't make intuitive sense and seem pretty useless, but as you go on learning about them in your formal classes, you will eventually understand their purpose. Our primary objective right now is familiarize you with the basic idea of what it is, just enough to answer the odd logarithm question that will come out in the UPCAT.

Logarithms come in the general notation of

$$log_b x = y$$

The notation simply means that b raised to y is equal to x. For example:

What is y in $log_2 8 = y$?

What is b? In here it's 2. And x is 8, meaning, 2 raised to y is equal to 8. From here on, just use your basic knowledge of exponents, and from that it is clear that 2 raised to 3 is equal to 8, therefore, y is equal to 8.

There are other examples of special logarithm notations that you have to take note of. They are the common logarithms and the natural logarithms.

Common logarithms are logarithms for which it is understood that the base *b* is equal to 10. Common logarithms are written as such:

log x=y, where the base is 10.

An easy example of this would be *log 100=x*. You simply make 10 the base, and think of what power you have to raise it to to get 100. The answer, clearly, is two, and you can apply the same method for a variety of cases.

Natural logarithms meanwhile make use of the constant e as the base instead of 10. They are written as:

ln x=y, where the base is e.

The constant e by mathematical convention has a set value, in much the same way that pi has a set (and never ending) value of 3.14… rounded off to 3.14 for simplicity. The value of *e* is roughly 2.718, and will come in handy as you go down the road in mathematics.

V. Trigonometry

Trigonometry involves a very special type of function which you will encounter time and again in college, the trigonometric function, which shows the relationship between two sides of triangle and an angle. Just like logarithms, it could be a bit difficult to comprehend. Even if you went through it in your review class, chances are, you just picked up some really simple (memorizable) concepts like the identities.

We think it's far too advanced at this stage for us to be teaching you trigonometry on very solid grounding, so again, we'll teach you the stuff you'll need to at least get the one or two trig questions on the UPCAT correctly.

A. Radians

This is, by far, the easiest concept to understand in trigonometry. If you have any experience converting between different measures in chemistry such as kilograms and pounds, this won't be too difficult.

Now, remember a bit of your chemistry. Let's say we have a bag that weighs 50 kilograms. How much does it weigh in pounds, if 1 kg is equal to 2.2 lbs (*lbs is shorthand for pounds. Fun fact: The Roman word for pound is 'Libra' and that's where we get the shortcut from, for both the weight, 'lb' and the British currency '£'*)?

So if we have 50kg, we can multiply it with a fraction that has 1 kg at the denominator, and 2.2 lbs at the numerator, so that the kg signs cancel out and we're left with lbs. It will look like this:

=(50 kg)[(2.2 lbs)/(1 kg)]
=(50)(2.2 lbs)
x=110 lbs

We apply essentially the same thing for radians. Radians are simply another way to express the degree of an angle, in the same way that both pounds and kilogram represent mass. The difference is that radians express it in terms of pi (π), which is equal to 180 degrees.

Just remember the conversions.

x degrees is x(π/180) radians.
y radians is y(180/π) degrees.

This makes sense because if something is expressed in radians, it has a pi attached to it. If we want to cancel it out, we need to have a pi at the denominator of what we're multiplying it with.

Example:

What is 360 degrees in radians?

x=(360)(π/180)
=2(π)
=2π

B. Basic Trigonometric Identities

This comes out more often in the ACET than the UPCAT, and harder questions will expect you to simplify rather daunting fractions with all those trig functions, but our objective here is to at least give you an idea of what the basic trigonometric identities are, not to help you solve those difficult problems. This is unlikely to happen if you haven't actually done trig yet because all of it looks really weird when you're used to algebra, so we'll stick with the easy to comprehend ones. You'll lose a point or two at most, and 99% of the rest probably did too.

Basically you just memorize these and play around with them until you find that you can cancel and simplify the expression. θ, or theta, by the way, is like the '*x*' in trigonometry. It's a sort of variable that represents the degree value of something that you input in your trig functions.

Basic:

$\tan \theta = \sin \theta / \cos \theta$
$\cot \theta = \cos \theta / \sin \theta$
$\sin \theta = 1 / \csc \theta$
$\cos \theta = 1 / \sec \theta$
$\tan \theta = 1 / \cot \theta$
$\csc \theta = 1 / \sin \theta$

Now that's quite a handful isn't it? Believe me you'll find it hard now, but you'll be memorizing all of them, including the ones coming up, when you take trig.

Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$
$\tan^2\theta + 1 = \sec^2\theta$
$\cot^2\theta + 1 = \csc^2\theta$

We'll stop here and give you examples instead, so you can comprehend what we mean by 'simplifying expressions' using these identities.

Example 1: Simplify $\sin^2\theta + \csc \theta$.

$\sin^2\theta + 1/\sin\theta$
$\sin \theta + 1$
The final answer is $\sin \theta$.

VI. Special Pre-Requisite Skills

The number one skill that most high school students miss out on come test time, and the only skill we will actually teach here is factoring. Never mind the discriminants, the quadratic formula and radicals, which can easily be memorized (You *should* look them up though!). This skill is used in so many problems you will encounter in entrance tests and college, that you'd best familiarize yourself with it by practicing.

1. DOTS, or Difference of Two Squares: $x^2 - y^2 = (x + y)(x - y)$
2. DOTC, or Difference of Two Cubes: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
3. Sum of Two Cubes: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
4. Quadratic Equations

a. No coefficient for x.

This is generally expressed in the form $x^2 + bx + c$, where x is the variable, b is a coefficient constant, and c is a constant. Unlike the next type of equation, note that x^2 does not have a coefficient a attached to it.

You can factor this our by looking for two numbers that add up to b, and, when multiplied, result in c.

For example:
$x^2 + 8x + 12 = (x + 2)(x + 6)$

Why?
2+6= b or 8
2*6 = c or 8

b. Coefficient for x.

This one is expressed in the form $ax^2 + bx + c$, where a and b are coefficients and c is a constant. It's best to explain this through an example though. For example, factor $6x^2 + 5x - 6$.

AC test first. a=6, c=6, so a times c is equal to 36.

Now look for two factors of ac that sum up to b, or 5. These two numbers would be 9 and -4. You then use these to numbers independently to represent bx. It will look like this:

$6x^2 + 9x - 4x - 6$. As you can see, you 'split up' 5x into 9x and -4x. It's just the same as $6x^2 + 5x - 6$ if you add them up.

What you do know is factor out the x's from the pairs on either side. For $6x^2 + 9x$, you can take out 3x, which will result in the following:

$[6x^2 + 9x] -4x - 6$
$[3x(2x + 3)] - 4x -6$

You can do the same for the other side, -4x - 6. For ease, let's factor out -1 first. Then we can factor out 2 from 4x+6, which will result in 2x+3. It will result in this:

$[3x(2x + 3)] + [-4x - 6]$
$[3x(2x + 3)] + (-1)[4x + 6]$
$[3x(2x + 3)] + (-1)[2(2x+3)]$
$3x(2x + 3) - 2(2x+3)$
You can factor out $(2x+3)$ easily now, and you're done!
$(2x+3)(3x-2)$ *is your final answer!*