1. The frequency distribution of total annual emoluments of a sample of 100 executives working in multinational companies is given below:

Total Emoluments (in Rs Lac)	02–06	06 –10	10-14	14 -18	18-22	22-26	26-30
Number of Executives	10	15	30	18	12	09	06

Find a) median emoluments, b) mode emoluments, c) 3rd quartile, and d) 9th decile.

(1)

Solution :-

The data is presented in the following format for calculation of the above measure

C.I	f_i (friquency)	x_i (class interval)	$f_i x_i$	C.f (Cumualitive frequency)
02 - 06	10	4	40	10
06 - 10	15	8	120	25
10 - 14	30	12	360	55
14 - 18	18	16	288	73
18 - 22	12	20	240	85
22 - 26	09	24	216	94
26 - 30	06	28	168	100

$\Sigma f_i = 100$ $\Sigma f_i x_i = 1432$

1). Arithmetic mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$= \dfrac{1432}{100} = 14.32$

(a) Median =

$$= L_m + \frac{\left(\frac{n}{2} - f_c\right)}{f_m} \times W_m$$

$$= 10 + \frac{\left(\frac{100}{2}\right) - 25}{30} \times 4$$

$$= 10 + \frac{50 - 25}{30} \times 4$$

$$= 10 + \frac{25}{30} \times 4$$

$$= 10 + \frac{10}{3} = \frac{40}{3} = 13.33$$

Hence, L_m = lower limit of the median class interval observation

f_m = frequency of the median class interval

f_c = cumulative frequency up to just before the median class-interval

W_m = width of class-interval

n = number of total observation

(b) mode (m)

$$\text{mode} = (3 \times \text{median} - 2 \times \text{mean})$$
$$= (3 \times 13.33 - 2 \times 14.32)$$
$$= (39.99 - 28.64)$$
$$= 11.35 \quad \underline{\text{Ans}}$$

(c) 3rd Quartile.

$$Q_3 = LQ_3 + \frac{\left(\frac{3n}{4}\right) - fc}{fQ_3} \times WQ_3$$

$$= 18 + \frac{\left(\frac{3 \times 100}{4}\right) - 73}{12} \times 4$$

$$= 18 + \frac{75 - 73}{12} \times 4$$

$$= 18 + \frac{2}{12} \times 4$$

$$= 18 + \frac{1}{6} \times 4$$

$$= 18 + \frac{2}{3} = \frac{54 + 2}{3}$$

$$= \frac{56}{3} = 15.5$$

(D). 9^{th} **DECILE**;

$$D_9 = L_{D9} + \frac{\left(\frac{9}{10}+n-f_c\right)}{f_{D90}} \times W_{D90}$$

$$= 14 + \frac{60-65}{16} \times 4$$

$$= 14 + \frac{5}{16} \times 4$$

$$= 14 + \frac{5}{4}$$

$$= \frac{56+5}{4} = \frac{61}{4} = 15.2$$

Q2. A population consisting of a certain proportion of defective items has mean μ = 2. If a sample of 4 items is examined and repeated 200 times, obtain a) probability of an item being defective, b) probability of getting 2 defective items in the sample, c) expected frequency of getting 2 defective items, and d) expected frequency of getting at the most 2 defective items. Is the resultant distribution skewed?

Ans:

we have,

$$\text{Mean}\ (\mu) = 2$$

$$\text{mean} = np$$

$$\mu = 4 \cdot p$$

$$p = \frac{\cancel{2}}{\cancel{4}} = \frac{1}{2}$$

$$q = 1 - p$$

$$= 1 - \tfrac{1}{2} = \frac{1}{2}$$

Let x denote the two of defective item

a) $P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

$$P(X=4) = {}^{4}C_{4}\left(\frac{1}{2}\right)^{4}\left(\frac{1}{2}\right)^{4-4}$$

$$= 1 \times \left(\frac{1}{2}\right)^{4} \times 1$$

$$= \frac{1}{16}$$

b) Probability of getting 2 defective items in the sample.

$$P(\bar{X}=2) = {}^{4}C_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{4-2}$$

$$= 6 \cdot \left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{2}$$

$$= \cancel{6}^{\,3} \times \frac{1}{\cancel{4}_{\,2}} \times \frac{1}{4} = \frac{3}{2} \times \frac{1}{4}$$

$$= \frac{3}{}$$

(c). expected frequency of getting 2 defective items and:

$$P(\bar{x}=3) + P(x=4)$$

$$= 4c_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{4-3} + 4c_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{4-4}$$

$$= \cancel{4}\times\frac{1}{\cancel{8}_2}\times\left(\frac{1}{2}\right) + 1\times\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0$$

$$= \frac{1}{4} + \frac{1}{16}$$

$$= \frac{4+1}{16} = \frac{5}{16}$$

(d). expected frequency of getting at the most 2 defective items. 1

$$P(x=0) + P(x=1) + P(x=2)$$

$$= 4c_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{4-0} + 4c_1\left(\frac{1}{2}\right)^{4-1}\left(\frac{1}{2}\right)^1 + 4c_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4$$

$$= 1\times1\times\left(\frac{1}{2}\right)^4 + 4\times\left(\frac{1}{2}\right)^3\times\frac{1}{2} + 4c_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4$$

$$= \left(\frac{1}{2}\right)^4\left[1+4+6\right]$$

$$= \frac{1}{16}\times11 = \frac{11}{16}$$

Resultant distribution:

$$\text{(bionomial distribution)} = \frac{1-2p}{\sqrt{np(1-p)}}$$

$$= \frac{1-\cancel{2}\frac{1}{\cancel{2}}}{\sqrt{\overset{2}{\cancel{4}}\cdot\frac{1}{\cancel{2}}\left(1-\frac{1}{2}\right)}}$$

$$= \frac{1-1}{\sqrt{\cancel{2}\left(\frac{1}{\cancel{2}}\right)}}$$

$$= \frac{1-1}{\sqrt{1}}$$

$$= \frac{0}{1} = 0$$

The resultant distribution skewed = 0

Q3.A research organization claims that the monthly wages of industrial workers in district X exceeds that of those in district Y by more than Rs 150. Two different samples drawn independently from the two districts yielded the following results:

District X:

District Y:

Verify at 0.05 level of significance whether the sample results support the claim of the organization.

Let

District X	District Y
$\bar{x}_1 = 648$	$x_2 = 495$
$s_1^2 = 120$	$s_2^2 = 140$
$n_1 = 100$	$n_2 = 90$

Let, $H_0 : \bar{x}_1 - \bar{x}_2 = 150$

& $H_A : \bar{x}_1 - \bar{x}_2 > 150$

using Z - statistics

$$|t| = \left| \frac{\bar{x}_1 - \bar{x}_2}{S.E_{(\bar{x}_1 - \bar{x}_2)}} \right|$$

$$= \left| \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \right|$$

$$= \left| \frac{648 - 495}{\sqrt{\frac{120}{100} + \frac{140}{90}}} \right| = \left| \frac{153}{\sqrt{1.2 + 1.56}} \right|$$

$$= \left| \frac{153}{\sqrt{2.76}} \right| = \left| \frac{153}{1.66} \right| = 92.169$$

Conclusion :-

At, $\alpha = 0.05$

t_{cal} (92.169) > $t_{tabulated}$ (1.645)

$\therefore$ H_0 is rejected and we conclude that the monthly wages of industrial workers in district X exceeds that of district Y by more than Rs. 150.

The following data relate to marketing expenditure in Rs lac and the corresponding sales of a product in Rs crores. Estimate the marketing expenditure to attain a sales target of Rs 40 crores.

Marketing expenditure(lakh)	10	12	15	20	23
Product sales(crores)	14	17	23	21	25

Q4.

Soln:

(4)

THE TWO REGRESSION EQUATION FOR THE FOLLOWING DATA

Marketing expenditure(x) (Rs. lac):	10	12	15	20	23
Sales(Y) Rs. crore	14	17	23	21	35

obtain the two regression eqn for the following data

X:	10	12	15	20	23
Y:	14	17	23	21	35

X	A = 15 dx	dx^2	Y	A = 23 dy	dy^2	dxdy
10	-5	25	14	+9	81	-45
12	-3	9	17	+6	36	-18
15	0	0	23	0	0	0
20	5	25	21	2	4	10
23	8	64	35	-12	144	-96
N = 5 $\Sigma x = 80$	$\Sigma dx = 5$	$\Sigma dx^2 = 123$	$\Sigma Y = 110$	$\Sigma dy = 5$	$\Sigma dy^2 = 265$	$\Sigma dxdy = -149$

$dx = x - A$

$dy = y - A$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{80}{5} = 16$$

$$\bar{Y} = \frac{\Sigma Y}{N} = \frac{110}{5} = 22$$

Since the actual means of X and Y are in function, we should take deviations from assumed mean to simplify the calculations.

$$byx = \frac{N \times \Sigma dxdy - \Sigma dx \cdot \Sigma dy}{N \cdot \Sigma dx^2 - (\Sigma dx^2)}$$

$$= \frac{5 \times (-149) - (5) \times 5}{5 \times 123 - (123)}$$

$$= \frac{-745 - 25}{615 - 123} = \frac{-770}{492}$$

$$= -1.565$$

$$= -1.57$$

$$b_{xy} = \frac{N \times \sum dxdy - \sum dx \cdot \sum dy}{N \cdot \sum dy^2 - (\sum dy)^2}$$

$$= \frac{5 \times (-149) - (5) \times 5}{5 \times 265 - (5)^2}$$

$$= \frac{-745 - 25}{1325 - 25}$$

$$= \frac{-770}{1300} = \frac{-77}{130}$$

$$= -0.592$$

$$= -0.59$$

when,

Regression equation of X on Y

$$X-\bar{X} = b_{xy}(Y-\bar{Y})$$

$$X-16 = +0.59(Y-22)$$

$$X-16 = +0.59Y - 12.98$$

$$X = +0.59Y + 16 - 12.98$$

$$X = 0.59Y + 3.02$$ Ans

when sales, Y = 40 crores

$$Y = 40$$

$$X = +0.59Y + 3.02$$

$$= 0.59(40) + 3.02$$

$$\therefore X_{40} = 23.6 + 3.02$$

$$= 26.62$$ Ans

Q5.Write short notes on:

i)Adjoint of a matrix

Ans:

In linear algebra, the adjugate, classical adjoint, or adjunct of a square matrix is the transpose of the cofactor matrix.

The adjugate has sometimes been called the "adjoint", but today the "adjoint" of a matrix normally refers to its corresponding adjoint operator, which is its conjugate transpose.

The adjugate of A is the transpose of the cofactor matrix C of A:

$$\operatorname{adj}(\mathbf{A}) = \mathbf{C}^{\mathsf{T}}.$$

In more detail: suppose R is a commutative ring and **A** is an n×n matrix with entries from R.

- The (i,j) minor of **A**, denoted $\mathbf{A}_{ij}$, is the determinant of the (n − 1)×(n − 1) matrix that results from deleting row i and column j of **A**.
- The cofactor matrix of **A** is the n×n matrix **C** whose (i,j) entry is the (i,j) cofactor of **A**:

$$\mathbf{C}_{ij} = (-1)^{i+j}\mathbf{A}_{ij}.$$

- The adjugate of **A** is the transpose of **C**, that is, the n×n matrix whose (i,j) entry is the (j,i) cofactor of **A**:

$$\operatorname{adj}(\mathbf{A})_{ij} = \mathbf{C}_{ji} = (-1)^{i+j}\mathbf{A}_{ji}.$$

The adjugate is defined as it is so that the product of A and its adjugate yields a diagonal matrix whose diagonal entries are det(**A**):

$$\mathbf{A}\operatorname{adj}(\mathbf{A}) = \det(\mathbf{A})\,\mathbf{I}.$$

A is invertible if and only if det(**A**) is an invertible element of R, and in that case the equation above yields:

$$\operatorname{adj}(\mathbf{A}) = \det(\mathbf{A})\mathbf{A}^{-1},$$
$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})}\operatorname{adj}(\mathbf{A}).$$

ii)Skewness and its measures

Ans:

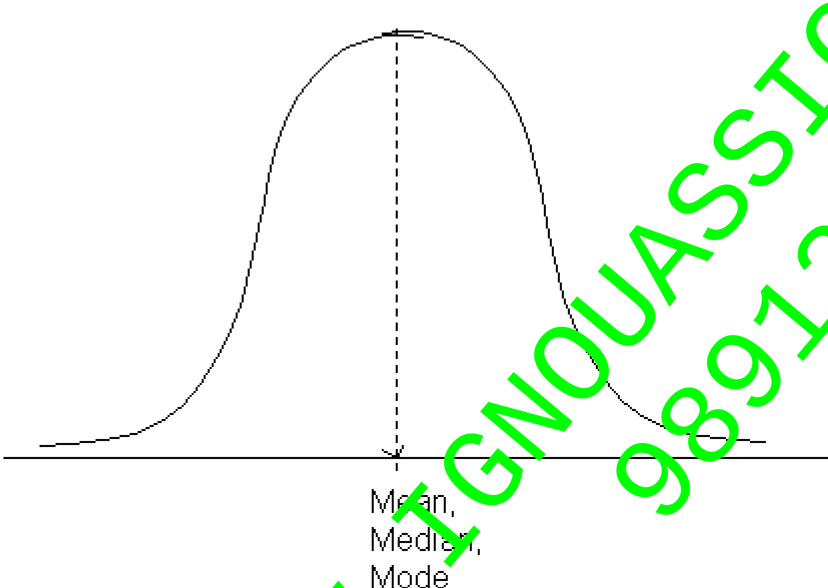
In probability theory and statistics, skewness is a measure of the asymmetry of the probability distribution of a real-valued random variable about its mean. The skewness value can be positive or negative, or even undefined.

The qualitative interpretation of the skew is complicated. For a unimodal distribution, negative skew indicates that the tail on the left side of the probability density function is longer or fatter than the right side – it does not distinguish these shapes. Conversely, positive skew indicates that the tail on the right side is longer or fatter than the left side. In cases where one tail is long but the other tail is fat, skewness does not obey a simple rule. For example, a zero value indicates that the tails on both sides of the mean balance out, which is the case both for a symmetric distribution, and for asymmetric distributions where the asymmetries even out, such as one tail being long but thin, and the other being short but fat. Further, in multimodal distributions and discrete distributions, skewness is also difficult to interpret. Importantly, the skewness does not determine the relationship of mean and median.

Its measures

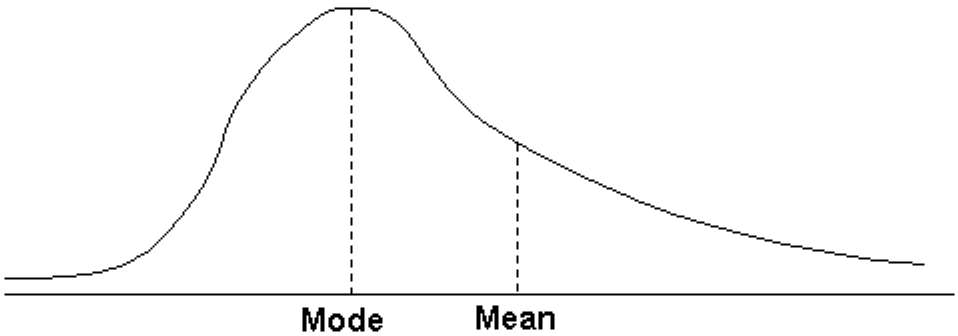
Skewness is a measure of symmetry, or more precisely, the lack of symmetry. A distribution, or data set, is symmetric if it looks the same to the left and right of the center point.

A normal distribution is a bell-shaped distribution of data where the mean, median and mode all coincide. A frequency curve showing a normal distribution would look like this:



In a normal distribution, approximately 68% of the values lie within one standard deviation of the mean and approximately 95% of the data lies within two standard deviations of the mean.

If there are extreme values towards the positive end of a distribution, the distribution is said to be **positively skewed**. In a positively skewed distribution, the mean is greater than the mode. For example:



A negatively skewed distribution, on the other hand, has a mean which is less than the mode because of the presence of extreme values at the negative end of the distribution.

There are a number of ways of measuring skewness:

$$\text{Pearson's coefficient of skewness} = \frac{\text{mean} - \text{mode}}{\text{Standard deviation}} = \frac{3(\text{mean} - \text{median})}{\text{Standard deviation}}$$

$$\text{Quartile measure of skewness} = \frac{Q_3 - 2Q_2 + Q_1}{Q_3 - Q_1}$$

iii)Business forecasting

Ans:

There are a number of different methods by which a business forecast can be made. All the methods fall into one of two overarching approaches: qualitative and quantitative.

Qualitative Models
Qualitative models have generally been successful with short-term predictions, where the scope of the forecast is limited. Qualitative forecasts can be thought of as expert-driven, in that they depend on market mavens or the market as a whole to weigh in with an informed consensus. Qualitative models can be useful in predicting the short-term success of companies, products and services, but meets limitations due to its reliance on opinion over measurable data. Qualitative models include:

- Market Research Polling a large number of people on a specific product or service to predict how many people will buy or use it once launched.
- Delphi Method: Asking field experts for general opinions and then compiling them into a forecast. (For more on qualitative modeling, read *Qualitative Analysis: What Makes A Company Great?*)

Quantitative Models
Quantitative models discount the expert factor and try to take the human element out of the analysis. These approaches are concerned solely with data and avoid the fickleness of the people underlying the numbers. They also try to predict where variables like sales, gross domestic product, housing prices and so on, will be in the long-term, measured in months or years. Quantitative models include:

- The Indicator Approach: The indicator approach depends on the relationship between certain indicators, for example GDP and unemployment rates, remaining relatively unchanged over time. By following the relationships and then following indicators that are leading, you can estimate the performance of the lagging indicators, by using the leading indicator data.
- Econometric Modeling: This is a more mathematically rigorous version of the indicator approach. Instead of assuming that relationships stay the same, econometric modeling tests the internal consistency of data sets over time and the significance or strength of the relationship between data sets. Econometric modeling is sometimes used to create custom indicators that can be used for a more accurate indicator approach. However, the econometric models are more often used in academic fields to evaluate economic policies. (For a basic explanation on applying econometric models, read *Regression Basics For Business Analysis*.)
- Time Series Methods: This refers to a collection of different methodologies that use past data to predict future events. The difference between the time series methodologies is usually in fine details, like giving more recent data more weight or discounting certain outlier points. By tracking what happened in the past, the forecaster hopes to be able to give a better than average prediction about the future. This is the most common type of business forecasting, because it is cheap and really no better or worse than other methods.