

Based on Sthi) Maharasitra Board Syllahus

Whincmatics andi Science

## Written as per the revised syllabus prescribed by the Maharashtra State Board

 of Secondary and Higher Secondary Education, Pune.
# SSC $^{2}$ HOTS 

## Salient Features

- Written as per the new textbook.
- Exhaustive coverage of entire syllabus.
- Facilitates complete and thorough preparation of HOTS section.
- Mark-wise segregation of each lesson.
- Constructions drawn with accurate measurements.
- Board questions of HOTS with solutions updated till the latest year (March 2016).
- Self evaluative in nature.


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## Preface

HOTS stands for Higher Order Thinking Skills. As the name implies, this book is filled to the brim with questions that challenge your Thinking Skills. Unlike the traditional methods that focus on drill and repetition as a mode of education, HOTS brings out the problem solving ability of a child.

Inclusion of HOTS in a Question Paper is to attest if the students factually have an in-depth understanding of the said topic. It is not just how much but how well you understand the subject.

These twisted and brain challenging questions (HOTS) that form a part of the Question Paper demand an out of the box thinking. Students are assessed through this format of Questions in terms of skills pertaining to their analyzing, reasoning, comprehending, application and evaluation of a subject.

This book is full of such thought provoking and curiosity crunching questions across the subjects of Algebra, Geometry and Science. The Questions throughout are framed in an innovative format including flowcharts. The emphasis in this courseware lies not just on the subject knowledge but also its applications in real life.

With a progressive thought we have thoroughly followed the new syllabus pattern while composing this book. There is no doubt that in the long run this book would be a proven aid for students to encounter questions pertaining to Higher Order Thinking Skills.

We wish all the aspirants all the best and hope this book turns out to be a formidable aid in your tryst with success.

## Best of luck to all the aspirants!

## Yours faithfully

Publisher

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## 01 Arithmetic Progression

## 1 Mark Questions

1. If $S_{n}=n P+\frac{1}{2} n(n-1) Q$, where $S_{n}$ denotes the sum of first $n$ terms of an A.P., find the common difference of the A.P.

## Solution:

Sum of first n terms of an A.P. is given by
$\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]=\mathrm{na}+\frac{1}{2} \mathrm{n}(\mathrm{n}-1) \mathrm{d}$
Comparing with $\mathrm{S}_{\mathrm{n}}=\mathrm{nP}+\frac{1}{2} \mathrm{n}(\mathrm{n}-1) \mathrm{Q}$
$\therefore \quad \mathbf{d}=\mathbf{Q}$
2. Find $t_{n}$ of the following A.P.

$$
\frac{5}{6}, 1,1 \frac{1}{6}, \ldots ., t_{n}
$$

## Solution:

The given A.P. is $\frac{5}{6}, 1,1 \frac{1}{6} \ldots \ldots$
Here, $a=\frac{5}{6}, \mathrm{~d}=\frac{1}{6} \quad---\left[\because \mathrm{d}=\mathrm{t}_{2}-\mathrm{t}_{1}\right]$
$\therefore \quad \mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}$
$=\frac{5}{6}+(n-1) \times \frac{1}{6}$

$$
=\frac{5}{6}+\frac{\mathrm{n}-1}{6}
$$

$\therefore \quad t_{n}=\frac{n+4}{6}$
3. Find the first term of the following sequence, if

$$
S_{n}=\frac{(4 n-3) 3^{n}+3}{2}
$$

## Solution:

$$
S_{n}=\frac{(4 n-3) 3^{n}+3}{2}
$$

$\therefore \quad$ When $\mathrm{n}=1$,

$$
\begin{aligned}
S_{1} & =\frac{(4 \times 1-3) \times 3^{1}+3}{2} \\
& =\frac{(4-3) \times 3+3}{2}=\frac{1 \times 3+3}{2}=\frac{6}{2}
\end{aligned}
$$

$\therefore \quad \mathrm{S}_{1}=3$
$\therefore \quad$ The first term is $\mathbf{3}$.
4. Find the value of $\mathbf{d}$ for an A.P., if $\mathbf{t}_{\mathbf{5}}=\mathbf{1 1}$ and $\mathrm{t}_{6}=13$.
Solution:

$$
\begin{array}{ll} 
& \mathrm{d}=\mathrm{t}_{6}-\mathrm{t}_{5}=13-11=2 \\
\mathbf{d}=\mathbf{2}
\end{array}
$$

## 2 Marks Questions

5. Find the sum of all three digit numbers which leave the remainder 3 when divided by 5 .

## Solution:

The sequence is $103,108,113, \ldots ., 998$
The sequence is an A.P. with
$\mathrm{a}=103, \mathrm{~d}=5, \mathrm{t}_{\mathrm{n}}=998$
Since, $\mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}$
$\therefore \quad 998=103+(n-1) 5$
$\therefore \quad 998=103+5 n-5$
$\therefore \quad 5 \mathrm{n}=900$
$\therefore \quad \mathrm{n}=180$
$\therefore \quad S_{n}=\frac{n}{2}[2 a+(n-1) d]$
$=\frac{180}{2}[2 \times 103+(180-1) 5]$
$=90[206+179 \times 5]=90 \times 1101$
$\therefore \quad \mathbf{S}_{\mathbf{n}}=\mathbf{9 9 0 9 0}$
6. If for an A.P., $S_{n}=0.02\left(2^{n}-1\right)$, find $t_{n}$.

## Solution:

$\mathrm{S}_{\mathrm{n}}=0.02\left(2^{\mathrm{n}}-1\right)$
We know that,
$\mathrm{t}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}-\mathrm{S}_{\mathrm{n}-1}$
$\mathrm{t}_{\mathrm{n}}=0.02\left(2^{\mathrm{n}}-1\right)-\left[0.02\left(2^{\mathrm{n}-1}-1\right)\right]$ $=0.02\left(2^{\mathrm{n}}-1\right)-\left[0.02\left(2^{\mathrm{n}} \cdot 2^{-1}-1\right)\right]$
$=0.02 \times 2^{\mathrm{n}}-0.02-\frac{0.02 \times 2^{\mathrm{n}}}{2}+0.02$
$=0.02 \times 2^{\mathrm{n}}-0.01 \times 2^{\mathrm{n}}$
$\therefore \quad \mathbf{t}_{\mathrm{n}}=\mathbf{0 . 0 1} \times \mathbf{2 n}^{\mathrm{n}}$

## 3 Marks Questions

7. Find the sum of all natural numbers lying between 50 and 500 , which are multiples of 5 .

## Solution:

Required numbers are $55,60,65,70$,
This is an A.P with
$\mathrm{a}=55, \mathrm{~d}=60-55=5$

Now, $\mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}=495$
$\therefore \quad 55+(n-1) \times 5=495$
$\therefore \quad 5(n-1)=495-55$
$\therefore \quad \mathrm{n}-1=\frac{440}{5}$
$\therefore \quad \mathrm{n}-1=88$
$\therefore \quad \mathrm{n}=89$
Here, $\mathrm{t}_{1}=55, \mathrm{t}_{\mathrm{n}}=495, \mathrm{n}=89$
$\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}\left(\mathrm{t}_{1}+\mathrm{t}_{\mathrm{n}}\right)=\frac{89}{2}(55+495)=\frac{89}{2} \times 550$
$\therefore \quad S_{n}=\mathbf{2 4 4 7 5}$
8. The first, second and last terms of an A.P. are $a, b$ and $2 a$. Find the number of terms in A.P.

## Solution:

$a, b, \ldots \ldots ., 2 a$ are in A.P.
Since, $t_{n}=a+(n-1) d$
$\therefore \quad 2 a=a+(n-1)(b-a)$
$\therefore \quad 2 a-a=(n-1)(b-a)$
$\therefore \quad a=(n-1)(b-a)$
$\therefore \quad \mathrm{n}-1=\frac{\mathrm{a}}{\mathrm{b}-\mathrm{a}}$
$\therefore \quad \mathrm{n}=\frac{\mathrm{a}}{\mathrm{b}-\mathrm{a}}+1=\frac{\mathrm{a}+\mathrm{b}-\mathrm{a}}{\mathrm{b}-\mathrm{a}}$
$\therefore \quad \mathrm{n}=\frac{\mathrm{b}}{\mathbf{b}-\mathbf{a}}$
9. How many terms of an A.P. $-6, \frac{-11}{2},-5, \ldots$ are needed to give the sum - 25 ? Explain the reason for getting two answers.

## Solution:

Here, $\mathrm{t}_{1}=\mathrm{a}=-6, \mathrm{~d}=\frac{1}{2} \quad---\left[\because d=\mathrm{t}_{2}-\mathrm{t}_{1}\right]$
Let -25 be the sum of $n$ terms of this A.P.
where, $\mathrm{n} \in \mathrm{N}$.
$\therefore \quad \mathrm{S}_{\mathrm{n}}=-25$
$\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]$
$\therefore \quad-25=\frac{\mathrm{n}}{2}\left[2 \times-6+(\mathrm{n}-1) \frac{1}{2}\right]$
$\therefore \quad-25=\frac{\mathrm{n}}{2}\left[\frac{\mathrm{n}-25}{2}\right]$
$\therefore \quad-25=\frac{\mathrm{n}^{2}-25 \mathrm{n}}{4}$
$\therefore \quad-25 \times 4=n^{2}-25 n$
$\therefore \quad-100=n^{2}-25 n$
$\therefore \quad n^{2}-25 n+100=0$
$\therefore \quad(n-5)(n-20)=0$
$\therefore \quad \mathrm{n}-5=0$ or $\mathrm{n}-20=0$
$\therefore \quad n=5$ or $n=20$
Both the values of $n$ are natural numbers and hence two answers are obtained.
10. Divide 45 into three terms which are in A.P. in such a way that the product of the last two terms is 255.

## Solution:

Let the three terms in A.P. be $\mathrm{a}-\mathrm{d}, \mathrm{a}, \mathrm{a}+\mathrm{d}$.
$\therefore \quad a-d+a+a+d=45$
$\therefore \quad 3 a=45$
$\therefore \quad a=15$
$\therefore \quad$ The three terms are $15-\mathrm{d}, 15,15+\mathrm{d}$
It is given that the product of last two terms is 255 .
$\therefore \quad a \times(a+d)=255$
$\therefore \quad \mathrm{a}+\mathrm{d}=\frac{255}{\mathrm{a}}$
$\therefore \quad 15+\mathrm{d}=\frac{255}{15}$
$---[\because a=15]$
$\therefore \quad 15+\mathrm{d}=17$
$\therefore \quad \mathrm{d}=2$
$\therefore \quad$ Three terms are 13, 15 and 17.
11. If the $p^{\text {th }}$ term of an A.P. is $q$ and the $q^{\text {th }}$ term is $p$, prove that $n^{\text {th }}$ term is $(p+q-n)$.

## Solution:

Let 'a' be the first term and ' $d$ ' be the common difference of the given A.P.
$\mathrm{t}_{\mathrm{p}}=\mathrm{q}$
----[Given]
$\therefore \quad a+(p-1) d=q \quad---(i)\left[\because t_{n}=a+(n-1) d\right]$
and $\quad \mathrm{t}_{\mathrm{q}}=\mathrm{p} \quad---$ [Given]
$\therefore \quad a+(q-1) d=p \quad---(i i)\left[\because t_{n}=a+(n-1) d\right]$
Subtracting equation (ii) from (i), we get
$(a-a)+(p-1) d-d(q-1)=q-p$
$\therefore \quad \mathrm{d}(\mathrm{p}-1-\mathrm{q}+1)=\mathrm{q}-\mathrm{p}$
$\therefore \quad d(p-q)=-(p-q)$
$\therefore \quad \mathrm{d}=-1$
Substituting value of $d$ in equation (i), we get
$a+(p-1)(-1)=q$
$\therefore \quad \mathrm{a}-\mathrm{p}+1=\mathrm{q}$
$\therefore \quad \mathrm{a}=\mathrm{p}+\mathrm{q}-1$
$t_{n}=a+(n-1) d$
$=(\mathrm{p}+\mathrm{q}-1)+(\mathrm{n}-1)(-1)$
$=\mathrm{p}+\mathrm{q}-1-\mathrm{n}+1$
$\therefore \quad \mathbf{t}_{\mathbf{n}}=\mathbf{p}+\mathbf{q}-\mathbf{n}$
12. Find $31^{\text {st }}$ term of A.P. whose $11^{\text {th }}$ term is 38 and $16{ }^{\text {th }}$ term is 73.

## Solution:

Let ' $a$ ' be the first term and ' $d$ ' be the common difference.
$\therefore \quad \mathrm{t}_{11}=\mathrm{a}+10 \mathrm{~d}=38$
and $\quad t_{16}=a+15 d=73$
Subtracting equation (ii) from (i), we get
$a+10 d=38$
$a+15 d=73$
$\frac{-\quad-}{-5 \mathrm{~d}=-35}$
$\therefore \quad \mathrm{d}=7$
Substituting $d=7$ in equation (i), we get
$\mathrm{t}_{11}=\mathrm{a}+10 \mathrm{~d}=38$
$\therefore \quad a+10 \times 7=38$
$\therefore \quad a=38-70$
$\therefore \quad a=-32$
$\mathrm{t}_{31}=\mathrm{a}+30 \mathrm{~d}$
Substituting the value of a and $d$, we get
$\mathrm{t}_{31}=-32+30 \times 7$
$\therefore \quad \mathrm{t}_{31}=-32+210$
$\therefore \quad \mathrm{t}_{31}=178$
$\therefore \quad 31^{\text {st }}$ term $=178$
13. Which term of A.P. 3, $15,27,39, \ldots$ will be 132 more than its $54^{\text {th }}$ term?
Solution:
Given series $3,15,27,39, \ldots$
$\mathrm{a}=3, \mathrm{~d}=12 \quad\left[\because \mathrm{~d}=\mathrm{t}_{2}-\mathrm{t}_{1}=15-3=12\right]$
$\because \quad \mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}$
$\therefore \quad \mathrm{t}_{54}=3+(54-1) 12$

$$
=3+(53 \times 12)
$$

$\therefore \quad \mathrm{t}_{54}=639$
Let us consider the required term as $\mathrm{t}_{x}$.
$\mathrm{t}_{x}=132+\mathrm{t}_{54}$
---- [Given]
$\therefore \quad \mathrm{t}_{x}=132+639$
$\therefore \quad \mathrm{t}_{x}=771$
$\because \quad \mathrm{t}_{x}=\mathrm{a}+(x-1) \mathrm{d}$
$\therefore \quad 771=3+(x-1) 12$
$\therefore \quad 771=3+12 x-12$
$\therefore \quad 771=12 x-9$
$\therefore \quad 771+9=12 x$
$\therefore \quad 780=12 x$
$\therefore \quad x=65$
$\therefore \quad$ The $65^{\text {th }}$ term of A.P. 3, 15, $27 \ldots$ will be 132 more than its $54^{\text {th }}$ term.
14. There are 20 rows of seats in a concert hall with 20 seats in first row, 21 seats in second row, 22 seats in third row and so on. Calculate the total number of seats in that concert hall.

## Solution:

As per the given condition, the sequence of number of seats in a hall is an A.P. with $\mathrm{t}_{1}=20, \mathrm{n}=20$ and $\mathrm{d}=1$
$\therefore \quad \mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2} \times[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]$
$=\frac{20}{2} \times[2 \times 20+(20-1) \times 1]$
$=10 \times[40+19]=10 \times 59$
$\therefore \quad \mathrm{S}_{\mathrm{n}}=590$
$\therefore \quad$ There are 590 seats in the concert hall.
15. The sum of the first ten terms of an A.P. is three times the sum of the first five terms, then find ratio of the first term to the common difference.

## Solution:

We know that,

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}] \\
& \mathrm{S}_{10}=\frac{10}{2}[2 \mathrm{a}+(10-1) \mathrm{d}] \\
\therefore \quad & \mathrm{S}_{10}=5(2 \mathrm{a}+9 \mathrm{~d}) \\
& \mathrm{S}_{5}=\frac{5}{2}[2 \mathrm{a}+(5-1) \mathrm{d}] \\
\therefore & \mathrm{S}_{5}=\frac{5}{2}(2 \mathrm{a}+4 \mathrm{~d})
\end{aligned}
$$

According to the given condition,
$\mathrm{S}_{10}=3 \mathrm{~S}_{5}$
$5(2 a+9 d)=3 \times \frac{5}{2}(2 a+4 d)$
$\therefore \quad 10(2 a+9 d)=15(2 a+4 d)$
$\therefore \quad 2(2 a+9 d)=3(2 a+4 d)$
$\therefore \quad 4 a+18 d=6 a+12 d$
$\therefore \quad 2 a=6 d$
$\therefore \quad \frac{\mathbf{a}}{\mathbf{d}}=\frac{\mathbf{3}}{\mathbf{1}}$
16. There is an auditorium with 35 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row and so on. Find the number of seats in the twenty second row.
[Mar 15]
Solution:
The number of seats arranged row wise are as follows :
$20,22,24, \ldots$
This sequence is an A.P. with
$\mathrm{a}=20, \mathrm{~d}=22-20=2, \mathrm{n}=22$

Now, $\mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}$
$\therefore \quad \mathrm{t}_{22}=20+(22-1) 2$

$$
\begin{aligned}
& =20+21 \times 2 \\
& =62
\end{aligned}
$$

$\therefore \quad$ The number of seats in the twenty second row are 62.
17. Obtain the sum of the first 56 terms of an A.P. whose $18^{\text {th }}$ and $39^{\text {th }}$ terms are 52 and 148 respectively.
[July 15]

## Solution:

Given, $\mathrm{t}_{18}=52$ and $\mathrm{t}_{39}=148$
Now, $\mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}$
$\therefore \quad \mathrm{t}_{18}=\mathrm{a}+(18-1) \mathrm{d}$
$\therefore \quad 52=\mathrm{a}+17 \mathrm{~d}$
$\therefore \quad a+17 d=52$
Also, $\mathrm{t}_{39}=\mathrm{a}+(39-1) \mathrm{d}$
$\therefore \quad 148=a+38 d$
$\therefore \quad a+38 d=148$
Adding (i) and (ii), we get
$a+17 d=52$
$a+38 d=148$
$2 \mathrm{a}+55 \mathrm{~d}=200$
Also, $\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]$
$\therefore \quad \mathrm{S}_{56}=\frac{56}{2}[2 \mathrm{a}+(56-1) \mathrm{d}]$
$=28[2 \mathrm{a}+55 \mathrm{~d}]$
$=28$ (200)
$\ldots[$ From (iii) $]$
$\therefore \quad \mathrm{S}_{56}=5600$
$\therefore \quad$ Sum of the 56 terms of an A.P. is 5600 .

## 4 Marks Questions

18. A man repays a loan of $₹ 3250$ by paying $₹ 305$ in the first month and then decreases the payment by ₹ 15 every month. How long will it take to clear his loan?

## Solution:

Here, $a=305, \mathrm{~d}=-15, \mathrm{~S}_{\mathrm{n}}=3250$
Let the time required to clear the loan be $n$ months.
$\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]$
$\therefore \quad 3250=\frac{n}{2}[2 \times 305+(n-1)(-15)]$
$\therefore \quad 6500=\mathrm{n}(610-15 \mathrm{n}+15)$

$$
\begin{array}{ll}
\therefore & 6500=n(625-15 n) \\
\therefore & 6500=625 n-15 n^{2} \\
\therefore & 15 n^{2}-625 n+6500=0 \\
\therefore & 3 n^{2}-125 n+1300=0 \\
\therefore & 3 n^{2}-60 n-65 n+1300=0 \\
\therefore & 3 n(n-20)-65(n-20)=0 \\
\therefore & n-20=0 \quad \text { or } \quad 3 n-65=0 \\
\therefore & n=20 \quad \text { or } \quad n=\frac{65}{3}
\end{array}
$$

Since $n$ is a natural number,
$\therefore \quad \mathrm{n} \neq \frac{65}{3}$
$\therefore \quad \mathrm{n}=20$
$\therefore$ The time required to clear the loan is 20 months.
19. If in an A.P. the sum of $m$ terms is equal to $n$ and the sum of $n$ terms is equal to $m$, then show that sum of $(\mathbf{m}+\mathrm{n})$ terms is $-(m+n)$.
Solution:
Let 'a' be the first term and 'd' be the common difference of A.P.
$\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]$
$\therefore \quad$ According to the given condition,
$\mathrm{S}_{\mathrm{m}}=\mathrm{n}$
$\therefore \quad \mathrm{n}=\frac{\mathrm{m}}{2}[2 \mathrm{a}+(\mathrm{m}-1) \mathrm{d}]$
$\therefore \quad 2 \mathrm{n}=\mathrm{m}[2 \mathrm{a}+\mathrm{md}-\mathrm{d}]$
$\therefore \quad 2 \mathrm{n}=2 \mathrm{am}+\mathrm{m}^{2} \mathrm{~d}-\mathrm{md}$
Also, $\mathrm{S}_{\mathrm{n}}=\mathrm{m}$
$\therefore \quad \mathrm{m}=\frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]$
$\therefore \quad 2 \mathrm{~m}=\mathrm{n}[2 \mathrm{a}+\mathrm{nd}-\mathrm{d}]$
$\therefore \quad 2 \mathrm{~m}=2 \mathrm{an}+\mathrm{n}^{2} \mathrm{~d}-\mathrm{nd}$
Subtracting (ii) from (i), we get
$2 \mathrm{am}-2 \mathrm{an}+\mathrm{m}^{2} \mathrm{~d}-\mathrm{n}^{2} \mathrm{~d}-\mathrm{md}+\mathrm{nd}=2 \mathrm{n}-2 \mathrm{~m}$
$\therefore \quad 2 \mathrm{a}(\mathrm{m}-\mathrm{n})+\mathrm{d}\left(\mathrm{m}^{2}-\mathrm{n}^{2}\right)-\mathrm{d}(\mathrm{m}-\mathrm{n})=2(\mathrm{n}-\mathrm{m})$
$\therefore \quad(m-n)[2 a+(m+n) d-d]=-2(m-n)$
$\therefore \quad(m-n)[2 a+(m+n-1) d]=-2(m-n)$
$\therefore \quad[2 \mathrm{a}+(\mathrm{m}+\mathrm{n}-1) \mathrm{d}]=-2 \quad---(\mathrm{iii})$
$\mathrm{S}_{\mathrm{m}+\mathrm{n}}=\frac{\mathrm{m}+\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{m}+\mathrm{n}-1) \mathrm{d}]$
$=\frac{\mathrm{m}+\mathrm{n}}{2} \times(-2) \quad---[$ From (iii)
$\therefore \quad \mathbf{S}_{\mathbf{m}+\mathbf{n}}=-(\mathbf{m}+\mathbf{n})$
20. A contract on construction job specifies a penalty for delay of completion beyond a certain limit as follows:
₹ 200 for first day,
₹ 250 for second day,
₹ 300 for third day, etc.
If the contractor pays $₹ 27,750$ as penalty, find the numbers of days for which the construction work is delayed.

## Solution:

Here, $\mathrm{a}=200, \mathrm{~d}=50, \mathrm{~S}_{\mathrm{n}}=27750$
$\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]$
$\therefore \quad 27750=\frac{\mathrm{n}}{2}[2 \times 200+(\mathrm{n}-1) 50]$
$\therefore \quad 27750=\frac{n}{2}[350+50 n]$
$\therefore \quad 27750=175 \mathrm{n}+25 \mathrm{n}^{2}$
$\therefore \quad 25 n^{2}+175 n-27750=06$
$\therefore \quad n^{2}+7 n-1110=0$
$\therefore \quad n^{2}+37-30 n-1110=0$
$\therefore \quad n(n+37)-30(n+37)=0$
$\therefore \quad(\mathrm{n}+37)(\mathrm{n}-30)=0$
$\therefore \quad \mathrm{n} \neq-37$ or $\mathrm{n}=+30$
$\mathrm{n} \neq-37$ as no. of days cannot be negative.
$\therefore \quad \mathrm{n}=+30$
$\therefore \quad$ The construction work is delayed by $\mathbf{3 0}$ days.
21. The interior angles of a polygon are in arithmetic progression. The smallest angle is $52^{\circ}$ and the common difference is $8^{\circ}$. Find the number of sides of the polygon.

## Solution:

Let ' $n$ ' be the number of sides of the polygon.
Sum of all interior angles of polygon
$=(\mathrm{n}-2) \times 180^{\circ}$
Here, $a=52, d=8$
$\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]$
$\therefore \quad(\mathrm{n}-2) \times 180=\frac{\mathrm{n}}{2}[2 \times 52+(\mathrm{n}-1) 8]$
$\therefore \quad 180 \mathrm{n}-360=\frac{\mathrm{n}}{2}[104+8 \mathrm{n}-8]$
$\therefore \quad 180 \mathrm{n}-360=\frac{\mathrm{n}}{2}[96+8 \mathrm{n}]$
$\therefore \quad 360 n-720=96 n+8 n^{2}$
$\therefore \quad 8 n^{2}+96 n-360 n+720=0$
$\therefore \quad 8 n^{2}-264 n+720=0$
$\therefore \quad n^{2}-33 n+90=0$
$\therefore \quad(\mathrm{n}-30)(\mathrm{n}-3)=0$

$$
\begin{array}{llll}
\therefore & \mathrm{n}-30=0 & \text { or } & \mathrm{n}-3=0 \\
\therefore & \mathrm{n}=30 & \text { or } & \mathrm{n}=3
\end{array}
$$

But, when $\mathrm{n}=30$, the last angle is
$\mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}$
$\therefore \quad 52^{\circ}+(30-1) 8^{\circ}=284^{\circ}$, which is not possible as interior angle of a polygon cannot be more than $180^{\circ}$.
$\therefore \quad$ Number of sides of the given polygon are 3 .
22. A man set out on a cycle ride of 50 km . He covers 5 km in the first hour and during each successive hour his speed falls by $\frac{1}{4} \mathrm{~km} / \mathrm{hr}$. How many hours will he take to finish his ride?

## Solution:

Here, $a=5, S_{n}=50, d=-\frac{1}{4}$
Let number of hours required to finish the ride be ' $n$ '.

$$
\begin{array}{ll} 
& \mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}] \\
\therefore & 50=\frac{\mathrm{n}}{2}\left[2 \times 5+(\mathrm{n}-1)\left(-\frac{1}{4}\right)\right] \\
\therefore & 50=\frac{\mathrm{n}}{2}\left[10+\frac{1}{4}-\frac{\mathrm{n}}{4}\right] \\
\therefore & 100=\mathrm{n}\left[\frac{41}{4}-\frac{\mathrm{n}}{4}\right] \\
\therefore & 100=\mathrm{n} \times\left(\frac{41-\mathrm{n}}{4}\right) \\
\therefore & 400=41 \mathrm{n}-\mathrm{n}^{2} \\
\therefore & \mathrm{n}^{2}-41 \mathrm{n}+400=0 \\
\therefore & (\mathrm{n}-25)(\mathrm{n}-16)=0 \\
\therefore & \mathrm{n}-25=0 \quad \text { or } \quad \mathrm{n}-16=0 \\
\therefore & \mathrm{n}=25 \quad \text { or } \quad \mathrm{n}=16
\end{array}
$$

If $\mathrm{n}=25$, speed would be negative.
$\therefore \quad \mathrm{n}=16$
$\therefore \quad 16$ hours are required to finish the ride.
23. The $11^{\text {th }}$ term and the $21^{\text {st }}$ term of an A.P. are 16 and 29 respectively then find:
i. The first term and common difference.
ii. The $34^{\text {th }}$ term.
iii. ' $n$ ' such that $\mathrm{t}_{\mathrm{n}}=55$.
[Mar 16]

## Solution:

Given, $\mathrm{t}_{11}=16, \mathrm{t}_{21}=29$
i. Since, $t_{n}=a+(n-1) d$
$\therefore \quad \mathrm{t}_{11}=\mathrm{a}+(11-1) \mathrm{d}$
$\therefore \quad 16=a+10 d$

$$
\begin{equation*}
\therefore \quad a+10 d=16 \tag{i}
\end{equation*}
$$

Also, $\mathrm{t}_{21}=\mathrm{a}+(21-1) \mathrm{d}$
$\therefore \quad 29=a+20 d$
$\therefore \quad a+20 d=29$
Subtracting (i) from (ii), we get

$$
a+20 d=29
$$

$$
a+10 d=16
$$

$\frac{(-)(-) \quad(-)}{10 \mathrm{~d}=13}$
$\therefore \quad \mathrm{d}=\frac{13}{10}$
Substituting $d=\frac{13}{10}$ in (i), we get

$$
\begin{array}{ll} 
& a+10 \times \frac{13}{10}=16 \\
\therefore & a+13=16 \\
\therefore & a=16-13 \\
\therefore & a=3 \\
\therefore & a=3 \text { and } d=\frac{13}{10}=1.3
\end{array}
$$

$\therefore \quad$ The $1^{\text {st }}$ term is $\mathbf{3}$ and the common difference is $\mathbf{1 . 3}$
ii. $\quad \mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}$
$\therefore \quad \mathrm{t}_{34}=3+(34-1) 1.3$

$$
=3+33 \times 1.3
$$

$$
=3+42.9=45.9
$$

$\therefore \quad \mathbf{t}_{34}=45.9$
iii. $\quad \mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}$
$\therefore \quad 55=3+(n-1) 1.3$
$\therefore \quad 55=3+1.3 n-1.3$
$\therefore \quad 55-3+1.3=1.3 \mathrm{n}$
$\therefore \quad 53.3=1.3 \mathrm{n}$
$\therefore \quad \mathrm{n}=\frac{53.3}{1.3}$
$\therefore \quad n=41$

## 5 Marks Questions

24. Prove that the sequence $S_{n}=2 n^{2}+5 n$ is in A.P. Hence, find $t_{n}$.

## Solution:

$$
S_{n}=2 n^{2}+5 n
$$

For $\mathrm{n}=1$,
$S_{1}=2(1)^{2}+5(1)=2+5=7$
$\mathrm{S}_{1}=7=\mathrm{t}_{1}$
For $\mathrm{n}=2$,
$S_{2}=2(2)^{2}+5(2)=2 \times 4+10=18$
$\therefore \quad \mathrm{S}_{2}=18$
$\mathrm{t}_{2}=\mathrm{S}_{2}-\mathrm{S}_{1}=18-7=11$
For $\mathrm{n}=3$,

$$
\begin{aligned}
\mathrm{S}_{3} & =2(3)^{2}+5(3) \\
& =2 \times 9+15 \\
& =33 \\
\therefore \quad S_{3} & =33 \\
\mathrm{t}_{3} & =\mathrm{S}_{3}-\mathrm{S}_{2}=33-18=15
\end{aligned}
$$

Now, $\mathrm{t}_{2}-\mathrm{t}_{1}=11-7=4$
$\mathrm{t}_{3}-\mathrm{t}_{2}=15-11=4$
$\therefore \quad$ The sequence is an A.P.
Here, $\mathrm{a}=\mathrm{t}_{1}=\mathrm{S}_{1}=7$ and $\mathrm{d}=4$
$\therefore \quad \mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}=7+(\mathrm{n}-1) 4=7+4 \mathrm{n}-4$
$\therefore \quad t_{n}=4 n+3$
25. If the ratio of the sum of $m$ terms and $n$ terms of an A.P. be $m^{2}: n^{2}$, prove that the ratio of $m^{\text {th }}$ and $n^{\text {th }}$ terms is $(2 m-1):(2 n-1)$.

## Solution:

> It is given that $\frac{S_{m}}{S_{n}}=\frac{m^{2}}{n^{2}}$
> $\frac{\left(\frac{m}{2}\right)[2 a+(m-1) d]}{\left(\frac{n}{2}\right)[2 a+(n-1) d]}=\frac{m^{2}}{n^{2}}$
> $\therefore \quad \frac{2 a+(m-1) d}{2 a+(n-1) d}=\frac{m}{n}$
> $\left.\begin{array}{ll}\therefore \quad \begin{array}{ll}2 a+(m-1) d=k m & ---(i) \\ & 2 a+(n-1) d=k n\end{array} \quad--(i i)\end{array}\right\}[k$ is constant $]$

Subtracting equation (ii) from (i), we get
$(m-1) d-d(n-1)=k m-k n$
$\therefore \quad \mathrm{md}-\mathrm{d}-\mathrm{nd}+\mathrm{d}=\mathrm{k}(\mathrm{m}-\mathrm{n})$
$\therefore \quad \mathrm{d}(\mathrm{m}-\mathrm{n})=\mathrm{k}(\mathrm{m}-\mathrm{n})$
$\therefore \quad \mathrm{d}=\mathrm{k}$
Substituting value of d in equation (i), we get
$2 a+(m-1) k=k m$
$\therefore \quad 2 \mathrm{a}+\mathrm{mk}-\mathrm{k}=\mathrm{km}$
$\therefore \quad 2 \mathrm{a}=\mathrm{k}$
$\therefore \quad a=\frac{k}{2}$
Now, $\mathrm{t}_{\mathrm{m}}=\mathrm{a}+(\mathrm{m}-1) \mathrm{d}$ and $\mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}$
$\therefore \quad \frac{\mathrm{t}_{\mathrm{m}}}{\mathrm{t}_{\mathrm{n}}}=\frac{\mathrm{a}+(\mathrm{m}-1) \mathrm{d}}{\mathrm{a}+(\mathrm{n}-1) \mathrm{d}}$
Substituting value of a and d, we get
$\frac{\mathrm{t}_{\mathrm{m}}}{\mathrm{t}_{\mathrm{n}}}=\frac{\left(\frac{\mathrm{k}}{2}\right)+(\mathrm{m}-1) \mathrm{k}}{\left(\frac{\mathrm{k}}{2}\right)+(\mathrm{n}-1) \mathrm{k}}$
$=\frac{k\left(\frac{1}{2}+m-1\right)}{k\left(\frac{1}{2}+n-1\right)}$

$$
\begin{aligned}
& =\frac{\frac{2 m-1}{2}}{\frac{2 n-1}{2}} \\
\therefore \quad \frac{\mathbf{t}_{m}}{\mathbf{t}_{\mathrm{n}}} & =\frac{2 m-1}{2 \mathrm{n}-1}
\end{aligned}
$$

26. Find the number of terms common to A.P. $3,7,11, \ldots ., 407$ and A.P. 2, 9, 16, 709.

## Solution:

Let the number of terms in A.P. 3,7,11,....,407
be $m$ and the number of terms in A.P.
$2,9,16, \ldots ., 709$ be $n$.
$\therefore \quad \mathrm{t}_{\mathrm{m}}=407$ and $\mathrm{t}_{\mathrm{n}}=709$
As, $\mathrm{t}_{\mathrm{m}}=\mathrm{a}+(\mathrm{m}-1) \mathrm{d}$
$\therefore \quad 407=3+(m-1) 4$
$\therefore \quad 407=3+4 m-4$
$\therefore \quad 407=4 \mathrm{~m}-1$
$\therefore \quad 408=4 \mathrm{~m}$
$\therefore \quad \mathrm{m}=102$
$\mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}$
$\therefore \quad 709=2+(n-1) 7$
$\therefore \quad 709=2+7 n-7$
$\therefore \quad 709=7 n-5$
$\therefore \quad 714=7 n$
$\therefore \quad \mathrm{n}=102$
$\therefore \quad$ Each A.P. consists of 102 terms.
Let $p^{\text {th }}$ term of $1^{\text {st }}$ A.P. be equal to $q^{\text {th }}$ term of the $2^{\text {nd }} A . P$.
$\therefore \quad 3+(p-1) 4=2+(q-1) 7$
$\therefore \quad 3+4 p-4=2+7 q-7$
$\therefore \quad 4 p-1=7 q-5$
$\therefore \quad 4 p+4=7 q$
$\therefore \quad 4(p+1)=7 q$
$\therefore \quad \frac{p+1}{7}=\frac{q}{4}$
Let $\frac{\mathrm{p}+1}{7}=\frac{q}{4}=\mathrm{k} \quad(\mathrm{k} \neq 0)$
$\therefore \quad \mathrm{p}=7 \mathrm{k}-1 \quad$ and $\quad \mathrm{q}=4 \mathrm{k}$
Since each A.P. consists of 102 terms.
$\begin{array}{llll} & \mathrm{p} \leq 102 & \text { and } & \mathrm{q} \leq 102 \\ \therefore & 7 \mathrm{k}-1 \leq 102 & \text { and } & 4 \mathrm{k} \leq 102 \\ \therefore & \mathrm{k} \leq 14 \frac{5}{7} & \text { and } & \mathrm{k} \leq 25 \frac{1}{2}\end{array}$
$\therefore \quad \mathrm{k} \leq 14$
$\therefore \quad \mathrm{k}=1,2,3,4, \ldots, 14$
For each value of $k$, we get a pair of identical terms.
$\therefore \quad$ There are 14 common terms.
27. Insert five number between 4 and 8 so that the resulting sequence is an A.P.
Solution:
Let the required numbers be $t_{2}, t_{3}, t_{4}, t_{5}$ and $t_{6}$ Thus, $4, \mathrm{t}_{2}, \mathrm{t}_{3}, \mathrm{t}_{4}, \mathrm{t}_{5}, \mathrm{t}_{6}, 8$ are in A.P.
In this case, $\mathrm{t}_{7}=8$
$\mathrm{t}_{1}=\mathrm{a}=4, \mathrm{t}_{\mathrm{n}}=8, \mathrm{n}=7$
We know that,
$t_{n}=a+(n-1) d$
$\therefore \quad 8=4+(7-1) \mathrm{d}$
$\therefore \quad 8=4+6 d$
$\therefore \quad 4=6 d$
$\therefore \quad \mathrm{d}=\frac{2}{3}$
$\mathrm{t}_{2}=\mathrm{a}+(2-1) \mathrm{d}$
$=4+(1) \times \frac{2}{3}=4+\frac{2}{3}$
$\therefore \quad \mathrm{t}_{2}=\frac{14}{3}$
$\mathrm{t}_{3}=4+(3-1) \times \frac{2}{3}$
$=4+2 \times \frac{2}{3}=4+\frac{4}{3}$
$\therefore \quad \mathrm{t}_{3}=\frac{16}{3}$
$\mathrm{t}_{4}=4+(4-1) \times \frac{2}{3}=4+3 \times \frac{2}{3}=4+2$
$\therefore \quad \mathrm{t}_{4}=6$
$\mathrm{t}_{5}=4+(5-1) \times \frac{2}{3}$
$=4+4 \times \frac{2}{3}=4+\frac{8}{3}$
$\therefore \quad \mathrm{t}_{5}=\frac{20}{3}$
$\mathrm{t}_{6}=4+(6-1) \times \frac{2}{3}$
$=4+5 \times \frac{2}{3}=4+\frac{10}{3}$
$=\frac{12+10}{3}=\frac{22}{3}$
$\therefore \quad \mathrm{t}_{6}=\frac{22}{3}$
$\therefore$ The required numbers are $\frac{14}{3}, \frac{16}{3}, 6$,
$\frac{20}{3}, \frac{22}{3}$
28. If the sum of first $m$ terms of an A. P. is equal to the sum of first $n$ terms, then show that the sum of its first $(m+n)$ terms is zero, where $\mathbf{m} \neq \mathbf{n}$.
[Mar 14]
Solution:
The sum of first $n$ terms of an A.P. is given by $S_{n}=\frac{n}{2}[2 a+(n-1) d]$
$\therefore \quad \mathrm{S}_{\mathrm{m}}=\frac{\mathrm{m}}{2}[2 \mathrm{a}+(\mathrm{m}-1) \mathrm{d}]$
But, $\mathrm{S}_{\mathrm{m}}=\mathrm{S}_{\mathrm{n}} \quad---$ [Given]
$\therefore \quad \frac{\mathrm{m}}{2}[2 \mathrm{a}+(\mathrm{m}-1) \mathrm{d}]=\frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]$
$\therefore \quad \frac{\mathrm{m}}{2}[2 \mathrm{a}+\mathrm{md}-\mathrm{d}]=\frac{\mathrm{n}}{2}[2 \mathrm{a}+\mathrm{nd}-\mathrm{d}]$
$\therefore \quad \frac{\mathrm{m}}{2}[2 \mathrm{a}+\mathrm{md}-\mathrm{d}]-\frac{\mathrm{n}}{2}[2 \mathrm{a}+\mathrm{nd}-\mathrm{d}]=0$
$\therefore \quad \frac{1}{2}\left[2 a m+m^{2} d-m d-2 a n-n^{2} d+n d\right]=0$
$\therefore \quad 2 \mathrm{am}+\mathrm{m}^{2} \mathrm{~d}-\mathrm{md}-2 \mathrm{an}-\mathrm{n}^{2} \mathrm{~d}+\mathrm{nd}=0$
$\therefore \quad(2 a m-2 a n)+\left(m^{2} d-n^{2} d\right)-(m d-n d)=0$
$\therefore \quad 2 \mathrm{a}(\mathrm{m}-\mathrm{n})+\mathrm{d}\left(\mathrm{m}^{2}-\mathrm{n}^{2}\right)-\mathrm{d}(\mathrm{m}-\mathrm{n})=0$
$\therefore \quad 2 a+d(m+n)-d=0$
[Dividing by $(\mathrm{m}-\mathrm{n})$ ]
$\therefore \quad 2 \mathrm{a}+\mathrm{d}(\mathrm{m}+\mathrm{n}-1)=0$
Multiplying by $\frac{\mathrm{m}+\mathrm{n}}{2}$ on both sides, we get
$\frac{\mathrm{m}+\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{m}+\mathrm{n}-1) \mathrm{d}]=0$
But, $\mathrm{S}_{\mathrm{m}+\mathrm{n}}=\frac{\mathrm{m}+\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{m}+\mathrm{n}-1) \mathrm{d}]--$ (ii)
From (i) and (ii), we get
$\mathbf{S}_{\mathrm{m}+\mathbf{n}}=\mathbf{0}$
29. Babubhai borrows $₹ 4000$ and agrees to repay with a total interest of $₹ 500$ in 10 instalments, each instalment being less than the preceding instalment by ₹ 10 . What should be the first and the last instalment?
[Mar 15]

## Solution:

The instalments are in A.P.
Here, $\mathrm{S}_{10}=4000+500=4500$
Also, $\mathrm{n}=10, \mathrm{~d}=-10$
Now, $\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]$
$\therefore \quad \mathrm{S}_{10}=\frac{10}{2}[2 \mathrm{a}+(10-1)(-10)]$
$\therefore \quad 4500=5[2 \mathrm{a}+9 \times(-10)]$

$$
\begin{aligned}
& \therefore \quad \frac{4500}{5}=2 a-90 \\
& \therefore \quad 900+90=2 \mathrm{a} \\
& \therefore \quad 990=2 \mathrm{a} \\
& \therefore \quad a=\frac{990}{2} \\
& \therefore \quad a=495 \\
& \text { Also, } \mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d} \\
& \therefore \quad \mathrm{t}_{10}=495+(10-1)(-10) \\
& =495+9 \times(-10) \\
& =495-90 \\
& =405
\end{aligned}
$$

$\therefore \quad$ The first instalment is $₹ \mathbf{4 9 5}$ and the last instalment is ₹ 405 .
30. Find the sum of all numbers from 50 to 350 which are divisible by 6 . Hence find the $15^{\text {th }}$ term of that A.P.
[Mar 16]

## Solution:

The numbers from 50 to 350 which are divisible by 6 are $54,60,66, \ldots .348$
This sequence is an A.P. with
$a=54, d=60-54=6, t_{n}=348$
But $\mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}$
$\therefore \quad 348=54+(n-1) 6$
$\therefore \quad 348-54=(n-1) 6$
$\therefore \quad 294=(n-1) 6$
$\therefore \quad \frac{294}{6}=\mathrm{n}-1$
$\therefore \quad 49=n-1$
$\therefore \quad 49+1=n$
$\therefore \quad \mathrm{n}=50$
Now, $\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}\left[\mathrm{t}_{1}+\mathrm{t}_{\mathrm{n}}\right]$
$\therefore \quad \mathrm{S}_{50}=\frac{50}{2}[54+348]$
$=25(402)$
$\therefore \quad \mathrm{S}_{50}=10050$
Also, $\mathrm{t}_{15}=\mathrm{a}+(15-1) \mathrm{d}$
$=54+14 \times 6=54+84$
$\therefore \quad \mathrm{t}_{15}=138$
$\therefore \quad$ The sum of all numbers from 50 to 350 which are divisible by 6 is 10050 and $15^{\text {th }}$ term of the A.P. is 138.

## 13 Mapping our Genes

## 1 Mark Questions

1. Write the characteristics on the basis of which duck - billed platypus is considered as a link between reptiles and mammals.
Ans: The duck - billed platypus lays eggs like reptile and has hair and mammary glands like mammals.
2. Give the genotypic ratio for monohybrid cross.
Ans: $1: 2: 1$ is the genotypic ratio for monohybrid cross.
3. Which term is used to give a sequence of gradual changes over millions of years in which new species are produced?
Ans: Evolution
4. Which of the following combinations of sex chromosomes produce a male child: XX or XY?
Ans: XY
5. Tail fin of lobster, flukes of whale have the same basic function but different structure. What is the name given to these organs?
Ans: Homologous organ
6. What supports the theory of evolution?

Ans: Anatomical study of the bodily structure of plants and animals.
7. Name the organism which is structurally intermediate between two different groups.
Ans: Peripatus is a connecting link between Annelida and Arthropoda.
8. Name the plant which has lost its chlorophyll and become saprophytic in nature.
Ans: Indian pipe
9. Which plant was used by Mendel in most of his experiments?
Ans: Pea plant (Pisum sativum) was used by Mendel in most of his experiments.

## 2 Marks Questions

10. i. The number of pairs of sex chromosomes in the zygote of human is
a. one
b. two
c. twenty three
ii. A zygote which has inherited ' X ' chromosome from the father will develop into
a. a baby boy b. a baby girl
c. either a boy or a girl

Ans:
11. Fill in the blanks.
i. The science of heredity is known as $\qquad$ .
ii. Genes always work in $\qquad$ .
Ans: i. Genetics ii. pairs
12. In many parts of our country, a mother is held responsible for the birth of a girl child or boy child. Society is biased towards the sex of the child and harass the women on giving birth to a baby girl? Are they right or wrong? Give scientific reasons for your answer.
Ans: The belief that mother is responsible for the sex of her baby is absolutely wrong. It is the father who is responsible for the birth of a girl child. A human male has one X chromosome and one Y chromosome. A female has two X chromosomes. If father contributes X sex chromosome at fertilization, the baby born will be a girl.
13. Name the following:
i. Charles Darwin published a book in $\mathbf{1 8 0 9}$ with his theory, with evidence for 'mechanism of evolution'. Name the book. What is his theory called?
ii. "Organisms can pass on characteristics that it acquired during its lifetime to its offspring". Which scientist gave this idea of inheritance?
Ans: i. Book - On the origin of species, Theory - Theory of natural selection
ii. Jean Baptiste Lamarck
14. Which organ in man suggests that he is a descendent of herbivorous animals?
Ans: Vermiform appendix of man is the organ which could suggest that man was a descendent of herbivorous animals. Vermiform appendix is a vestigial organ which has no use. In other herbivorous plant eating animals, appendix helps in the digestion of cellulose.
15. If two pea plants with different trait for height are crossed, they produce 21 tall plants and 7 dwarf plants in $F_{1}$ generation.
i. What is the phenotypic ratio in $F_{1}$ generation?
ii. Fill the punnett square to map out the genotypes.


Ans: i. $3: 1$

16. If, Duck billed platypus = link between reptiles and mammals then, peripatus = ? Why?
Ans: Peripatus $=$ Connecting link between Annelida and Arthropoda.
Because Peripatus has thin cuticle and parapodia like appendages as in Annelida. Also, it has trachea and open circulation as in Arthropoda.
17. What type of cross would produce the following phenotypic ratios?
i. $3: 1$
ii. $\quad 9: 3: 3: 1$

Ans: i. Monohybrid cross
ii. Dihybrid cross

## 3 Marks Questions

18. In pea plants, if gene $T$ gives tall pea plants and gene $t$ gives short pea plants,
i. What will be the height of the plants having the following combination of genes?
a. $\mathbf{T t}$
b. $\quad \mathbf{t t}$
c. TT
ii. If pure tall pea plants are first crossed with pure dwarf pea plants, what do the plants of $F_{1}$ generation look like?
iii. The plants obtained in $F_{1}$ generation are then crossed to produce $F_{2}$ generation of pea plants.
a. What is the ratio of tall plants to dwarf plants in $F_{2}$ generation?
b. Which type of plants were missing in $F_{1}$ generation but reappeared in $F_{2}$ generation?
iv. The above experiment is an example of (test cross / dihybrid cross / dominant cross / monohybrid cross). Explain the cross.
v. Write the Phenotypic ratio and the Genotypic ratio of tall plants to dwarf plants in $F_{2}$ generation.
Ans: i. a. Tt - Tall, b. tt - Dwarf
c. TT - Tall
ii. All plants are Tall.
iii. a. $3: 1$
b. Dwarf pea plants
iv. Monohybrid cross - When two pea plants were crossed with only one pair of contrasting characters each, then it is called a monohybrid cross.
v. Phenotypic ratio - Tall plants : Dwarf plants $=3: 1$
Genotypic ratio - TT : Tt : tt $=1: 2: 1$
19. In human genetic traits, if $X$ is the gene for curly hair and $x$ is the gene for straight hair, then answer the following:
i. Both father and mother have the genes $X x$ in their cells. What is their hair type?
ii. Which combination of genes will produce children with Curly hair?
iii. Which combination of genes will produce children with Straight hair?
Ans: i. Curly hair ii. XX and Xx
iii. xx
20. Plants obtained from $F_{1}$ generation bearing dominant round seeds were crossed with self to produce $F_{2}$ generation.
i. Fill in the punnett square to map out the genotypes of the offsprings of $F_{2}$ generation plants.
ii. Analyse the number of offsprings of each type and write their phenotypes.
iii. Figure out the ratios of the offsprings.


Ans: i.


| ii. | Genotype | Phenotype |
| :---: | :---: | :--- |
| RR | Round seeds |  |
| Rr | Round seeds |  |
| Rr | Round seeds |  |
| rr | Wrinkled seeds |  |

iii. Genotypic ratio - $1: 2: 1$

Phenotypic ratio - $3: 1$
21. The body colour of rabbit is of two varieties, one with black hair and the other with brown hair colour. When these two pure varieties were crossed, all the offsprings were having black colour.
i. Give reasons why the brown hair colour rabbits do not appear.
ii. Choose suitable letters to represent the two genes. Give reasons.
iii. With your chosen letters, give the homozygous and heterozygous genotypes for them.
Ans: i. Because the brown colour fur is recessive, they do not appear. Black hair fur is dominant over brown, so all offsprings were of black colour.
ii. Black colour is represented by letter B and Brown colour by b . This is because dominant genes are represented by capital letters and recessive genes are represented by small letters.
iii. Heterozygous genotype -Bb

Homozygous genotype -BB , bb
22. Which of the following traits of pea plants are dominant traits? Also, give their recessive traits.
White flowers, Wrinkled seeds, Axial flowers, Green colour seeds, Green pea pods, Inflated (full) pea pods.
Ans: Dominant trait Recessive trait
Axial flowers Terminal flowers
Green pea pods Yellow pea pods
Inflated pea pods Constricted pea Pods
23. Mention the recessive traits of the following characters of garden pea plant:
i. Flower colour and Flower position
ii. Seed shape and Seed colour
iii. Pod shape and Pod colour

Ans: Recessive traits
i. White flower colour and Terminal flower position.
ii. Wrinkled seed and Green seed.
iii. Constricted pod shape and Yellow pod colour.
24. Consider the wing of an Eagle, wing of a Butterfly and the whale fin. Out of these, which two are analogous organs? Explain your answer.
Ans: Wings of Eagle and wings of butterfly are analogous. Wings of an eagle are made up of bones and feathers, whereas wings of butterfly have no bones and feathers. They are made up of membranes. But, their wings have same functions, both are used for flying.

## 5 Marks Questions

25. Study the given data and answer the questions following the data:

| $P_{1}$ Parental plants cross fertilised and seeds are collected | $F_{1}$ First generation offsprings | $\mathrm{F}_{2}$ Offsprings of self pollination of $\mathrm{F}_{1}$ |
| :---: | :---: | :---: |
| Pure bred plants with red flowers were crossed with pure bred white flower plants. | 100 seeds sown and observed. <br> All 100 gave red flowers. | Out of 44 seeds sown, 33 seeds gave plants with red flowers and 11 seeds gave plants with white flowers. |

i. What is the term for this type of cross?
ii. What does the data of the column marked $F_{1}$ indicate?
iii. Express the genotype of: (a) parents (b) $F_{1}$ progeny and (c) $F_{2}$ progeny

Ans: i. Monohybrid cross
ii. Red coloured flowers are dominant over White coloured ones.
iii. Genotypes

Parents - RR and rr
$\mathrm{F}_{1}$ progeny -Rr
$\mathrm{F}_{2}$ progeny $-\mathrm{RR}, \mathrm{Rr}$ and rr
26. Study the following diagram showing the forelimbs of different animals. They show the same structure of bones and point towards a common origin.

'The hand of a human

The front leg of a dog

The flipper of a whale

The wing of a bat

The wing of a bird
i. What do you conclude from this similarity?
ii. What is the term given to such structures? Explain the term.
iii. Write the functions of each of the organs.

Ans: i. The similarity shows that they have developed from a common ancestor.
ii. Homologous organs. These organs have same basic structure but different function.
iii. The hands of humans are used for grasping, the front leg of a dog is used for running, the flipper of a whale is used for swimming and the wings of bird and bat are used for flying.
27. Mendel obtained pea-plants with
a. Round and yellow seeds and
b. Wrinkled and green seeds

One parent
Round shaped seed (RR)
Yellow coloured seed (YY)
(Dominant traits)

Other parent
Wrinkled seed (rr)
Green coloured seed (yy)
(Recessive traits)

When the gametes of $F_{1}$ generation are formed, each pair of genes segregate independently. The results of the $F_{1}$ cross is shown in the chart given below:
$P_{1}$ generation
RRYY $\qquad$

i. Fill in the empty spaces of the chart showing $P_{1}$ generation, gametes and $F_{1}$ generation.
ii. Write the phenotypes of $F_{1}$ generation.
iii. If $F_{1}$ generation is crossed, which type of gametes will they form?
iv. When $F_{1}$ progeny are used to produce $F_{2}$ generation by self pollination, then four types are obtained.

Write the phenotypes and mention the ratio of these four types of progenies.
Ans: i. $\quad \mathrm{P}_{1}$ generation: rryy Gametes: ry, ry $\quad \mathrm{F}_{1}$ generation: RrYy, RrYy
ii. All the plants of $\mathrm{F}_{1}$ generation are with Round and Yellow seeds. (Phenotype)
iii. Gametes: YR, Yr, yR, yr and YR, Yr, yR, yr
iv. The four types of progeny are:
a. Plants with Round and Yellow seeds (9) b. Wrinkled and Yellow seeds (3)
c. Round and Green seeds (3)
d. Wrinkled and Green seeds (1)
Phenotypic Ratio-9:3:3:1
28. In human genetic traits, if $E$ is the gene for free ear lobes and $e$ is the gene for attached ear lobes,
i. Which gene is recessive and which is dominant?
ii. Both father and mother have the genes Ee, what is their ear lobe type?
iii. Which combination of genes will produce children with free ear lobe? Why?
iv. Which combination of genes will produce children with attached ear lobe?

Ans: i. $\quad \mathrm{E}$ is dominant gene for free ear lobe, e is the recessive gene for attached ear lobe.
ii. Free ear lobe
iii. EE and Ee. This is because in EE, both genes are dominant so, it will show free ear lobe character. In Ee , although gene e is present but it is recessive, so it cannot express itself. Thus, in Ee, E gene being dominant will show free ear lobe character.
iv. Children with gene combination ee will show attached ear lobes.
29. Using a Punnett square, workout the distribution of phenotypic features in the cross between green pod pea plant and yellow pod pea plant.
i. What will be the colour of pod formed in $F_{1}$ generation?
ii. If plants produced in $F_{1}$ generation are crossed to produce $F_{2}$ generation.
a. How much percentage of plants will be produced with yellow pods?
b. What will be the ratio of green pod plants to yellow pod plants?
iii. Fill in the punnett square to map out the genotype of offspring in $F_{2}$ generation.

iv. Write down the genotypic ratio of offsprings formed in $F_{\mathbf{2}}$ generation

Ans: i. Green
ii. a. $25 \%$
b. $3: 1$
iii.

| $\mathbf{G}$ |  |
| :---: | :---: |
| $\mathbf{G}$ |  |
| $\mathbf{G}$ | $\mathbf{G G}$ |
| $\mathbf{g}$ | $\mathbf{G g}$ |
|  | $\mathbf{G g}$ |
|  | $\mathbf{g g}$ |

iv. Genotypic ratio $\mathrm{GG}: \mathrm{Gg}: \mathrm{gg}$
= $1: 2: 1$
30. A boy who can roll his tongue has mother who can also roll her tongue, but his father cannot roll his tongue.
i. Why can the boy roll his tongue, if his father could not?
ii. How can you represent genotype for the trait: Rolling tongue and non rolling tongue.
iii. With your chosen letters, write genotypes for homozygous and heterozygous.
iv. What will be the genotype of the boy in the above case?

Ans: i. Father cannot roll his tongue because he is recessive for the character, whereas boy is heterozygous dominant for the character Hence, he can roll his tongue.
ii. Rolling tongue: RR

Non rolling tongue: rr
iii. Heterozygous genotype: Rr

Homozygous genotype: rr
iv. Boy's genotype: Rr
31. Some structures ' $X$ ' are found in animals and plants which are of no use to them. $Y$ is the vestigial structure found in man related to digestion.
i. Identify $\mathbf{X}$.
ii. Identify Y.
iii. What is the function of caecum and appendix in mammals?
iv. Give 4 examples of $X$ found in humans

Ans: i. Vestigial organs.
ii. Vermiform appendix.
iii. It helps to digest cellulose.
iv. a. Ear muscles b. Wisdom teeth
c. Plica semilunaris
d. Coccyx
32. If pure tall red flowered pea plant is crossed with pure dwarf white flowered plant, then
i. What will be the genotype of parents?
ii. What will be the phenotype and genotype of offsprings in $\mathrm{F}_{1}$ generation?
iii. Fill the punnett square to map out genotype of $F_{2}$ generation.

|  | TR | Tr | tR | tr |
| :--- | :--- | :--- | :--- | :--- |
| TR | TTRR |  |  | - |
| Tr | - | TTrr |  | - |
| tR | TtRR |  |  | - |
| tr |  | Ttrr |  |  |

iv. Name the type of cross.

Ans:
i. $\quad$ Pure tall red flower $=$ TTRR

Pure dwarf white flower = ttrr
ii. $\quad$ Phenotype $=$ Tall red flower; Genotype: $\operatorname{TrRr}$ iii.

| $\rho$ | $\mathbf{o}^{7}$ | TR | Tr | tR |
| :--- | :--- | :--- | :--- | :--- |
| tr |  |  |  |  |
| TR | TTRR | TTRr | TtRR | TtRr |
| Tr | TTRr | TTrr | TtRr | Ttrr |
| tR | TtRR | TtRr | ttRR | ttRr |
| tr | TtRr | Ttrr | ttRr | ttrr |

iv. Dihybrid cross.
33. i. How many pairs of autosomes and pairs of sex chromosomes are present in a human being?
ii. Which chromosomes are present in a Female?
iii. Which chromosomes are present in a Male?
iv. How does sex determination take place in human beings?
[Oct 14]
Ans: i. Pairs of autosomes in human being $=22$ Pair of sex chromosomes = 1
ii. XX
iii. XY
iv. In human beings, the sex of the offspring is determined by the type of chromosome ( X or Y ) inherited from the father.
34. Read the paragraph and answer the questions given below:
Sarika got married to Ramesh (a business man) in Delhi. After a year, she gave birth to a girl child. Sarika was happy and came home with a baby. Ramesh and his parents were expecting a male child. So, they advised her to kill that girl child or else to leave the house. When Sarika refused to kill that child, they threw her out of the house. They blamed Sarika for the birth of the girl child and held her responsible for the same. In India, this picture can be seen in many families. Girls are not only killed after birth but also before birth. Since 1991, $80 \%$ districts in India have recorded an increasingly masculine sex ratio. Punjab has the most masculine sex ratio. Pre-natal sex determination and sex selective abortion is also practiced in India.

## Questions:

i. Why is the number of females declining in India?
ii. Sarika was held responsible for birth of a girl child. Is it scientifically correct?
iii. What is the chromosome constitution of the female child?
iv. Which method is used to determine sex of the child before birth?

## Answers:

i. Number of females is declining in India because with the help of technique such as pre-natal sex determination, female foetuses are aborted and some girls are killed after birth.
ii. Sarika was held responsible for birth of a girl child which is scientifically incorrect. Biologically, all females have genetic constitution $44+\mathrm{XX}$ and males have $44+$ XY. Child gets X chromosome from mother and other chromosome from father. If X chromosome containing sperm fertilizes the egg, child will be girl and if Y chromosome containing sperm fertilizes the egg, child will be boy. So, father decides the sex of the child and not the mother.
iii. Chromosome constitution of female child is $44+$ XX.
iv. Pre-natal sex determination techniques are used to determine sex of the foetus.

