A Lesson in Formal Logic (for Logic Games and Logical Reasoning)

Understanding formal logic can improve your LSAT score by as much as 10-15 points. The following tutorial will help you learn formal logic for both the LSAT Analytical Reasoning Section (Games) and the LSAT Logical Reasoning Section.

Contents:

See also: Examples of IF-THEN statements for Analytical Reasoning questions

Formal logic is a set of rules for making deductions that seem self evident. In the context of the LSAT it relates primarily to IF-THEN statements. By learning to express IF-THEN statements symbolically and to quickly identify the contra-positive of an IF-THEN statement you can improve your score on the Games and Logical Reasoning sections of the LSAT.

Contra-Positive: The Contra-Positive of a conditional statement (IF-THEN statement) is formed by negating both the hypothesis and the conclusion, and then interchanging the resulting negations. In other words, the contra-positive negates and switches the parts of the sentence. Don't worry too much about the word Contra-Positive or what it means. For the LSAT all you need to know is how to find the Contra-Positive.

Basic IF-THEN Statements:

Here are some formal logic examples to help you with producing symbolic IF-THEN statements and the Contra-Positive.

If the team scores a goal then the crowd will cheer.

Let's use the variables A and B to represent the two parts of this conditional statement.

If A (the team scores) then B (the crowd will cheer).

Or

If A then B

All **If A then B** statements can then be converted into

If Not B then Not A (the Contra-Positive)

This is done by taking the second term (B) changing it to the opposite meaning (B becomes Not B) and placing it at the beginning of the If Then statement.

Then B becomes **If Not B**

Next we take the first term (A) change it to the opposite meaning (A becomes Not A) and placing it at the end of the If Them statement.

If A becomes **Then Not A**

Combining the two rules produces the following:

If A then B
Becomes…

If Not B then Not A
The above is referred to as "The Contra-Positive" – knowing this name is not important to the LSAT but understanding the principle behind it is essential to scoring high on the LSAT. Formal Logic is used in numerous Logical Reasoning questions as well as Analytical Reasoning questions. Understanding formal logic will make many of the Games questions far easier and allow you to come to the correct answer faster.

Common Mistake on the LSAT: Many students make the mistake of drawing some other information or conclusion from an IF-THEN or conditional statement. **Remember: the Contra- Positive is the only additional piece of information you can derive from an If Then Statement.**
For example if you are given:
If A then B
You also know
If Not B then Not A
But you know nothing else for sure. For example if you are given:
If A then B
You are not able to fill in any of the following blanks:
If B then ???
If Not A then ???
You can Only say:
If Not B then Not A
Sample Questions:
Get out a pencil and paper and try a few sample questions:

If you take an LSAT course then you will score high on the LSAT

Convert this into an If then Statement separating out two elements of the statement and identifying them as A and B.

Answer:

If A (take a course) Then B (score high)

If A then B

Question:

Now convert to the contra-positive:

Answer:

If Not B (do not score high) Then Not A (did Not take a course)

If Not B then Not A

This was done by taking the opposite of the second term B and changing it to the opposite meaning Not B, and then taking the opposite of the first term A and changing it to the opposite meaning Not A.

Try another one:

If the jury has a reasonable doubt the accused will be acquitted

Convert this into an IF-THEN Statement separating out two elements of the statement and identifying them as A and B.

Answer:

If A (reasonable doubt) then B (accused acquitted)

Now convert to the contra-positive:

Answer:

If Not B (Not Acquitted) then Not A (No Reasonable Doubt)

If A then B

If Not B then Not A

Question:

Given that an accused person was found guilty by a jury you can conclude which of the following?

1. The accused was not innocent
2. The jury had a reasonable doubt
3. The jury did not have a reasonable doubt

Answer:

The jury did not have a reasonable doubt

The question “Given that an accused person was found guilty by a jury you can conclude which of the following?” is asking “if we are given the folloing: If Not B (the accused was Not acquitted/ was found guilty) then what can we conclude?”

If Not B then we can conclude Not A or Not a reasonable Doubt

IF-THEN statements can come in a variety of forms. Here are some sample IF-THEN statements and their respective contra-positives

Sample IF-THEN Statements:
Original Statement: If A then B
Contra Positive: If Not B then Not A

Original Statement: If A then Not B
Contra Positive: If B then Not A
Original Statement: If Not A then B
Contra Positive: If Not B then A
Original Statement: If Not A then Not B
Contra Positive: If B then A
Original Statement: If A then B1 AND B2
Contra Positive: If Not B1 OR Not B2 then Not A
Original Statement: If A then B1 OR B2
Contra Positive: If Not B1 AND Not B2 then Not A
Statements like the above could be something like: If Sara puts bananas in her milkshake she will also add either oranges or blueberries. See below for more on multiple elements in IF-THEN statements.
Original Statement: Only if A then B = If B then A
Contra Positive: If Not A then Not B
See below for more on "Only If" statements.
Original Statement: A Only if B = If A then B
Contra Positive: If Not B then Not A
Original Statement: A If and Only If B = If A then B and If B then A
Contra Positive: If Not B then Not A, and If Not A then Not B
As you can see here, If and Only If actually gives you two IF-THEN statements.

See also: Examples of IF-THEN statements for Analytical Reasoning questions.

Only If statements:

If Then statements can be changed by using "Only If" instead of "If". This simply reverses the entire statement.

Only if A then B is the same as saying If B then A

Then the contra-positive of Only If A then B is:

If Not A then Not B

Because:
Only If A then B is the same as If B then A and the contra-positive of If B then A is If Not A then Not B
Only If A then B = If B then A

Contra-Positive: If Not A then Not B

Another way to look at "Only If" is that it is actually "THEN" so when you have A only if B it is just like IF A THEN B. The part that follows only if becomes the part of the IF THEN statement that follows then.

Get out your pencil and paper and try the following:

Convert the following into an If then Statement separating out two elements of the statement and identifying them as A and B.

Only if you go to law school can you become a lawyer

Answer:

If B (you become a lawyer) then A (you went to law school)

Question:

State the contra-positive

If Not A (you did not go to law school) then Not B (you can not become a lawyer)

Multiple Elements in IF-THEN Statements:
On the LSAT they may complicate questions by adding multiple elements to A and B.
For example:
Only if you go to law school and pass the bar exam can you become a lawyer.

Breaking this statement down into its elements it becomes:

If B (you become a lawyer) then A1 (you went to law school) AND A2 (you passed the Bar exam)

Remember that “Only if” means you simply switch the order of A and B in the If Then statement.

Both A1 and A2 are required in order for A to be true. If either A1 or A2 are missing it becomes Not A.

(A1 and A2) = A
If you know Not A1 then you know Not A
If you know Not A2 then you know Not A
For example if it is given that you did not pass the Bar exam then you can conclude that you will not become a lawyer.

Because:
If Not A2 (Not pass the Bar) then you know that you do not have A as to have A requires both A1 and A2.
Given that you have Not A you know that you have Not B because:

If Not A (did not go to law school or did not write the bar exam) then Not B (will not become a lawyer.

Only If A1 AND A2 then B = If B then A1 AND A2
Contra-Positive: If Not A1 OR A2 then Not B
Multiple Elements with OR separator:
If Then statements can also include OR as a addition within one of the A or B components.

For example:

If A1 OR A2 then B

For the purpose of finding the contra-positive the opposite of “A1 OR A2” is “Not A1 and Not A2” because if you have either A1 or A2 you satisfy the condition of “A1 OR A2”

So the contra-positive is as follows:

Original Statement: If A1 OR A2 then B

Contra-Positive: If Not B then Not A1 AND Not A2

So the Or becomes AND when you switch to the contra-positive.

Quiz: (Get out your pencils and try the following)
Convert these statements to symbolic IF-THEN statements and then convert to the contra-positive: (remember you don't have to use A and B, try using other variables to represent the different sentence parts. Pick letters that match the key words of the sentence e.g. "If you are smart" becomes "If S")

1. If you shoot tequila you will get drunk.

2. If you do not jump in you will never learn to swim.

3. All romantics are happy.

4. Anyone who smokes is healthy.

5. If you ace this quiz you will find true happiness and fortune.

6. People who do not read often become smarter.

7. Everyone who loves is happy and everyone who is happy is loved.

8. Given the information in number 7. above: If Michael is not loved then what conclusion can we draw?

9. Given the information in number 7. above: If Sara is loved then what conclusion can we draw?

Once you have completed the above quiz on paper check the answers below: (The answers are listed with the symbolic version of the original statement listed first and then the contra-positive)

1. If you shoot tequila you will get drunk.

If T then D

If Not D then Not T

2. If you do not jump in you will never learn to swim.

If Not J then Not S

If S then J

3. All romantics are happy.

If R then H

If Not H then Not R

Note that not all conditional statements include the words IF-THEN. Statements like this one have the same effect as an IF-THEN statement. “All” is often used on the LSAT in conditional statements.

4. Anyone who smokes is healthy.

If S then H

If Not H then Not S

Note: Do not get tied up in factual accuracy. It does not matter if the statement is correct, just that you understand the logic.

5. If you ace this quiz you will find true happiness and fortune.

If A then H and F

If Not H OR Not F then Not A

6. People who do not read often, become smarter.

If Not R then S

If Not S then R

Note: This one can be a bit tricky and demonstrates the need to memorize and rely upon the method for determining the contra-positive.

7. Everyone who loves is happy and everyone who is happy is loved.

There are actually two conditional statements here. Let’s set them both up separately and then combine them.

If GL (give love) then H

If Not H then Not GL

If H then RL (receive love)

If Not RL then Not H

Now we can combine the IF-THEN statements. More on how to do this coming soon. Check back next week.

If GL then H then RL

If Not RL then Not H then Not GL

8. Given the information in number 7. above: If Michael is not loved then what conclusion can we draw?

If Michael is not loved (Not RL) then he is Not Happy and does Not Give Love. From the combined statements: If Not RL then Not H then Not GL

9. Given the information in number 7. above: If Sara is loved then what conclusion can we draw?

If Sara is loved then we can draw no conclusion. The statements above and their contra-positives tell us nothing about someone who is loved.

See also: Examples of IF-THEN statements for Analytical Reasoning questions.