D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1.

VECTOR ANALYSIS

CLASS: II-B.Sc. MATHEMATICS **SUB. CODE:15CMA4B**

UNIT – I DIFFERENTIAL VECTOR CALCULUS **SECTION-A (2 MARKS)**

1. Prove $\vec{F}$ = sint $\vec{i}$ + cost $\vec{j}$ (a) $\frac{d\vec{F}}{dt}$ (b) $\frac{d^2\vec{F}}{dt^2}$.
2. Find $\frac{d}{dt}(\bar{A}.\bar{B})$ and $\frac{d}{dt}(\bar{A}.X\bar{B})$
3. Define a vector point function.
4. If $\bar{A}$ has a constant magnitude, show that $\bar{A}$ and $\frac{d\bar{A}}{dt}$ are perpendicular.
5. Define a vector point function with an example.
6. Find the values of φ(x,y,z)= $4yz^2+3xyz-z^2+2$ at the points (1,-1,2) and (0,-3,1).

SECTION-B 5 Marks

1. If $\vec{r}$ = $\vec{a}$ cosωt + $\vec{b}$ sinωt prove that (a) $\vec{r} \times \frac{d\vec{r}}{dt} = \omega\vec{a} \times \vec{b}$ (b) $\frac{d^2\vec{r}}{dt^2} + \omega^2\vec{r} = 0$.
2. Find the unit tangent vector at the point t=2 given $x=1+t^2$; y=4t-3, $z=2t^2-6t$.
3. If $\vec{F} = e^{-t}\vec{i}+\log(1+t^2)\vec{j}-\tan t\vec{k}$. Find(i) $\frac{d\vec{F}}{dt}$ (ii) $\frac{d^2\vec{F}}{dt^2}$ (iii) $\left|\frac{d^2\vec{F}}{dt^2}\right|$ at t=0.
4. Find the velocity & acceleration of a partial which moves along that curve x=2 sin3t, y=2 cos3t, z=8t at any time t. Find the also the magnitudes of the velocity and acceleration.
5. Find the velocity & acceleration of a partial which moves along that curve $x=e^{-t}$, y=2 cos3t, z=2 sin3t at any time t=0. Find the also the magnitudes of the velocity and acceleration.
6. Find (i) $\frac{d\vec{F}}{dt}$ (ii) $\frac{d^2\vec{F}}{dt^2}$ (ii) $\left|\frac{d\vec{F}}{dt}\right|$ (iii) $\left|\frac{d^2\vec{F}}{dt^2}\right|$. Given that $\vec{F}$=sint $\vec{i}$ + cost $\vec{j}$.
7. A partial moves along the curve $x=t^3+1$, $y=t^2$, z=2t+5 where t is the time. Find the components of its velocity and acceleration at t=1 in the direction $\vec{i}+\vec{j}+3\vec{k}$.
8. Find the acceleration x=2sin3t, y=2cos 3t, z=3t at t= $\frac{\pi}{2}$.
9. If $\bar{F}$ =sint$\bar{\imath}$ +cost$\bar{\jmath}$+t$\bar{k}$, find i) $\frac{d\bar{F}}{dt}$ ii)$\frac{d^2\bar{F}}{dt^2}$iii) $\left|\frac{d\bar{F}}{dt}\right|$ iv)$\left|\frac{d^2\bar{F}}{dt^2}\right|$.
10. Find the velocity and acceleration of a moving particle $\bar{r}(t)$ =2sin3t$\bar{\imath}$ +2cos3t $\bar{\jmath}$+8t$\bar{k}$.
11. If $\bar{A}=3t^2\bar{\imath}-(t+4)\bar{\jmath}+(t^2-2t)\bar{k}$ and $\bar{B}$ =sint$\bar{\imath}$ +3$e^t\bar{\jmath}$-3cost$\bar{k}$. Find $\frac{d^2}{dt^2}(\overline{\bar{A}X\bar{B}})$ at t=0.
12. If $\bar{r}$ =e^{-t}($\bar{A}$cos2t +$\bar{B}$sin2t) where $\bar{A}$ *and* $\bar{B}$ are constant vectors,

    Prove that $\frac{d^2\bar{r}}{dt^2} + 2\frac{d\bar{r}}{dt} + 5r = 0$.

13. Find the unit tangent vector at any point on the curve $\bar{r}$=(1+t²)$\bar{\imath}$ +(4t-3) $\bar{\jmath}$+(2t²-6t)$\bar{k}$. Also find the unit tangent vector at the point t=2.

SECTION-C 10 Marks

1. If $\vec{A}=t^2\vec{\imath}-t\vec{\jmath}+(2t+1)\vec{k}$, $\vec{B}=(2t-3)\vec{\imath}+\vec{\jmath}-t\vec{k}$

   (i) $\frac{d}{dt}(\vec{A}+\vec{B})$ (ii) $\frac{d}{dt}(\vec{A}.\vec{B})$ (iii) $\frac{d}{dt}(\vec{A}\times\vec{B})$ at the point t=1.
2. A particle moves along the curve $\bar{r}=2t^2\bar{\imath}+(t^2-4t)\bar{\jmath}+(3t-5)\bar{k}$ where t is the time. Find the component of velocity and acceleration at time t=1 in the direction of $\bar{\imath}-3\bar{\jmath}+2\bar{k}$.
3. If $\bar{r}=\bar{a}coswt+\bar{b}sinwt$ where $\bar{a},\bar{b}$ and w are constants prove that,

   i) $\bar{r}\text{x}\frac{d\bar{r}}{dt}=w\bar{a}\text{x}\bar{b}$ ii) $\frac{d^2\bar{r}}{dt^2}+w^2\bar{r}=0$.

UNIT – II GRADIENT, DIVERGENCE AND CURL

SECTION-A 2 Marks

1. Define scalar point function.
2. Define vector point function.
3. Gradient of a scalar point function.
4. Define Divergence of a vector point function.
5. Define Curl of a vector point function.
6. Find the maximum value of the direction derivate of the function $\emptyset=2x^2+3y^2+5z^2$ at (1,1,-4).
7. If $\vec{r}=x\vec{\imath}+y\vec{\jmath}+z\vec{k}$ & $|\vec{r}|=\sqrt{x^2+y^2+z^2}$ prove that (i) $\Delta r=\frac{\vec{r}}{r}$ (ii) $\Delta\log r=\frac{\vec{r}}{r^2}$.
8. Show that (i) grad $(\vec{r}.\vec{a})=\vec{a}$.
9. Show that (i) grad $[\vec{r},\vec{a},\vec{b}]=\vec{a}\times\vec{b}$.
10. Show that $\vec{F}=3y^4z^2\vec{\imath}+4x^3z^2\vec{\jmath}-3x^2y^2\vec{k}$. $\vec{F}$ is a solenoidal.
11. Show that $\vec{F}=(\sin y+z)\vec{\imath}+(x\cos y-z)\vec{\jmath}+(x-y)\vec{k}$ is irrotational.
12. Define $\nabla\varphi$ *and* $\nabla^2\varphi$.
13. If $\bar{r}=x\bar{\imath}+y\bar{\jmath}+z\bar{k}$, find div$\bar{r}$ and curl$\bar{r}$.
14. If $\bar{A}$ *and* $\bar{B}$ are irrotational, Prove that $\bar{A}X\bar{B}$ is solenoidal.
15. If $\varphi=x^2yz^3$ find $\nabla\varphi$ at (1,1,1).
16. Find the unit vector normal to the surface $x^2+3y^2+2z^3=6$ at the pint (2,0,1).
17. Show that $\bar{F}=3x^2y\bar{\imath}-4xy^2\bar{\jmath}+2xyz\bar{k}$ is solenoidal.
18. Find a so that $\bar{F}=(axy-z^2)\bar{\imath}+(x^2+2yz)\bar{\jmath}+(y^2-axz)\bar{k}$ is irrotational.
19. Prove that grad$(\bar{r}.\bar{a})=\bar{a}$ when $\bar{a}$is a constant vector.

SECTION-B 5 Marks

1. If $\emptyset(x,y,z)=x^2y+y^2x+z^2$ find $\Delta\emptyset$ at the point (1,1,1).
2. If $\vec{F}=xy^2\vec{i}+2x^2yz\vec{j}-3yz\vec{k}$ find divergence $\vec{F}$ and curl of $\vec{F}$ at (1,-1,1).
3. Find the directional derivates of $\emptyset(x,y,z) = xyz-xy^2z^3$. The point (1,2,-1) in the directional of $\vec{i}-\vec{j}-\vec{k}$.
4. Find the unit normal vector to the surface $\emptyset(x,y,z)=x^2+3y^2+2z2=6$ at the point (2,0,1).
5. If $\nabla\emptyset=(6xy+z^3)\vec{i}+(3x^2-z)\vec{j}+(3xz^2-y)\vec{k}$. Find scalar potential.
6. Find $\emptyset$ if $\nabla\emptyset=x(2yz+1)\vec{i}+x^2z\vec{j}+x^2y\vec{k}$.
7. Find the equation of the tangent plane and normal line to the surface xyz=4 at the point (1,2,2).
8. Find the directional derivate of $\emptyset(x,y,z)=x^2yz+4xz^2$ at the point(1,2,-1) in the direction of $2\vec{i}-\vec{j}-2\vec{k}$.
9. Find the directional derivative of $\emptyset(x,y,z)=x^2-2y^2+4z^2$ at the point (1,1,-1) in the direction at $2\vec{i}-\vec{j}-\vec{k}$.
10. If $\vec{a}$ is a constant vector and $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ prove that $\nabla x(\vec{a}\times\vec{r}) = 2\vec{a}$.
11. Show that (i) grad $(\vec{r}.\vec{a})=\vec{a}$ (ii) grad $[\vec{r},\vec{a},\vec{b}]=\vec{a}\times\vec{b}$ where $\vec{a}$ & $\vec{b}$ are constant vectors.
12. Show that the surface $5x^2-2yz-9x=0$ and $4x^2y+z^3-4=0$ are orthogonal at (1,-1,2).
13. Find a & b so that $ax^2-byz=(a+2)x$ will be orthogonal to the surface $4x^2y+z^3=4$ at the point (1,-1,2).
14. .Prove that $\nabla.\left(\frac{\emptyset}{\Psi}\right)=\frac{\Psi\nabla\emptyset-\emptyset\nabla\Psi}{\Psi^2}$.
15. If $\vec{F}=(x^2-y^2+2xz)\vec{i}+(xz-xy+yz)\vec{j}+(z^2+x^2)\vec{k}$. Find the $(i)\nabla.\vec{F}$, (ii)$\nabla(\nabla.\vec{F})$, (iii) $\nabla\times\vec{F}$, (iv)$\nabla.(\nabla\times\vec{F})$ &(v) $\nabla\times[\nabla\times\vec{F}]$ at the point (1,1,1).
16. .Find $\nabla\ (\frac{1}{r}.\vec{r})$ prove $\frac{2}{r}$.
17. If u=x+y+z, $v=x^2+y^2+z^2$ and w=xy+yz+zx prove that grad u.(grad v × grad w)=0.
18. Let $\vec{F}(x,y,z)=3xy\vec{i}+4yx\vec{j}+3z\vec{k}$. Find $\nabla\times\vec{F}$ at the point (1,1,1).
19. Find a,b and c so that $\bar{F}=(x+2y+az)\bar{\imath}+(bx-3y-z)\bar{\jmath}+(4x+cy+2z)\bar{k}$ is irrotational.
20. P.T $\nabla x(r^n\bar{r})=0$.
21. Find the directional derivative of $\varphi=xy+yz+zx$ is the direction of vector $2\bar{\imath}+3\bar{\jmath}+6\bar{k}$ at the point (3,1,2).
22. Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2,-1,2).
23. Show that surfaces $5x^2-2yz-9x=0$ and $4x^2y+z^3-4=0$ are orthogonal at the point (1,-1,2).

SECTION-B 10 Marks

1. Find the angle between in normal to the surface $\emptyset(x,y,z)=xy-z^2$ at the points (1,4,-2) & (-3,-3,3).
2. If $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ & $|\vec{r}|=\sqrt{x^2+y^2+z^2}$ prove that
   a. (i) $\nabla r = \frac{\vec{r}}{r}$ (ii) $\nabla\left(\frac{1}{x}\right)=\frac{-\vec{r}}{r^3}$
   b. (iii) $\nabla r^n = nr^{n-2}\vec{r}$ (iv) $\nabla f(r) = f'(r)\frac{\vec{r}}{r}$
   c. (v) $\nabla \log r = \frac{\vec{r}}{r^2}$ (vi) $\nabla f(x) \times \vec{r}=0$.
3. Find the equation of the tangent plane and normal line to the surface $x^2+y^2+z^2=25$ at (4,0,3).
4. If $\bar{v} = \overline{w}\ X\ \bar{r}$, Prove that $\overline{w}=1/2$ curl $\bar{v}$ where $\overline{w}$ is the constant vector and $\bar{r}$ is the position vector.
5. If $\bar{r}=x\bar{\imath}+y\bar{\jmath}+z\bar{k}$, prove that
   i) $\nabla x[f(r)\bar{r}]=0$
   ii) $\nabla.(r^n\bar{r})=(n+3)r^n$ deduce that n=-3 when $r^n\bar{r}$ is solenoidal.

UNIT – III VECTOR IDENTITIES 2 Marks

1. Define vector identities.
2. Prove that $\nabla.(\vec{u}+\vec{v}) = \nabla.\vec{u}+\nabla.\vec{v}$.
3. Prove that $\nabla\times(\vec{u}+\vec{v}) = (\nabla\times\overrightarrow{u})+(\nabla\times\vec{v})$
4. Prove that $.\nabla(\emptyset\vec{u}) = (\nabla.\emptyset)\vec{u}+\emptyset(\nabla.\vec{u})$.
5. Prove that $\nabla(\emptyset\vec{u}) = \nabla(\emptyset\times\vec{u})+\emptyset(\nabla\times\vec{u})$.
6. Prove that $.\nabla\emptyset = \nabla^2\emptyset$.
7. Prove that $\nabla\times\nabla\emptyset = 0$.
8. If $\vec{A}$ *and* $\vec{B}$ are irrotational prove that $\vec{A}\times\vec{B}$ is a solenoidal.
9. Prove that if $\vec{F}$ is a solenoidal curl curl curl (curl $\vec{F}$)=$\nabla^4\vec{F}$.
10. If $\vec{a}$ is a constant vector prove that $\text{div}[r^n[\vec{a}\times\vec{r}]] = 0$.
11. Prove that $\nabla\times(\nabla r^n) = 0$.
12. Prove that $\nabla[(\vec{r}\times\vec{a})\times\vec{b}] = -2(\vec{b}.\vec{a})$.
13. Prove that $\nabla.(\nabla x\bar{F})=0$.
14. Prove that $\nabla.(\nabla\phi)=\nabla^2\boldsymbol{\phi}$

SECTION-B 5 Marks

1. Prove that (*i*) $\nabla.(\emptyset\vec{u}) = (\nabla.\emptyset)\vec{u}+\emptyset(\nabla.\vec{u})$
   (ii) $\nabla(\emptyset\vec{u}) = \nabla(\emptyset\times\overrightarrow{u})+\emptyset(\nabla\times\vec{u})$.

2. Prove that $\nabla.(\vec{u}\times\vec{v}) = \vec{v}.\nabla\times\vec{u} - \vec{u}.\nabla\times\vec{v} = \vec{v}.curl\,\vec{u} - \vec{u}.curl\,\vec{v}$.

3. Prove that $(\vec{u}.\vec{v}) = (\vec{v}.\nabla)\vec{u} - (\vec{u}.\nabla)\vec{v} - [(\vec{u}.\nabla).\vec{v} - (\nabla.\vec{v})\vec{u}]$.

4. Prove that $\nabla(\vec{u}.\vec{v}) = (\vec{v}.\nabla)\vec{u} + (\vec{u}.\nabla)\vec{v} + \vec{v}\times(\nabla\times\vec{u}) + \vec{u}\times(\nabla\times\vec{v})$.

5. Prove that $\nabla.(\nabla\times\vec{F}) = 0$.

6. Prove that $(\vec{v}\times\nabla)\times\vec{r} = -2\vec{v}$.

7. Prove that $\nabla\left[\frac{f(r)}{r}\vec{r}\right] = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 f)$.

8. If $\vec{u} = \frac{1}{r}\vec{r}\; find\; div\; grad\;(\vec{u}) = {}^2/_r$.

9. Prove that . $\left[r\nabla\left(\frac{1}{r^3}\right)\right] = {}^3/_{r^4}$.

10. Prove that . $(\nabla.r^n) = n(n+1)r^{n-2}$.

11. Show that $\nabla^2(e^r) = e^r + {}^2/_r\, e^r$.

12. Using the result $\nabla^2 f(r) = \frac{d^2r}{dr^2} + \frac{2}{r}\frac{dF}{dr}$ prove that $\nabla^4(e^r) = e^r + \frac{4}{r}e^r$.

13. Prove that $(\nabla\emptyset\Psi - \Psi\nabla\emptyset) = \emptyset\nabla^2\Psi - \Psi\nabla^2\emptyset$.

14. Prove that curl curlcurl (curl$\bar{F}$)= $\nabla^4\bar{F}$.

15. Prove that ∇x[($\bar{r}X\bar{a}$)x$\bar{b}$]=$\bar{b}$x$\bar{a}$.

16. If $\bar{u}=\frac{\bar{r}}{r}$ find div $\bar{u}$.

17. Prove that $\nabla X(\nabla r^n)=0$.

SECTION-C 10 Marks

1. Prove that $\nabla\times(\nabla\times\vec{F}) = \nabla(\nabla.\vec{F}) - \nabla^2\vec{F}$.

2. If $\vec{a}$ *and* $\vec{b}$ are constant vector prove that (i) $\nabla[(\vec{r}\times\vec{a})\times\vec{b}] = -2(\vec{b}.\vec{a})$

 (ii) $\nabla[(\vec{r}\vec{a})\times\vec{b}] = \vec{b}\times\vec{a}$.

3. Prove that $\nabla\times(\nabla\times\vec{F}) = \nabla(\nabla.\vec{F}) - \nabla^2 F^2$.

4. Prove that ∇.[f(r)$\frac{\bar{r}}{r}$]=f '(r)+(2/r) f(r).

5. Prove that ∇^2.[f(r)]=f "(r)+(2/r) f '(r).

6. Using the result, ∇^2.[f(r)]=f "(r)+(2/r) f '(r), Prove that $\nabla^4(e^r)= e^r+(4/r)\, e^r$.

7. Prove that $\nabla^2[\nabla(\frac{\bar{r}}{r2})]=2/r^4$.

UNIT – IV VECTOR INTEGRATION 2 Marks

1. Define line integral.
2. Define conservative vectors field.
3. Find integral $\int_2^3 f(\vec{t})\,dt\; where\; f(\vec{t}) = (3t^2-1)\vec{\imath} + (2-6t)\vec{\jmath} - 4t\vec{k}$.

4. Show that $\int_0^{\frac{\pi}{2}}(3sin\, t\vec{\imath} + 4cost\vec{\jmath}\ )dt$.

5. Shoe that $\int_0^1 (e^t\vec{\imath} + e^{-t}\vec{\jmath} + \vec{k})$dt .

6. Show that $\int_1^2 (t - t^2)\vec{\imath} + 2t^3\vec{\jmath}$-$3\vec{k}$ dt .

7. Define surface integral.

8. Statement Gauss divergence theorem.

9. Evaluate $\int_s$ (ax$\vec{\imath}$+by$\vec{\jmath}$+cz$\vec{k}$) . $\vec{n}$ ds where s is the surface of the sphere $x^2 + y^2 + z^2 = 1$.

10. Find the $\int_s\int \vec{F}.\vec{n}$ ds for the vector $\vec{F} = x\vec{\imath}$-y$\vec{\jmath} + 2z\vec{k}$ over the sphere $x^2 + y^2(z - 1)^2$=1.

11. Find $\int_0^{\pi/2}(3sint\bar{\imath} + 2cost\bar{\jmath})dt$.

12. .

13. If $\overline{f(t)}$=(t-t^2) $\bar{\imath} + 2t^3\bar{\jmath} - 3\bar{k}$, find $\int_1^2 \overline{f(t)}$ dt.

14. Prove $\int \bar{a}.\frac{d\bar{r}}{dt}$dt=$\bar{a}$.$\bar{r}$+$\bar{c}$ where $\bar{a}$ *and* $\bar{c}$ are constants.

SECTION-B **5 Marks**

1. If $\vec{F} = (3x^2 + 6y)\vec{\imath} - 14yz\vec{\jmath} + 20xz^2\vec{k}. Eveluate \int_c \vec{F}\ d\vec{r}$ from (0,0,0) to (1,1,1) along the curve x=t, y=t^2, z=t^3.

2. Find the work done the moving a particle in the force field $\vec{F} = 3x\vec{\imath} + 12xz - y\vec{\jmath} - z\vec{k}$ from the t=0 to t=1 along the curve x=2t, y=t, z=4t^3.

3. If $\vec{F} = 3x^2\vec{\imath} + (2xz - y)\vec{\jmath} + z\vec{k}. Evaluate\ the \int_c \vec{F}.d\vec{r}$ where c is the straight lint from A(0,0,0) to B(2,1,3) or find the work done in a moving particle in the force field $\vec{F} = 3x^2\vec{\imath} + (2xz - y)\vec{\jmath} + z\vec{k}$ along the curve defined by $x^2 = 4y, 3x^3 = 8z$ from x=0, x=2.

4. If $\vec{F} = [4xy - 3x^2z^2]\ \vec{\imath} + 2x^2\vec{\jmath} - 2x^3z\vec{k}$. Check wheather $\int_c . d\vec{r}$ is a independent of the path c.

5. Find $\int \vec{F} = 3xy\vec{\imath} - y^2\vec{\jmath}$. C is the parabola y=2x^2 from (0,0) to (1,2) & x=0.

6. $\vec{F} = (x^2 + y^2)\vec{\imath} - 2xy\vec{\jmath}$. Rectangle x=±a, y=0, y=b.

7. $\vec{F} = (x^2 - y^2)\vec{\imath} + 2xy\vec{\jmath}$ c is rectangle m=0, a, y=0,b.

8. $\vec{F} = x^2\vec{\imath} + xy\vec{\jmath}$ (square x=±1, y=±1).

9. Find the work done by $\vec{F} = (2x - y + 2z)\vec{\imath} + (x + y - z^2)\vec{\jmath} + (3x - 2y - 5z)\vec{k}$ *c is the* circle xy plane $x^2 + y^2 = 4, dz = 0$.

10. $\vec{a} = e^{-t}\vec{\imath} - 6(t + 1)\vec{\jmath} + 3sin\vec{k}$. *Find* $\vec{v}$ *and* $\vec{r}$ *at* $t = 0$.

11. $\vec{a} = 18cos3t\vec{\imath} - 8sin2t\vec{\jmath} + 6t^2\vec{k}$. Find $\vec{v}$ *and* $\vec{r}$ *at* $t = 0$.

12. Find $\int_c \vec{F}.d\vec{r}$ *where* $\vec{F} = z\vec{\imath} + x\vec{\jmath} + y\vec{k}$ *along c is* $\vec{r} = cost\vec{\imath} + sin\vec{\jmath} + t\vec{k}$.
x =cost, y=sint, z=t from t=o to 2π.

13. Find the work done of $\vec{F} = (2x - y + z)\vec{\imath} + (x + y - z^2)\vec{\jmath} + (3x - 2y + 4z)\vec{k}$. c is the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

14. Evaluate $\iint_s (y^2z^2\vec{\imath} + z^2x^2\vec{\jmath} + z^2y^2\vec{k})\vec{n}.ds\ where$ s is the part of the sphere. $x^2 + y^2 + z^2$=1 above the xy plane boundered the surface s.

15. Evaluate $\int_s$ (ax$\vec{\imath}$+by$\vec{\jmath}$+cz$\vec{k}$) . $\vec{n}$ ds where s is the surface of the sphere $x^2 + y^2 + z^2 = 1$.

16. Show that $\iint_s \vec{F}.\vec{n}\ ds = \frac{12}{5}\ \pi R^5$ where s is the sphere, center and origin at R and $\vec{F} = x^3\vec{\imath} + y^3\vec{\jmath} + z^3\vec{k}$.

17. If $\vec{n}\ is$ the Unit outward drawn normal to any closed surface area s show that $\iint_S \frac{\vec{r}.\vec{n}}{r^2} ds = \iiint_v \frac{dv}{r^2}$.

18. Find the $\int_s\int \vec{F}.\vec{n}$ ds for the vector $\vec{F} = x\vec{\imath}$-y$\vec{\jmath} + 2z\vec{k}$ over the sphere $x^2 + y^2(z-1)^2$=1.

19. Shoe that $\int_s \vec{F}.\vec{n}ds = {}^{12}/_5\ \pi R^5$ where s is the sphere and radius R and $\vec{F} = x^3\vec{\imath} + y^3\vec{\jmath} + z^3\vec{k}$.

SECTION-C 10 Marks

1. The acceleration of a moving partial at the time t is given by $\vec{a} = 9t\vec{\imath} - 24t^2\vec{\jmath} + 4sint\vec{k}$. If $\vec{r} = 2\vec{\imath} + \vec{\jmath}, \vec{v} = \frac{d\vec{r}}{dt} = -\vec{\imath} - 3\vec{k}\ at\ the\ t = 0. Find\ \vec{r}.$
2. If $\vec{F} = (3x^2 + 6y)\vec{\imath} - 14yz\vec{\jmath}20xz^2\vec{k}$. Find $\int \vec{F}.d\vec{r}$ from (0,0,,0) to (1,0,0) along the following path (i)The straight line from (0,0,0) to (1,0,0) then to (1,1,0) and then to (1,1,1) (ii) The straight line joining (0,0,0) and (1,1,1).
3. Find $\int_c \vec{F}.d\vec{r}.where\ (i)\vec{F} = (x^2 - y^2)\vec{\imath} + 2xy\vec{\jmath}$. in c:square bowoled by x=0, y=0, x=a, y=a.
4. If $\vec{F} = yz\vec{\imath} + zx\vec{\jmath} - xy\vec{k}$. Find the $\int_c \vec{F}.d\vec{r}$ wheren x=t, y=$t^2, z = t^3$ from P(0,0,0) to Q(2,4,8).
5. Evaluate $\iint_s \vec{F}.\hat{n}\ ds$ where $\vec{F} = yz\vec{\imath} + zx\vec{\jmath} + xy\vec{k}$. s is the part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant.
6. Evaluate $\iint_s \vec{F}.\hat{n}$ ds where $\vec{F} = z\vec{\imath} + x\vec{\jmath} - y\vec{k}$ and s is the first optional between the planes z=0 & z=2.
7. Verify Gauss theorem $\vec{F} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ taken over the region boundered by the plane x=0, x=a, y=0, y=a, z=0, z=a.
8. Verify the divergence theorem $\vec{F} = 4x\vec{\imath} - 2y^2\vec{\jmath} + z^2\vec{k}$ taken over the region the boundered by $x^2 + y^2 + z^2 = 4$, z=0, z=3.
9. Evaluate $\iint_s (y^2z^2\vec{\imath} + z^2x^2\vec{\jmath} + z^2y^2\vec{k})\vec{n}.ds$ where s is the part of the sphere $x^2 + y^2 + z^2 = 1$ above the xy plane boundered the surface s.
10. Evaluate $\iint_s (x^3\vec{\imath} + y^3\vec{\jmath} + z^3\vec{k})\vec{n}ds$ where s is the surface of the sphere $x^2 + y^2 + z^2 = 16$.
11. Evaluate $\iint_s (y^2z^2\vec{\imath} + z^2x^2\vec{\jmath} + x^2y^2\vec{k})\ d\vec{s}$ where s is the upper part of the sphere $x^2 + y^2 + z^2 = 9$ above the xy plane.

UNIT – V SECTION-A 2 Marks

1. Define stoke`s theorem.
2. Define green`s theorem.
3. Find the area of the ellipse x= a cosθ, y=sinθ .
4. Find the area of the ellipse using Green's theorem.
5. Prove that area bounded by simple closed curve C is $\int_C xdy - ydx$
6. If **ϕ**is the scalar function, using Stoke's theorem, Prove that curl(grad**ϕ**)=0

SECTION-B 5 Marks

1. Verify stoke`s stheorem for $\vec{F} = (2x - y)\vec{\imath} - yz^2\vec{\jmath} - y^2z\vec{k}$ $s\ upper\ half\ sphere\ x^2 + y^2 + z^2 = 1$ and c is the bbounded.
2. Show that $\int_c \vec{A} \times \vec{r}\, d\vec{r} = 2\iint_s \vec{n}\vec{A}$ dr where A is a constant vector.
3. Prove that $\int_c \vec{r}.d\vec{r} = 0$.
4. If $\emptyset$ is a scalar point function use stoke`s theorem prove that curl (grad $\emptyset$) = 0
5. Use stoke`s theorem to evaluate $\iint_s \nabla \overline{\times \vec{F}}.\vec{n}\, ds$ where s is the upper half of the hemisphere of radius a center at the origin and $\vec{F}$=2y$\vec{\imath} - x\vec{\jmath} + z\vec{k}$.
6. Find the area for the four leafed rose r=3 sin2θ .
7. Find the area of the curve $x^{2/3} + a^{2/3}$ using green`s theorem.
8. Find the area between the parabola $y^2 = 4x$ and $x^2 = 4y$.
9. Compute $\int_c (xy - x^2)dx + x^2ydy$ over the triangle boundered by the lines y=0, x=1, y=x, and verify the green`s theorem.
10. Find the area between the parabola, y^2=4x and x^2=4y

SECTION-C 10 Marks

1. Verify the stokes theorem for $x^2\vec{\imath} + xy\vec{\jmath}$ taken round the square in the xy plane whose rides are x=0, x=a, y=0, y=a.
2. Verify the stoke`s theorem when $\vec{F} = y\vec{\imath} + z\vec{\jmath} + x\vec{k}$ and surface s is the part of the sphere $x^2 + y^2 + z^2 = 1$.
3. Verify the st0ke`s theorem for $\vec{F} = (y - z + 2)\vec{\imath} + (yz + 4)\vec{\jmath} - xz\vec{k}$ over the surface of a cube x=0, y=0, z=0, x=2, y=2, above xoy plane (open at the bottom).
4. Verify the stokes theorem for the function over the $\vec{F} = (x^2 + y - 4)\vec{\imath} + 3xy\vec{\jmath} + (2xz + z^2)\vec{k}$ over the surface of the hemisphere $x^2 + y^2 + z^2 = 16$ above the xy plane.

5. Verify the stoke`s theorem for the function $\vec{F} = (x+y)\vec{i} + (2x-z)\vec{j} + (y+z)\vec{k}$ taken over the triangle ABC cut from the plane 3x+2y+z=6 by the co-ordinate plane.
6. Verify green`s theorem in the plane for $\oint (xy+y^2)\,dx + x^2 dy$ where c is the chord curve of the region bounded by y=x and y=x^2.
7. Verify the green`s theorem in the plane for $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the bounded of the region defined by y= $\sqrt{x}$ and $y = x^2$.
8. Verify green`s theorem in the plane $\int_c [(x^2 - 2xy)dx + (x^3 y + 1)dy]$ where c is the boundary given by $y^2 = 8x$ and $x = 2$.
9. Verify Stoke's theorem for $\bar{F}$ =(y-z+2)$\bar{i} - (yz+4)\bar{j} - xz\bar{k}$ over the cube x=0,y=0,z=0,x=2,y=2,z=2.
10. Verify Green's theorem for $\int_C (xy - x^2)dx + x^2 y\,dy$ over the triangle bounded by the lines x=1,x=y, y=0.