KINEMATICS

PREVIOUS EAMCET BITS

ENGINEERING PAPER

1. A body is projected vertically upwards at time t = 0 and is seen at a height 'H' at time t_1 and t_2 seconds during its flight. The maximum height attained is [g = acceleration one to gravity]

[EAMCET 2009E]

1) $\frac{g(t_2-t_1)^2}{8}$ 2) $\frac{g(t_1+t_2)^2}{4}$ 3) $\frac{g(t_1+t_2)^2}{8}$ 4) $\frac{g(t_2-t_1)^2}{4}$

Ans: 2

Sol: When a body is projected vertically upwards it occupies the same position while going up and coming down after time of t_1 and t_2

$\therefore \quad H = ut - \frac{1}{2}gt^2$

$gt^2 - 2ut + 2H = 0$

Sum of roots = $t_1 + t_2 = \frac{2u}{g} \Rightarrow u = \frac{g(t_1+t_2)}{2}$

$\therefore \; H_{max} = \frac{u^2}{2g} = \frac{g[t_1+t_2]^2}{8}$

2. A body thrown vertically up to reach its maximum height in 't' seconds. The total time from the time of projection to reach a point at half of its maximum height while returning (in seconds) is

[EAMCET 2008 E]

1) $\sqrt{21}$ 2) $\left(1+\frac{1}{\sqrt{2}}\right)t$ 3) $\frac{3t}{2}$ 4) $\frac{t}{\sqrt{2}}$

Ans: 2

Sol: Time taken to reach the maximum height = time of ascent = t

$\therefore$ as $h \propto t^2 \Rightarrow \frac{h_1}{h_2} = \left(\frac{t_1}{t_2}\right)^2 \Rightarrow \frac{h}{h/2} = \left(\frac{t}{t_2}\right)^2$

$\Rightarrow t_2 = \frac{t}{\sqrt{2}}$

$\therefore$ Time taken to reach half distance of its maximum height $= \frac{t}{\sqrt{2}}$

$\therefore$ Total time $t + \frac{t}{\sqrt{2}} = \left(1+\frac{1}{\sqrt{2}}\right)t$

3. A body is projected from the earth at angle 30° with the horizontal with some velocity. If its range is 20m, the maximum height reached by it is [in metres] **[EAMCET 2006 E]**

1) $5\sqrt{3}$ 2) $\frac{5}{\sqrt{3}}$ 3) $\frac{10}{\sqrt{3}}$ 4) $10\sqrt{3}$

Ans 2:

Sol: $\frac{h_{max}}{R} = \frac{u^2 \sin^2\theta}{2g \times \frac{2u^2 \sin\theta\cos\theta}{g}} = \frac{\tan\theta}{4}$

$\frac{h_{max}}{20} = \frac{\tan 30°}{4} \Rightarrow h_{max} = \frac{5}{\sqrt{3}}m$

4. A body is projected vertically upwards at time t = 0 and is seen at a height 'H' at time t_1 and t_2 seconds during its flight. The maximum height attained is [g = acceleration due to gravity]

[EAMCET 2005 E]

1) $\frac{g(t_2 - t_1)^2}{8}$ 2) $\frac{g(t_1 + t_2)^2}{4}$ 3) $\frac{g(t_1 + t_2)^2}{8}$ 4) $\frac{g(t_2 - t_1)^2}{4}$

Ans: 3

Sol: When a body is projected vertically upwards it occupies the same position while going up and coming down after time of t_1 and t_2

$\therefore\ H = ut - \frac{1}{2}gt^2$

$gt^2 - 2ut + 2H = 0$

Sum of roots = $t_1 + t_2 = \frac{2u}{g} \Rightarrow u = \frac{g(t_1 + t_2)}{2}$

$\therefore\ H_{max} = \frac{u^2}{2g} = \frac{g[t_1 + t_2]^2}{8}$

5. The horizontal and vertical displacement x and y of a projectile at a give time 't' are given by x = 6t metres and y = 8t – 5t^2 metres. The range of the projectile in metres is

[EAMCET 2004 E]

1) 9.6 2) 10.6 3) 19.2 4) 38.4

Ans: 1

Sol: Comparing x = 6t with $x = (u\cos\theta)t, u\cos\theta = 6$

$y = 8t - 5t^2$ with $y = (u\sin\theta)t - \frac{1}{2}gt^2, u\sin\theta = 8$

$\therefore \text{Range} = \frac{2[u\cos\theta][u\sin\theta]}{g} = \frac{2\times6\times8}{10} = 9.6m$

6. The equations of motion of a projectile are given by x = 36t metre and 2y = 96t- 9.8t^2 metre. The angle of projection is **[EAMCET 2003 E]**

1) $\sin^{-1}\left(\frac{4}{5}\right)$ 2) $\sin^{-1}\left(\frac{3}{5}\right)$ 3) $\sin^{-1}\left(\frac{4}{3}\right)$ 4) $\sin^{-1}\left(\frac{3}{4}\right)$

Ans: 1

Sol: Comparing x = 36t with $x = (u\cos\theta)t,\ u\cos\theta = 36$ …..(1)

$2y - 96t - 9.8t^2 \Rightarrow y = 48t - 4.9t^2$

Comparing the equation with $y = (u\sin\theta)t - \frac{1}{2}gt^2$

$u\sin\theta = 48$ ………(2)

$\therefore \frac{u\sin\theta}{u\cos\theta} = \frac{48}{36} \Rightarrow \tan\theta = \frac{4}{3}$ (or) $\sin\theta = \frac{4}{5}$ (or) $\theta = \sin^{-1}\left(\frac{4}{5}\right)$

7. The horizontal and vertical displacement of a projectile at time 't' are x = 36t and y = 48t-4.9t^2 respectively. Initial velocity of the projectile in m/s is **[EAMCET 2002 E]**

1) 15 2) 30 3) 45 4) 6 0

Ans: 4

Sol: comparing the equation x = 36t with $x = (u\cos\theta)t$

$u\cos\theta = 36$ …………(1)

$y = 48t - 4.9t^2$ with $y = (u\sin\theta)t - \frac{1}{2}gt^2$

$\therefore$ $u\sin\theta = 48$ ……..(2)

Squaring (1) and (2) and adding u = 60 ms^{-1}

8. An object is projected with a velocity of 20 m/s making an angle of 45° with horizontal. The equation for the trajectory is 'h = Ax – Bx^2' where 'h' is height , 'x' is horizontal distance, A and B are constants. The ratio A : B is (g = 10 m/s^2) **[EAMCET 2001 E]**

1) 1 : 5 2) 5 : 1 3) 1 : 40 4) 40 : 1

Ans: 4

Sol: $h = Ax - Bx^2$ compared with $h = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$\tan\theta = A,\ B = \frac{g}{2u^2\cos^2\theta}$

$\therefore \frac{A}{B} = \frac{\tan\theta}{\frac{g}{2u^2\cos^2\theta}} \Rightarrow \frac{A}{B} = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{40}{1}$

9. Four bodies P, Q, R and S are projected with equal velocities having angles of projection 15°, 30°, 45° and 60° with the horizontal respectively. The body having shortest range is **[EAMCET 2000 E]**

1) P 2) Q 3) R 4) S

Ans: 1

Sol: As Range = $\frac{u^2\sin 2\theta}{g} \Rightarrow R \propto \sin 2\theta$

$\therefore$ As θ increases sin2θ also increases

$\therefore$ Shortest range is for P .

10. A body is thrown horizontally from the top of a tower of 5m height. It touches the ground at a distance of 10 m from the foot of the tower. The initial velocity of the body is : (g = 10 ms^{-2}) **[EAMCET 2000 E]**

1) 2.5 ms^{-1} 2) 5 ms^{-1} 3) 10 ms^{-1} 4) 20 ms^{-1}

Ans: 3

Sol: $x^2 = \left(\frac{2u^2}{g}\right)y$ given x = 10 m, y = 5 m, g = 10 ms^{-2}

$\therefore u = 10 ms^{-1}$

11. A body is thrown vertically upwards with an initial velocity 'u' reaches maximum height in 6 seconds. The ratio of distances travelled by the body in the first second and seventh second is **[EAMCET 2000 E]**

1) 1 : 1 2) 11 : 1 3) 1 : 2 4) 1 : 11

Ans: 2

Sol: As time of ascent = 6 s

$\therefore t = \frac{u}{g} \Rightarrow u = gt = 6g$

$\therefore h_1$ = distance travelled by the body in the first

Second $ut - \frac{1}{2}gt^2 = 6g(1) - \frac{1}{2}g(1)^2 = \frac{11g}{2}$

h_2 = distance travelled in the seventh second is same as distance travelled in 1st second in downward motion = g/2

$\therefore \frac{h_1}{h_2} = 11:1$

MEDICAL

12. A body of mass 2 kg is projected from the ground with a velocity 20 ms^{-1} at an angle 30°C with the vertical. If t_1 is the time in seconds at which the body is projected and t_2 is the time in seconds at which it reaches the ground, the change in momentum in kg ms^{-1} during the time $(t_2 - t_1)$ is **[EAMCET 2006 M]**

1) 40 2) $40\sqrt{3}$ 3) $50\sqrt{3}$ 4) 60

Ans :2

Sol: Initial velocity $\vec{u} = (u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$

Final velocity $\vec{v} = (u\cos\theta)\hat{i} + (u\sin\theta)(-\hat{j})$

$\therefore$ Change in velocity = $\vec{v} - \vec{u} = 2u\sin\theta(-\hat{j})$

Magnitude of change in momentum = 2musinθ

as θ = 30° with the vertical. $\therefore$ with the horizontal θ = 60°

$\Delta p = 2 \times 2 \times 20 \times \frac{\sqrt{3}}{2} = 40\sqrt{3}$

13. A body is projected vertically upwards with a velocity 'u'. It crosses a point in its journey at a height 'h' twice, just after 1 and 7 seconds. The value of u in ms^{-1} is : (g = 10 ms^{-2}) [EAMCET 2006 M]

1) 50 2) 40 3) 30 4) 20

Ans: 2

Sol: As the body is projected upwards we can write the equation as

$h = ut - \frac{1}{2}gt^2$

$\Rightarrow gt^2 - 2ut + 2h = 0$

$\therefore t_1 + t_2$ are the roots of the equation

$\therefore t_1 + t_2 = \left(\frac{2u}{g}\right) \Rightarrow u = \frac{g}{2}(t_1 + t_2)$

$\therefore u = 40ms^{-1}$

14. Two balls are projected simultaneously in the same vertical plane from the same point with velocities V_1 and V_2 with angles θ_1 and θ_2 respectively with the horizontal. If $V_1 \cos\theta_1 = V_2 \cos\theta_2$, the path of one ball as seen from the position of other ball is **[EAMCET 2006 M]**

1) Parabola
2) Horizontal straight line
3) Vertical straight line
4) Straight line making 45° with the vertical

Ans:3

Sol: It is given that $v_1 \cos\theta_1 = v_2 \cos\theta_2$. Hence the relative velocity between them is zero. The path of one ball as seen from the position of the other ball is vertical straight line.

15. The maximum height reached by a projectile is 4 metres. The horizontal range is 12 metres. Velocity of project in ms^{-1} is (g – acceleration due to gravity) **[EAMCET 2004 M]**

1) $5\sqrt{\frac{g}{2}}$
2) $3\sqrt{\frac{g}{2}}$
3) $\frac{1}{3}\sqrt{\frac{g}{2}}$
4) $\frac{1}{5}\sqrt{\frac{g}{2}}$

Ans: 1

Sol: $\frac{h_{max}}{R} = \frac{u^2 \sin^2\theta}{2g \times \frac{2u^2 \sin\theta\cos\theta}{g}} = \frac{\tan\theta}{4}$

$\frac{4}{12} = \frac{\tan\theta}{4} \Rightarrow \tan\theta = \frac{4}{3}$ (or) $\sin\theta = \frac{4}{5}$

$\therefore h_{max} = 4 = \frac{u^2\left[\frac{4}{5}\right]^2}{2g} \Rightarrow u = 5\sqrt{\frac{g}{2}}$

16. Two stones are projected with the same speed but making different angles with the Horizontal. Their horizontal ranges are equal. The angle of projection of one is π/3 and the maximum height reached by it is 102 metres. Then the maximum height reached by the other in metre is **[EAMCET 2003 M]**

1) 336
2) 224
3) 56
4) 34

Ans: 4

Sol: As ranges are equal, the angles of projection are θ and 90 – θ

One angle is 60° $\therefore$ the other angle is 30°

$$\frac{h_1}{h_2} = \frac{u^2 \sin^2 60°}{2g \times \frac{u^2 \sin^2 30°}{2g}} = \frac{\sin^2 60}{\sin^2 30}$$

$$\frac{10^2}{h_2} = \frac{3}{4} \times \frac{4}{1} \Rightarrow h_2 = 34\text{m}$$

17. A projectile has initially the same horizontal velocity as it would acquire if it had moved from rest with uniform acceleration of 3 ms^{-2} for 0.5 minutes. If the maximum height reached by it is 80 m then the angle of projection is : (g = 10 ms^{-2}) **[EAMCET 2002 M]**

1) $\tan^{-1} 3$ 2) $\tan^{-1}\left(\frac{3}{2}\right)$ 3) $\tan^{-1}\left(\frac{4}{9}\right)$ 4) $\sin^{-1}\left(\frac{4}{9}\right)$

Ans: 3

Sol: $v = u + at = 0 + 3 \times 30 = 90ms^{-1}$

$u\cos\theta = 90ms^{-1}$ ....(1)

$h_{max} = 80 = \frac{u^2 \sin^2\theta}{2 \times 10} \Rightarrow u\sin\theta = 40$ ....(2)

dividing (2) and (1)

$$\frac{u\sin\theta}{u\cos\theta} = \tan\theta = \frac{4}{9}$$

$$\theta = \tan^{-1}\left(\frac{4}{9}\right)$$

18. It is possible to project particle with a given speed in two possible ways so that it has the same horizontal range 'R'. The product of the times taken by it in the two possible ways is (g = acceleration due to gravity) **[EAMCET 2001 M]**

1) $\frac{R}{g}$ 2) $\frac{2R}{g}$ 3) $\frac{3R}{g}$ 4) $\frac{4R}{g}$

Ans: 2

Sol: For same range two angles of projection are θ and 90 - θ

$$\therefore t_1 = \frac{2u\sin\theta}{g}, t_2 = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\therefore t_1 t_2 = \frac{2}{g}\left[\frac{2u^2 \sin\theta\cos\theta}{g}\right] = \frac{2R}{g}$$

19. The initial velocity of a particle, $\bar{u} = 4\bar{i} + 3\bar{j}$. It is moving with uniform acceleration, $\bar{a} = 0.4\bar{i} + 0.3\bar{j}$. Its velocity after 10 seconds is **[EAMCET 2001 M]**

1) 3 units 2) 4 units 3) 5 units 4) 10 units

Ans: 4

Sol: From $\vec{v} = \vec{u} + \vec{a}t = (4\hat{i} + 3\hat{j}) + (0.4\hat{i} + 0.3\hat{j}) \times 10$

$= 8\hat{i} + 6\hat{j}$

$\therefore |\vec{v}| = 10$ units

20. A body of mass m_1 projected vertically upwards with an initial velocity u reaches a maximum height h. Another body of mass m_2 is projected along an inclined plane making an angle 30° with the horizontal and with speed u. The maximum distance travelled along the incline is

[EAMCET 2001 M]

1) 2h 2) h 3) h/2 4) h/4

Ans: 1

Sol: $h = \frac{u^2}{2g}$ …………(1)

From the equation $v^2 - u^2 = 2as$

$(0)^2 - u^2 = -2(g\sin\theta).h_1$

$\therefore h_1 = \frac{u^2}{2g\sin\theta} = \frac{h}{\sin 30°} = 2h$ [from (1)]

21. A stone projected with a velocity u at an angle θ with the horizontal reaches maximum height H_1. When it is projected with velocity u at an angle $\left(\frac{\pi}{2} - \theta\right)$ with the horizontal reaches maximum height H_2. The relation between horizontal range, R of the projectile H_1 and H_2 is

[EAMCET 2000 M]

1) $R = 4\sqrt{H_1H_2}$ 2) $R = 4(H_1 - H_2)$ 3) $R = 4(H_1 + H_2)$ 4) $\frac{H_1^2}{H_2^2}$

Ans: 1

Sol: $H_1 = \frac{u^2\sin^2\theta}{2g}, H_2 = \frac{u^2\sin^2(90-\theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$

but $R = \frac{2u^2\sin\theta\cos\theta}{g}$

$\therefore$ on solving $R = 4\sqrt{H_1H_2}$

22. For a projectile the ratio of maximum height reached to the square of time of flight is [g = 10 ms^{-2}]

[EAMCET 2000 M]

1) 5 : 4 2) 5 : 2 3) 5 : 1 4) 10 : 1

Ans: 1

Sol: $\frac{h_{max}}{T^2} = \frac{u^2\sin^2\theta}{2g} \times \frac{g^2}{4u^2\sin^2\theta} = \frac{g}{8} = \frac{10}{8} = \frac{5}{4}$

23. The average velocity of a body moving with uniform acceleration after traveling a distance of 3.06m is 0.34 ms^{-1}. If the change in velocity of the body is 0.18 ms^{-1} during this time its uniform acceleration is **[EAMCET 2000 M]**

1) 0.01 m/s^2 2) 0.02 ms^{-2} 3) 0.03 ms^{-2} 4) 0.04 ms^{-2}

Ans: 2

Sol: $s = \left(\frac{u+v}{2}\right)t \Rightarrow 3.06 = \left(\frac{u+v}{2}\right).t$

$= 3.06 = (0.34) \times t \Rightarrow t = \frac{306}{34} = 93$

but $a = \frac{v-u}{t} = \frac{0.18}{9} = 0.02\,ms^{-2}$