Dhirubhai Ambani Institute of Information and Communication Technology (DA-IICT)

Ph.D. Entrance Examination Sample Question Paper (Part-II)

No. of Printed Pages: 13 *Time: 2 hours* No. of Answer Booklets used: 1+

Application No. **Name:**

Instructions

1. Write your **Application Number and Name** clearly in the space provided at the top of the question paper. **The booklet has to be returned at the end of the examination.**
2. This examination has five sections viz. A. Mathematics, B. Physics C. Computer Science and Information Technology, D. Electronics and Embedded Systems, E. Communication and Signal Processing.
3. All question carry equal marks. You should attempt questions from only one section (out of five sections). You may answer any number of questions from this section.
4. Show all your work and explain how you arrived at your answers, unless explicitly told to do otherwise.
5. You have two hours to complete the test.

FOR EVALUATOR'S USE ONLY

Question No.	Section A	Section B	Section C	Section D	Section E
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
Section-Total					

Total

Section A : Mathematics

1. Is it possible to find real functions f and g, both continuous at $x = 0$, for which the composite $f \circ g$ is discontinuous at $x = 0$? Why or why not?

2. Consider the integral $\int_{-1}^{1} \int_{y^2}^{1} \int_{0}^{1-x} f(x, y, z) dz dx dy$. Rewrite the integral as an equivalent integral in each of the five other orders of integration.

3. Is $u(r, \theta) = r^2 \cos 2\theta$ harmonic? Prove your claim. If harmonic, then find its conjugate harmonic function and the corresponding analytic function.

4. A professor teaching a Discrete Mathematics course gives a multiple choice quiz that has ten questions, each with four possible responses: a, b, c, d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical? (Assume that no answers are left blank.) How did you arrive at this number?

5. Suppose r and s are any positive integers. For what values of r and s does there exist a graph G with the property that G has vertices of degrees r and s and of no other degrees? Explain.

6. Let n be a positive integer and A, B be two $n \times n$ complex matrices. Prove or disprove

$$\mathrm{tr}(AB^*)|^2 \leq \mathrm{tr}(AA^*)\mathrm{tr}(BB^*)$$

7. Suppose that a linear operator $A : V \to V$ has n distinct eigenvalues (where n is the dimension of V) then what can be said about the independence of the eigenvectors corresponding to these eigenvalues? Justify. Further, in this case state whether A can or cannot be diagonalized.

8. Let $\langle \mathbb{Z}_n, \oplus_n \rangle$ be the cyclic group on $\{0, 1, ..., n-1\}$ with operation addition modulo n. Let $\mathbb{Z}_n^*$ be the integers that are less than n (excluding zero) and are relatively prime to n. For example $\mathbb{Z}_8^* = \{1, 3, 5, 7\}$. Then $\langle \mathbb{Z}_n^*, \odot_n \rangle$ is a group with $\odot_n$ denoting multiplication modulo n. Is Z_9^* isomorphic to $\mathbb{Z}_6$? Why or why not?

9. British English and American English spellings are rigour and rigor, respectively. A man staying at a hotel writes this word, and a letter taken at random from this spelling is found to be a vowel. If 40 percent of the English-speaking men staying at the hotel are British and 60 percent are Americans, what is the probability that the writer is a British national? Justify.

10. Trains headed for destination A arrive at 15-minute time intervals starting at 7 a.m. whereas trains headed for destination B arrive at 15-minute time intervals starting 7:05 a.m. If a passenger arrives at the train station at a time that is uniformly distributed between 7 a.m. and 8 a.m. and then gets on the first train that arrives, what proportion of times does the passenger go to destination A? Justify.

Section B : Physics

Electromagnetic Theory

1. Two concentric spherical shells of radius $r = a$ and $r = b$ have a uniform charge distribution trapped in the region $a \leq r \leq b$. Find the general solutions of the electric field, $\mathbf{E}$, and the potential, V. What is the solution of $\mathbf{E}$ if there were no charges trapped between the two shells? In this case also find the unique solution of V under the boundary condition $V = 0$ at $r = a$, and $V = V_0$ at $r = b$.

2. For a magnetic field, $\mathbf{B} = B_0 \cos(kz - \omega t)\hat{y}$ (with B_0 being a constant), find the respective magnitudes and directions of $\mathbf{E}$, $\mathbf{D}$ and $\mathbf{H}$ in free space. With the obtained expressions of $\mathbf{E}$ and $\mathbf{B}$, show that $\nabla \cdot \mathbf{B} = 0$ and $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$. If the space were to be filled with a dielectric material of electric susceptibility, χ_{e}, then by what factor would the magnitude of $\mathbf{D}$ change?

Mathematical Physics

3. (a) Find the unit normal to the surface $z^3 + xz = x^2 + y^2$ at the point (1,1,1).
(b) Show that $F = (2xy + z^3)\mathbf{i} + (x^2)\mathbf{j} + 3xz^2\mathbf{k}$ is a conservative force field.

Classical Mechanics

4. (a) A particle of mass "m" is subjected to a force $F(t) = me^{-bt}$. The initial position and speed are 0. Find $x(t)$.
(b) A particle (arbitrary mass) moves under the influence of the potential $V(x) = (k/2)x^2 + mgx$. Find the frequency of small oscillations around the equilibrium point.

5. (a) For the setup with a mass on the end of a spring; Newtonian mechanics gives $m\frac{d^2x}{dt^2} = -kx$. Write down the Lagrangian for this system.
(b) Obtain the Hamiltonian $\mathbf{H}$ and the Hamilton's equations of motion of a simple pendulam.

Thermodynamics and Statistical Physics

6. The Helmholtz function may be expressed by $\mathrm{d}A = -S\,\mathrm{d}T - P\,\mathrm{d}V$. Show that entropy and pressure are derivatives of the Helmholtz function. Then obtain a relevant Maxwell relation.

Quantum Mechanics

7. A photon whose energy equals the rest energy of the electron, undergoes a Compton collision with an electron. If the electron moves at an angle of 40° with the original photon, what is the energy of the scattered photon?

8. A particle of mass m moves in a potential, $V(x) = F_0|x|$, in which F_0 is a positive constant. Use the Heisenberg uncertainty principle to estimate the minimum total energy of the particle.

Nuclear Physics

9. The most probable energy of a thermal neutron is 0.025 eV at room temperature. Over what distance will half of a beam of 0.025 eV neutrons have decayed? Neutron half-life is 10.3 min.

10. The binding energy of neon isotope ($Z = 10$ and $A = 20$) is 160.647 MeV. Find its atomic mass.

Useful Physical Constants and Conversion Factor

$G = 6.67 \times 10^{-11}\text{m}^3/\text{kg s}^2$,
$g = 9.8\text{m/s}^2$,
$\epsilon_0 = 8.85 \times 10^{12}\text{F/m}$,
electron charge $e = 1.6 \times 10^{-19}$ coulombs
Plancks constant $h = 6.626 \times 10^{-34}$ joule-sec,
Speed of light $c = 3 \times 10^8$ m/sec,
Boltzmanns constant $k = 1.38 \times 10^{-23}$ joule/K ,
Gas constant $R = 8.31441$ joule/mole K,
Avogadro number $N = 6.023 \times 10^23$.
Proton mass $m_p = 1.673 \times 10^{-27}kg = 1.007277\text{u}$,
Neutron mass $m_n = 1.675 \times 10^{-27}kg = 1.008665\text{u}$,
Electron mass $m_e = 9.109 \times 10^{-31}kg = 0.00055\text{u}$,
$1u = 931$ MeV,
1 Gauss = .0001 Tesla,
$1eV = 1.6 \times 10^{-19}$ joule
$1\text{A}^\circ = 10^{-10}\text{m}$

Section C : Computer Science and Information Technology

1. Let us consider sorting a list on n distinct numbers. We denote the reverse of a list L by L'. Supposing a sorting algorithm takes a long time to sort L, but sorts L' significantly faster. Then which of the following two sorting algorithms is being used: (a) Insertion sort, (b) Merge sort. Explain your answer.

2. Consider the array $A[\,] = \{1, 2, 3, 4, 5, 6, 7\}$. Which is the only element in this array that will be detected faster by linear search than by binary search. Explain your answer.

3. Given L = {w|w is a binary string which has equal number of 0's and 1's}, determine whether L is regular or not.

4. Explain how will you convert a non deterministic finite state automaton to an equivalent deterministic finite state automaton. Also determine the time incurred in this operation.

5. Consider relation R and set of functional dependencies, given below, and answer following questions-

   R(A,B,C,D,E,F)
   A $\rightarrow B$
   B $\rightarrow$ {D, E, F}

   (a) Compute key of R.
   (b) Determine Normal form of R.
   (c) If not already in BCNF, decompose R into BCNF relations using BCNF Decomposition Algorithm. Properly comment each step of your decomposition solution.

6. Consider a relation `registers(course_no, student_id, grade)`. The relation stores facts of students registering in various courses offered in an institute. The grade is stored as number and draws a value from {`0,3,4,5,6,7,8,9,10`}. Assume that {`course_no, student_id`} is key of the relation `registers`.

   Write relational algebra expressions to answer query - "List IDs of students who have taken courses `CS205` and `CS305`, and have passed both of them with grade `7` and higher".

7. State True or False

   (a) If a thread is CPU-bound, it makes sense to give it higher priority for disk I/O than an I/O bound thread.
   (b) Virtual addresses must be same size as physical addresses.
   (c) Page offsets in virtual addresses must be the same size as page offsets in physical addresses.
   (d) All operating system code is run in the same address space
   (e) A context switch can occur in the middle of an instruction.

8. A set of synchronization primitives are given below.

   - Counting semphore
   - Lock w/condition

- Variable Locks
- R/W Locks
- Monitors

For each of the problems described below, indicate which of the synchronization primitives listed is BEST suited for it. Explain briefly why you chose the primitive you did.

(a) Problem A: You have been given a set of self-synchronizing chairs. The set supports the operations: allocate-chair, free-chair, spin-all-chairs, throw-chair-down-stairs. What primitive do you suppose they used to implement the chair interfaces?

(b) Problem B: All students in a class want to schedule a meeting with their Professor. What primitive did the professor pick to schedule the meetings?

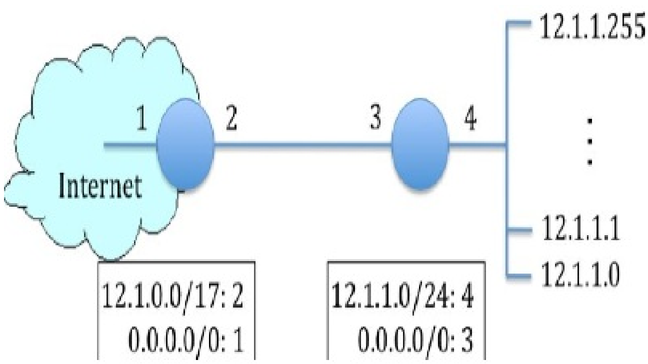
9. Host A and B are communicating over a TCP connection, and Host B has already received from A all bytes up through byte 126. Suppose Host A then sends two segments to Host B back-to-back. The first and second segments contain 80 and 40 bytes of data, respectively. In the first segment, the sequence number is 127, the source port number is 302, and the destination port number is 80. Host B sends an acknowledgment whenever it receives a segment from Host A. Answer the following:

   (a) In the second segment sent from Host A to B, what are the sequence number, source port number, and destination port number?

   (b) If the first segment arrives before the second segment, in the acknowledgment of the first arriving segment, what is the acknowledgment number, the source port number, and the destination port number?

   (c) If the second segment arrives before the first segment, in the acknowledgment of the first arriving segment, what is the acknowledgment number?

10. A small university campus is assigned a large address block 12.1.0.0/17, but is only using a portion of these addresses (in 12.1.1.0/24) to number its computers. This picture below shows the forwarding tables on the ISPs router (on the left) and the campus edge router (on the right):



For example, the ISP forwards all packets with destination addresses in 12.1.0.0/17 out link 2 toward the campus edge router. Both routers include a default forwarding entry 0.0.0.0/0 that can match any destination IP address. Answer the following:

(a) How many IP addresses does the campus own in its 12.1.0.0/17 block? You can represent your answer as a power of two.

(b) What are the smallest and largest IP addresses that the campus owns, whether or not the campus is currently using the address?

(c) Suppose the ISP router receives a packet with destination IP address 12.1.1.1. What path does this packet follow?

(d) Suppose the ISP router receives a packet with destination IP address 12.1.20.1 What path does this packet follow?

Section D : Electronics and Embedded Systems

1. If $R_1 = R_2 = R_3 = R$ and $R_4 = 1.1R$ in the bridge circuit shown in figure 1, then find the reading in the ideal voltmeter connected between a and b.

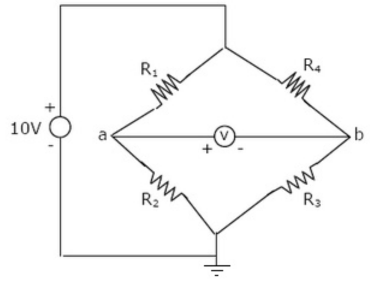


Figure 1:

2. In the circuit shown below in figure 2, find the power supplied by the voltage source with 10V.

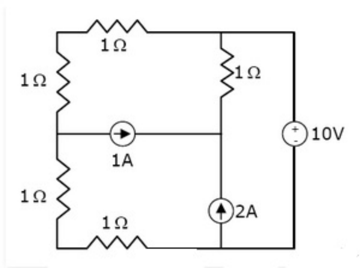


Figure 2:

3. In the circuit shown below in Figure 3, $R = 1\Omega$, $L = \frac{1}{6}H$, $C = 2F$ and $V(t) = sin(2t)$. Find the steady state current $i(t)$ supplied by the source.

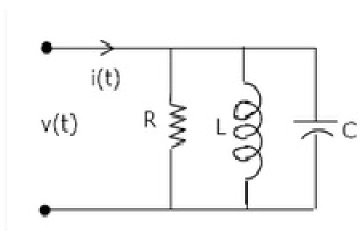


Figure 3:

4. Consider the circuit shown below in figure 4. If $R_1 = 10K\Omega$, $R_2 = 2K\Omega$, $R_C = 1K\Omega$, $R_E = 100\Omega$, $V_{CC} = 10V$, $\beta = 100$(forward current gain) and $I_{CO} = 0$, then find the current I_C.

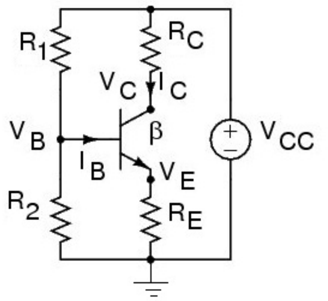


Figure 4:

5. For the circuit shown in the figure 5 below, sketch the waveform for the output v.

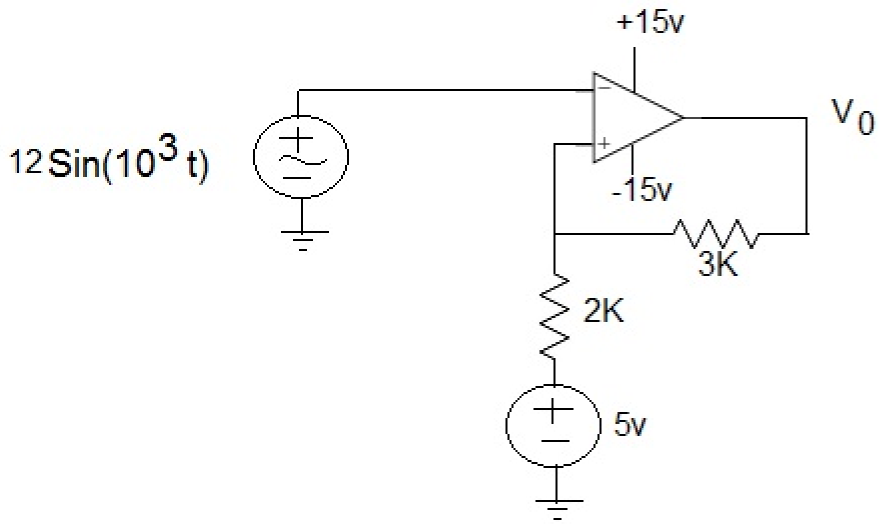


Figure 5:

6. A Boolean function $F(A,B,C)$ is expressed in SOP form as $F = \sum m(0,2,3,4,6)$.
   a) Implement the function F by $8:1$ MUX.
   b) Find the minimized SOP for F using K-map.

7. What is the optimal number of stages to use in a multi-stage CMOS buffer with a load of $10\ pF$. Design this with an initial stage having $(\frac{W}{L})n_1{=}4\mu m/2\mu m$, and $(\frac{W}{L})p_1{=}10\mu m/2\mu m$. C_{ox} is $690e^{-18}aF/\mu m^2$. Draw and find the capacitance of each stage.

8. Draw I_D versus V_{DS} characteristics of CMOS transistor showing all the regions.
   a) Write the equations for all the regions
   b) Sketch $I_D - V_{GS}$ characteristic of n-channel and p-channel depletion and enhancement type operating in saturation.
   c) What is the basic difference between a JFET and MOSFET?

9. Draw the static and dynamic characteristic of a CMOS inverter
   a) For static: v_o/v_i (all regions specified)
   b) For dynamic: i_{DN} versus v_o

10. Design a safety system for a one seater car given:
a) an indicator which provides an output of $+5V$ when someone is sitting in the car and $0V$ otherwise
b) an indicator which provides an output which is $+5V$ when the seat belt is connected properly across the occupant and $0V$ otherwise
c) an indicator showing that the door is shut ($+5V$ output when the door is shut and $0V$ otherwise)
d) a system safety condition indicator for the brakes and suspension, with output +5V when the condition is unsafe and $0V$ otherwise

The safety system should provide a signal f which can be used to allow the car to proceed (f =1 to allow progress and 0 to prevent progress). The car should not proceed when a driver is not sat in it or when the driver is not wearing a seat belt or the system condition is unsafe. The car should not proceed when the door is open.

a) draw up a truth table for the signal f
b) extract the logical function which gives the signal f
c) implement the system using NOT, AND and OR gates
d) implement it again using NOT and $NAND$ or NOR gates.

Section E : Communications and Signal Processing

Analog and Digital Communications

1. Explain the motivation for using single sideband (SSB) AM scheme? How is a lower SSB signal (i.e., a signal which only has lower sideband in it) generated from DSB-SC? Give the time domain representation of a lower SSB signal. **(10 points)**

2. Consider two 4-ary signal constellations. For constellation 1 the four points are located at $(0,0)$, $(-\sqrt{2}a, \sqrt{2}a)$, $(-2\sqrt{2}a, 0)$ and $(-\sqrt{2}a, -\sqrt{2}a)$. For constellation 2 the four points are located at $(a,0)$, $(0,a)$, $(-a,0)$ and $(0,-a)$. Assume that all four points in the two constellation diagrams to be equiprobable. Given the above:

   (a) Compute the ratio of average energy of Constellation 1 to the average energy of Constellation 2. **(5 points)**

   (b) If both these constellations are used in digital communications then which of them is most likely to give a lower probability of symbol error? Justify. **(5 points)**

Electromagnetic and Transmission Lines Theory

3. Electromagnetic Theory:

   (a) Write Gauss law for electric field in integral form. Assume D is electric flux density and ρ_v is volume charge density. **(2 points)**

   (b) Write Amperes law in differential form. Assume H is magnetic field intensity, J is current density and D is electric flux density. **(2 points)**

   (c) Name four wave polarizations of electromagnetic waves. **(2 points)**

   (d) State frequency ranges of X-band and Ku-band. **(2 points)**

   (e) What is represented by the magnitude and the direction of Poynting vector? **(2 points)**

4. Microwaves and Transmission Lines:

   (a) For a two-port network, a_1 and b_1 are incident and scattered wave amplitudes of port 1. Also, a_2 and b_2 are incident and scattered wave amplitudes of port 2. Define S_{11} and S_{21}. **(2 + 2 = 4 points)**

   (b) In transmission line theory, given inductance per unit length (L'), capacitance per unit length (C'), resistance per unit length (R'), conductance per unit length (G') and angular frequency (ω), state expressions for characteristic impedance (z_0) and complex propagation constant (γ). **(2+2 = 4 points)**

   (c) State any two advantages of microwaves in industrial heating applications. **(2 points)**

Signals and Systems

5. What is the difference between discrete time Fourier transform (DTFT), discrete Fourier transform (DFT) and fast Fourier transform (FFT)? Explain clearly using mathematical expressions, if required. **(10 points)**

6. For the following signals, (a) sketch the signals, (b) determine if they are periodic (if yes, give the fundamental period):

   (a) $x(t) = u(t) - 1/2$ where $u(t)$ is the unit step function such that $u(t) = 1, t \geq 0$ and 0 otherwise. **(2 points)**

   (b) $x(t) = cos(3\pi t) + 2\,cos(4\pi t)$ **(2 points)**

   (c) $x(t) = c$ where c is a constant. **(2 points)**

   (d) $x(t) = 2\,cos(3\pi t + \pi/2) + 4\,cos(10t - \pi/2)$ **(2 points)**

   (e) $x(t) = cos(2\omega t) + cos(3\omega t)$ where ω denotes a certain frequency. **(2 points)**

Digital Signal Processing

7. LSI Systems:

   (a) When does the ROC of Z Transform extend outward from the outermost pole? When does it extend inward from the innermost pole? Support your answers with explanations. **(2 points)**

   (b) What is the requirement for an LSI system to ensure that its phase response is linear? **(2 points)**

   (c) What is the Z Transform $X(z)$ of $x(n) = a^n u(n)$? **(2 points)**

   (d) Name three advantages of FIR filters over IIR filters and one advantage of IIR filters over FIR filters. **(2 points)**

   (e) What are the eigenfunctions of the LSI systems? Explain in words why they are "special"? **(2 points)**

8. A commonly used numerical operation called the first backward difference is defined as follows:

$$y(n) = \bigtriangledown(x(n)) = x(n) - x(n-1),$$

   where $x(n)$ is the input and $y(n)$ denotes the output of the system that performs this operation.

   (a) Is this system (i) linear? (ii) time invariant? (iii) causal and (iv) stable? **(2 points)**

   (b) Determine the impulse response of this system. **(2 points)**

   (c) Find and plot the frequency response (magnitude and phase) of the system. **(2 points)**

   (d) Show that if $x(n) = f(n) * g(n)$,

$$y(n) = \bigtriangledown(x(n)) = \bigtriangledown(f(n)) * g(n) = f(n) * \bigtriangledown(g(n)).$$

   (2 points)

   (e) Find the impulse response of the system that can be cascaded with this system to recover the input $x(n)$, i.e., find $h_{inv}(n)$ such that $h_{inv}(n) * \bigtriangledown(x(n)) = x(n)$. **(2 points)**

Statistics for Engineers

9. Following problem was given to students and doctors at a medical school. Assume that there exits a test to detect a disease, say D, whose prevalence is 0.001, i.e., the probability $P[D]$ that a person picked at random is suffering from D is 0.001. The test has a false positive rate of 0.005, i.e., if you don't have the disease, the probability that your test will still come out positive is 0.005. The test has a correct detection rate of 1, i.e., if you have D, the test will say with probability 1 that you have D. You take the test and the test result comes out positive. What is the probability that you actually have D? Many of the medical students/doctors answerd this probability is 0.95. Show that your knowledge of probability is greater by getting the right answer of 0.17. **(10 points)**

10. Two random variables X and Y have a joint PDF given as follows:

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{16}, & |x| \leq 4; \quad 2 \leq y \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

(a) Are X and Y independent? **(5 points)**

(b) Are X and Y uncorrelated? **(5 points)**