

# Sample Question Paper - II 

## Mathematics - Class XII

Time: 3 Hours
Max. Marks : 100

## General Instructions

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections $A, B$ and $C$. Section $A$ comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

## SECTION - A

Q.1. If $A$ is a square matrix of order 3 such that $|\operatorname{adj} A|=64$, find $|A|$.
Q.2. If $A, B, C$ are three non zero square matrices of same order, find the condition on $A$ such that $A B=A C \Rightarrow B=C$
Q.3. Give an example of two non zero $2 \times 2$ matrices $\mathrm{A}, \mathrm{B}$ such that $\mathrm{AB}=0$.
Q.4. If $f(1)=4 ; f^{\prime}(1)=2$, find the value of the derivative of $\log f\left(e^{x}\right)$ w.r.t $x$ at the point $x=0$.
Q.5. Find a, for which $f(x)=\mathrm{a}(x+\sin x)+\mathrm{a}$ is increasing.
Q.6. Evaluate, $\int_{0}^{1.5}[x] \mathrm{d} x$ (where $[\mathrm{x}]$ is greatest integer function)
Q.7. Write the order and degree of the differential equation, $y=x \frac{\mathrm{~d} y}{\mathrm{~d} x}+\mathrm{a} \sqrt{1+\frac{\mathrm{d} y}{\mathrm{~d} x}}{ }^{2}$
Q.8. If $\overrightarrow{\mathrm{a}}=\hat{i}+\hat{\mathrm{j}} ; \overrightarrow{\mathrm{b}}=\hat{\mathrm{j}}+\hat{\mathrm{k}} ; \quad \overrightarrow{\mathrm{c}}=\hat{\mathrm{k}}+\hat{i}$, find a unit vector in the direction of $\overrightarrow{\mathrm{a}}+\overrightarrow{\mathrm{b}}+\overrightarrow{\mathrm{c}}$
Q.9. A four digit number is formed using the digits $1,2,3,5$ with no repetitions. Find the probability that the number is divisible by 5 .
Q.10. The probability that an event happens in one trial of an experiment is 0.4 . Three independent trials of the experiment are performed. Find the probability that the event happens at least once.

## SECTION - B

Q.11. Find the value of $2 \tan ^{-1}\left(\frac{1}{5}\right)+\sec ^{-1}\left(\frac{5 \sqrt{2}}{7}\right)+2 \tan ^{-1} \frac{1}{8}$
Q.12. If $A=\left[\begin{array}{ll}0 & 1 \\ 0 & 0\end{array}\right]$, prove that $(a I+b A)^{n}=a^{n} . I+n a^{n-1} b A$ where $I$ is a unit matrix of order 2 and $n$ is a positive integer

## OR

Using properties of determinants, prove that

$$
\left|\begin{array}{ccc}
a+b+2 c & a & b \\
c & b+c+2 a & b \\
c & a & c+a+2 b
\end{array}\right|=2(a+b+c)^{3}
$$

Q.13. If $x=\mathrm{a} \sin \mathrm{pt}$ and $y=\mathrm{b} \cos \mathrm{pt}$, find the value of $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$ at $\mathrm{t}=0$.
Q.14. Find the equations of tangent lines to the curve $y=4 x^{3}-3 x+5$ which are perpendicular to the line $9 y+x+3=0$.
Q.15. Show that the function $f(x)=|x+2|$ is continuous at every $x \in \mathrm{R}$ but fails to be differentiable at $x=-2$.
Q.16. Evaluate $\int \frac{x^{2}+4}{x^{4}+x^{2}+16} d x$
Q.17. Evaluate $\int_{0}^{\pi / 2} \frac{x+\sin x}{1+\cos x} d x$

## OR

$$
\text { Evaluate } \int \frac{\mathrm{e}^{\mathrm{x}}}{\sqrt{5-4 \mathrm{e}^{x}-\mathrm{e}^{2 \mathrm{x}}}} d x
$$

Q.18. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices $A, B, C$ of $a \Delta A B C$ respectivily. Find an expression for the area of $\triangle \mathrm{ABC}$ and hence deduce the condition for the points $\mathrm{A}, \mathrm{B}, \mathrm{C}$ to be collinear.

OR
Show that the points A, B, C with position vectors $2 \hat{i}-\hat{j}+\hat{k}, \hat{i}-3 \hat{\mathrm{j}}-5 \hat{\mathrm{k}}$ and $3 \hat{i}-4 \hat{\mathrm{j}}-4 \hat{\mathrm{k}}$ respectively, are the vertices of a right triangle. Also find the remaining angles of the triangle.
Q.19. Evaluate, $\int_{0}^{\pi} \frac{x}{\mathbf{a}^{2} \cos ^{2} x+\mathrm{b}^{2} \sin ^{2} x}, \mathrm{~d} x \quad \mathrm{a}, \mathrm{b}>0$
Q.20. Solve the differential equation, $\frac{\mathrm{d} y}{\mathrm{~d} x}+y \sec ^{2} x=\tan x \sec ^{2} x ; y(0)=1$

OR

$$
2 x y+y^{2}-2 x^{2} \frac{\mathrm{~d} y}{\mathrm{~d} x}=0 ; y(1)=2
$$

Q.21. In a bolt factory machines, A, B and C manufacture respectively $25 \%, 35 \%$ and $40 \%$ of the total bolts. Of their output 5,4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product.
(i) What is the probability that the bolt drawn is defective?
(ii) If the bolt is found to be defective find the probability that it is a product of machine $B$.
Q.22. Two dice are thrown simultaneously. Let X denote the number of sixes, find the probability distribution of X . Also find the mean and variance of $X$, using the probability distribution table.

## SECTION - C

Q.23. Let X be a non-empty set. $\mathrm{P}(\mathrm{x})$ be its power set. Let '*' be an operation defined on elements of $\mathrm{P}(\mathrm{x})$ by,

$$
A * B=A \cap B \forall A, B \in P(X)
$$

Then,
(i) Prove that * is a binary operation in $\mathrm{P}(\mathrm{X})$.
(ii) Is * commutative ?
(iii) Is * associative?
(iv) Find the identity element in $\mathrm{P}(\mathrm{X})$ w.r.t. *
(v) Find all the invertible elements of $\mathrm{P}(\mathrm{X})$
(vi) If $o$ is another binary operation defined on $P(X)$ as $A \circ B=, A \cup B$ then verify that o distributes itself over *.

## OR

Consider $\mathrm{f}: \mathrm{R}_{+} \rightarrow[-5, \infty)$ given by $f(x)=9 x^{2}+6 x-5$. Show that $f$ is invertible. Find the inverse of $f$.
Q.24. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m , find the dimensions of the rectangle that will produce the largest area of the window.
Q.25. Make a rough sketch of the region given below and find it's area using integration
Q.26. Every gram of wheat provides 0.1 gm of proteins and 0.25 gm of carbohydrates. The corresponding values for rice are 0.05 gm and 0.5 gm respectively. Wheat costs Rs. 4 per kg and rice Rs. 6 per kg. The minimum daily requirements of proteins and carbohydrates for an average child are 50 gms and 200 gms respectively. In what quantities should wheat and rice be mixed in the daily diet to provide minimum daily requirements of proteins and carbohydrates at minimum cost. Frame an L.P.P. and solve it graphically.
Q.27. font $\mathrm{A}=\left[\begin{array}{ccc}1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1\end{array}\right]$, find $\mathrm{A}^{-1}$ and hence solve the system of equations

$$
\begin{aligned}
& x+2 y+z=4 \\
& -x+y+z=0 \\
& x-3 y+z=2
\end{aligned} \quad\left\{(x, y) ; y^{2} \leq 4 x, 4 x^{2}+4 y^{2} \leq 9\right\}
$$

Q.28. Find the equation of the plane containing the lines,

$$
\overrightarrow{\mathrm{r}}=i+\mathrm{j}+\lambda(\hat{i}+2 \hat{\mathrm{j}}-\hat{\mathrm{k}}) \quad \text { and } \quad \overrightarrow{\mathrm{r}}=\hat{i}+\hat{\mathrm{j}}+\mu(-\hat{i}+\hat{\mathrm{j}}-2 \hat{\mathrm{k}})
$$

Find the distance of this plane from origin and also from the point $(1,1,1)$
OR
Find the equation of the plane passing through the intersection of the planes, $2 x+3 y-z+1=0 ; x+y-2 z+3=0$ and perpendicular the plane $3 x-y-2 z-4=0$. Also find the inclination of this plane with the $x y$ plane.
Q.29. Prove that the image of the point $(3,-2,1)$ in the plane $3 x-y+4 z=2$ lies on the plane, $x+y+z+4=0$.

