SOLUTION & ANSWER FOR ISAT-2012
SET – E

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART A – PHYSICS

1. In a closed container filled with air at a pressure p_0 there is an air bubble ------

Ans : $\frac{1}{24}p_0R$

Sol: $p_i - p_0 = \frac{4T}{R}$ —(i)

$p_iV_i = p_i'V_i'$ (θ is constant)

$\Rightarrow p_i' = \frac{p_iV_i}{V_i'} = \frac{p_i}{8}$ (∵ R ⇒ 2R)

$p_i' - \frac{p_0}{16} = \frac{4T}{2R} \Rightarrow \frac{p_i}{8} - \frac{p_0}{16} = \frac{4T}{2R}$ —(ii)

Solving (i) and (ii) $\Rightarrow p_i = \frac{28T}{R} \Rightarrow T = \frac{p_0R}{24}$

2. A solid hemisphere of radius R of some material is attached on top of a solid cylinder of ------

Ans : $\frac{9}{20}MR^2$

Sol: $M_1 = \frac{4}{3}\pi R^3\rho \times \frac{1}{2} = \frac{2}{3}\pi R^3\rho$

$M_2 = \pi R^2 . \frac{2}{3}R\rho = \frac{2}{3}\pi R^3\rho$

$M_1 = M_2$ and $M_1 + M_2 = M$

$\Rightarrow M_1 = M_2 = \frac{M}{2}$

$I_1 = \frac{2}{5}.\frac{M}{2}.R^2 = \frac{MR^2}{5}$;

$I_2 = \frac{1}{2}.\left(\frac{M}{2}\right)R^2 = \frac{MR^2}{4}$

$I = I_1 + I_2 = \frac{MR^2}{5} + \frac{MR^2}{4} = \frac{9}{20}MR^2$

3. For the prism shown in the figure, the angle of incidence is adjusted such that------

Ans : $\frac{\left(\sqrt{3}+1\right)}{2}$

Sol: A = 90° (from figure)

D_{min} = 60° (Data)

$$n = \frac{\sin\left(\frac{A+D_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$= \frac{\sin 45^\circ \cos 30^\circ + \cos 35^\circ \sin 30^\circ}{\sin 45^\circ}$$

$$= \left(\frac{\sqrt{3}+1}{2}\right)$$

4. Two physicists `A' and `B' calculate the efficiency of a Carnot engine running between two heat reservoirs by measuring------

Ans : $\frac{10}{9}$ and $\frac{10}{3}$

Sol: **For A**

$\frac{d\eta}{\eta} = \frac{\Delta T_A}{T_A} \times 100 = \frac{10}{900} \times 100$

$= \frac{10}{9}$

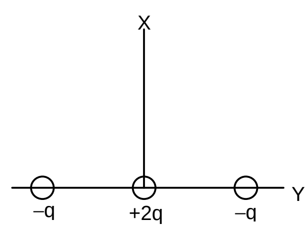
For B

$\frac{d\eta}{\eta} = \frac{\Delta T_B}{T_B} \times 100 = \frac{10}{300} \times 100$

$= \frac{10}{3}$

5. Two positive and two negative charges of magnitude q are kept on the x-y plane as shown ------

Ans :



Sol: Resultant field of the given configuration is zero along any point on Z-axis. In configuration (B), the dipole moments of two dipoles (–q, +q), (–q, +q) are in opposite directions

⇒ field along Z-direction is zero.

6. Two hemispheres made of glass (μ = 1.5) are kept as shown in the figure. The radius ------

Ans : $\frac{2R}{3}$

Sol: Normal shift = $t\left[\frac{\mu-1}{\mu}\right]$

$= (R+R)\left[\frac{1.5-1}{1.5}\right]$

$= \frac{2R}{3}$

7. A right-angled prism ABC ($\angle C < \angle B$) made of a material of refractive index μ_0 is immersed------

Ans : $\sin^{-1}\left(\frac{\mu}{\mu_0}\right)$

Sol: $r_1 + r_2 = \phi$ for prism

$r_1 = 0$ (Data)

$\Rightarrow r_2 = \phi$, should be the critical angle

$\Rightarrow \sin\phi = \frac{\mu}{\mu_0} \Rightarrow \phi = \sin^{-1}\left(\frac{\mu}{\mu_0}\right)$

8. Four screw gauges are to be calibrated to the standard thickness `t_{st}' of a wire. Series of measurements ------

Ans : Screw gauge 1 is less precise but more accurate than screw gauge 4.

Sol: Distributions symmetric about t_{st} are more accurate. Distributions with smaller widths about t_{st} are more precise.

$\Rightarrow$ Statement (A) is correct.

9. Velocity $\bar{v}$ (m/s) versus time graph of a cyclist moving along the ------

Ans : $\frac{3}{4}\hat{i}, \frac{15}{4}$

Sol: $\bar{S}_1 = \left(\frac{4+6}{2}\right)\times 5 = 25$ m

$\bar{S}_2 = \left(\frac{4+8}{2}\right)\times(-5) = -30$ m

$\bar{S}_3 = \left(\frac{2+6}{2}\right)\times 5 = 20$ m

$\therefore S = S_1 + S_2 + S_3 = 75$ m

$\bar{S} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = 15$ m

$\bar{v}_{Av} = \frac{\bar{S}}{t} = \frac{15}{20} = \frac{3}{4}\hat{i}$

$|\bar{v}_{Av}| = \frac{S}{20} = \frac{75}{20} = \frac{15}{4}$

10. A sphere of mass M and radius R is surrounded by a shell of the same mass and radius 2R. A small hole ------

Ans : $\sqrt{\frac{3GM}{R}}$

Sol: $U_{shell} = -\frac{GMm}{2R}$

$U_{solid\ sphere} = -\frac{GMm}{R}$

$\therefore U_i = -\frac{GMm}{2R} - \frac{GMm}{R} = -\frac{3GMm}{2R}$

i.e. $\frac{1}{2}mv_e^2 = \frac{3GMm}{2R}$

$\Rightarrow v_e = \sqrt{\frac{3GM}{R}}$

11. A particle of mass m is projected in the vertical plane (taken to be the x-y plane) with speed v at an angle ------

Ans : $-0.5mv\cos\theta gt^2\hat{k}$

Sol: $\bar{J} = \bar{F}dt = -mgdt\bar{j}$

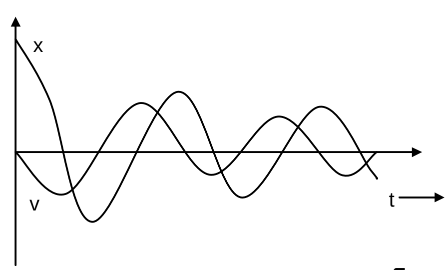
$\bar{L} = \bar{r}\times\bar{J} = \int_0^t (u\cos\theta t)\hat{i}\times -mgdt\hat{j}$

$= -mg\,u\cos\theta\int tdt$

$= -0.5mgu\cos\theta t^2$

12. Consider a damped simple harmonic oscillator given by the equation of motion ------

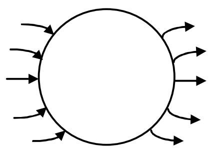
Ans :



Sol: x and v have phase difference of $\frac{\pi}{2}$ rad and since starting is from extreme position, x and v must be in opposite directions.

13. A metal sphere is kept in a uniform electric field as shown. What is the correct------

Ans :



Sol: No electric field inside sphere and field lines are normal to surface.

14. An electron travelling with velocity $\bar{v} = 3\hat{i} + 5\hat{j}$ in an electric field ------

Ans : $5\hat{i} - 3\hat{j} + 18\hat{k}$

Sol: $q\bar{E} + q(\bar{v} \times \bar{B}) = 0$

$\Rightarrow \bar{E} = -(\bar{v} \times \bar{B})$

$$= -\begin{vmatrix} i & j & k \\ 3 & 5 & 0 \\ 6 & 4 & -1 \end{vmatrix}$$

$= 5\hat{i} - 3\hat{j} + 18\hat{k}$

15. A small block is kept on a frictionless horizontal table. A wooden plank pivoted at O, but otherwise free to rotate, pushes the block by applying a constant ------

Ans : 0.2 rad/s

Sol: τ constant $\Rightarrow$ FR constant

$\Rightarrow$ F constant

Impulse = Ft = 10 F

But impulse = Δp = mv

$\Rightarrow 10\text{ F} = mv \Rightarrow F = \frac{mv}{10}$

$F_{centripetal}$ = friction = $\mu F = mv\omega$

$\Rightarrow \frac{\mu mv}{10} = mv\omega$

$\Rightarrow \omega = \frac{\mu}{10} = 0.02 \text{ rad s}^{-1}$

16. A model potential between two molecules A and B in a solid is shown in the figure, where x gives the distance of B with respect------

Ans : $\frac{1}{T}$

Sol: PE = KE = $\frac{1}{2}kx^2$ —(i)

KE = constant $\times k_BT$ —(ii)

$\Rightarrow \frac{1}{2}kx^2 \propto k_BT \Rightarrow x^2 \propto T$

$\Rightarrow 2xdx \propto dT \Rightarrow \frac{dx}{dT} \propto \frac{1}{x}$

$\frac{dx}{xdT} \propto \frac{1}{x^2} \propto \frac{1}{T}$

17. The mass density of a dusty planet of radius R is seen to vary from its center as ------

Ans : $\frac{45}{16}$

Sol: $dm = 4\pi r^2\rho_0\left(1 - \frac{r}{R}dr\right)$

$\Rightarrow M_1 = \int_0^{R/2} dm = \frac{5\pi\rho_0R^3}{48}$

$M = \int_0^R dm = \frac{\pi\rho_0R^3}{3}$

$E_1 = \frac{GM_1}{\left(\frac{R}{2}\right)^2} = \frac{5}{12}\pi\rho_0GR$;

$E_2 = \frac{GM}{\left(\frac{3}{2}R\right)^2} = \frac{4}{27}\pi\rho_0GR$

$\Rightarrow \frac{E_1}{E_2} = \frac{45}{16}$

18. A particle is moving in a force field given by $\bar{F} = y^2\hat{i} - r^2\hat{j}$. Starting from A the particle has to reach ------

Ans : (–1, 1)

Sol: $dW = \bar{F}.d\bar{r}$

$W = \int_{path} dW$; For AB, $d\bar{r} = dx\hat{i}$;

For BC, $d\bar{r} = dy\hat{j}$, for AD, $d\bar{r} = dy\hat{j}$ and for DC, $d\bar{r} = dx\hat{i}$

$\Rightarrow W_{ABC} = \int_{AB} \bar{F}.d\bar{r} + \int_{BC} \bar{F}.d\bar{r}$

$= \int_{(0,0)}^{(1,0)} \bar{F}.dx\hat{i} + \int_{(1,0)}^{(1,1)} \bar{F}.dy\hat{j} = -1$

$W_{ADC} = \int_{AD} \bar{F}.d\bar{r} + \int_{DC} \bar{F}.d\bar{r}$

$\int_{(0,0)}^{(0,1)} \bar{F}.dy\hat{j} + \int_{(0,1)}^{(1,1)} \bar{F}.dx\hat{i} = +1$

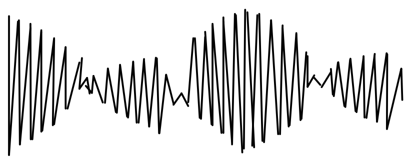
$\therefore$ (–1, 1) is the answer.

19. A capacitor made of two parallel circular plates of area A holds a charge Q_0 initially. Suppose that it discharges as ------

Ans : $\frac{1}{16\pi}\mu_0\varepsilon_0\left(A\lambda^2\right)$

20. Three sinusoidal oscillations A sin(21t), and A sin(19t) are superposed. Which of ------

Ans :



Sol: $y_1 = A \sin 19t$

$y_2 = A \sin 20t$

$y_3 = A \sin 21t$

$y = y_1 + y_2 + y_3$

$\left(\because \sin C + \sin D = 2\sin\frac{C+D}{2}.\cos\frac{C-D}{2}\right)$

$\Rightarrow y = A[2\cos 2\pi t + 1]\sin 20t$

$\Rightarrow$ There will be intermediate maxima with smaller amplitude.

21. A vertical resonance pipe is filled with water and resonates with a tuning fork at minimum air column length of 30 cm ------

Ans : 4 : 1

Sol: $V_{air} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4}{28\times10^{-3}}}\times\sqrt{RT}$

$\lambda_{air} = 30 \times 4 = 120$ cm = 0.12 m

f of tuning fork = $\frac{V_{air}}{\lambda_{air}}$

$= \sqrt{\frac{1.4}{28\times10^{-3}}}\times\frac{\sqrt{RT}}{0.12}$ —(i)

$\lambda_{mixture} = 42 \times 4 = 168$ cm = 0.168

$V_{mixutre} = \lambda_{mixture} \times f$]

$= 0.168\times\sqrt{\frac{1.4}{28\times10^{-3}}}\times\frac{\sqrt{RT}}{0.12}$ —(ii)

But $v_{mixture} = \sqrt{\frac{\gamma_{mixture}RT}{M_{mixture}}}$ —(iii)

$\Rightarrow \sqrt{\frac{\gamma_{mixture}}{M_{mixture}}} = \frac{0.168}{0.12}\times\sqrt{\frac{1.4}{28\times10^{-3}}}$

$= \frac{0.168}{0.12}\times\sqrt{50}$

$\gamma_{mixture} = \frac{5}{3}$ ($\because$ both monoatomic)

$\Rightarrow M_{mixture} = \frac{\gamma_{mixture}\times(0.12)^2}{(0.168)^2\times50}$

$= \frac{5}{3}\times\frac{(0.12)^2}{(0.168)^2\times50}$ = 0.017 kg

= 17 gram

$\frac{n_1m_1+n_2m_2}{(n_1+n_2)} = 17$

$n_1 \propto V_1, \quad n_2 \propto V_2$

$\frac{4V_1+20V_2}{(V_1+V_2)} = 17$

$4V_1 + 20V_2 = 17V_1 + 17V_2$

$3V_2 = 13V_1$

$\therefore \frac{V_1}{V_2} = \frac{13}{3}$

= 4 : 1

22. The minimum repulsive energy between the two electrons would ------

Ans : 27.2 eV

23. If the Hydrogen atom ionization temperature is T, the temperature at which He atoms ------

Ans : 6T

Sol: Energy needed for ionizing H-atom is 13.6 eV $\propto$ T. Energy needed for ionizing singly ionized He is 54.4 eV. Energy needed for remaining 2 electrons for He will be less than 54.4 $\times$ 2 = 108.8 eV but more than 54.4 eV

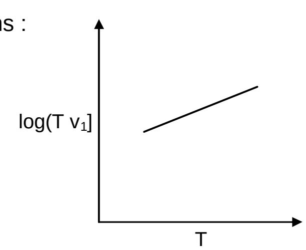
i.e. 54.4 eV < E < 108.8 eV

i.e. 4T < xT < 8T

Only acceptable choice is 6T.

24. The coefficient of viscosity of a fluid is known to vary with temperature ------

Ans :



Sol: $v_T = \frac{2}{9}\frac{r^2g(\rho-\sigma)}{\eta}$

$\Rightarrow v_T\eta$ = constant

$\Rightarrow v_T.CTe^{-\frac{T}{T_0}}$ = constant

$\Rightarrow\ v_T T \propto e^{\frac{T}{T_0}}$

$\ell n\ v_T T = k\frac{T}{10}$, where k is a constant.

$\Rightarrow$ graph of $\ell n\ v_T T$ will be a straight line with positive slope.

25. A particle moves in a force field of the form $\bar{F} = k\frac{\bar{r}\times\bar{L}}{r^2}$, where $\bar{r}$ is the position vector from the ------

Ans: Magnitude of angular momentum decreases exponentially, but its direction remains unchanged.

Sol: $\bar{F}$ is $\perp$ to $\bar{r}$, $\perp$ to $\bar{L}$ and $\perp^r$ to plane containing $\bar{r}$ and $\bar{L}$

$\Rightarrow$ $\bar{F}$ is in the plane of motion and opposing the motion.

$\bar{\tau}$ is not zero $\Rightarrow$ $\bar{L}$ is decreasing

$\bar{F}$ is also not constant as $\bar{L}$ is changing

$\Rightarrow$ Variation is exponential.

PART B – CHEMISTRY

26. The approximate standard enthalpies of formation-----

Ans : ΔH (octane) is more negative than ΔH (methanol)

Sol: Combustion of C_8H_8 involves more number of carbons and hydrogens compared to CH_4O

27. The Boyle temperatures of three gases are------

Ans : I-hydrogen, II-oxygen, III-ethene

Sol: More the compressibility factor greater is the negative deviation from ideal behaviour.

28. The reduction potentials of M^{2+} / M follow the trend------

Ans : V < Fe < Ni < Cu

Sol: $E^{\circ}_{M^{2+}/M}$ for V = – 1.18 V

Fe = – 0.44 V

Ni = – 0.25 V

Cu = 0.34 V

29. The total number of isomers expected for------

Ans : 9

Sol: ± cis $[Pt(en)_2(CNS)_2]$

± cis $[Pt(en)_2(NCS)_2]$

± cis $[Pt(en)_2(NCS)\ (CNS)]$

trans $[Pt(en)_2(CNS)_2]$

trans $[Pt(en)_2(NCS)_2]$

trans $[Pt(en)_2(NCS)\ (CNS)]$

30. In the conversion of dinitrogen to hydrazine, -----

Ans : 4 and 4

Sol: $N_2 + 4H^+ + 4e^- \rightarrow N_2H_4$

31. The temperature of dependence of the e.m.f ------

Ans : -5.8×10^{-5}

Sol: $E = -4 \times 10^{-5}\ T - 9 \times 10^{-7} \times T^2 + 3.6 \times 10^{-5}T$

$\frac{dE}{dT} = -0.4 \times 10^{-5} - 9 \times 10^{-7} \times 2\ T$

$= -0.4 \times 10^{-5} - 5.4 \times 10^{-5}$

$= -5.8 \times 10^{-5}$

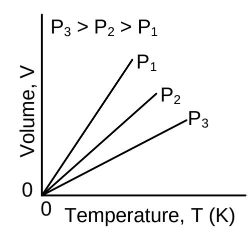
32. Lithium nitrate when heated gives------

Ans : Li_2O, NO_2 and O_2

Sol: $4LiNO_3 \rightarrow 2Li_2O + 2NO_2 + O_2$

33. For a fixed mass of an ideal gas the correct------

Ans :



Sol: Plot of V α T is a straight line and the slope of the isobar decreases with increase of pressure

34. Match each one with the correct method ------

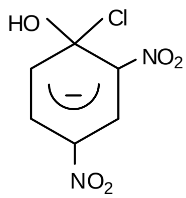
Ans : (a) → (ii), b → (iv), c → (iii), (d) → (i)

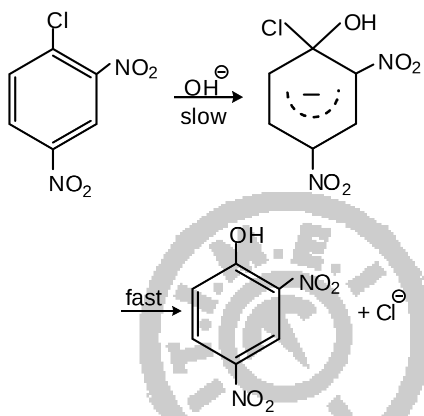
Sol: Cr_2O_3 – Al reduction

$Fe_2O_3 \rightarrow$ CO reduction

$Cu_2S \rightarrow$ self reduction
ZnS $\rightarrow$ Roasted to ZnO and then CO reduction

35. The intermediate formed in the following ------

Ans : 

Sol: 

36. The crystal field splitting energy (Δ_0), of------

Ans : I < II < III < IV

Sol: The arrangement of ligands in the spectrochemical series is
$CN^- > NCS^- > F^- > Br^-$

37. The compounds that form stable hydrates are-----

Ans : II and IV

Sol: II is Indane-1,2,3-trione. It forms a stable hydrate known as ninhydrin which is stabilized by intramolecular hydrogen bonding. IV is chloral which also forms stable chloral hydrate

38. The symbols F, H, S, V_m and E^0 denote ------

Ans : F, H, S, are extensive; V_m and E^0 intensive

Sol: F, H and S are extensive properties, as they depend on the quantity of the system
V_m and E^0 are intensive properties

39. For a 1st order reaction of the form------

Ans : I and IV

Sol: $\ln A/A_0 = -kt$
i.e., A/A_0 decreases with increase of kt

$$\frac{A}{A_0} = \frac{1}{e^{kt}}$$

$\frac{A}{A_0}$ decreases with increase of kt

40. Consider the reaction $2A \rightleftharpoons B$.------

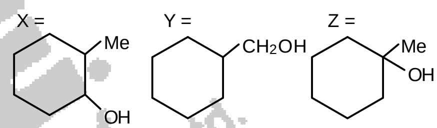
Ans : 0.05

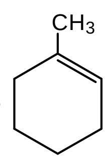
Sol: $2A \rightleftharpoons B$

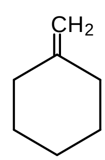
$$K = \frac{(x/2)}{(a-x)^2}$$

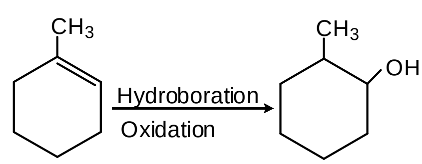
on solving, x = 0.15 and 0.1
x = 0.15 is not possible
$\therefore$ Amount of B at equilibrium = 0.05

41. Two isomeric alkenes A and B on hydrogenation in the presence ------

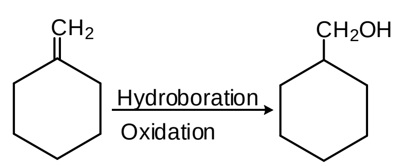
Ans : 

Sol: Alkenes (A) and (B) are  and

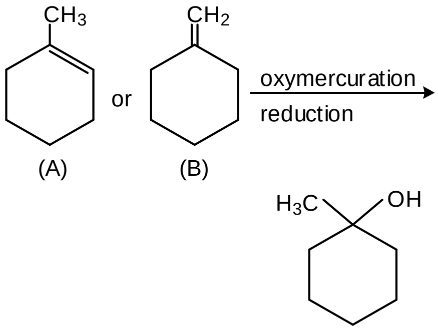
 respectively



(A) (X)



(B) (Y)



(A) (B)

(Z)

42. The major product of the following reaction is-----

Ans : $C_6H_5CH{=}CHCH{=}NNHCONH_2$

Sol: $C_6H_5\text{–}CH{=}CH\text{–}CHO$ +

$\underset{\text{(Semicarbazide)}}{H_2N-NH-CO-NH_2} \longrightarrow$

$C_6H_5\text{–}CH{=}CH\text{–}CH{=}N\text{–}NH\text{–}CO\text{–}NH_2$

The – NH_2 group away from the $-\overset{\overset{\displaystyle O}{\|}}{C}-$ group of semicarbazide reacts with aldehydes and ketones

43. At 100 K, a reaction is 30% complete in 10 minutes, while at 200 K------

Ans : 1150 J

Sol: $\log \frac{K_2}{K_1} = \frac{E_a}{2.303\,R}\frac{T_2 - T_1}{T_1T_2}$

$\log 2 = \frac{E_a \times 100}{2.303 \times 8.314 \times 100 \times 200}$

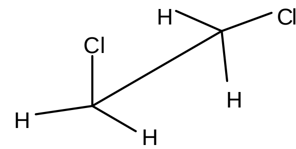
E_a = 1150 J

44. The stability order of the following carbocation is-----

Ans : II > III > I > IV

Sol: II is the most stable carbocation because positive charge is at allylic position with respect to two double bonds.
IV is the least stable carbocation as it is antiaromatic

45. For the following Newman projection------

Ans : 

Sol: (b)

46. For bromoalkanes------

Ans : I and III

Sol: Statements I and III are correct

47. The number of unpaired electrons present ------

Ans : 0 and 4

Sol: $[Co(C_2O_4)_3]^{3-}$ is a spin paired complex with d^2sp^3 hybridisation where as $[CoF_6]^{3-}$ is a spin free complex with sp^3d^2 hybridisation

48. The correct statement regarding the functioning of a catalyst is that it------

Ans : II and IV

Sol: Statements II & IV are correct

49. The relationship among the following pairs of isomers is------

Ans : I – A, II – A, III – B, IV – B

Sol: Geometrical and optical isomers are known as configurational isomers

50. In the oxidation of sulphite using permanganate, the number of protons ------

Ans : 3

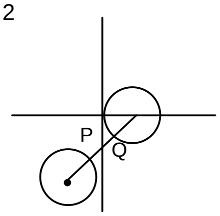
Sol: $2MnO_4^- + 5SO_3^{2-} + 6H^+ \rightarrow 2Mn^{2+} + 5SO_4^{2-} + 3H_2O$

PART C – MATHEMATICS

51. Let the line segment joining the centers of the circles $x^2 - 2x + y^2 = 0$ ------

Ans : $5x^2 + 5y^2 + 2x + 16y + 8 = 0$

Sol: Centre of $x^2 + y^2 - 2x = 0$ (1, 0) radius = 1
centre and radius of
$x^2 + y^2 + 4x + 8y + 16 = 0$
is (–2, –4) radius = 2



Distance between the centers = 5
$\therefore$ PQ = 5 – 1 – 2 = 2
Clearly centre of the required circle lies the third quadrant and radius of the required circle, is 1 which is
$5x^2 + 5y^2 + 2x + 16y + 8 = 0$

52. If the angle between the vectors $\bar{a}$ and $\bar{b}$ is $\frac{\pi}{3}$ ------

Ans : $2\sqrt{3}$

Sol: Area of the triangle = $\frac{1}{2}\left|\bar{a}\times\bar{b}\right| = 3$

$\Rightarrow \left|\bar{a}\times\bar{b}\right| = 6$

$\Rightarrow ab\sin\theta = 6$

$\Rightarrow ab\sin\frac{\pi}{3} = 6$

$\Rightarrow ab = \frac{12}{\sqrt{3}}$

$\therefore\ \bar{a}\bullet\bar{b} = ab\cos\frac{\pi}{3}$

$= \frac{12}{\sqrt{3}}\times\frac{1}{2} \Rightarrow \frac{6}{\sqrt{3}} = 2\sqrt{3}$

53. Let P = $\begin{bmatrix}\cos\frac{\pi}{9} & \sin\frac{\pi}{9}\\ -\sin\frac{\pi}{9} & \cos\frac{\pi}{9}\end{bmatrix}$-and α, β, γ -----

Ans : 1

Sol: $\alpha p^6 + \beta p^3 + \gamma I = 0$

$$\Rightarrow \alpha\begin{bmatrix}-\frac{1}{2} & \frac{\sqrt{3}}{2}\\ \frac{-\sqrt{3}}{2} & \frac{-1}{2}\end{bmatrix} + \beta\begin{bmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ \frac{-\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}$$

$$+\gamma\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$\Rightarrow \frac{-\alpha}{2}+\frac{\beta}{2}+\gamma = 0$ and $\frac{\sqrt{3}}{2}\alpha+\frac{\sqrt{3}}{2}\beta = 0$

$\Rightarrow -\alpha + \beta + 2\gamma = 0$ and $\alpha + \beta = 0 \Rightarrow \alpha = -\beta$

$\Rightarrow \therefore\ 2\beta + 2\gamma = 0 \Rightarrow \beta + \gamma = 0 \Rightarrow \beta = -\gamma$

$\therefore\ \alpha = -\beta = r$

$\therefore\ \alpha - \gamma = 0$

$\therefore\ (\alpha^2 + \beta^2 + \gamma^2)^{(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)}$

$= (\alpha^2 + \beta^2 + \gamma^2)^0 = 1$

54. A random variable X takes values –1, 0, 1, 2 with probabilities------

Ans : $\frac{-1}{16}$ and $\frac{5}{4}$

Sol: x : –1 0 1 2

$P(x) = \frac{1+3p}{4}\quad \frac{1-p}{4}\quad \frac{1+2p}{4}\quad \frac{1-4p}{4}$

$\therefore\ \bar{x} = \Sigma xp(x)$

$= \frac{2-9p}{4}$; But $p\in R$ and from the given probabilities since $0< p(x) <1$

we get $p\in\left(\frac{-1}{3},\frac{1}{4}\right)$.

Hence $\bar{x}\in\left(\frac{-1}{16},\frac{5}{4}\right)$

55. Let f(x) = log(sinx + cosI), $I\in\left(\frac{-\pi}{4}-\frac{\pi}{4}\right)$------

Ans : $\left(\frac{-\pi}{4},\frac{\pi}{4}\right)$

Sol: $f(x) = \log\sqrt{2}\sin\left(\frac{\pi}{4}+x\right)$

$f'(x) = -\cot\left(x+\frac{\pi}{4}\right)$

$0 < x+\frac{\pi}{4} < \frac{\pi}{2}$ or $\pi < x+\frac{\pi}{4} < \frac{3\pi}{2}$

$\frac{3\pi}{4} < x < \frac{5\pi}{4}$

56. The number of distinct real values of λ for which the vectors ------

Ans : 1

Sol: $\begin{vmatrix}\lambda^3 & 0 & 1\\ 1 & -\lambda^3 & 0\\ 1 & 2\pi-\sin\lambda & -\lambda\end{vmatrix} = 0$

$$\begin{vmatrix} \lambda^3 & 0 & 1 \\ 1 & -\lambda^3 & 0 \\ \lambda^4 & 2\pi - \sin & \lambda 0 \end{vmatrix}$$

$2\lambda - \sin\lambda + \lambda^7 = 0$

$X^7 + 2\lambda = \sin\lambda$

$f(x) = \lambda^7 - 2\lambda \quad f'(x) = 7\lambda^6 + 2$

$f'(x) > 0 \quad \therefore f(x)$ is increasing

So it intersects with $\sin\lambda$ only once

57. -The minimum value of $|z_1 - z_2|$ as z_1 and z_2 vary over the curves-----

Ans : $\dfrac{5\sqrt{7}}{2\sqrt{3}}$

Sol: z_1 lies on the circle $\left|z \quad -\frac{1}{2} \quad -\frac{i}{\sqrt{3}}\right| = \sqrt{\frac{7}{3}}$

(i.e) $|z - z_0| = \sqrt{\frac{7}{3}}$ where z_0

$= \left(\frac{1}{2} + \frac{i}{\sqrt{3}}\right)$

Z_2 lies on $\left|z \quad +1+ \quad \frac{2i}{\sqrt{3}}\right| = \left|z - 9 - \frac{18i}{\sqrt{3}}\right|$

(i.e) z_2 lies on the perpendicular bisector of $\left(-1, \frac{-2}{\sqrt{3}}\right)$ and $\left(9, \frac{18}{\sqrt{3}}\right)$

$\Rightarrow z_2$ passes through $\left(4, \frac{8}{\sqrt{3}}\right) = 8z_0$

$\therefore$ Minimum of $|z_1 - z_2|$

$= |z_2 - z_0| - |z_1 - z_0|$

$= |8z_0 - z_0| - \sqrt{\frac{7}{3}}$

$= 7\,|z_0| - \sqrt{\frac{7}{3}}$

$= \dfrac{5\sqrt{7}}{2\sqrt{3}}$

58. Let $f(\theta) = \dfrac{1}{\tan^9\theta}\,(1+\tan\theta)^{10} + (2+\tan\theta)^{10} +$ -----$+(20+\tan\theta)^{10}) - 20\tan\theta$-----

Ans : 2100

Sol: Put $t = \tan\theta$

$$= \frac{(1+t)^{10} + (2+t)^{10} + -- + (20+t)^{10} - 20t^{10}}{t^9}$$

$$= \frac{(1+t)^{10} - t^{10}}{1+t-t} + \frac{(2+t)^{10} - t^{10}}{2+t-t} + --- + \frac{1}{t^9}$$

$$= \frac{1}{t^9}\left[\frac{(1+t)^{10} - t^{10}}{1+t-t}\right] + 2\left[\frac{(2+t)^{10} - t^{10}}{(2+t) - t}\right] ---$$

$= \dfrac{1}{t^9}\,[10.\,t^9 + 2 \times 10\,.\,t^9 + ----+20]$

$= 10\,[\,1 + 2 + -------+20]$

$= 10\,\dfrac{[20 \times 21]}{2} = 2100$

59. Let $r > 1$ and $n > 2$ be integers. Suppose L and M are coefficients ------

Ans : $n = 2r+1$

Sol: Let $\ell = 3r \quad m = r+2$

Given ℓ^{th} term = L & m^{th} term is M

$L = {}^{(2n-1)}C_{\ell-1}$ & $M = {}^{(2n-1)}C_{m-1}$

$$\therefore \frac{(2n-1)!}{(\ell-1)!\;(2n-\ell)!} = \frac{(2n-1)!}{(m-1)!\;(2n-m)!}$$

$$\frac{2n\,!}{\ell!\;(2n-\ell)!} = \frac{2n!}{m!\,(2n-m)!}$$

$${}^{2n}C_{\ell} = {}^{2n}C_m$$

$\Rightarrow \ell = m$ OR $\ell + m = 2n$

$\Rightarrow 3r = r+2$ OR $2r = 4r+1$

$\therefore n = 2r+1$

60. The value of the integral $\displaystyle\int_0^2 \frac{\log(x^2+2)}{(x+2)^2}$ dr is------

Ans : $= \dfrac{\sqrt{2}}{3}\tan^{-1}\sqrt{2} + \dfrac{1}{12}\log 3 - \dfrac{5}{12}\log 2$

Sol: $\displaystyle\int_0^2 \frac{\log(x^2+2)}{(x+2)^2}\,dx =$

$= \displaystyle\int_0^2 \log(x^2+2)(x+2)^{-2}\,dx$

$= \log(x^2+2)\left(\dfrac{-1}{x+2}\right)^2$

$\displaystyle -\int \frac{2x}{x^2+2}\left(\frac{-1}{x+2}\right)d\,x$

$= -\left(\dfrac{1}{4}\log(3\times 2) - \dfrac{1}{2}\log 2\right) + \dfrac{2}{3}\displaystyle\int \frac{xdx}{x^2+2}$

$+ \dfrac{2}{3}\displaystyle\int \frac{dx}{x^2+2} - \dfrac{2}{3}\int \frac{dx}{x+2}$

(By partial fractions)

$= -\dfrac{1}{4}\log 3 + \dfrac{1}{4}\log 2 - \dfrac{1}{2}\log 2$

$+ \dfrac{2}{3}\log(x^2+2)_0^2 + \dfrac{2}{3}\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{x}{\sqrt{2}}\right)$

$- \dfrac{2}{3}\log(x+2)$

$= \left(-\frac{1}{4}+\frac{1}{3}\right)\log 3+\frac{\sqrt{2}}{3}\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$

$-\frac{5}{2}\log 2$

$= \frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}+\frac{1}{12}\log 3-\frac{5}{12}\log 2$

61. The age distribution of 400 persons in a colony having median age 32 is given below------

Ans : –10

Sol: below 25 110
below 30------110 + x
below 35------185 + x
below 40------240 + x
below 45------240 + x + y
below 50-----240 + x + y
$\therefore$ 270 + × + y = 400 $\Rightarrow$ x + y = 130 -----(1)

Medians = $\ell + \frac{\left(\frac{N}{2}-c.f\right)h}{f}$

$= 30 + \frac{200-(110+x)}{7.5}\times 5 = 32$

$\Rightarrow$ x = 60 $\therefore$ y = 70
$\therefore$ x – y = –10

62. The probability that a randomly selected calculator from a store is of brand r is proportional to r, ------

Ans : $\frac{8}{63}$

Sol: Let n be the total no of calculators.
Since p (r) α r and Σp(r) = 1

$\Rightarrow \frac{k}{n}+\frac{2k}{n}+----+\frac{6k}{n}=1$

$\Rightarrow$ n = 21 k

$\Rightarrow$ p(r) = $\frac{r}{21}$, r= 1, 2, -------, 6

Again p(D_r) = $\frac{6}{21},\frac{5}{21},-----,\frac{1}{21}$ when r =1, 2, -----,6

P (Defective calculator) $\sum_{r=1}^{6} p(r)\times p(D_r)$

$= \frac{1}{21}\times\frac{6}{21}+\frac{2}{21}\times\frac{5}{21}+---$

$+ \frac{6}{21}\times\frac{1}{21}=\frac{56}{21^2}=\frac{8}{63}$

63. There are 10 girls and 8 boys in a class room including Mr. Ravi, Ms. Rani and Ms. Radha.-----

Ans : 308

Sol: 10g 8b
8g 6b

Ravi is in : ${}^9C_8\times{}^7C_5 = 9\times\frac{7\times 6}{2}=189$

Rani is in : ${}^7C_6\times{}^8C_7$ = 7 × 8 = 56

Both Ravi & Rani are out:
${}^9C_8\times{}^7C_6$ = 9 × 7 = 63

Total = 189 +56+ 63 = 308

64. Let f: [0, 4] $\to$ R be a continuous functions such that |f(x)| $\le$ 2 for all x $\in$ [0, 4]------

Ans : [–6+2x.,10 –2x]

Sol: $-2 \le f(x) \le 2$

$\therefore\ -2x \le \int_0^x f(x)dx \le 2x$

$\therefore\ \int_0^x f(x)dx = 2x-k,\quad k>0$

When x = 4, $\int_0^4 f(x)dx = 8-k=2$

$\Rightarrow$ k = 6 $\Rightarrow$ –6 +2x

$\therefore\ \int_0^x f(x)dx = k-2x,\quad k>0$

When x = 4, $\int_0^4 f(x)dx = k-8=2$

$\Rightarrow$ k = 10 $\Rightarrow$ 10 – 2x

$\therefore\ \int_0^x f(x)dx \in [-6+2x,\ 10-2x]$

65. The number of solutions of the equation------

Ans : 2

Sol: $\cos^2\left(x+\frac{\pi}{6}\right)-2\cos\left(x+\frac{\pi}{6}\right)\cos\frac{\pi}{6}$

$=\sin^2\frac{\pi}{6}-\cos^2 x$

$\cos\left(x+\frac{\pi}{6}\right)\left(\cos\left(x+\frac{\pi}{6}\right)-2\cos\frac{\pi}{6}\right)$

$= -\left(\cos^2 x-\sin^2\frac{\pi}{6}\right)$

$= \cos\left(x+\frac{\pi}{6}\right)\left(\cos\left(x+\frac{\pi}{6}\right)-2\cos\frac{\pi}{6}\right)$

$= -\left(\cos\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)\right)$

$\cos\left(x+\frac{\pi}{6}\right)$

$\left(\cos\left(x+\frac{\pi}{6}\right)-2\cos\frac{\pi}{6}+\left(\cos\left(x-\frac{\pi}{6}\right)\right)\right) = 0$

$\Rightarrow \cos\left(x+\frac{\pi}{6}\right)\ \left(2\cos x\cos\frac{\pi}{6}-2\cos\frac{\pi}{6}\right)$

$= 0$

$\Rightarrow 2\cos\left(x+\frac{\pi}{6}\right)\ \cos\frac{\pi}{6}\ (\cos x - 1) = 0$

$\Rightarrow \cos\left(x+\frac{\pi}{6}\right) = 0$ or $\cos x - 1 = 0$

$\Rightarrow x+\frac{\pi}{6} = \pm\frac{\pi}{2}$ or $\cos x = 1$

$\Rightarrow x = \frac{\pi}{3}$ or $\frac{-2\pi}{3}$ or $x = 0 \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\therefore$ But $\frac{-2\pi}{3} \notin \left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

$\therefore$ Number of solutions = 2

66. The equation of the circle which cuts each of the three circles $x^2 + y^2 = 4$,------

Ans : $x^2 + y^2 - x - 2y + 4 = 0$

Sol: Radical axis of S_1 and S_2 is $2x - 1 = 0$ and that of S_2 and S_3 is $y - 1 = 0$

$\therefore$ Center of the orthogonal circle is $\left(\frac{1}{2},1\right)$.

Clearly option (C)

67. Suppose an ellipse and a hyperbola have the same pair of f------

Ans : $\sqrt{\frac{7}{3}}$

Sol: $e = \frac{1}{2}$ for ellipse

i = $\frac{1}{2} = \sqrt{1-\frac{b^2}{a^2}}$

$\frac{1}{4} = 1-\frac{b^2}{a^2}$

$\therefore \frac{b^2}{a^2} = 1-\frac{1}{4} = \frac{3}{4}$

$\therefore b^2 = 7$ and $a^2 = \frac{28}{3}$

$\therefore$ Foci = ae = $\left(\pm\frac{7}{3},\ 0\right)$

Equation of the hyperbola that passes through (2, 2) is

$\frac{x^2}{a^2} - \frac{y^2}{\frac{7}{3}-a^2} = 1 \Rightarrow a = 1$

Hence $a^2e^2 = \frac{7}{3} \Rightarrow e = \sqrt{\frac{7}{3}}$

68. -Let a be on a on – zero real number and α, β be the root of the equation $ax^2 + 5x + 2 = 0$-----

Ans : $|\alpha^2 - \beta^2|$

Sol: Let α', β' be the roots of $a^3(x+5)^2 - 25a(x+5) + 50 = 0$

$\therefore |\alpha' - \beta'|$ will remain the same for

$a^3y^2 = 25ay + 50 = 0$

$\Rightarrow a\left(\frac{ay}{5}\right)^2 - 5\left(\frac{ay}{5}\right) + 2 = 0$

$\Rightarrow ay^2 - 5y + 2 = 0$. Whose roots are α and β

$\therefore \alpha = \frac{-a\alpha'}{5}$ and $\beta = \frac{-a\beta'}{5}$

$|\alpha' - \beta'| = \frac{-5}{a}\,|\alpha - \beta| = |\alpha^2 - \beta^2|$

since $\alpha + \beta = \frac{-5}{a}$

69. The set of all 2 $\times$2 matrices which commute with the matrix $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$------

Ans : $\left\{\begin{bmatrix}p & q\\q & p-q\end{bmatrix} : p, q \in R\right\}$

Sol: Clearly matrix A = $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$ commute with matrix B = $\begin{bmatrix}p & q\\q & p-q\end{bmatrix}$

i AB = BA

70. Let f: (0, 1) $\to$ (0, 1) be a differentiable function such that $f'(x) \neq 0$------

Ans : $\left\{\frac{\sqrt{7}}{4},\frac{\sqrt{15}}{4}\right\}$

Sol: Using L' H rule, we get

$\frac{\sqrt{1-f(x)^2}}{f'(x)} = f(x)$

$\therefore dx = \frac{y^{dy}}{\sqrt{1-y^2}}$

$x + \sqrt{1-y^2} \Rightarrow C \Rightarrow C = 1$

$\because f\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$

$\therefore f\left(\frac{1}{9}\right) = \pm\frac{\sqrt{7}}{4}$

71. In the interval $\left[0, \frac{\pi}{2}\right]$, the equation $\cos^2 x - \cos x - x = 0$ has ------

Ans : Exactly one solution

Sol: $f(x) = \cos^2 x - \cos x - x = 0$

$f'(x) = -\sin 2x + \sin x - 1$

< 0 in $\left(0, \frac{\pi}{2}\right)$ which is decreasing

f(x) = one solution

72. The points with position vectors ------

Ans : $\Rightarrow (1-\alpha)(\beta+1) = 0$

Sol: Vectors are denoted by A, B, C, D then

$\overline{AB} = (\alpha - 1)\, i + 2j + 2k$

$\overline{AC} = (\alpha - 1)\, i - j + 2k$

$\overline{AD} = (\alpha - 1)\, i + (1-\beta)\, k$

$\therefore \left[A\overline{B}\ \ A\overline{C}\ \ A\overline{D}\right] = 0$

$$\begin{vmatrix} \alpha-1 & 2 & 2 \\ \alpha-1 & -1 & 2 \\ \alpha-1 & 0 & 1-\beta \end{vmatrix} = 0$$

$\Rightarrow (\alpha - 1)(\beta + 1) = 0$

$\Rightarrow (1-\alpha)(\beta - 1) = 0$

$\Rightarrow (1-\alpha)(\beta + 1) = 0$

73. For a real number x, let [x] denote the greatest integer less than or equal to x------

Ans : one – one but NOT onto

Sol: f(x) = 2x + [x] +sinxcosx

$= 3x\{x\} + \frac{1}{2}\sin 2x$

$\therefore f'(x) = 3 - 1 + \cos 2x > 0$

$\therefore$ f(x) is strictly increasing

$\Rightarrow$ One – one function

But due to the presence of [x] f(x) jumps at integral points.

$\Rightarrow$ f(x) is NOT onto

74. Let M be a 3×3 non singular matrix with det (M) = a ------

Ans : α

Sol: $\text{adj}(\text{adj} M) = |m|^{3-2} M = M\alpha$

$\therefore M^{-1}\, \text{adj}(\text{adj} n) = M^{-1} m\alpha = \alpha I$

$\therefore k = \alpha =$

75. If $y^x - xy = 1$ then the value of $\frac{dy}{dx}$ at x = 1 is------

Ans : 2(1 – log2)

Sol: $y^x = u \quad x^y = v$

$\Rightarrow u + v = 1$

$\Rightarrow \frac{dy}{dx} = \frac{dv}{dx}$

$y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = x^y\left(\frac{y}{x} + \log x \frac{dy}{dx}\right)$

$$\therefore \frac{dy}{dx} = \left(\frac{x^y\left(\frac{y}{x}\right) - y^x \log y}{y^x\left(\frac{x}{y}\right) - x^y \log x}\right)$$

$x = 1 \quad y = 2$

$\therefore \frac{dy}{dx} = \frac{2 - 2\log 2}{1} = 2(1 - \log 2)$