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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

Third Semester

Civil Engineering

CE 6302 – MECHANICS OF SOLIDS

(Regulations 2013)

(Common to Environmental Engineering)

Time : Three Hours

Maximum : 100 Marks

(1): Example:

For suddenly applied load — (1) Mark.

→ Hitting ball with bat.

For Impact load. — (1) Mark.

→ Hammer strike on a metal plate from certain height.

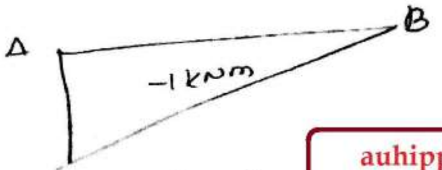
(2). Resilience : — 2 marks

It is the ability of a material to absorb energy when it is deformed elastically, and release that energy upon unloading.

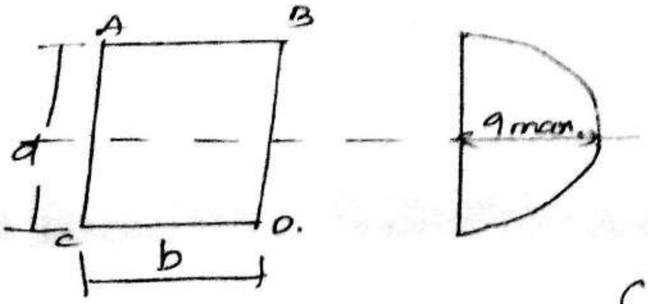
(or)

The total strain energy stored in the body.

(3) — 2 Marks



BMD

(4). 

(2) marks

$$q_{max} = \frac{2}{3} \times q_{avg} \qquad \left[q_{avg} = \frac{F}{b \times d} \right].$$

(5). Simply supported beam with udl.

$$\theta_A = \theta_B = \frac{wl^3}{24EI}$$

$w \rightarrow$ load in kN

$E \rightarrow$ Youngs modulus N/mm²

$I \rightarrow$ Moment of Inertia mm⁴

(6) Moment area method: [2 marks.]

* It is one of the most effective methods for obtaining the bending displacement in beams and frames. In this method, the area of the bending moment diagram is utilized for computing the slope & deflections at particular points along the axis of the beam.
* The change in slope b/w any two points on the elastic curve equals the area of the M/EI diagram b/w these points.

(7) Elastic theory of torsion: [2 marks]

When the shaft is subjected to equal & opposite end couples whose axes coincide with the axis of the shaft is said to be in pure torsion.

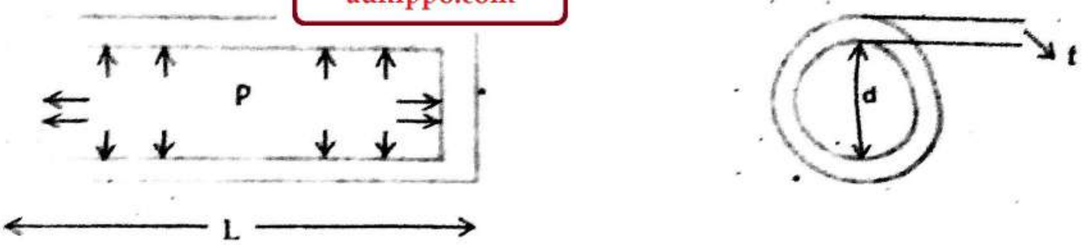
$$\frac{\tau_{max} f_s}{R} = \frac{T}{J} = \frac{C\theta}{l}.$$

$\tau_{max} f_s \rightarrow$ Max Shear Stress N/mm²

$J \rightarrow$ Polar moment of Inertia

$R \rightarrow$ Radius of Shaft.

11)a) ii) Mathematical expression for the closed thin cylinder subjected to internal fluid pressure
When the thickness of the wall of the cylinder is less than 1 10 to 1 15 of the diameter of cylinder then the cylinder is considered as thin cylinder. Otherwise it is termed as thick cylinder.



L = Length of the cylinder d= Diameter of cylinder t = thickness of cylinder P= Internal Pressure due to fluid

Due to the fluids inside a cylinder, these are subjected to fluid pressure or internal pressure (Say P). Hence at any point on the wall of the cylinder, three types of stresses are developed in three perpendicular directions.

These are:-

1. Circumferential Stress or Hoop Stress (σ_h)
2. Longitudinal Stress (σ_L)
3. Radial Stress (σ_r)

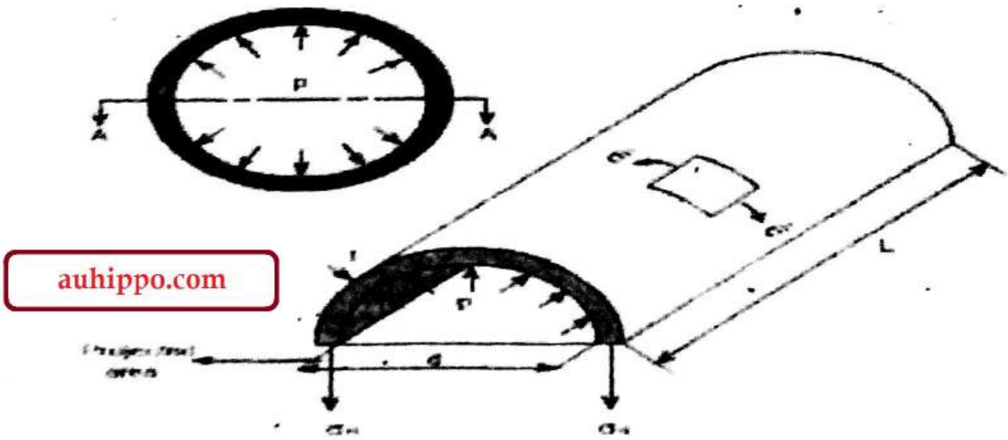
Assumptions in Thin Cylinders

1. It is assumed that the stresses are uniformly distributed throughout the thickness of the wall.
2. As the magnitude of radial stresses is very small in thin cylinders, they are neglected while analyzing thin cylinders i.e. $\sigma_r=0$

3 marks

Stresses in Thin Cylinder

1. Circumferential Stress (σ_h):- This stress is directed along the tangent to the circumference of the cylinder. This stress is tensile in nature. This stress tends to increase the diameter.



The bursting in the cylinder will takes place if the force due to internal fluid pressure(P) acting vertically upwards and downwards becomes more than the resisting force due to circumferential stress (σ_h) developed in the cylinder.

Total diametrical Bursting force= P * Projected area of the curved surface = P * d * L

Resisting force due to circumferential stress= 2 * h* t * L

Under equilibrium, Resisting force = Total diametrical Bursting force 2 * h* t * L = P * d * L

Circumferential stress, σ_h = Pd /2t

2. Longitudinal Stress (σ_L) :- This stress is directed along the length of the cylinder. This stress is also tensile in nature. This stress tends to increase the length.

Total longitudinal bursting force (on the ends of cylinder) = P *(Π /4)* d^2

Area of crossection where longitudinal stress is developed= Π * d * t

Resisting force due to longitudinal stress = L * Π * d * t

Under equilibrium, Resisting force = Total longitudinal Bursting force

$f * \Pi * d * t = P * (\Pi/4) * d^2$

Longitudinal stress, $\sigma_l = Pd/4t$

Note: Due to the presence of longitudinal stress and hoop stress, there is shear stress developed in the cylinder.

Maximum in plane shear stress is given by $\tau_{max,inplane} = (\sigma_h - \sigma_l)/2 = Pd/8t$ [(3) marks]

[OR]

11(b) To find:

Change in length (δl).

$P_1 = 50\ kN$

$P_2 = 100\ kN$

$P_3 = 50\ kN$

$$\delta l_1 = \frac{P_1 l_1}{A_1 E} = \frac{50 \times 10^3 \times 1}{400 \times 2.1 \times 10^5}$$

$$\delta l_1 = 5.952 \times 10^{-4}\ mm$$

$$\delta l_2 = \frac{P_2 l_2}{A_2 E_2} = \frac{100 \times 10^3 \times 1}{2500 \times 2.1 \times 10^5}$$

$$\delta l_2 = 1.904 \times 10^{-4}\ mm$$

$$\delta l_3 = \frac{P_3 l_3}{A_3 E_3} = \frac{50 \times 10^3 \times 1}{\pi/4 \times 20^2 \times 2.1 \times 10^5}$$

$$\delta l_3 = 7.578 \times 10^{-4}\ mm.$$

$$\delta l = 15.434 \times 10^{-4}\ mm.$$

[(13) marks]

Part B [5 × 13 = 65 Marks]

11 (a)

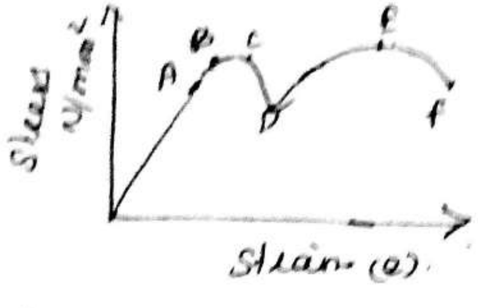
Mild steel: → Ductile in nature [2 marks]

→ It obeys hooks law upto elastic limit.

→ Specific yeild point (Upper & lower yeild point)

→ Necking takes place at ultimate point

→ Failure takes place at breaking point



A → limit of proportionality
B → Elastic limit
C → Upper yeild point
D → Lower yeild point
E → Ultimate point
F → Breaking point

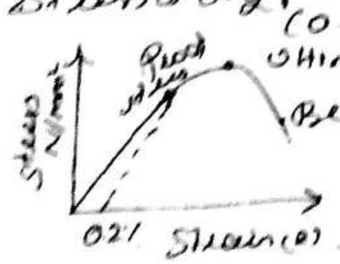
Tor Steel: [2 marks]

→ No specific yeild point.

→ Necking takes place at ultimate point.

→ Failure takes place at breaking point.

→ Proof stress 0.2 percent of strain (0.2%)



Concrete: [2 marks]

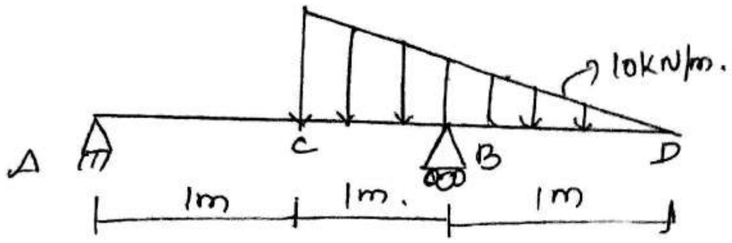
→ Do not have particular yield point.

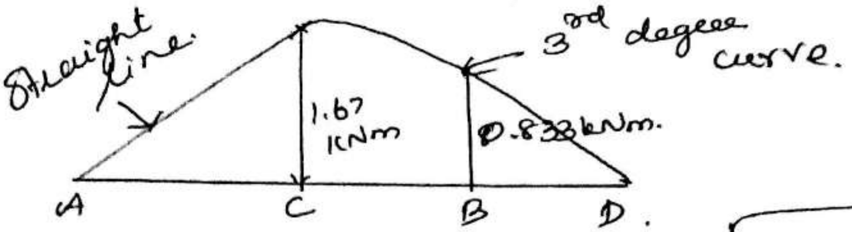
→ Necking donot occur.

→ Ultimate & Breaking point are same

→ Brittle in nature

12) a)





BM Diagram.

4 marks

To find reaction:

Taking moment About 'A'.

$$R_B \times 2 - \left(\frac{1}{2} \times 2 \times 10\right)\left(\tfrac{1}{3} \times 2 + 1\right) = 0.$$

$$R_B = 8.33 \text{ kN}$$

$$R_A + R_B = \frac{1}{2} \times 2 \times 10$$

$$\boxed{R_A = 1.67 \text{ kN.}}$$
$$\boxed{R_B = 8.33 \text{ kN}}$$

3 marks

Bending moment calculation.

BM @ A = 0

BM @ C = $R_A \times 1$ = 1.67 kNm.

BM @ B $= \left(\frac{1}{2} \times 1 \times 5\right) \times \left(\frac{1}{3} \times 1\right)$

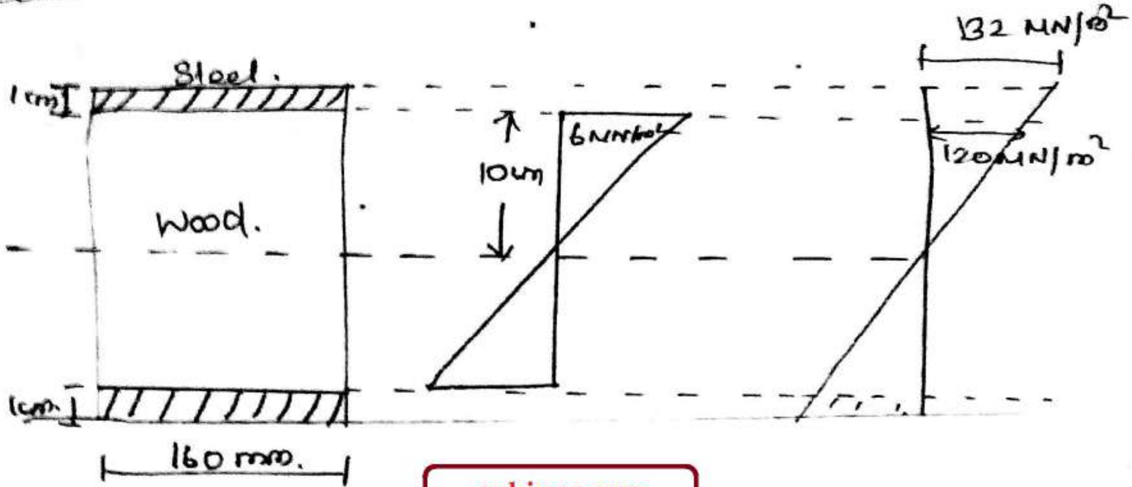
$= 0.833$ kNm.

BM @ D = 0.

6 Marks

12(b).

Case (i) - Flitches attached to top & bottom.



Stress in wood at level of steel.

$$6_{\omega} = \frac{6}{10} \times 11 = 6.6 \text{ MN/m}^2.$$

As they are rigidly connected,

Stress of steel at level of wood $= \frac{132}{11} \times 10 = 120$

$$6_s = \frac{E_s}{E_{\omega}} 6_{\omega} = \frac{20 E_{\omega}}{E_{\omega}} \times 6.6$$

$$6_s = 132 \text{ MN/m}^2.$$

Total resisting moment $M = M_{\omega} + M_s$

$$M_{\omega} = 6_{\omega} Z_{\omega} = \frac{6.6 \times 0.16 \times 0.2^2}{6} = 6.4 \times 10^{-3} \text{ MNm}$$

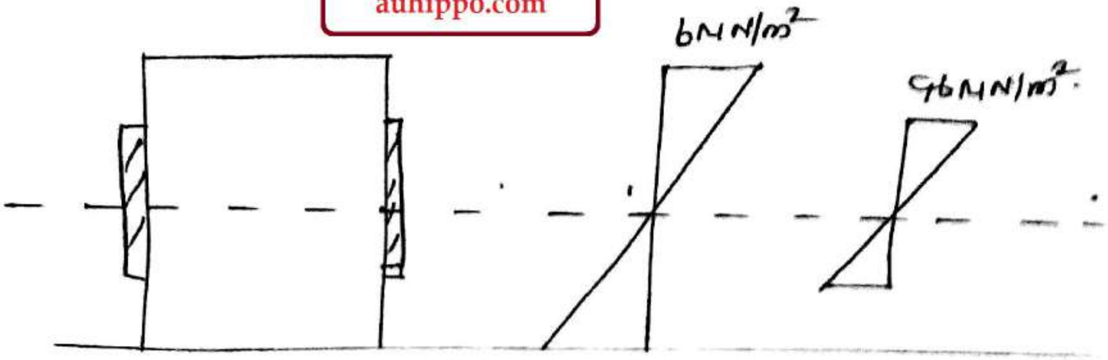
$$M_s = 6_s Z_s = \frac{132 \times 0.16 \times 0.22^2}{6} - \frac{120 \times 0.16 \times}{6}$$

$$= 0.04237 \text{ MNm}$$

Total moment $= 0.0487$ MN-m.

6 Marks

Case ii Flitches attached at sides.



Stress in wood at level of Steel.

$$\sigma_\omega = \frac{6}{10} \times 8 = 4.8 \ MN/m^2 .$$

$$\sigma_S = \frac{E_S}{E_\omega} \sigma_\omega = \frac{20 E_\omega}{E_\omega} \times 4.8$$

$$= 96 \ MN/m^2 .$$

Total Moment = $M_\omega + M_S$.

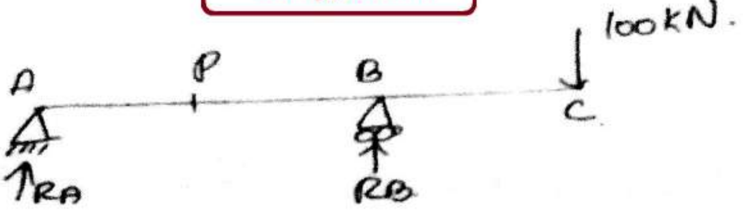
$$M_S = \sigma_S Z_S = 96 \times \frac{0.02 \times (0.16)^2}{6} = 0.00819 \ MNm$$

$$M_\omega = \sigma_\omega Z_\omega = \frac{4.8 \times 0.16 \times (0.2)^2}{6}$$

$$M = 0.01459 \ MNm.$$

$\boxed{\text{9 marks}}$

13 (a).



To find reaction:

Taking Moment abt 'A' = 0

$\Sigma M_A = 0.$

$100 \times 6 - R_B \times 4 = 0$

$R_B = 150$ kN

$R_A = -50$ kN

Moment @ B = -200 kN-m.

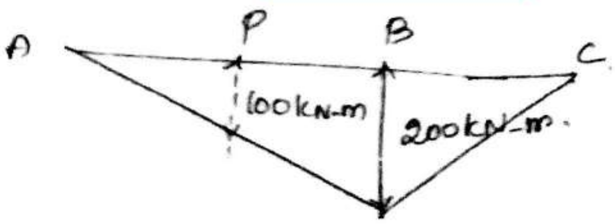
Moment @ P = -100 kN-m

Moment @ A = 0

Moment @ C = 0.

$\boxed{\text{(6) Marks}}$

BMD:



$$y_{max} \underset{@B.}{} = \frac{A\bar{x}}{EI}$$

$$y_{max} @ B = \left(\frac{1}{2} \times 2 \times 200\right) \times \frac{2}{3} \times 2 = \frac{266.67}{EI}$$

$$\text{Deflection} @ P = \left(\frac{1}{2} \times 2 \times 100\right) \times \frac{1}{3} \times 2 = \frac{66.67}{EI}$$

$\boxed{\text{(7) marks}}$

13(b).

To find reaction:

$R_A = 100\,kN$

To find moment:

Moment about A $= 300\,kN\text{-}m$

Moment about B $= 100 \times 2 = 200\,kN\text{-}m$

Moment about C $= 100 \times 1 = 100\,kN\text{-}m$.

Moment about D $= 0$.

$\boxed{\text{3 marks.}}$

Conjugate beam about A.

$$R_A \times 3 = \left(\frac{300}{2EI} \times 1\right)\frac{1}{2} + \left(\frac{200}{EI} \times 1\right)\left(\frac{1}{2} * 1\right) + \left(\frac{1}{2} \times \frac{100}{2EI} \times 1\right)\left(\frac{1}{3} \times 1 +\right.$$

$$3R_A = \frac{408.33}{EI}$$

$$R_A = \frac{131.11}{EI}.$$

Slope at E.

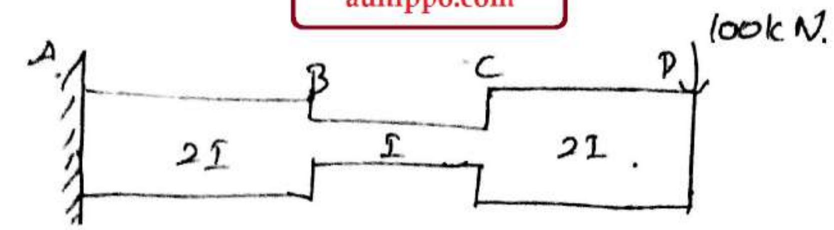
$\theta_E = R_A -$ load at ABE.

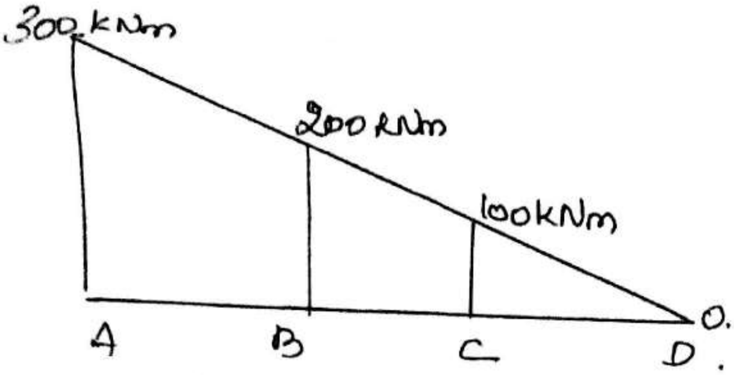
$$\theta_E = \frac{131.11}{EI} - \frac{200}{EI}$$

$$\theta_E = \frac{-68.89}{EI}$$

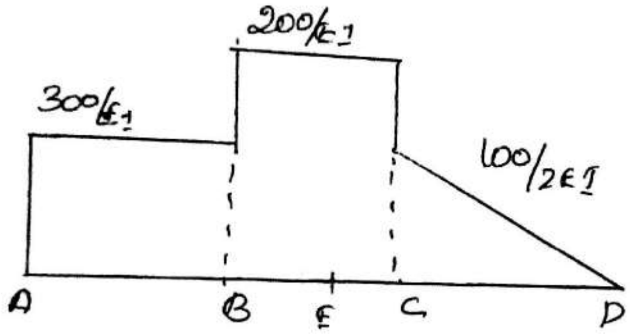
$\boxed{\text{6 Marks.}}$

(−) indicates downward.





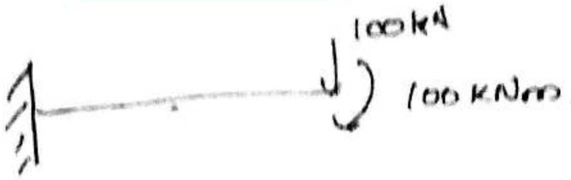
BMD.



Conjugate Beam diagram.

4 marks

11)a)



Given

$d = 30\ mm$

$T = 100\ kNm$

load $= 100\ kN$

Soln:

Torsional eqn.

$$\frac{T}{J} = \frac{\tau}{R}$$

$$J = \frac{\pi}{32} \times d^4 = \frac{\pi}{32} \times (30)^4 = 7.95 \times 10^4\ mm^4$$

$$R = \frac{30}{2} = 15\ mm.$$

Maximum Shear Stress:

$$\tau = \frac{T}{J} \times R$$

$$= \frac{100 \times 10^6}{7.95 \times 10^4} \times 15$$

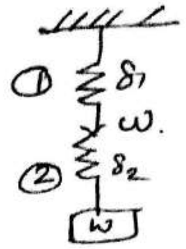
$$\tau = 18891.6\ N/mm^2$$

$$\tau = 18.89\ kN/mm^2.$$

13 marks

14 (b).

(i) Springs in series:



W = load applied

K_1 = Stiffness of spring 1

K_2 → Stiffness of spring 2

δ_1 → Extension of spring 1

δ_2 → Extension of spring 2

K → Stiffness of composite spring.

Let us consider that each spring will be subjected to load W and the total extension produced will be the sum of extensions of two springs,

Total extension $\delta = \delta_1 + \delta_2$

$$\delta = \frac{W}{K}$$

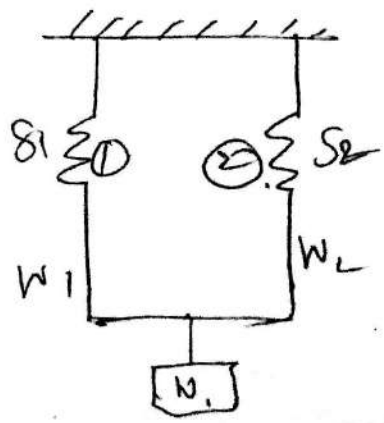
$$\frac{W}{K} = \frac{W}{K_1} + \frac{W}{K_2}$$

(6) Marks.

(or) $$\boxed{\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}.}$$

(ii) Springs in parallel:

When subjected to load W, they will extend equally say by an amount δ. The load will be shared such that.



$W = W_1 + W_2$

$\delta k = \delta k_1 + \delta k_2$

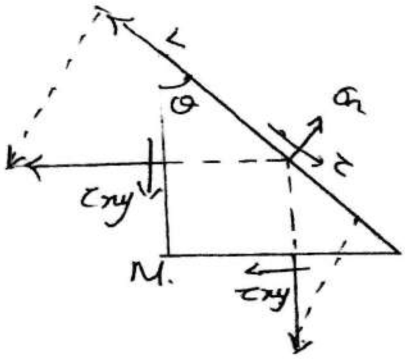
(or) $$\boxed{K = k_1 + k_2}$$

(7) marks

15)a) 10

When the stresses at some point are considered to be acting on a small triangular block at point, the a stress system will consist of direct & shearing stresses acting across the faces of the block.

Consider some plane LN at angle θ to the plane of the stress.



Resolving normal to LN:

$$\sigma_n \times LN = \tau_{xy} LM \sin\theta + \sigma_x LM \cos\theta + \tau_{xy} MN \cos\theta + \sigma_y MN \sin\theta.$$

$$\sigma_n = 2\tau_{xy} \sin\theta \cos\theta + \sigma_x \cos^2\theta + \sigma_y \sin^2\theta.$$

$$\sigma_n = \tau_{xy} \sin 2\theta + \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta \quad \text{—} (1).$$

Resolving along LN:

$$\tau \times LN = \tau_{xy} \times MN \sin\theta + \sigma_x \times LM \sin\theta - \tau_{xy} \times LM \cos\theta - \sigma_y \times MN \cos\theta.$$

$$\tau = \tau_{xy} (\sin^2\theta - \cos^2\theta) + \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta.$$

$$\tau = \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta - \tau_{xy} \cos 2\theta. \quad \text{—} (2)$$

Differentiating σ_n w.r.to θ.

$$\frac{d(\sigma_n)}{d\theta} = 2\tau_{xy} \cos 2\theta - 2\frac{(\sigma_x - \sigma_y)}{2} \sin 2\theta = 0.$$

$$\tau = 0$$

$$\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta = \tau_{xy} \cos 2\theta.$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

differentiating eqn ②. w.r.to θ & equate = 0.

$$\frac{dc}{d\theta} = \left(\frac{\sigma_x - \sigma_y}{2}\right) 2\cos 2\theta + \tau_{xy} \sin 2\theta \times 2 = 0.$$

$$\tan 2\theta = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}}.$$

$\boxed{\text{6 marks.}}$

$$\cot(180^\circ - 2\theta_1) = -\cot 2\theta_1$$

$$\cot(360^\circ - 2\theta_2) = -\cot 2\theta_2.$$

$$\cot(180^\circ - 2\theta_1) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}.$$

$$\cot(360^\circ - 2\theta_2) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}.$$

Resolving normal to LN:

$$\sigma_x \times LM + \tau_{xy} \times MN = \sigma_n\, LN \cos\theta.$$

$\div LN$ $\quad \sigma_x + \tau_{xy}\tan\theta = \sigma_n$

Resolving parallel to LM

$$\sigma_y \times MN + \tau_{xy} \times LM = \sigma_n\, LN \sin\theta.$$

$\div LN$ $\quad \sigma_y + \tau_{xy}\cot\theta = \sigma_n.$

$$\sigma_y + \tau_{xy}\cot\theta = \sigma_n$$

$$\tau_{xy}\tan\theta = \sigma_n - \sigma_x.$$

$$\tau_{xy}\cot\theta = \sigma_n - \sigma_y.$$

$$\tau^2_{xy} = (\sigma_n - \sigma_x)(\sigma_n - \sigma_y).$$

$$\tau^2_{xy} = \sigma_n^2 - (\sigma_x + \sigma_y)\sigma_n + \sigma_x\sigma_y.$$

Solving. $\quad \sigma_n = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x + \sigma_y)^2 - 4\sigma_x\sigma_y + 4\tau_{xy}^2}$

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

Major principal stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Minor Principal stress

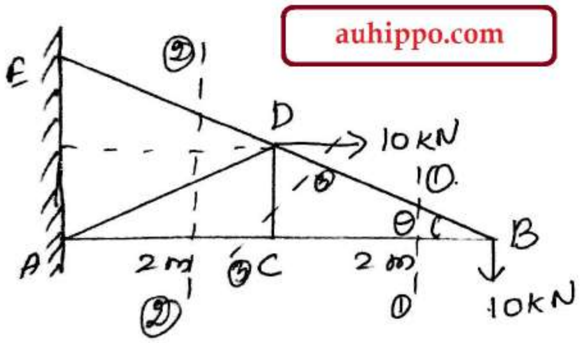
$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$;

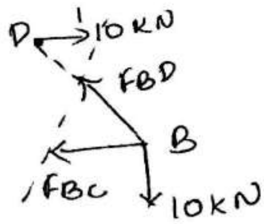
$\boxed{\text{(4) marks.}}$

15) b)



$\theta = \tan^{-1}\left(\frac{1.5}{4}\right)$

$\theta = 20.55'$



Consider section (1) - (1)

Consider the right portion of section is in equilibrium.

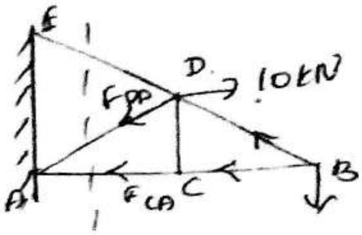
$\Sigma M_D = 0$

$10 \times 2 - F_{BC} \times 0.75 - F_{BD} \sin 20.55 = 0.$

$0.75\, F_{BC} - 0.35\, F_{BD} = 0.$

$\Sigma M_C = 0$; $10 \times 2 - F_{BD} \sin 20.55 \times 2 + 10 \times 0.7 = 0$

$$\boxed{\begin{array}{l} F_{BD} = 39.17\,kN \\ F_{BC} = 18.28\,kN \end{array}}$$ T T



$\Sigma M_D = 0$

$10 \times 2 + 18.28 \times 0.75 - 39.17 \sin 20.55 \times 2$

$+ 39.17 \cos 20.55 \times 0.75 + F_{CA}\, 0.75 = 0$

$\boxed{F_{CA} = -45.20 \text{ kN}}$ C

$\Sigma M_C = 0$

$10 \times 2 - 39.17 \sin 20.55 \times 2 + 10 \times 0.75 = 0$

$- F_{DE} \cos\theta \times 0.75 - F_{DA} \cos\theta \times 0.75$

$0.7 F_{DE} + 0.70 F_{DA} = 13.750$ — (1)

$\Sigma M_A = 0$

$10 \times 4 - 39.17 \sin 20.55 + 10 \times 0.75 - F_{DA} \sin\theta$

$- F_{DE} \sin\theta \times 2 - F_{DE} \cos\theta \times 0.75 = 0$

$1.4 F_{DE} + 0.75 F_{DA} = 33.75$ — (2).

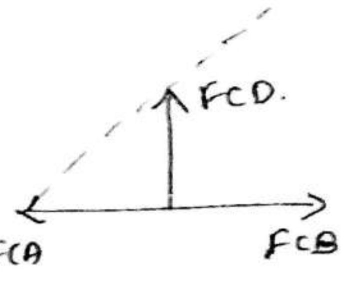
Solving (1) & (2)

$\boxed{F_{DE} = 28.57 \text{ kN}}$ T

$\boxed{F_{DA} = -6.25 \text{ kN}}$ C

$\boxed{\text{(10) marks}}$

consider section (3)-(3)



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$\Sigma M_B = 0.$

$- F_{CD} \times 2 + 10 \times 0.75$

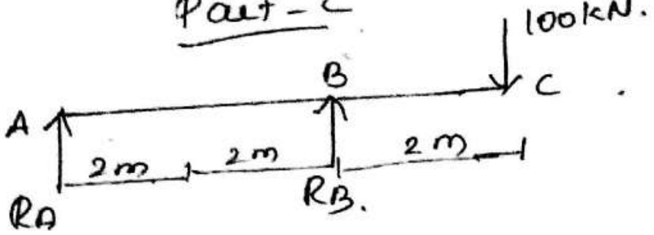
$+ 39.17 \cos\theta \times 0.75 + 39.17 \sin\theta \times 2 = 0$

$\boxed{F_{CD} = 31.25 \text{ KN}}$ T

$\boxed{\text{(3) marks}}$

16(a) Part – C $1 \times 15 = 15$ Marks.

(12)



To find reaction.

$100 \times 2 - R_A \times 4 = 0$

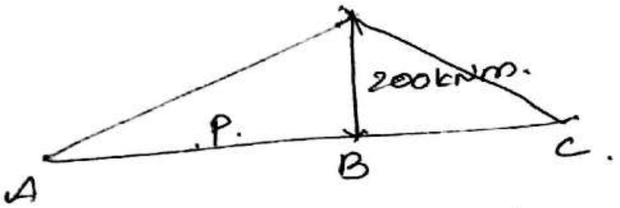
$\boxed{R_A = 50\ kN}$

$\boxed{R_B = 50\ kN.}$

(5) marks:

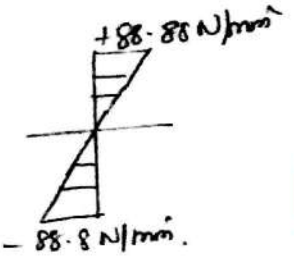
Maximum moment occurs at 'B'.

BM_{max} @ $B = 100 \times 2 = 200\ kNm.$



(5) marks

Maximum Bending stress at B. $= \frac{M}{Z}$

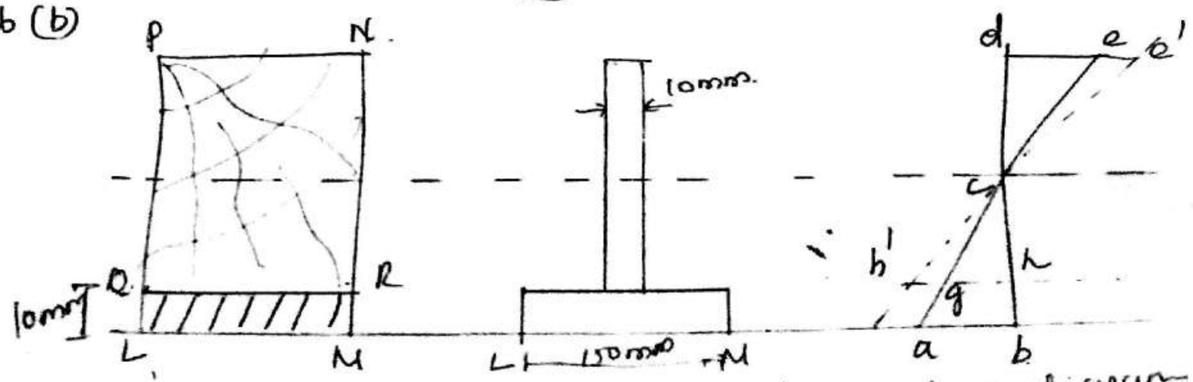


$= \frac{200 \times 10^6}{\left(\frac{150 \times 300^2}{6}\right)}$

$\sigma_{max} = \underline{88.88}\ N/mm^2$ (5) marks.

(OR)

16(b)



Given section. Equivalent section Stress diagram steel section equivalent to the given wooden section shall be 20 cm deep &

$\frac{15}{m} = \frac{15}{15} = 1$ cm wide $\quad \left(m = \frac{E_s}{E_w} = 15\right)$

To find centroid for the equivalent section.

Component	Area (a) cm^2	Distance 'y' of the centroid from LM (cm)	ay (cm^3)
Hzl rectangle	$15\times1=15$	$\frac{1}{2}=0.5$	$15\times0.5=7.5$
Vertical □le	$20\times1=20$	$\frac{20}{2}+1=11$	$20\times11=220$
Total	$\Sigma a=35$	—	$\Sigma ay=227.5$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{227.5}{35} = 6.5 \text{ cm}.$$

$$I_{xx} = \left[\frac{15\times1^3}{12} + 15\times1\times(6.5-0.5)^2\right] + \left[\frac{1\times20^3}{12} + 1\times20\times(14.5-10)^2\right]$$

$$= 541.25 + 1071.66 \quad = 1612.9 \text{ cm}^4$$

$$\boxed{I_{xx} = 1612.9\times10^{-8}\ m^4} \qquad \boxed{(9)\ \text{Marks.}}$$

$\sigma_w = 6\ MN/m^2 \quad (QR)$

Permissible stress in steel:

$\sigma_s = m\times6 = 15\times6 = 90\ MN/m^2 \quad (hh')$

$ch = 5.5$ cm & $cd = 14.5$ cm.

$de' = \frac{14.5}{5.5}\times90 = 237.3\ MN/m^2$.

Stress in timber at PN is $6\ MN/m^2$ &

Stress in timber at PN is $6\ MN/m^2$ & stress in top surface of equivalent section = 90 M

Stress d_a in top surface of equivalent section

Stress in steel at QR ⇒ $gh = \frac{de}{cd}\times ch = \frac{90}{14.5}\times5.5$

$= 34.14\ MN/m^2$

Stress in steel at LN is

$$ab = \frac{de}{cd} \times bc = \frac{90}{14.5} \times 6.5$$

$$= 40.34 \ MN/m^2.$$

Moment of resistance.

$$M = 90 \times \frac{I}{y}$$

$$= 90 \times \frac{1612.9 \times 10^{-8}}{0.21 - 0.065}$$

$$\boxed{M = 0.01 \ MNm \ \text{(or)} \ 10 kNm.}$$

8 Marks