EAMCET 2014

ENGINEERING

Question Paper with Solutions

CODE-A

MATHS

1. If R is the set of all real numbers and if $f: R-\{2\} \to R$ is defined by $f(x)=\frac{2+x}{2-x}$ for $x \in R-\{2\}$, then the range of f is

1) R 2) $R-\{1\}$ 3) $R-\{-1\}$ 4) $R-\{-2\}$

Key: 3

Sol: $\frac{y}{1}=\frac{2+x}{2-x}$

$2+x=2y-xy$

$x+xy=2(y-1)$

$x(1+y)=2(y-1)$

$2=2\left[\frac{(y-1)}{y+1}\right]$

$\therefore$ Range $=R-\{-1\}$

2. Let Q be the set of all rational numbers in $[0,1]$ and $f:[0,1]\to[0,1]$ be defined by

$$f(x)=\begin{cases} x & \text{for } x \in Q \\ 1-x & \text{for } x \notin Q \end{cases}.$$

Then the set $S=\{x \in [0,1] : (fof)(x)=x\}$ is equal to

1) Q 2) $[0,1]-Q$ 3) $(0,1)$ 4) $[0,1]$

Key: 4

Sol: $f=f^{-1}$

3. $\sum_{k=1}^{2n+1}(-1)^{k-1}.k^2=$

1) $(n+1)(2n+1)$ 2) $(n+1)(2n-1)$ 3) $(n-1)(2n+1)$ 4) $(n-1)(2n-1)$

Key: 1

Sol: $.\sum_{k=1}^{3}(-1)^{k-1}k^2$

$=1+(-1)^{2-1}(4)+(-1)^{3-1}3^2$

$=1-4+9=6$

$2.3=6$

4. If a,b,c and d are real numbers such that $a^2+b^2+c^2=d^2=1$ and if $A=\begin{bmatrix} a+ib & c+id \\ -c+id & a-ib \end{bmatrix}$ then

$A^{-1}=$

1) $\begin{bmatrix} a-ib & c+id \\ -c+id & a+ib \end{bmatrix}$ 2) $\begin{bmatrix} a-ib & -c-id \\ c-id & a+ib \end{bmatrix}$ 3) $\begin{bmatrix} a+ib & c+id \\ c-id & a-ib \end{bmatrix}$ 4) $\begin{bmatrix} a+ib & -c-id \\ c-id & a-ib \end{bmatrix}$

Key: 2

Sol: . $|A| = \frac{1}{(a+ib)(a-ib)+(c-id)(c+id)}$

$= \frac{1}{a^2+b^2+c^2+d^2} = 1$

$A^{-1}\begin{bmatrix} a-ib & -c-id \\ c-id & a+ib \end{bmatrix}$

5. **If the matrix** $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & \alpha \end{bmatrix}$ **is of rank 3, then** $\alpha =$

1) 5 2) 4 3) 1 4) -5

Key: 1

Sol: $|A| \neq 0$ then $\alpha = 5$

6. **If** $k > 1$**, and the determinant of the matrix** A^2**, where** $A = \begin{bmatrix} k & k\alpha & \alpha \\ 0 & \alpha & k\alpha \\ 0 & 0 & k \end{bmatrix}$**, is** k^2 **then** $|\alpha| =$

1) k 2) k^2 3) $\frac{1}{k}$ 4) $\frac{1}{k^2}$

Key: 3

Sol: $|A| = k(k\alpha - 0) = k^2\alpha$

$\begin{bmatrix} k & k\alpha & \alpha \\ 0 & \alpha & k\alpha \\ 0 & 0 & k \end{bmatrix}\begin{bmatrix} k & k\alpha & \alpha \\ 0 & \alpha & k\alpha \\ 0 & 0 & k \end{bmatrix}$

7. **The number of solutions for** $z^3 + \bar{z} = 0$ **is**

1) 1 2) 2 3) 3 4) 5

Key: 4

Sol: $z^3 + \bar{z} = 0$

$(x+iy)^3 = -x+iy$

$x^3 = i^3y^3 + 3x^2iy + 3xi^2y^2 = -x+iy$

$x^3 - iy^3 + 3x^2iy - 3xy^2 = -x+iy$

$\left(x^3 - 3xy^2\right) + i\left(3x^2y - y^3\right) = -x+iy$

$x^3 - 3xy^2 = -x$

$x^2 - 3y^2 = -1$

$x^3 - 3(3x^2 - 1) = -1$

$x^2 - 9x^2 + 3 + 1 = 0$

$-8x^2 + 4 = 0$

$x^2 = \frac{1}{2}$

$x = \pm\frac{1}{2}$

$3x^2y - y^3 = y$

$3x^2 - y^2 = 1$

$y^2 = 3x^2 - 1$

$x = \frac{1}{\sqrt{2}}$

$y^2 = \frac{3}{2} - 1 = \frac{1}{2}$

$y = \pm\frac{1}{\sqrt{2}}$

$x = -\frac{1}{\sqrt{2}}$

$y^2 = \frac{3}{2} - 1 = \frac{1}{2}$

$y = \pm\frac{1}{\sqrt{2}}$

8. The least positive integer n for which $(1+i)^n = (1-i)^n$ holds is

1) 2 2) 4 3) 6 4) 8

Key: 2

Sol: $(1+i)^n = (\sqrt{2})^n\left(\frac{1}{\sqrt{2}} + i - \frac{1}{\sqrt{2}}\right)^n = (\sqrt{2})^n \operatorname{cis}\frac{n\pi}{4}$

$= 2^{\frac{n}{2}} \operatorname{cis}\left(-\frac{n\pi}{4}\right)$

$\operatorname{cis}\frac{n\pi}{4} = \operatorname{cis}\left(\frac{-n\pi}{4}\right)$

$2i\sin\frac{n\pi}{4} = 0$

$\sin\frac{n\pi}{4} = 0$

$n = 4$

9. If $x = p + q, y = pw + qw^2$ and $z = pw^2 + qw$ where w is a complex cube root of unity then $xyz =$

1) $p^2 - pq + q^2$ 2) $1 + p^3 + q^3$ 3) $p^3 - q^3$ 4) $p^3 + q^3$

Key:4

Sol: $p = q = 1$

$x = 2$

$y = w + w^2$

$z = w^2 + w$

$xyz = 2(-1)(-1) = 2$

10. **If** $Z_r = \cos\left(\frac{\pi}{2^r}\right) + i\sin\left(\frac{\pi}{2^r}\right)$ **for** $r = 1, 2, 3,$ ..... **then** $Z_1 Z_2 Z_3 ..... \infty =$

1) 1 2) 2 3) -1 4) -2

Key:10

Sol: $z_r = cis\left(\frac{\pi}{2^r}\right)$

$z_1.z_2 ....\infty = cis\left(\frac{\pi}{2} + \frac{\pi}{2^2} + ...\infty\right)$

$cis\pi = -1$

11. **If** x_1 **and** x_2 **are the real roots of the equation** $x^2 - kx + c = 0$ **then the distance betwen the points** $A(x_1, 0)$ **and** $B(x_1, 0)$ **is**

1) $\sqrt{k^2 - c}$ 2) $\sqrt{c - k^2}$ 3) $\sqrt{k^2 - 4c}$ 4) $\sqrt{k^2 + 4c}$

Key: 3

Sol: $x_1 + x_2 = k$

$x_1 x_2 = c$

$|x_1 - x_2| = \sqrt{(x_1 + x_2)^2 - 4x_1 x_2}$

$= \sqrt{k^2 - 4c}$

12. **If** x **is real, then the minimum value of** $y = \frac{x^2 - x + 1}{x^2 + x + 1}$ **is**

1) $\frac{1}{3}$ 2) $\frac{1}{2}$ 3) 2 4) 3

Key: 1

Sol: $.\frac{y}{1} = \frac{x^2 - x + 1}{x^2 + x + 1}$

$x^2 - x + 1 = x^2 y + xy + y$

$(1 - y)x^2 (-1 - y)x + (1 - y) = 0$

$(1 + y)^2 - 4(1 - y)^2 > 0$

$1 + y^2 + 2y - 4(1 + y^2 - 2y) > 0$

$1 + y^2 + 2y - 4 - 4y^2 + 8y \geq 0$

$-3y^2 + 10y - 3 \geq 0$

$3y^2 - 10y + 3 < 0$

$3y^2 - 9y - y + 3 < 0$

$3y(y-3) - 1(y-3) \le 0$

$y \in \left[\frac{1}{3}, 3\right]$

$\therefore 3$

13. If p and q are distinct prime numbers and if the equation $x^2 - px + q = 0$ has positive integers as its roots then the roots of the equation are

1) 2, 3 2) 1, 2 3) 3, 1 4) 1, -1

Key: 2

Sol: .Sum of the roots $\alpha + \beta = p$

Product of the roots $\alpha\beta = q$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1, x = 2$

14. The cubic eqation whose roots are the squares of the roots of $x^3 - 2x^2 + 10x - 8 = 0$ is

1) $x^3 + 8x^2 + 68x - 64 = 0$ 2) $x^3 + 16x^2 - 68x - 64 = 0$

3) $x^3 - 16x^2 + 68x - 64 = 0$ 4) $x^3 + 16x^2 + 68x - 64 = 0$

Key: 3

Sol: .Let $y = \sqrt{x}$

$x\sqrt{x} - 2x + 10\sqrt{x} - 8 = 0$

$\sqrt{x}(x+10) = (2x+8)$

$\Rightarrow x(x+10)^2 = (2x+8)^2$

$\Rightarrow x(x^2 + 20x + 100) = 4x^2 + 64 + 32x$

$\Rightarrow x^3 + 20x^2 + 100x - 4x^2 - 32x - 64 = 0$

$\Rightarrow x^3 + 16x^2 + 68x - 64 = 0$

15. Out of thirty points in a plane, eight of them are collinear. The number of straight lines that can be formed by joining these points is

1) 540 2) 408 3) 348 4) 296

Key: 2

Sol: $30c_2 - 8c_2 + 1$

$= \frac{30 \times 29}{2} - \frac{8 \times 7}{2} + 1$

$= 15 \times 29 - 28 + 1$

$= 435 - 28 + 1$

$= 436 - 28$

$= 408$

16. If n is an integer with $0 \le n \le 11$ then the minimum value of $n!(11-n)!$ is attained when a value of $n =$

1) 5 2) 7 3) 9 4) 11

Key: 1

Sol: . $^{11}C_n = \frac{11!}{(11-n)!n!}$

minimum value $= \frac{^nC_{n-1}}{2}$

$^{11}C_n = {}^{11}C_5$

$\therefore \; n = 5$

17. If $(a+bx)^{-3} = \frac{1}{27} + \frac{1}{3}x + ...$ then the ordered pair $(a,b) =$

1) $\left(1, \frac{1}{3}\right)$ 2) $(3,9)$ 3) $(3,-9)$ 4) $(3,-27)$

Key: 3

Sol: . $\frac{1}{a^3}\left[1+\frac{b}{a}x\right]^{-3} = \frac{1}{27} + \frac{x}{3} + ...$

$\Rightarrow \frac{1}{a^3}\left[1-\frac{3b}{a}x+......\right] = \frac{1}{27} + \frac{x}{3}$

$\Rightarrow \frac{1}{a^3} = \frac{1}{27} = a = 3$

$-\frac{3b}{a^4}x = \frac{x}{3} = -\frac{3b}{27} = \frac{1}{3} \quad b = -9$

$\therefore (3,-9)$

18. The term independent of x in the expansion of $\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)^{18}$ is

1) $\binom{18}{9}2^{12}$ 2) $\binom{18}{6}2^{6}$ 3) $\binom{18}{6}2^{8}$ 4) $-\binom{18}{9}2^{9}$

Key:4

Sol: . $\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)^{18}$

independent term exist in T_{r+1}

where $r = \frac{np}{p+q} = \frac{18 \times \frac{1}{2}}{1} = 9$

$T_{9+1} = 18C_9(-2)^9$

19. $\frac{2x^3+x^2-5}{x^4-25}=\frac{Ax+B}{x^2-5}+\frac{Cx+1}{x^2+5}\Rightarrow(A,B,C)=$

1) $(1,1,0)$ 2) $(1,0,1)$ 3) $(1,2,1)$ 4) $(1,1,1)$

Key: 2

Sol: . $\frac{2x^3+x^2-5}{x^4-25}=\frac{(Ax+B)(x^2+y^2)+(cx+1)(x^2-5)}{x^4-25}$

$A+C=2\rightarrow(1)$

$5A-5C=0\rightarrow(2)$

$A+C=2$

$\underline{A-C=0}$

$2A=2, A=1,\ C=1$

Compare x^2

$B+1=1$ $B=0$

$(1,0,1)$

20. **If** $\cos x=\tan y, \cot y=\tan z$ **and** $\cot z=\tan x$ **; then** $\sin x=$

1) $\frac{\sqrt{5}-1}{4}$ 2) $\frac{\sqrt{5}+1}{2}$ 3) $\frac{\sqrt{5}-1}{2}$ 4) $\frac{\sqrt{5}+1}{4}$

Key: 3

Sol: . $\cos x=\tan y=\frac{1}{\cot y}=\frac{1}{\tan z}$

$\cos x=\tan x$

$\cos x-\tan x=0$

$\cos x-\frac{\sin x}{\cos x}=0$

$\cos x-\sin x=0$

$1-\sin x-\sin x=0$

$\sin x+\sin x-1=0$

$\sin x=\frac{-1+\sqrt{1+4}}{2}=\frac{\sqrt{5}-1}{2}$

21. $\tan 81^0-\tan 63^0-\tan 27^0+\tan 9^0=$

1) 0 2) 2 3) 4 4) 6

Key: 3

Sol: . $(\cot 9^0+\tan 9^0)-(\cot 27^0+\tan 27^0)$

$=\frac{2}{\sin 18^0}-\frac{2}{\sin(54^0)}=4$

22. If x and y are acute angles such that $\cos x+\cos y=\frac{3}{2}$ and $\sin x+\sin y=\frac{3}{4}$ then $\sin(x+y)=$

1) $\frac{3}{4}$ 2) $\frac{3}{5}$ 3) $\frac{4}{5}$ 4) $\frac{2}{5}$

Key: 3

Sol: . $\frac{\sin x+\sin y}{\cos x+\cos y}=\frac{\frac{3}{4}}{\frac{3}{2}}=\frac{3}{4}\times\frac{2}{3}=\frac{1}{2}$

$$\Rightarrow \frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}=\frac{1}{2}$$

$$\tan\left(\frac{x+y}{2}\right)=\frac{1}{2}$$

$$\sin(x+y)=\frac{2\tan\left(\frac{x+y}{2}\right)}{1+\tan^2\left(\frac{x+y}{2}\right)}$$

$$=\frac{2\times\frac{1}{2}}{1+\frac{1}{4}}=\frac{1}{\left(\frac{5}{4}\right)}=\frac{4}{5}$$

23. The sum of the solutions in $(0,2\pi)$ of the equation $\cos x\cos\left(\frac{\pi}{3}-x\right)\cos\left(\frac{\pi}{3}+x\right)=\frac{1}{4}$ is

1) π 2) 2π 3) 3π 4) 4π

Key: 2

Sol: . $\frac{1}{4}\cos(3x)=\frac{1}{4}$

$\cos 3x=1$

$\Rightarrow \cos 3x=\cos(2n\pi)$

$3x=2n\pi$

$x=\frac{2n\pi}{3}$

Solutions $\frac{2\pi}{3},\frac{4\pi}{3}$

Sum $= =\frac{6\pi}{3}=2\pi$

24. **If** $x > 0, y > 0, z > 0, xy + yz + zx < 1$ **and if** $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$ **then** $x + y + z =$

1) xyz 2) $3xyz$ 3) $\sqrt{xyz}$ 4) 0

Key: 1

Sol: .Given $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$

$x + y = z = xyz$

25. $\sec h^{-1}\left(\frac{1}{2}\right) - \operatorname{cosech}^{-1}\left(\frac{3}{4}\right) =$

1) $\log_e\left(\frac{1+\sqrt{3}}{3}\right)$ 2) $\log_e\left(\frac{2+\sqrt{3}}{3}\right)$ 3) $\log_e\left(\frac{2-\sqrt{3}}{3}\right)$ 4) $\log_e\left(3\left(2+\sqrt{3}\right)\right)$

Key: 2

Sol: . $\cosh^{-1}(2) - \sin^{-1}\left(\frac{4}{3}\right)$

$\log\left(2+\sqrt{3}\right) - \log\left(\frac{4}{3} + \sqrt{\frac{16}{9} + 1}\right)$

$\log\left(2+\sqrt{3}\right) - \log\left(\frac{4}{3} + \frac{5}{3}\right)$

$= \log\left(2+\sqrt{3}\right)\log 3$

$= \log\left(\frac{2+\sqrt{3}}{3}\right)$

26. **If any** $\Delta ABC, \frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2} =$

1) $\cos^2 A$ 2) $\cos^2 B$ 3) $\sin^2 A$ 4) $\sin^2 B$

Key: 3

Sol: . $\frac{(25)2(s-a)\times 2(s-b)\times 2(s-c)}{4b^2c^2}$

$= \frac{8s(s-a)(s-b)(s-c)}{4b^2c^2}$

$= \frac{8\Delta^2}{4b^2c^2}$ $\Delta = \frac{1}{2}bc\sin A$

$= \frac{8 \times \frac{1}{2}b^2c^2\sin^2 A}{4b^2c}$

$= \sin^2 A$

27. **If the angles of triangle are in the ratio** $1:1:4$ **then the ratio of the perimeter of the triangle to its largest side is**

1) $3:2$ 2) $\sqrt{3}+2:\sqrt{2}$ 3) $\sqrt{3}+2:\sqrt{3}$ 4) $\sqrt{2}+2:\sqrt{3}$

Key: 2

Sol: . $\alpha+\alpha+4\alpha=180^0$

$6\alpha=180^0$

$\alpha=30^0$

$30^0,30^0,120^0$

$a=2R\sin 30^0=R$

$b=2R\sin 30^0=R$

$c=2R\sin\left(120^0\right)=\sqrt{3}R$

$2S:C=2R+\sqrt{3}R:\sqrt{3}R$

$=2+\sqrt{3}:\sqrt{3}$

28. **If in a triangle ABC,** $r_1=2, r_2=3$ **and** $r_3=6$ **then** $a=$

1) 1 2) 2 3) 3 4) 4

Key: 3

Sol: . $\frac{1}{a}=\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}=1$

$\Delta=1$

$\Delta=\sqrt{r_1r_2r_3r}=6$

$S=\frac{\Delta}{a}=6$

$\Delta_1=\frac{\Delta}{S-a}\Rightarrow a=3$

29. **Three non-zero non-collinear vectors** $\bar{a},\bar{b},\bar{c}$ **are such that** $\bar{a}+3\bar{b}$ **is collinear with** $\bar{c}$**, while** $3\bar{b}+2\bar{c}$ **is** collinear with $\bar{a}$. Then $3\bar{a}+2\bar{c}=$

1) $2\bar{a}$ 2) $3\bar{b}$ 3) $4\bar{c}$ 4) $\bar{0}$

Key: 4

Sol: . $a+3\bar{b}=\lambda\left(\bar{c}\right)\rightarrow(1)$

$3\bar{b}+2\bar{c}=\mu\left(\bar{a}\right)\rightarrow(2)$

$\bar{a}+3\bar{b}+2\bar{c}=\lambda\bar{c}+2\bar{c}=\left(\lambda+2\right)\bar{c}$

$\bar{a}+\mu\bar{a}=\bar{a}+3\bar{b}+2\bar{c}$

$\left(\lambda+2\right)\bar{c}=\left(1+\mu\right)\bar{a}$

$\lambda+2=0, \mu+1=0$

$\lambda=-2, \mu=-1$; $\Rightarrow\bar{a}+3\bar{b}+2\bar{c}=\bar{0}$

30. **If** $\bar{a},\bar{b}$ **and** $\bar{c}$ **are non-coplanar vectors and if** $\bar{d}$ **is such that** $\bar{d}=\frac{1}{x}\left(\bar{a}+\bar{b}+\bar{c}\right)$ **and** $\bar{a}=\frac{1}{y}\left(\bar{b}+\bar{c}+\bar{d}\right)$ **where** x **and** y **are non-zero real numbers, then** $\frac{1}{xy}\left(\bar{a}+\bar{b}+\bar{c}+\bar{d}\right)=$

1) $-\bar{a}$ 2) $\bar{0}$ 3) $2\bar{a}$ 4) $3\bar{c}$

Key: 2

Sol: By eliminating $\bar{d}$

Find x & y

31. **The angle between the lines** $\bar{r}=\left(2\bar{i}-3\bar{j}+\bar{k}\right)+\lambda\left(\bar{i}+4\bar{j}+3\bar{k}\right)$ **and** $\bar{r}=\left(\bar{i}-\bar{j}+2\bar{k}\right)+\mu\left(\bar{i}+2\bar{j}-3\bar{k}\right)$ **is**

1) $\cos^{-1}\left(\frac{9}{\sqrt{91}}\right)$ 2) $\cos^{-1}\left(\frac{7}{\sqrt{84}}\right)$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

Key: 4

Sol: . $\cos = \dfrac{1+8-9}{\sqrt{1+16+9}\sqrt{1+4+9}} = 0$

$\theta = \dfrac{\pi}{2}$

32. If $\bar{a}, \bar{b}$ and $\bar{c}$ are vectors with magnitudes 2, 3 and 4 respectively then the best upper bound of $|\bar{a}-\bar{b}|^2 + |\bar{b}-\bar{c}|^2 + |\bar{c}-\bar{a}|^2$ among the given values is

1) 97 2) 87 3) 90 4) 93

Key:2

Sol: $|\bar{a}| = 2, |\bar{b}| = 3, |\bar{c}| = 4$

$2\left(|\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 - (\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a})\right)$

$= 2(29 - \{6\cos A + 12.\cos B + 8\cos C\})$

$= 2[29 - (26)]$

33. If x, y, z are non-zero real numbers, $\bar{a} = x\bar{i} + 2\bar{j}, \bar{b} = y\bar{j} + 3\bar{k}$ and $\bar{c} = x\bar{i} + y\bar{j} + z\bar{k}$ are such that $\bar{a} \times \bar{b} = 2\bar{i} - 3\bar{j} + \bar{k}$ then $[\bar{a}\,\bar{b}\,\bar{c}] =$

1) 10 2) 9 3) 6 4) 3

Key: 2

Sol: . $\bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x & 2 & 0 \\ 0 & y & 3 \end{vmatrix}$

$2\bar{i} - 3\bar{j} + \bar{k} = 6\bar{i} + 3x\bar{j} + \bar{k}(xy)$

$2 = 6, x = 1, y = 1$

$[\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 1 & 1 & 6 \end{vmatrix}$

$= 1(6-3) - 2(-3)$

$= 3 + 6 = 9$

34. The shortest distance between the skew lines $\bar{r} = (\bar{i} + 2\bar{j} + 3\bar{k}) + t(\bar{i} + 3\bar{j} + 2\bar{k})$ and $\bar{r} = (4\bar{i} + 5\bar{j} + 6\bar{k}) + t(2\bar{i} + 3\bar{j} + \bar{k})$ is

1) 3 2) $2\sqrt{3}$ 3) $\sqrt{3}$ 4) $\sqrt{6}$

Key: 3

Sol: . $\frac{\left[\bar{a}-\bar{c}\ \bar{b}\ \bar{d}\right]}{\left|\bar{b}\times\bar{d}\right|}$

$$\bar{b}\times\bar{d}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & 3 & 2\\ 2 & 3 & 1\end{vmatrix}$$

$=\bar{i}(-3)-\bar{j}(-3)+\bar{k}(-3)$

$=-3\bar{i}+3\bar{j}-3\bar{k}$

$\left|\bar{b}\times\bar{d}\right|=\sqrt{9+9+9}=\sqrt{27}-3\sqrt{3}$

$\left[\bar{a}-\bar{c}\ \bar{b}\ \bar{d}\right]=\left(-3\bar{i}-3\bar{j}-3\bar{k}\right).\left(-3\bar{i}+3\bar{j}-3\bar{k}\right)$

$=9-9+9=9$

$\therefore \text{S.D.}=\frac{9}{3\sqrt{3}}=\frac{3}{\sqrt{3}}=\sqrt{3}$

35. The mean of four observations is 3. If the sum of the squares of these observations is 48 then their standard deviation is

1) $\sqrt{2}$ 2) $\sqrt{3}$ 3) $\sqrt{5}$ 4) $\sqrt{7}$

Key: 2

Sol: $x_1+x_2+x_3+x_4=12$ -----(1)

$x_1^2+x_2^2+x_3^3+x_4^4=48$

$\text{S.D.}=\sqrt{\frac{\Sigma x_i^2}{n}-\mu^2}$

$=\sqrt{\frac{48}{4}-9}$

$=\sqrt{12-9}$

$=\sqrt{3}$

36. If $x_1, x_2, ...x_n$ are n observations such that $\sum_{i=1}^{n} x_i^2=400$ and $\sum_{i=1}^{n} x_i=80$ then the least value of n is

1) 12 2) 15 3) 16 4) 18

Key:3

Sol: Variance ≥ 0 and mean > variance

variance $=\Sigma\frac{Xi^2}{n}-\bar{x}^2$, mean $=\Sigma\frac{xi}{n}$

37. If A, B and C are mutually exclusive and exhaustive events of a random experiment such that $P(B)=\frac{3}{2}P(A)$ and $P(C)=\frac{1}{2}P(B)$ then $P(A\cup C)=$

1) $\frac{3}{13}$ 2) $\frac{6}{13}$ 3) $\frac{7}{13}$ 4) $\frac{10}{13}$

Key:3

Sol: $P(A)+P(B)=P(C)=1$

38. A six-faced unbaised die is thrown twice and the sum of the numbers appearing on the upper face is observed to be 7. The probability that the number 3 has appeared at least once is

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) $\frac{1}{5}$

Key:2

Sol: Use conditional probability

$$P\left(\frac{A}{B}\right) = P\left(\frac{A \cap B}{B}\right)$$

39. A candidate takes three tests in succession and the probability of passing the first test is p. The probability of passing each succeeding test is p or $\frac{p}{2}$ according as he passes or fails in the preceding one. The candidate is selected if he ;passes at least two tests. The probability that the candidate is selected is

1) $p(2-p)$ 2) $p+p^2+p^3$ 3) $p^2(1-p)$ 4) $p^2(2-p)$

Key:4

Sol: Required events can occured in the following mutually exclusive ways.

SSS, SSF, SFS, FSS

40. A random variable X has the probability distribution given below. Its variance is

X	1	2	3	4	5
$P(X=x)$	K	2k	3k	2k	k

1) $\frac{4}{3}$ 2) $\frac{5}{3}$ 3) $\frac{10}{3}$ 4) $\frac{16}{3}$

Key:1

Sol: Variance = $\Sigma xi^2 P(x = xi^2) - (\bar{x})^2$

41. If the mean and variance of a binomial variate X are 8 and 4 respectively then $P(X<3)=$

1) $\frac{137}{2^{16}}$ 2) $\frac{697}{2^{16}}$ 3) $\frac{265}{2^{16}}$ 4) $\frac{265}{2^{15}}$

Key:1

Sol: $npq = 4$, $np = 8$

$q = \frac{1}{2}, p = \frac{1}{2}$

$n = 16$

$$P(x<3) = \frac{^{16}C_0 + {}^{16}C_1 + {}^{16}C_2}{2^{16}}$$

42. The locus of the centroid of the triangle with vertices at $(a\cos\theta, a\sin\theta), (b\sin\theta, -b\cos\theta)$ and $(1,0)$ is (Here θ is a parameter)

1) $(3x-1)^2 + 9y^2 = a^2 - b^2$ 2) $(3x-1)^2 + 9y^2 = a^2 + b^2$

3) $(3x+1)^2 + 9y^2 = a^2 - b^2$ 4) $(3x+1)^2 + 9y^2 = a^2 + b^2$

Key:2

Sol: $a\cos\theta + b\sin\theta + 1 = 3x$,

$a\sin\theta - b\cos\theta = 3y$

eliminate θ.

43. The point $P(1,3)$ undergoes the following transformations successively

(i) Reflection with respect to the line $y = x$

(ii) Translation through 3 units along the positive direction of the X-axis

(iii) Rotation through an angle of $\frac{\pi}{6}$ about the origin in the clockwise direction

The final position of the point P is

1) $\left(\frac{7}{\sqrt{2}}, \frac{-5}{\sqrt{2}}\right)$ 2) $\left(\frac{6+\sqrt{3}}{2}, \frac{1-6\sqrt{3}}{2}\right)$ 3) $\left(\frac{6\sqrt{3}-1}{2}, \frac{6+\sqrt{3}}{2}\right)$ 4) $\left(\frac{6\sqrt{3}+1}{2}, \frac{\sqrt{3}-6}{2}\right)$

Key:3

Sol: Image of $(1,3)$ w.r.t. x-axis is $(3,1)$

Translation through 3 units along the positive directiion of the X-axis is $(6,1)$

$$\left(r\cos\left(\theta-\frac{\pi}{6}\right), r\sin\left(\theta-\frac{\pi}{6}\right)\right)$$

$$r = \sqrt{37}, \cos\theta = \frac{6}{\sqrt{37}}, \sin\theta = \frac{1}{\sqrt{37}}$$

44. The equation of a straight line, perpendicular to $3x-4y=6$ and forming a triangle of area 6 squares units with coordinate axes, is

1) $4x+3y=12$ **2)** $4x+3y+24=0$ **3)** $3x+4y=12$ **4)** $x-2y=6$

Key:1

Sol: Perpendicular line is $4x+3y+k=0$

$$=\frac{1}{2}\frac{|c^2|}{|ab|}=6$$

45. If the image of $\left(-\frac{7}{5}, -\frac{6}{5}\right)$ in a line is $(1,2)$, then the equation of the line is

1) $3x-y=0$ 2) $4x-y=0$ 3) $3x+4y=1$ 4) $4x+3y=1$

Key:3

Sol: Find perpendicular bisector of line segment joining of two points

46. If a line l passes through $(k,2k),(3k,3k)$ and $(3,1), k \neq 0$, then the distance from the origin to the line l is

1) $\frac{4}{\sqrt{5}}$ 2) $\frac{3}{\sqrt{5}}$ 3) $\frac{2}{\sqrt{5}}$ 4) $\frac{1}{\sqrt{5}}$

Key:4

Sol: .Find 'K' using colliniarity

47. The area (in square units) of the triangle formed by the lines $x^2-3xy+y^2=0$ and $x+y+1=0$

1) $\frac{\sqrt{3}}{2}$ 2) $5\sqrt{2}$ 3) $\frac{1}{2\sqrt{5}}$ 4) $\frac{2}{\sqrt{3}}$

Key:3

Sol: Area $=\left|\frac{n^2\sqrt{h^2-ab}}{am^2-2hlm+bl^2}\right|$

48. **If** $x^2+\alpha y^2+2\beta y=a^2$ **represents a pair of perpendicular lines, then** $\beta=$

1) a
2) 2a
3) 3a
4) 4a

Key:1

Sol: $\alpha+1=0$

$\Delta=0$

49. **A circle with centre at (2, 4) is such that the line $x+y+2=0$ cuts a chord of length 6. The radius of the circle is**

1) $\sqrt{11}$
2) $\sqrt{21}$
3) $\sqrt{31}$
4) $\sqrt{41}$

Key:4

Sol: Length of Chord = $2\sqrt{r^2-d^2}=6$

$d=$perpendicular distance from centre to chord.

50. **The point at which the circles** $x^2+y^2-4x-4y+7=0$ **and** $x^2+y^2-4x-4y+7=0$ **touch each other is**

1) $\left(\frac{2}{5},\frac{5}{6}\right)$
2) $\left(\frac{14}{5},\frac{13}{6}\right)$
3) $\left(\frac{12}{5},2+\frac{\sqrt{21}}{5}\right)$
4) $\left(\frac{13}{5},\frac{14}{5}\right)$

Key:2

Sol: Point of contact divides c_1c_2 in the ratio $r_1:r_2$ internally

51. **The condition for the lines** $lx+my+n=0$ **and** $l_1x+m_1y+n_1=0$ **to be conjugate with aspect to the circle** $x^2+y^2=r^2$ **is**

1) $r^2(ll_1-mm_1)=nn_1$
2) $r^2(ll_1+mm_1)+nn_1=0$
3) $r^2(lm_1+l_1m)=nn_1$
4) $r^2(ll_1+mm_1)=nn_1$

Key:4

Sol: Conceptual

52. **The length of the common chord of the two circles** $x^2+y^2-4y=0$ **and** $x^2+y^2-8x-4y+11=0$ **is**

1) $\frac{\sqrt{11}}{2}$
2) $\sqrt{135}$
3) $\frac{\sqrt{135}}{4}$
4) $\frac{\sqrt{145}}{4}$

Key:3

Sol: Common chord $S-S^1=0$

then apply $2\sqrt{r^2-d^2}$

53. **The locus of the centre of the circle which cuts the circle** $x^2+y^2-20x+4=0$ **orthogonally and touches the line x = 2 is**

1) $y^2=4x$
2) $y^2=16x$
3) $x^2=4y$
4) $x^2=16y$

Key:2

Sol: $d^2 = r_1^2 + r_2^2$

$r_1 = 96,\ r_2 = |x_1 - 2|$

$\therefore d = \sqrt{(x_1 - 10)^2 + y_1^2}$

54. **If a normal chord at a point t on the parabola $y^2 = 4ax$ subtends a right angle at the vertex, then t =**

1) $\sqrt{2}$ 2) 2 3) $\sqrt{3}$ 4) 1

Key:1

Sol: $t_1 t_2 = -4$

$t_2 = -t_1 - \frac{2}{t_1}$

55. **The slopes of the focal chords of the parabola $y^2 = 32x$ which are tangents tot he circle $x^2 + y^2 = 4$ are**

1) $\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$ 2) $\frac{1}{\sqrt{15}}, \frac{-1}{\sqrt{15}}$ 3) $\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}$ 4) $\frac{1}{2}, \frac{-1}{2}$

Key:2

Sol: $2x - (t_1 + t_2)y - 2a = 0$

where $a = 8$

apply tangency conditions

56. **If tangents are drawn from any point on the circle $x^2 + y^2 = 25$ to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ then the angle between the tangents is**

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) $2\frac{\pi}{3}$

Key:3

Sol: $x^2 + y^2 = 25$ is the director circle

57. **An ellipse passing through $(4\sqrt{2}, 2\sqrt{6})$ has foci at (–4, 0) and (4, 0). Its eccentricity is**

1) $\frac{1}{2}$ 2) $\frac{1}{\sqrt{2}}$ 3) $\frac{1}{\sqrt{3}}$ 4) $\sqrt{2}$

Key:1

Sol: $SS^1 = 2ae$

$SC + S'C = 2a$

58. **A hyperbola pases through a focus of the ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$. its transverse and conjugate axes coincide respectively with the major amd ,ompr axes of the ellipse. The product of eccentricities is 1. Then the equation of the hyperbola is**

1) $\frac{x^2}{169} - \frac{y^2}{25} = 1$ 2) $\frac{x^2}{144} - \frac{y^2}{25} = 1$ 3) $\frac{x^2}{25} - \frac{y^2}{9} = 1$ 4) $\frac{x^2}{144} - \frac{y^2}{9} = 1$

Key:2

Sol: $e_1 = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$

$e_2 = \sqrt{1 + \frac{9}{144}} = \frac{13}{12}$

$e_1 . e_2 = 1$

59. If the line joining A(1, 3, 4) and B is divided by the point (–2, 3, 5) in the ratio 1:3, then B is

1) (–11, 3, –8) 2) (–8, 12, 20) 3) (13, 6, –13) 4) (–11, 3, 8)

Key: 4

Sol: A———————————B

$\lambda = x_1 ; x_2 - \lambda = 1:3$

$(-2-1); x_2 + 2 = 1:3$

$\frac{-3}{x_2 + 2} = \frac{1}{3}$

$x_2 + 2 = -9$

$x_2 = -11$

$\frac{\beta - y_1}{y_2 - \beta} = \frac{1}{3}$

$\frac{3-3}{y_2 - 3}$

$y_2 = 3$

$\left(-11, 3, \frac{r - Z_1}{Z_2 - r}\right) = \frac{1}{3}$

$\frac{5-4}{Z_2 - 5} = \frac{1}{3}$

$3 = Z_2 - 5$

$Z_2 = 8$

60. If the direction cosines of two lines are given by $l + m + n = 0$ **and** $l^2 - 5m^2 + n^2 = 0$ **then the angle between them is**

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

Key:1

Sol: . $l = -m - n$

$(m+n)^2 - 5m^2 + n^2 = 0$

$m^2 + n^2 + mn - 5m^2 + n^2 = 0$

$-4m^2 + 2mn + 2n^2 = 0$

$$2m^2 - mn - n^2 = 0$$
$$2m^2 - 2mn + mn - n^2 = 0$$
$$2m(m-n) + n(m-n) = 0$$
$$m = n$$
$$l = -2n$$
$$l : m : n = -2n; n; n$$
$$= 2,1,1$$
$$m = \frac{n}{2}$$
$$l = -\frac{n}{2} - n = \frac{-3}{2}n$$
$$l : m : n$$
$$= \frac{-3n}{2} : \frac{n}{2} : n$$
$$= \frac{-3}{2}, \frac{1}{2}, 1$$
$$-2\left(\frac{-3}{2}\right) + \frac{1}{2} + 1$$
$$\cos\theta = \frac{3 + \frac{1}{2} + 1}{\sqrt{6}\sqrt{\frac{9}{4} + \frac{1}{4} + 1}}$$
$$= \frac{\frac{9}{2}}{\sqrt{6}\frac{\sqrt{15}}{2}} = \frac{9}{3\sqrt{10}}$$

61. If A (3, 4, 5), B(4, 6, 3), C(–1, 2, 4) and D(1, 0, 5) are such that the angle between the lines $\overleftrightarrow{DC}$ and $\overleftrightarrow{AB}$ is θ then cos θ =

1) $\frac{2}{9}$ 2) $\frac{4}{9}$ 3) $\frac{5}{9}$ 4) $\frac{7}{9}$

Key: 2

Sol: . DC = (2, –2, 1)

AB = (1, 2, –2)

$$\cos\theta = \frac{|2-4-2|}{3\times 3} = \frac{4}{9}$$

62. $\lim\limits_{x\to 0} \frac{\sqrt{1+x^2} - \sqrt{1-x+x^2}}{3^x - 1} =$

1) $\log 9$ 2) $\frac{1}{\log 9}$ 3) $\log 3$ 4) $\frac{1}{\log 3}$

Key: 2

Sol: . $\lim_{x\to0}\frac{\frac{\sqrt{1+x^2}}{x}-\frac{\sqrt{1-x+x^2}}{x}}{\left(\frac{3x-1}{x}\right)}$

$$=\frac{1}{\log 3}\lim_{x\to0}\frac{\sqrt{1+x^2}-\sqrt{1-x+x^2}}{x}$$

$$=\frac{1}{\log 3}\frac{1}{2\sqrt{1+x^2}}-\frac{1}{2\sqrt{1-x+x^2}}(2n-1)$$

$$=\frac{l}{\log 3}\left[0+\frac{1}{2\sqrt{1}}\right]$$

$$=\frac{1}{\log 9}$$

63. **If f:[–2, 2] $\to$ R is defiend by**

$$f(x)=\begin{cases}\frac{\sqrt{1+cx}-\sqrt{1-cx}}{x} & \text{for } -2\le x<0\\ \frac{x+3}{x=1} & \text{for } 0\le x\le 2\end{cases}$$

is continuous on [–2, 2], then c =

1) 3 2) $\frac{3}{2}$ 3) $\frac{3}{\sqrt{2}}$ 4) $\frac{2}{\sqrt{3}}$

Key: 3

Sol: . $f(0-)=\underset{x\to0}{Lt}\frac{1}{2\sqrt{1+cx}}c-\frac{1}{2\sqrt{1-cx}}(-c)$

$$=\frac{c}{2}+\frac{c}{2}=c\ ;\ f(0+)=3$$

$C=3$

64. **If** $f(x)=x\tan^{-1}x$ **then** $\lim_{x\to1}\frac{f(x)-f(1)}{x-1}=$

1) $\frac{\pi}{4}$ 2) $\frac{\pi+1}{4}$ 3) $\frac{\pi+2}{4}$ 4) $\frac{\pi+3}{4}$

Key:3

Sol: .Apply L - hospital rule

65. $y=\tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)\Rightarrow(1+a^2x^2)y''+2a^2xy'=$

1) a^2 2) $2a^2$ 3) 0 4) $-2a^2$

Key:3

Sol: Put $ax=\tan\theta$

66. **If** $f(x)=\frac{x}{1+x}$ **and** $g(x)=f(f(x))$ **then** $g'(x)=$

1) $\frac{1}{(x+1)^2}$ 2) $\frac{1}{x^2}$ 3) $\frac{1}{(2x+1)^2}$ 4) $\frac{1}{(2x+3)^2}$

Key:3

Sol: $g(x)=\frac{\frac{x}{1+x}}{1+\frac{x}{1+x}}=\frac{x}{1+2x}$

67. **If the curves** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ **and** $\frac{x^2}{25}+\frac{y^2}{16}=1$ **cut each other othogonally, then** $a^2-b^2=$

1) 400 2) 75 3) 41 4) 9

Key: 4

Sol: Product of tangents slopes = -1

68. **The condition that** $f(x)=ax^3+bx^2+cx=d$ **has no exterme value is**

1) $b^2=4ac$ 2) $b^2=3ac$ 3) $b^2<3ac$ 4) $b^2>3ac$

Key:3

Sol: $\frac{3ax^2+bx+c}{b^2-4ac<0}$

$(2b)^2-4(3a)c<0$

$4b^2-12ac<0$

$b^2-3ac<0 \Rightarrow b^2<3ac$

69. **If there is an error of** $\pm 0.04\text{cm}$ **in the measurement of the diameter of a sphere then the approximate percentage error in its volume, when the radius is 10 cm, is**

1) ± 0.06 2) ± 0.006 3) ± 0.6 4) ± 1.2

Key:4

Sol: . $v=\frac{4}{3}\pi r^3$

70. **The value of c in the Lagrange's mean - value theorem for** $f(x)\sqrt{x-2}$ **in the itnerval [2, 6] is**

1) $\frac{5}{2}$ 2) 3 3) 4 4) $\frac{9}{2}$

Key:2

Sol: $f'(c)=\frac{f(b)-f(a)}{b-a}$

71. $\int\frac{dx}{\sqrt{\sin^3 x\cos x}}=g(x)+c\Rightarrow g(x)=$

1) $\frac{-2}{\sqrt{\tan x}}$ 2) $\frac{2}{\sqrt{\cot x}}$ 3) $\frac{2}{\sqrt{\tan x}}$ 4) $\frac{-2}{\sqrt{\cot x}}$

Key: 1