SYLLABUS
MICROWAVES AND RADAR

Subject Code	: **10EC54**	IA Marks	: 25
No. of Lecture Hrs/Week	: 04	Exam Hours	: 03
Total no. of Lecture Hrs	: 52	Exam Marks	: 100

PART - A

UNIT - 1

MICROWAVE TRANSMISSION LINES: Introduction, transmission lines equations and solutions, reflection and transmission coefficients, standing waves and SWR, line impedance and line admittance. Smith chart, impedance matching using single stubs, Microwave coaxial connectors.

7 Hours

UNIT - 2

MICROWAVE WAVEGUIDES AND COMPONENTS: Introduction, rectangular waveguides, circular waveguides, microwave cavities, microwave hybrid circuits, directional couplers, circulators and isolators.

7 Hours

UNIT - 3

MICROWAVE DIODES,

Transfer electron devices: Introduction, GUNN effect diodes – GaAs diode, RWH theory, Modes of operation, Avalanche transit time devices: READ diode, IMPATT diode, BARITT diode, Parametric amplifiers Other diodes: PIN diodes, Schottky barrier diodes.

7 Hours

UNIT - 4

Microwave network theory and passive devices. Symmetrical Z and Y parameters, for reciprocal Networks, S matrix representation of multi port networks.

6 Hours

PART - B

UNIT - 5

Microwave passive devices, Coaxial connectors and adapters, Phase shifters, Attenuators, Waveguide Tees, Magic tees.

4 Hours

UNIT - 6

STRIP LINES: Introduction, Microstrip lines, Parallel strip lines, Coplanar strip lines, Shielded strip Lines.

6 Hours

UNIT - 7

AN INTRODUCTION TO RADAR: Basic Radar, The simple form of the Radar equation, Radar block diagram, Radar frequencies, application of Radar, the origins of Radar.

8 Hours

UNIT - 8

MTI AND PULSE DOPPLER RADAR: Introduction to Doppler and MTI Radar, delay line Cancellers, digital MTI processing, Moving target detector, pulse Doppler Radar.

7 Hours

TEXT BOOKS:

1. **Microwave Devices and circuits**- Liao / Pearson Education.
2. **Introduction to Radar systems**-Merrill I Skolnik, 3rd Ed, TMH, 2001.
3. **Microwave Engineering** – Annapurna Das, Sisir K Das TMH Publication, 2001.

REFERENCE BOOK:

1. **Microwave Engineering –** David M Pozar, John Wiley, 2e, 2004

INDEX SHEET

UNIT – 1

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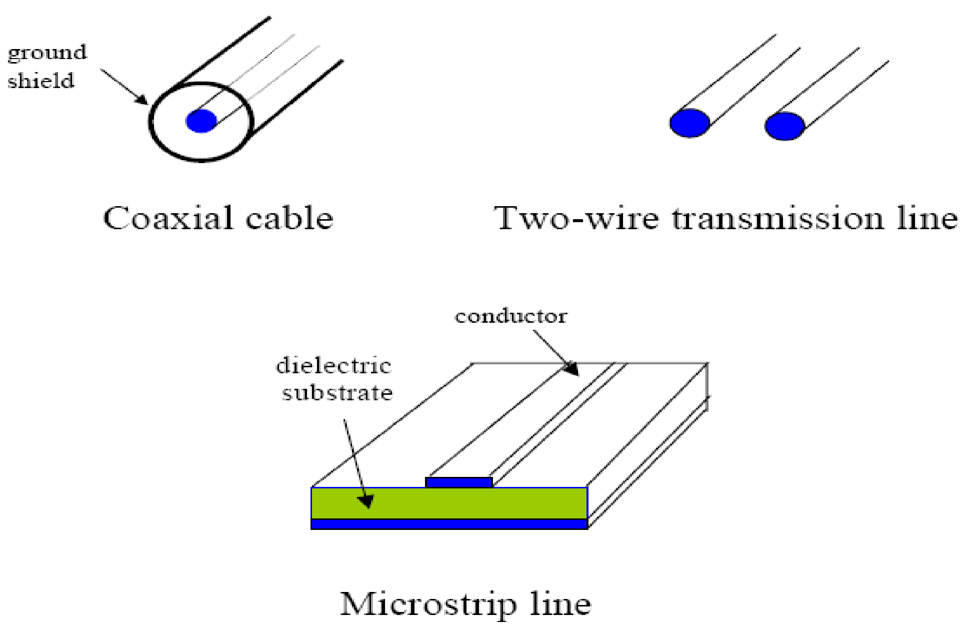
UNIT- 1

MICROWAVE TRANSMISION LINES

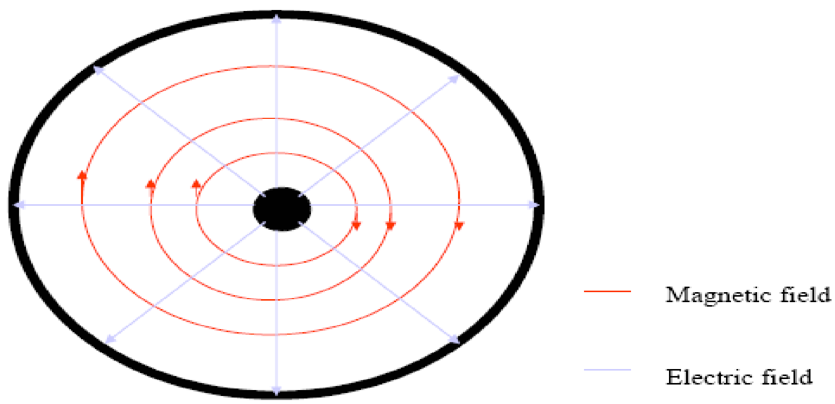
INTRODUCTION:

Any pair of wires and conductors carrying currents in opposite directions form transmission lines. Transmission lines are essential components in any electrical/communication system. They include coaxial cables, two-wire lines, microstrip lines on printed-circuit-boards (PCB).

The characteristics of transmission lines can be studied by the electric and magnetic fields propagating along the line. But in most practical applications, it is easier to study the voltages and currents in the line instead.



Different types of transmission lines

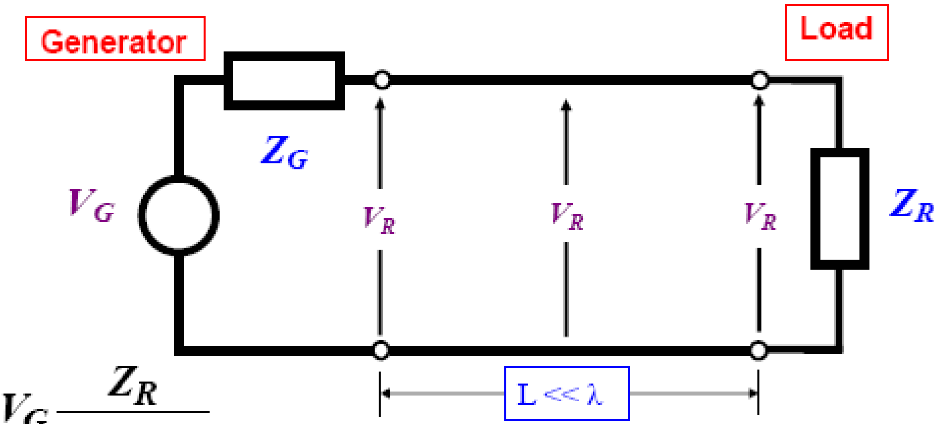


Cross-section of a coaxial cable showing the electric and magnetic fields

A typical engineering problem involves the transmission of a signal from a generator to a load. A transmission line is the part of the circuit that provides the direct link between generator and load.

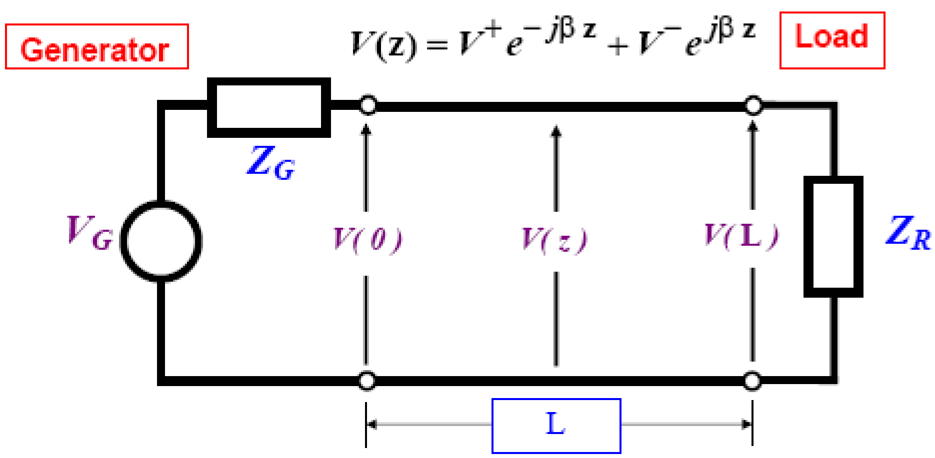
Transmission lines can be realized in a number of ways. Common examples are the parallel-wire line and the coaxial cable. For simplicity, we use in most diagrams the parallel-wire line to represent circuit connections, but the theory applies to all types of transmission lines.

If you are only familiar with low frequency circuits, you are used to treat all lines connecting the various circuit elements as perfect wires, with no voltage drop and no impedance associated to them (lumped impedance circuits). This is a reasonable procedure as long as the length of the wires is much smaller than the wavelength of the signal. At any given time, the measured voltage and current are the same for each location on the same wire.



$$V_R = V_G \frac{Z_R}{Z_G + Z_R}$$

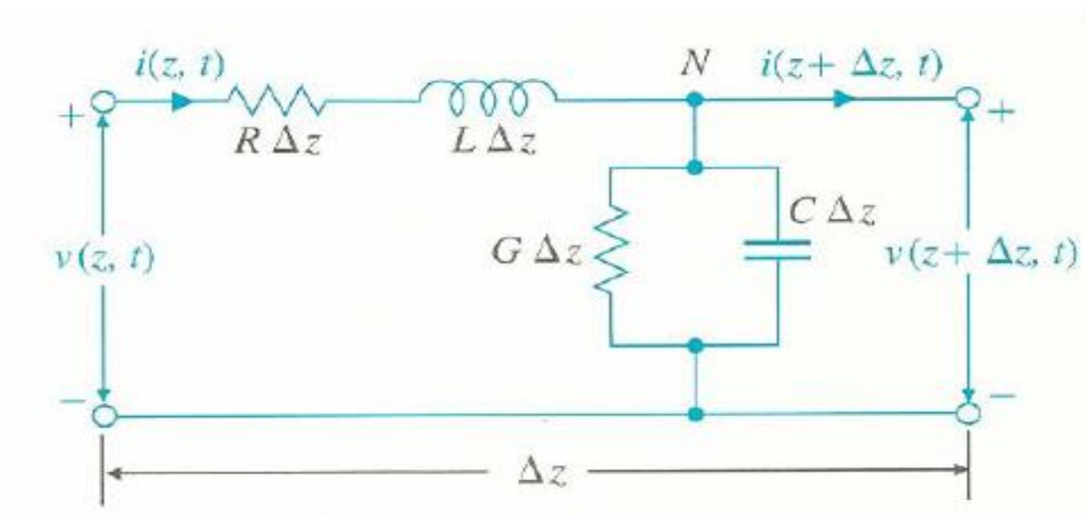
For sufficiently high frequencies the wavelength is comparable with the length of conductors in a transmission line. The signal propagates as a wave of voltage and current along the line, because it cannot change instantaneously at all locations. Therefore, we cannot neglect the impedance properties of the wires.



TRANSMISSION LINE EQUATIONS AND SOLUTIONS:

A transmission line can be analyzed wither by solution of Maxwells field equations or by distributed circuit theory which involves only one space variable in addition to the time variable.

Voltage and Current Waves in general transmission lines



Equivalent circuit of an element section (length Δz) of the transmission line: *L*, *R* are the distributed inductance and resistance (per unit length) of the conductor; *C*,*G* are the distributed capacitance and conductance (per unit length) of the dielectric between the conductors.

Relation between instantaneous voltage *v* and current *i* at any point along the line:

$$\frac{\partial i}{\partial z} = -Gv - C\frac{\partial v}{\partial t}$$

$$\frac{\partial v}{\partial z} = -Ri - L\frac{\partial i}{\partial t}$$

For periodic signals, Fourier analysis can be applied and it is more convenient to use **phasors** of voltage *V* and current *I*.

$$\frac{\partial V}{\partial z} = -\left(R + j\omega L\right)I$$

$$\frac{\partial I}{\partial z} = -(G + j\omega C)V$$

Decoupling the above equations, we get

$$\frac{\partial^2 V}{\partial^2 z} = \gamma^2 V$$

$$\frac{\partial^2 I}{\partial^2 z} = \gamma^2 I$$

where γ is called the propagation constant, and is in general complex.

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \alpha + j\beta$$

α is the attenuation constant, β is the phase constant

The general solutions of the second-order, linear differential equation for *V*, *I* are :

$$V = V^+ e^{-\gamma z} + V^- e^{+\gamma z}$$

$$I = I^+ e^{-\gamma z} + I^- e^{+\gamma z}$$

V+, *V*-, *I*+, *I*- are constants (complex phasors). The terms containing e-γz represent waves traveling in +z direction; terms containing e+γz represent waves traveling in *–z direction.*

$$e^{-\gamma z} = e^{-\alpha z} e^{-j\beta z}$$

It can be shown that the ratio of voltage to current is given by:

$$\frac{V^+}{I^+} = Z_o \qquad \frac{V^-}{I^-} = -Z_o$$

where *Zo* is the characteristic impedance of the line, given by

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

The current *I* can now be written as:

$$I = \frac{V^+}{Z_o} e^{-\gamma z} - \frac{V^-}{Z_o} e^{+\gamma z}$$

Lossless transmission lines:

In lossless transmission lines, the distributed conductor resistance *R* and dielectric conductance *G* are both zero. In this case the characteristic impedance is real and is equal to:

$$Z_o = \sqrt{\frac{L}{C}}$$

The propagation constant γ is also imaginary with:

$$\alpha = 0$$

$$\gamma = j\beta = j\omega\sqrt{LC}$$

Expressing the waves in time-domain

$$v(t,z) = \left|V^{+}\right| \cos(\omega t - \beta z) + \left|V^{-}\right| \cos(\omega t + \beta z)$$

$$i(t,z) = \frac{\left|V^{+}\right|}{Z_o} \cos(\omega t - \beta z) - \frac{\left|V^{-}\right|}{Z_o} \cos(\omega t + \beta z)$$

The velocity with which a front of constant phase travels is called the phase velocity u_p.

In any transmission line

$$u_p = \frac{\omega}{\beta} \qquad \beta = \frac{2\pi}{\lambda}$$

In lossless transmission line

$$\beta = \omega\sqrt{LC}$$

Therefore

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

In a coaxial cable,

$$C = \frac{2\pi\varepsilon_o\varepsilon_r}{\ln\left(\frac{b}{a}\right)}$$

$$L = \frac{\phi}{I} = \frac{\mu_o}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$u_p = \frac{1}{\sqrt{\varepsilon_o\varepsilon_r\mu_o}}$$

εo – permittivity of vacuum
εr – relative permittivity (dielectric constant) of dielectric
μo – permeability of vacuum

Example: Calculate the characteristic resistance *Ro* of a RG-58U coaxial cable which has a inner conductor of radius *a*=0.406 mm and a braided outer conductor with radius *b*=1.553 mm. Assume the dielectric is polyethylene with dielectric constant of 2.26.

Solution: The distributed capacitance and inductance of the cable can be calculated to be:

L = 0.268 μH/m

C = 93.73 pF/m

Ro = *L* / *C* = 53.47Ω

Reflection and Transmission:

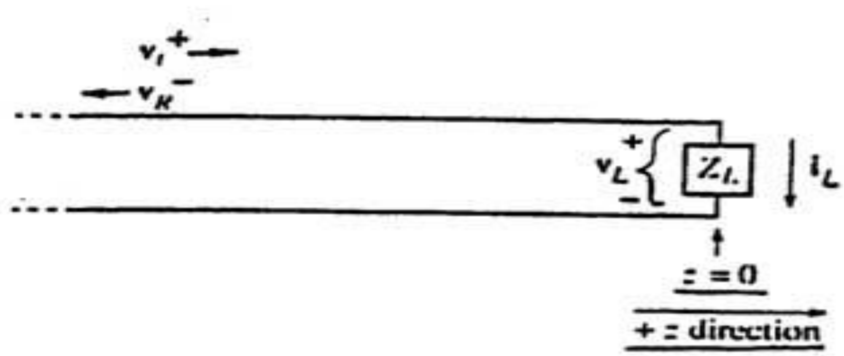
Reflection co-efficient is defined as the ratio of amplitudes of reflected voltage wave to the incident voltage wave at the receiving end.

$$\boxed{\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$$

Transmission co-efficient is defined as the ratio of transmitted voltage or current to the incident voltage or current .

$$\boxed{\tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1}}$$

A transmission line terminated in its characteristic impedance is called a properly terminated line. According to the principle of conservation of energy, the incident power minus the reflected power must be equal to the power transmitted to the load.



$$\underline{v}(z=0) = \underline{v}_i^+(z=0) + \underline{v}_r^-(z=0)$$

$$\underline{i}(z=0) = \underline{i}_i^+(z=0) + \underline{i}_r^-(z=0)$$

$$= \frac{1}{Z_o}[\underline{v}_i^+(z=0) - \underline{v}_r^-(z=0)]$$

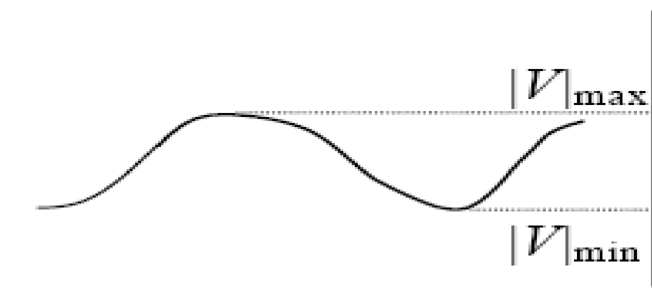
$$\frac{\underline{v}(z=0)}{\underline{i}(z=0)} = Z_o \frac{\underline{v}_i^+(z=0) + \underline{v}_r^-(z=0)}{\underline{v}_i^+(z=0) - \underline{v}_r^-(z=0)} = Z_L$$

Standing wave ratio:

In a lossless line, the amplitude of the forward (or backward) voltage remains constant as the wave propagates along *z*, only with a shift in the phase angle. The superimposition of the forward wave and backward wave results in a standing wave pattern.

$$\boxed{S = \frac{\left|\overline{E}_1(z)\right|_{\max}}{\left|\overline{E}_1(z)\right|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

• In a standing wave, there are positions at the line where the amplitude of the resultant voltage has maximum and minimum.



The voltage standing wave ratio (VSWR) is the ratio of the maximum and minimum voltage magnitudes. The distance between two successive maximums is equal to λ/2.

$$VSWR = \frac{|V|_{\max}}{|V|_{\min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

VSWR is useful to find the maximum voltage magnitude on the line due to reflection from the load. If *V*inc is the incident voltage on the load,

$$|V|_{\max} = 2|V_{inc}|\frac{VSWR}{VSWR+1}$$

SMITH CHART:

Smith Chart is a convenient graphical means of determining voltages along transmission lines. It is essentially a plot of the complex reflection coefficient Γ(-*l*) at a point with input impedance *Z*in(-*l*) looking into the end of the transmission line.

$$\Gamma(-l) = \frac{Z_{in}(-l) - Z_o}{Z_{in}(-l) + Z_o}$$

Let the real and imaginary parts of Γ(-*l*) be Γr , Γi respectively,and *z* be the input impedance normalized by *Z*o.

$$z = \frac{Z_{in}(l)}{Z_o} = r + jx$$

$$\Gamma = \frac{z-1}{z+1}$$

In a lossless transmission line, there is no attenuation and a wave traveling along the line will only have a phase shift. So the reflection coefficient Γ(-*l*) at a point of distance *l* from the load at the end of the line is related to the load reflection coefficient ΓL by:

$$\Gamma(-l) = \Gamma_L e^{-j2\beta l}$$

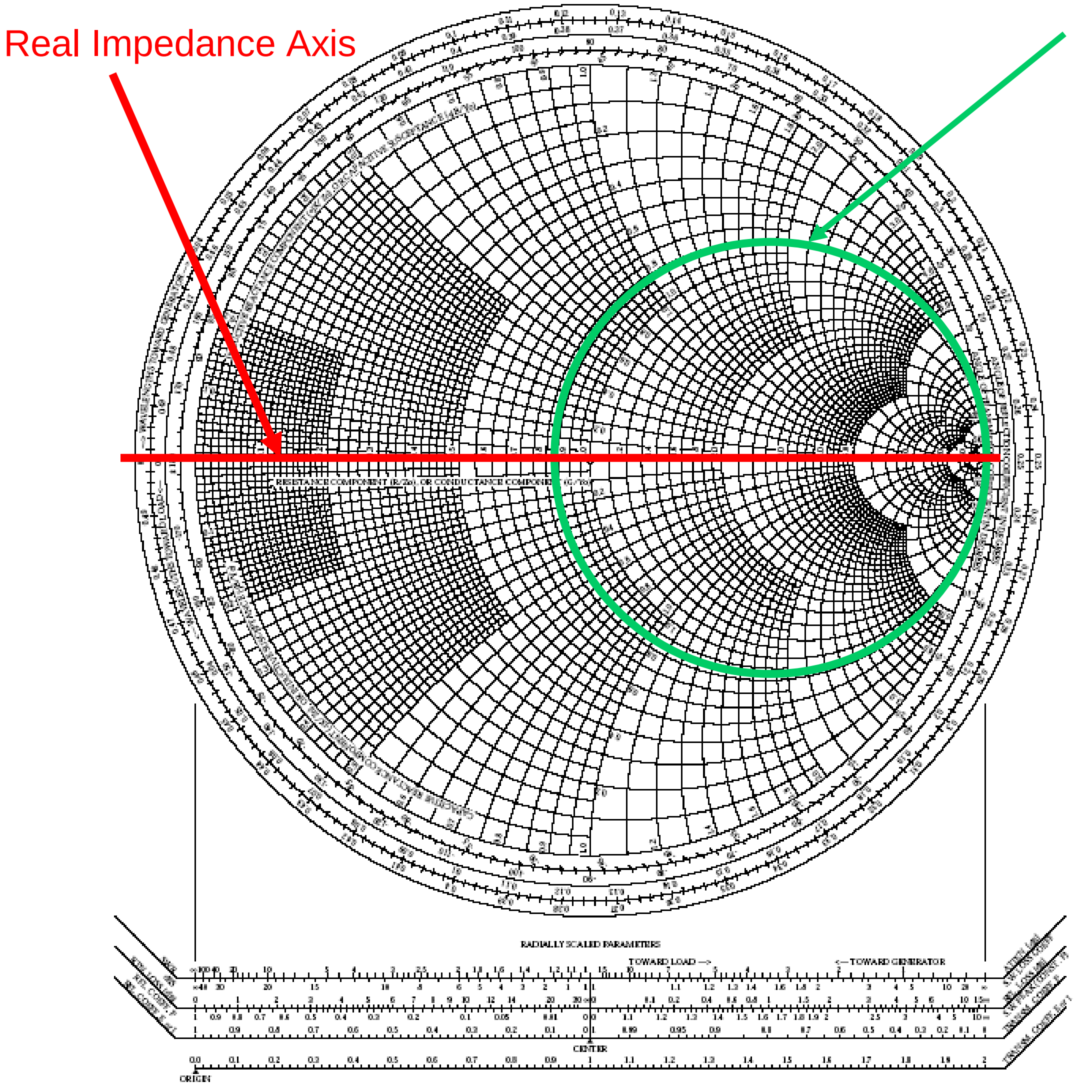
It means the reflection coefficient has same magnitude but only a phase shift of 2 β *l* if we move a length *l* along the line (Γ rotates clockwise on the Smith Chart when moving away from the load and anti-clockwise when moving towards the load).

- The Smith Chart is a clever tool for analyzing transmission lines
- The outside of the chart shows location on the line in wavelengths

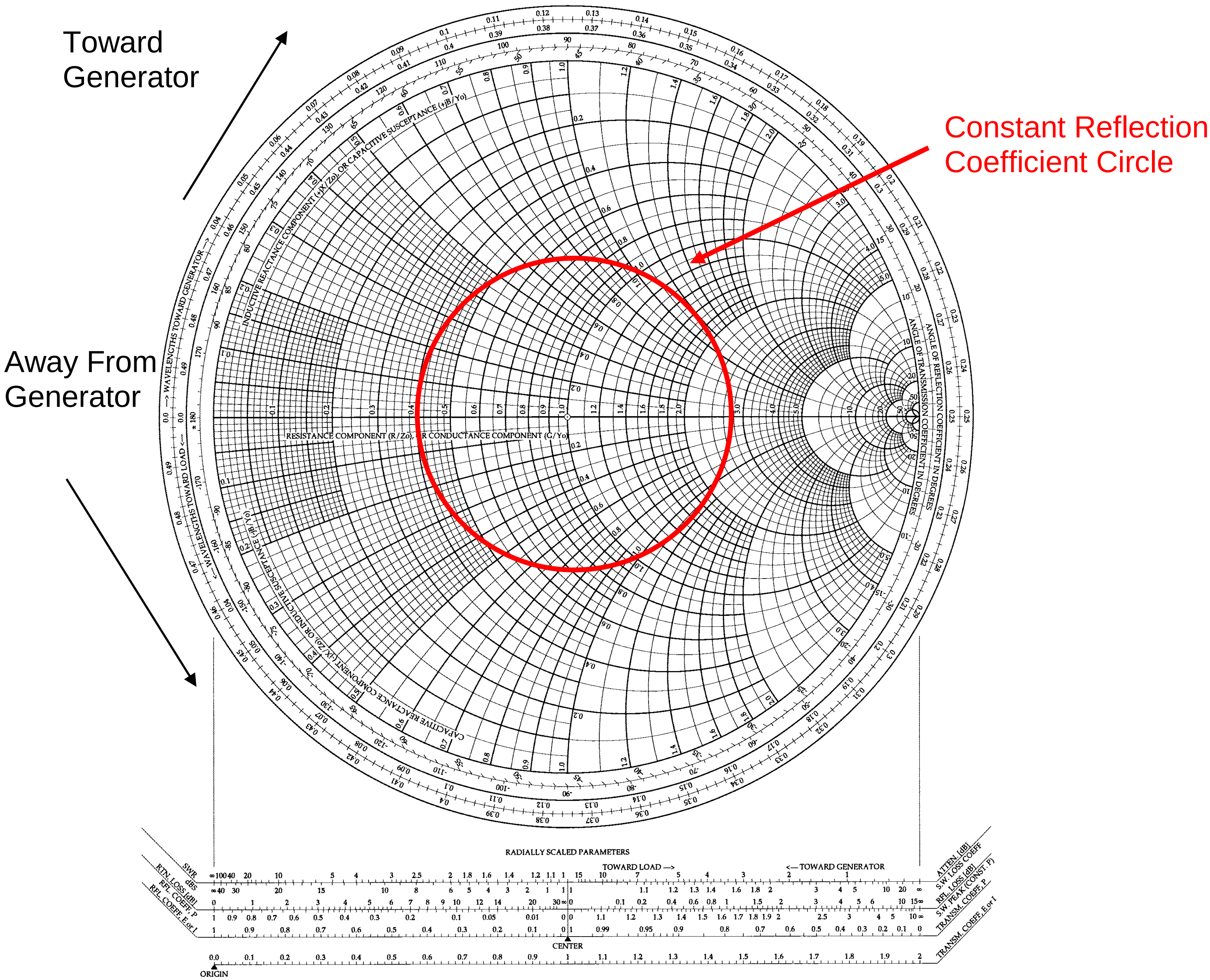
- The combination of intersecting circles inside the chart allow us to locate the normalized impedance and then to find the impedance anywhere on the line
- Impedances, voltages, currents, etc. all repeat every half wavelength
- The magnitude of the reflection coefficient, the standing wave ratio (SWR) do not change, so they characterize the voltage & current patterns on the line
- If the load impedance is normalized by the characteristic impedance of the line, the voltages, currents, impedances, etc. all still have the same properties, but the results can be generalized to any line with the same normalized impedances

Explanaiton of smith chart:

Imaginary Impedance Axis



- Thus, the first step in analyzing a transmission line is to locate the normalized load impedance on the chart
- Next, a circle is drawn that represents the reflection coefficient or SWR. The center of the circle is the center of the chart. The circle passes through the normalized load impedance
- Any point on the line is found on this circle. Rotate clockwise to move toward the generator (away from the load)
- The distance moved on the line is indicated on the outside of the chart in wavelengths



- First, locate the normalized impedance on the chart for ZL = 50 + j100
- Then draw the circle through the point
- The circle gives us the reflection coefficient (the radius of the circle) which can be read from the scale at the bottom of most charts
- Also note that exactly opposite to the normalized load is its admittance. Thus, the chart can also be used to find the admittance. We use this fact in stub matching

- Now the line is matched to the left of the stub because the normalized impedance and admittance are equal to 1
- Note that the point on the Smith Chart where the line is matched is in the center (normalized z=1) where also the reflection coefficient circle has zero radius or the reflection coefficient is zero.
- Thus, the goal with the matching problem is to add an impedance so that the total impedance is the characteristic impedance.

PROBLEMS:

A transmission line has the following parameters:

$$R = 2\ \Omega/\text{m} \qquad G = 0.5\ \text{mmho/m} \qquad f = 1\ \text{GHz}$$

$$L = 8\ \text{nH/m} \qquad C = 0.23\ \text{pF}$$

Calculate: **(a)** the characteristic impedance; **(b)** the propagation constant.

Solution

a. From Eq. (3-1-25) the line characteristic impedance is

$$\mathbf{Z}_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{2 + j2\pi \times 10^9 \times 8 \times 10^{-9}}{0.5 \times 10^{-3} + j2\pi \times 10^9 \times 0.23 \times 10^{-12}}}$$

$$= \sqrt{\frac{50.31\underline{/87.72^\circ}}{15.29 \times 10^{-4}\underline{/70.91^\circ}}} = 181.39\underline{/8.40^\circ} = 179.44 + j26.50$$

b. From Eq. (3-1-18) the propagation constant is

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{(50.31\underline{/87.72^\circ})(15.29 \times 10^{-4}\underline{/70.91^\circ})}$$

$$= \sqrt{769.24 \times 10^{-4}\underline{/158.63^\circ}}$$

$$= 0.2774\underline{/79.31^\circ} = 0.051 + j0.273$$

A transmission line has a characteristic impedance of $50 + j0.01\ \Omega$ and is terminated in a load impedance of $73 - j42.5\ \Omega$. Calculate: (**a**) the reflection coefficient; (**b**) the standing-wave ratio.

Solution

a. From Eq. (3-2-8) the reflection coefficient is

$$\Gamma = \frac{\mathbf{Z}_\ell - \mathbf{Z}_0}{\mathbf{Z}_\ell + \mathbf{Z}_0} = \frac{73 - j42.5 - (50 + j0.01)}{73 - j42.5 + (50 + j0.01)} = 0.377\underline{/-42.7^\circ}$$

b. From Eq. (3-3-17) the standing-wave ratio is

$$\rho = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.377}{1 - 0.377} = 2.21$$

A lossless line has a characteristic impedance of 50 Ω and is terminated in a load resistance of 75 Ω. The line is energized by a generator which has an output impedance of 50 Ω and an open-circuit output voltage of 30 V (rms). The line is assumed to be 2.25 wavelengths long. Determine:

a. The input impedance
b. The magnitude of the instantaneous load voltage
c. The instantaneous power delivered to the load

Solution

a. From Eq. (3-4-26) the line that is 2.25 wavelengths long looks like a quarter-wave line. Then

$$\beta d = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

From Eq. (3-4-26) the input impedance is

$$\mathbf{Z}_{in} = \frac{R_0^2}{R_\ell} = \frac{(50)^5}{75} = 33.33\ \Omega$$

b. The reflection coefficient is

$$\Gamma_\ell = \frac{R_\ell - R_0}{R_\ell + R_0} = \frac{75 - 50}{75 + 50} = 0.20$$

Then the instantaneous voltage at the load is

$$\mathbf{V}_\ell = \mathbf{V}_+ e^{-j\beta\ell}(1 + \Gamma_\ell) = 30(1 + 0.20) = 36\ \mathrm{V}$$

c. The instantaneous power delivered to the load is

$$\mathbf{P}_\ell = \frac{(36)^2}{75} = 17.28\ \mathrm{W}$$

RECOMMENDED QUESTIONS FOR UNIT 1

1. Discuss different types of transmission lines used in communication and the frequencies at which they are preferred.
2. Starting from basics obtain the solution of the transmission line equations.
3. Define and derive expressions for attenuation and phase constants, wavelength and velocity of propagation in a transmission line.
4. What is meant by " relative phase velocity factor? Obtain the expression fro the same.
5. Derive an expression for input impedance of microwave transmission line.
6. What is reflection co-efficient? Obtain an expression for the same? How is it related to SWR?
7. What is transmission co-efficient ? obtain an expression for the same.
8. What are standing waves? How are they formed? Obtain the expression for VSW.
9. What is line impedance? Derive an expression for line impedance at any point on the line.
10. Obtain an expression for line impedance in terms of reflection co-efficient.
11. Explain the steps involved in calculation of standing wave ratio
12. What is smith chart ? how is it constructed?
13. Discuss applications and properties of smith chart.
14. Explain how impedance can be converted to admittance using smith chart.
15. Explain the steps involved in single stub matching using smith chart

UNIT - 2

MICROWAVE WAVEGUIDES AND COMPONENTS: Introduction, rectangular waveguides, circular waveguides, microwave cavities, microwave hybrid circuits, directional couplers, circulators and isolators.

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1. **Microwave Engineering –** David M Pozar, John Wiley, 2e, 2004

UNIT- 2

MICROWAVE WAVEGUIDES AND COMPONENTS

INTRODUCITON

A waveguide consists of a hollow metallic tube of either rectangular or circular cross section used to guide electromagnetic wave. Rectangular waveguide is most commonly used as waveguide. waveguides are used at frequencies in the microwave range.

At microwave frequencies (above 1GHz to 100 GHz) the losses in the two line transmission system will be very high and hence it cannot be used at those frequencies . hence microwave signals are propagated through the waveguides in order to minimize the losses.

Properties and characteristics of waveguide:

1. The conducting walls of the guide confine the electromagnetic fields and thereby guide the electromagnetic wave through multiple reflections .
2. when the waves travel longitudinally down the guide, the plane waves are reflected from wall to wall .the process results in a component of either electric or magnetic fields in the direction of propagation of the resultant wave.
3. TEM waves cannot propagate through the waveguide since it requires an axial conductor for axial current flow .
4. when the wavelength inside the waveguide differs from that outside the guide, the velocity of wave propagation inside the waveguide must also be different from that through free space.

5. if one end of the waveguide is closed using a shorting plate and allowed a wave to propagate from other end, then there will be complete reflection of the waves resulting in standing waves.

APPLICATION OF MAXWELLS EQUATIONS TO THE RECTANGULAR WAVEGUIDE:

Let us consider waves propagating along O*z* but with restrictions in the *x* and/or *y* directions. The wave is now no longer necessarily transverse.

The wave equation can be written as

$$\nabla^2 \vec{H} + k^2 \vec{H} = 0 \quad \text{where} \quad k = \frac{\omega}{c}$$

In the present case this becomes

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - k_z^2 + k^2 \right) \vec{H} = 0$$

and similarly for .electric field.

There are three kinds of solution possible

TEM	$H_z = E_z = 0,$	i.e. the familiar transverse EM waves
TE	$E_z = 0$	
TM	$H_z = 0$	

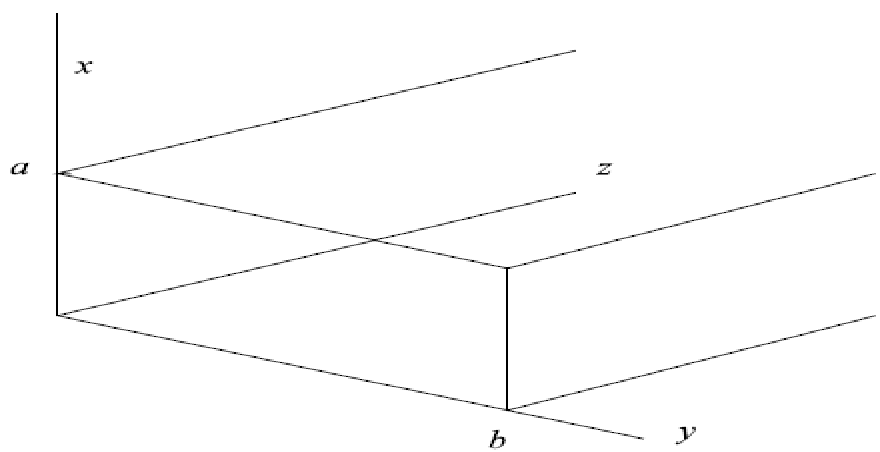
Boundary conditions:

We assume the guides to be perfect conductors so = 0 inside the guides. Hence, the continuity of *Et* at a boundary implies that *Et* = 0 in the wave guide at the boundary.

En is not necessarily zero in the wave guide at the boundary as there may be surface charges on the conducting walls (the solution given below implies that there are such charges)

It follows from Maxwell's equation that because = 0, is also zero inside the conductor (the time dependence of is exp(-iT*t*)). The continuity of *Hn* implies that *Hn* = 0 at the boundary.

There are currents induced in the guides but for perfect conductors these can be only surface currents. Hence, there is no continuity for *Ht*. This is to be contrasted with the boundary condition used for waves reflecting off conducting surfaces with finite conductivity.



The standard geometry for a rectangular wave guide is given fig 1. A wave can be guided by two parallel planes for which case we let the planes at $x = 0$, a extend to $y = \pm 4$.

TE Modes: By definition, $Ez = 0$ and we start from

$$H_z = H_0 X(x) Y(y) e^{ik_z z}$$

as the wave equation in Cartesian coordinates permits the use of the separation of variables.

TM Modes: By definition, *Hz* = 0 and we start from

$$E_z = E_0 X(x) Y(y) e^{ik_z z}$$

It is customary in wave guides to use the longitudinal field strength as the reference. For the parallel plate wave guide there is no *y* dependence so just set *Y* =

TE modes

Using the above form for the solution of the wave equation, the wave equation can be rewritten as

$$\frac{X''}{X} + \frac{Y''}{Y} = k_z^2 - k^2$$

$$\text{Let} \quad \frac{X''}{X} = -k_x^2 \quad \text{and} \quad \frac{Y''}{Y} = -k_y^2 , \quad k_x^2 + k_y^2 + k_z^2 = k^2$$

the minus signs being chosen so that we get the oscillatory solutions needed to fit the boundary conditions.

Now apply the boundary conditions to determine the restrictions on *Hz*.

At *x* = 0, *a*: *Ey* = 0 and *H x* = 0 (*Ez* is zero everywhere)

For the following Griffith's writes down all the Maxwell equations specialized to propagation along 0*z*. I will write just those needed for the specific task and motivate the choice.

We need to relate *Ey*, *Hx* to the reference *Hz*. Hence, we use the *y* component of ME2 (which has 2 *H* fields and 1 *E* field)

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = -i\omega\epsilon_0 E_y$$

The first term is *ikzHx* which is zero at the boundary.

Consequently, $\frac{\partial H_z}{\partial x} = \mathbf{0}$ at $x = 0,\ a$ and $X = \cos k_x x$ with

$$k_x = \frac{m\pi}{a}$$

The absence of an arbitrary constant upon integration is justified below.

At $y = 0$, b: $Ex = 0$ and $Hy = 0$ and we now use the x component of ME2

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = -i\omega\epsilon_0 E_x$$

As the second term is proportional *Hy* we get

$$\frac{\partial H_z}{\partial y} = \mathbf{0} \text{ at } y = 0,\ b \text{ and } Y = \cos k_y y \text{ with } k_y = \frac{n\pi}{b}$$

The general solution is thus

$$H_x = H_0 \cos(k_x x)\cos(k_y y)\, e^{ik_z z}$$

$$= H_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) e^{ik_z z}$$

However, $m = n = 0$ is not allowed for the following reason.

When $m = n = 0$, *Hz* is constant across the waveguide for any *xy* plane. Consider the integral version of Faraday's law for a path that lies in such a plane and encircles the wave guide but in the metal walls.

$$\int \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$$

As $E = 0$ in the conducting walls and the time dependence of is given by exp(-*i*T*t*) this equation requires that . We need only evaluate the integral over the guide as = 0 in the walls.

For constant *Bz* this gives *Bzab* = 0. So *Bz* = 0 as is *Hz*. However, as we have chosen *Ez* = 0 this implies a TEM wave which cannot occur inside a hollow waveguide. Adding an arbitrary constant would give a solution like

$$H_x = H_0\left[\cos\left(\frac{m\pi x}{a}\right) + \text{Const}\right]\cos\left(\frac{n\pi y}{b}\right) e^{ik_z z}$$

which is not a solution to the wave equation ... try it. It also equivalent to adding a solution with either *m* = 0 or *n* = 0 which is a solution with a different

Cut off frequency

This restriction leads to a minimum value for *k*. In order to get propagation *kz*2 > 0. Consequently

$$k^2 > k_x^2 + k_y^2$$

$$\text{i.e.} \qquad \omega^2 > c^2\pi^2\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]$$

Suppose *a* > *b* then the minimum frequency is *c*B/*a* and for a limited range of T (dependent on *a* and *b*) this solution (*m* = 1, *n* = 0, or TE10) is the only one possible.

Away from the boundaries

$$ik_z H_x + k_x H_z^x = -i\omega\epsilon_0 E_y$$

where *Hzx* means that cos *k xx* has been replaced by sin *kxx*.

We need another relation between *Ey* and either *Hx* or *Hz*, which must come from the other Maxwell equation (ME1). We have to decide which component of ME1 to use. If we choose the *z* component, the equation involves *Ex* and *Ey*, introducing another unknown field (*Ex*). However, the *x* component involves *Ey* and *Ez*. As *Ez* = 0, this gives the required relation.

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = i\mu_0\omega H_x$$

i.e. $$-ik_zE_y = i\mu_0\omega H_x\ , \quad \text{or} \quad k_zE_y = -\mu_0\omega H_x$$

Substituting in the above gives

$$-\frac{ik_z^2E_y}{\mu_0\omega} + i\omega\epsilon_0E_y = -k_xH_z^x\ , \qquad E_y = \frac{i\mu_0\omega k_x}{k_x^2 + k_y^2}H_z^x\ , \text{ etc}$$

$$-k_yH_z^y - ik_zH_y = -i\omega\epsilon_0E_x$$

and the y component of ME1

$$ik_zE_x = i\mu_0\omega H_y$$

we get

$$-\frac{ik_z^2E_x}{\mu_0\omega} + i\omega\epsilon_0E_x = k_yH_z^y\ , \qquad E_x = -\frac{i\mu_0\omega k_y}{k_x^2 + k_y^2}H_z^y$$

Velocity

The phase velocity v_p is given by

$$v_p = \frac{\omega}{k_z} = \frac{ck}{k_z} = \frac{ck}{\sqrt{k^2 - k_x^2 - k_y^2}} > c$$

However the group velocity is given by

$$v_g = \frac{\partial\omega}{\partial k_z} = c\frac{\partial k}{\partial k_z} = c\frac{k_z}{k} < c \quad \text{and} \quad v_pv_g = c^2$$

TM modes

The boundary conditions are easier to apply as it is *Ez* itself that is zero at the boundaries.

Consequently, the solution is readily found to be

$$E_z = E_0 \sin(k_x x)\sin(k_y y)e^{ik_z z}$$

Note that the lowest TM mode is due to the fact that *Ez* . 0. Otherwise, along with *Hz* = 0, the solution is a TEM mode which is forbidden. The details are not given here as the TM wave between parallel plates is an assignment problem.

It can be shown that for ohmic losses in the conducting walls the TM modes are more attenuated than the TE modes.

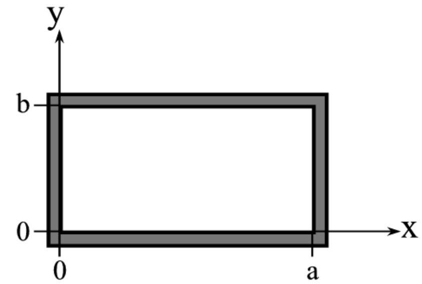
MAXWELL EQUATIONS

$$ME1 \quad \vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \qquad ME2 \quad \vec{\nabla} \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

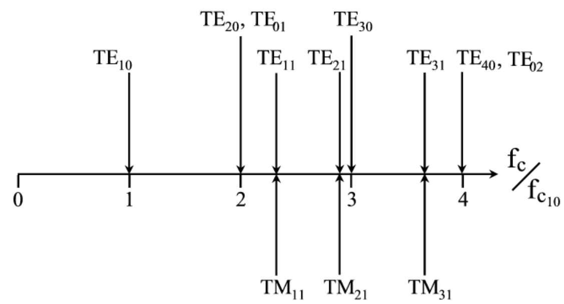
Rectangular Waveguide:

- Let us consider a rectangular waveguide with interior dimensions are a x b,
- Waveguide can support TE and TM modes.
  - In TE modes, the electric field is transverse to the direction of propagation.
  - In TM modes, the magnetic field that is transverse and an electric field component is in the propagation direction.
- The order of the mode refers to the field configuration in the guide, and is given by m and n integer subscripts, TEmn and TMmn.
  - The m subscript corresponds to the number of half-wave variations of the field in the x direction, and
  - The n subscript is the number of half-wave variations in the y direction.
- A particular mode is only supported above its cutoff frequency. The cutoff frequency is given by

Rectangular Waveguide



Location of mod



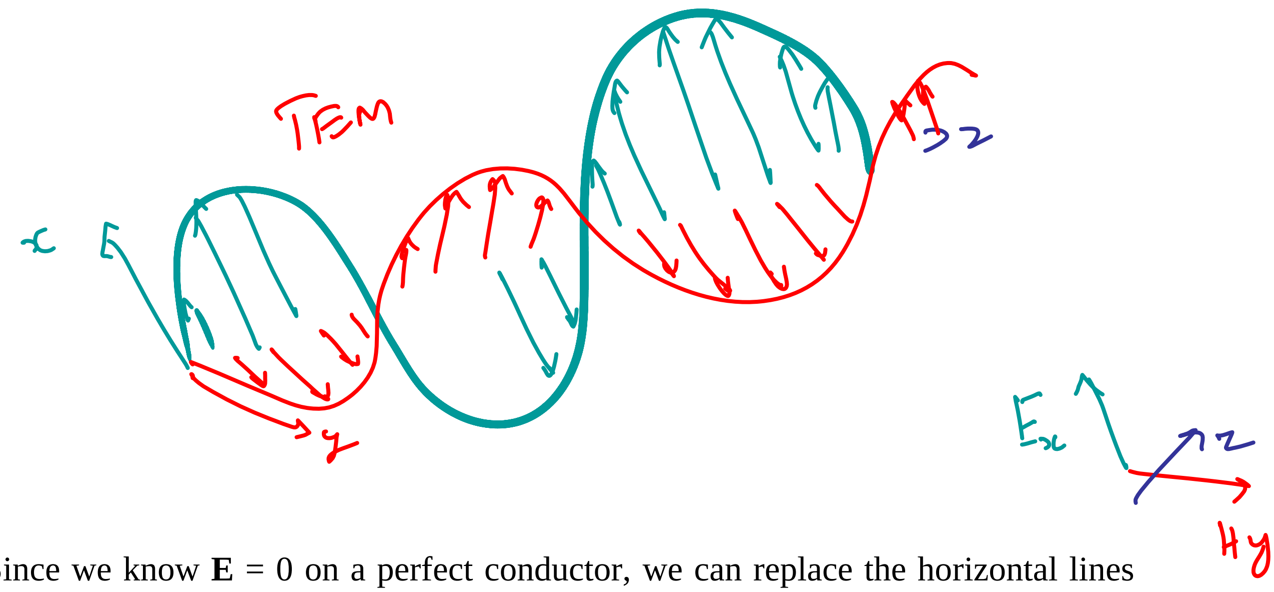
$$f_{c_{mn}} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{c}{2\sqrt{\mu_r\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$u = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_o\mu_r\varepsilon_o\varepsilon_r}} = \frac{1}{\sqrt{\mu_o\varepsilon_o}}\frac{1}{\sqrt{\mu_r\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

We can achieve a qualitative understanding of wave propagation in waveguide by considering the wave to be a superposition of a pair of TEM waves.

Let us consider a TEM wave propagating in the z direction. Figure shows the wave fronts; bold lines indicating constant phase at the maximum value of the field (+Eo), and lighter lines indicating constant phase at the minimum value (-Eo).

The waves propagate at a velocity uu, where the u subscript indicates media unbounded by guide walls. In air, uu = c.



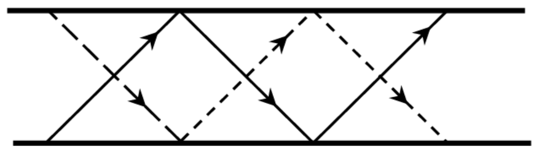
Since we know **E** = 0 on a perfect conductor, we can replace the horizontal lines of zero field with perfect conducting walls. Now, u+ and u- are reflected off the walls as they propagate along the guide.

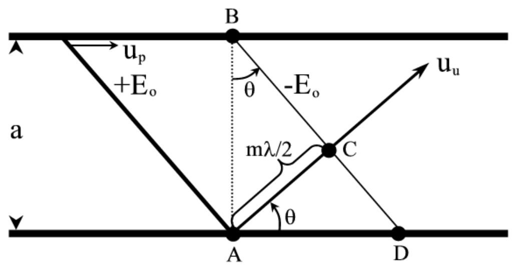
The distance separating adjacent zero-field lines in Figure (b), or separating the conducting walls in Figure (a), is given as the dimension a in Figure (b).

The distance *a* is determined by the angle θ and by the distance between wavefront peaks, or the wavelength λ. For a given wave velocity uu, the frequency is f = uu/λ.

If we fix the wall separation at a, and change the frequency, we must then also change the angle θ if we are to maintain a propagating wave. Figure (b) shows wave fronts for the u+ wave.

The edge of a +Eo wave front (point A) will line up with the edge of a –Eo front (point B), and the two fronts must be λ/2 apart for the m = 1 mode.





For any value of m, we can write by simple trigonometry

$$\sin\theta = \frac{m\lambda/2}{a} \quad \Rightarrow \quad \lambda = \frac{2a}{m}\sin\theta = \frac{u_u}{f}$$

The waveguide can support propagation as long as the wavelength is smaller than a critical value, λc, that occurs at θ = 90°, or

$$\lambda_c = \frac{2a}{m} = \frac{u_u}{f_c}$$

Where *fc* is the cutoff frequency for the propagating mode.

We can relate the angle θ to the operating frequency and the cutoff frequency by

$$\sin\theta = \frac{\lambda}{\lambda_c} = \frac{f_c}{f}$$

The time *tAC* it takes for the wavefront to move from A to C (a distance *lAC*) is

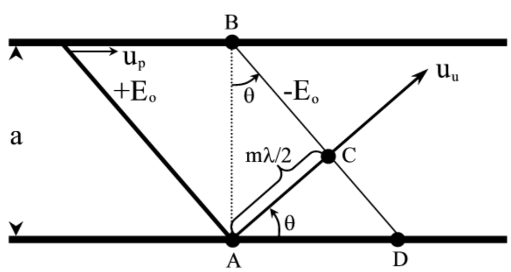
$$t_{AC} = \frac{\text{Distance from A to C}}{\text{Wavefront Velocity}} = \frac{l_{AC}}{u_u} = \frac{m\lambda/2}{u_u}$$

A constant phase point moves along the wall from A to D. Calling this phase velocity *up*, and given the distance *lAD* is

$$l_{AD} = \frac{m\lambda/2}{\cos\theta}$$

Then the time *tAD* to travel from A to D is

$$t_{AD} = \frac{l_{AD}}{u_p} = \frac{m\lambda/2}{\cos\theta\, u_p}$$



Since the times *tAD* and *tAC* must be equal, we have

$$u_p = \frac{u_u}{\cos\theta}$$

The *Wave velocity* is given by

$$u_u = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_o\mu_r\varepsilon_o\varepsilon_r}} = \frac{1}{\sqrt{\mu_o\varepsilon_o}}\frac{1}{\sqrt{\mu_r\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

The *Phase velocity* is given by

$$u_p = \frac{u_u}{\cos\theta}$$

$$\cos\theta = \sqrt{\cos^2\theta} = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(f_c/f\right)^2}$$

The *Group velocity* is given by $u_G = u_u \cos\theta$

The phase constant is given by

$$\beta = \beta_u \sqrt{1-\left(f_c / f \right)^2}$$

The guide wavelength is given by

$$\lambda = \frac{\lambda_u}{\sqrt{1-\left(f_c / f \right)^2}}$$

The ratio of the transverse electric field to the transverse magnetic field for a propagating mode at a particular frequency is the *waveguide impedance*.

For a TE mode, the wave impedance is

$$\boxed{Z_{mn}^{TE} = \frac{\eta_u}{\sqrt{1-\left(\frac{f_c}{f} \right)^2}}},$$

For a TM mode, the wave impedance is

$$\boxed{Z_{mn}^{TM} = \eta_u \sqrt{1-\left(\frac{f_c}{f} \right)^2}}.$$

General Wave Behaviors:

The wave behavior in a waveguide can be determined by

Mode	Wave Impedance, Z	Guide Wavelength, λ_g
TEM	$\eta = \sqrt{\frac{\mu}{\epsilon}}$	$\lambda = \frac{1}{f\sqrt{\mu\epsilon}}$
TM	$\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$	$\frac{\lambda}{\sqrt{1-(f_c/f)^2}}$
TE	$\frac{\eta}{\sqrt{1-(f_c/f)^2}}$	$\frac{\lambda}{\sqrt{1-(f_c/f)^2}}$

$$H_x^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial H_z^0}{\partial x} - j\omega\epsilon\frac{\partial E_z^0}{\partial y}\right),$$

$$H_y^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial H_z^0}{\partial y} + j\omega\epsilon\frac{\partial E_z^0}{\partial x}\right),$$

$$E_x^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial E_z^0}{\partial x} + j\omega\mu\frac{\partial H_z^0}{\partial y}\right),$$

$$E_y^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial E_z^0}{\partial y} - j\omega\mu\frac{\partial H_z^0}{\partial x}\right),$$

$$H_x^0 = \frac{j\omega\epsilon}{h^2}\frac{\partial E_z^0}{\partial y},$$

$$H_y^0 = -\frac{j\omega\epsilon}{h^2}\frac{\partial E_z^0}{\partial x},$$

$$E_x^0 = -\frac{\gamma}{h^2}\frac{\partial E_z^0}{\partial x},$$

$$E_y^0 = -\frac{\gamma}{h^2}\frac{\partial E_z^0}{\partial y}.$$

(1) TM mode phase velocity always faster than the light speed in the medium
(2) TM mode group velocity always slower than the light speed in the medium
(3) Depends on frequency dispersive transmission systems
(4) Propagation velocity (velocity of energy transport) = group velocity.

Modes of propagation:

Using phasors & assuming waveguide filled with

- lossless dielectric material and
- walls of perfect conductor,

the wave inside should obey…

$$\nabla^2 E + k^2 E = 0$$

$$\nabla^2 H + k^2 H = 0$$

$$\text{where} \quad k^2 = \omega^2 \mu \varepsilon_c$$

Then applying on the z-component

$$\nabla^2 E_z + k^2 E_z = 0$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} + k^2 E_z = 0$$

Solving by method of Separation of Variables :

$$E_z(x, y, z) = X(x)Y(y)Z(z)$$

from where we obtain :

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -k^2$$

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -k^2$$

$$-k_x^2 - k_y^2 + \gamma^2 = -k^2$$

which results in the expressions :

$$X'' + k_x^2 X = 0$$

$$Y'' + k_y^2 Y = 0$$

$$Z'' - \gamma^2 Z = 0$$

From Faraday and Ampere Laws we can find the remaining four components

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$H \quad = \frac{j\omega\varepsilon}{\ } \frac{\partial E_z}{\ } - \frac{\gamma}{\ } \frac{\partial H_z}{\ }$$

Modes of propagation:

From the above equations we can conclude:

- TEM (Ez=Hz=0) can't propagate.

- TE (Ez=0) transverse electric
  - In TE mode, the electric lines of flux are perpendicular to the axis of the waveguide

- TM (Hz=0) transverse magnetic, Ez exists
  - In TM mode, the magnetic lines of flux are perpendicular to the axis of the waveguide.
- HE hybrid modes in which all components exists.

TM Mode:

$$E_z = E_o \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta z}$$

$$H_z = 0$$

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} \qquad E_x = -\frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right)E_o\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} \qquad E_y = -\frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right)E_o\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

- The *m* and *n* represent the mode of propagation and indicates the number of variations of the field in the *x* and *y* directions

TM Cutoff:

$$\gamma = \sqrt{\left(k_x^2 + k_y^2\right) - k^2}$$
$$= \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu\varepsilon}$$

- The cutoff frequency occurs when

$$\text{When } \omega_c^{\,2}\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad \text{then} \quad \gamma = \alpha + j\beta = 0$$

$$\text{or} \quad f_c = \frac{1}{2\pi}\frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

No propagation, everything is attenuated

$$\text{When} \quad \omega^2\mu\varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad \gamma = \alpha \text{ and } \beta = 0$$

Propagation:

$$\text{When} \quad \omega^2\mu\varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad \gamma = j\beta \ \text{ and } \ \alpha = 0$$

Cutoff

- The cutoff frequency is the frequency below which attenuation occurs and above which propagation takes place. (High Pass)

$$f_{c\,mn} = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

The phase constant becomes

$$\beta = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} = \beta'\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

Phase velocity and impedance

- The phase velocity is defined as

$$u_p = \frac{\omega}{\beta'} \qquad \lambda = \frac{2\pi}{\beta} = \frac{u_p}{f}$$

- intrinsic impedance of the mode is

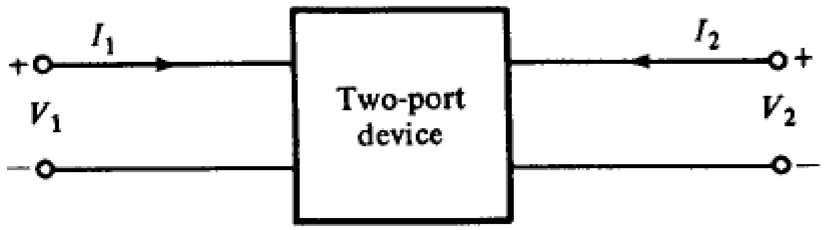
$$\eta_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \eta'\sqrt{1 - \left[\frac{f_c}{f}\right]^2}$$

MICROWAVE HYBRID CIRCUITS:

A microwave circuit is formed when several microwave components and devices such as microwave generators , microwave amplifiers, variable attenuators, cavity resonators, microwave filters, directional couplers, isolators are coupled together without any mismatch for proper transmission of a microwave signal.

Scattering matrix :

Let us consider a two port network which represents a number of parameter



$$H \text{ parameters: } \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \qquad \begin{aligned} V_1 &= h_{11}I_1 + h_{12}V_2 \\ I_2 &= h_{21}I_1 + h_{22}V_2 \end{aligned}$$

$$Y \text{ parameters: } \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_1 \end{aligned}$$

$$Z \text{ parameters: } \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned}$$

$$ABCD \text{ parameters: } \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad \begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}$$

All the above listed parameters can be represented as the ratio of either voltage to current or current or voltage under certain conditions of input or output ports.

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} \qquad \text{(short circuit)}$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} \qquad \text{(open circuit)}$$

At microwave frequencies it is impossible to measure :

1. total voltage and current as the required equipment is not available.
2. Over a broad band region, it is difficult to achieve perfect open and short circuit conditions.
3. The active devices used inside the two port network such as microwave power transistors will tend to become unstable under open and short circuit conditions.

WAVE GUIDE TEE JUNCTIONS:

A waveguide Tee is formed when three waveguides are interconnected in the form of English alphabet T and thus waveguide tee is 3-port junction. The waveguide tees are used to connects a branch or section of waveguide in series or parallel with the main waveguide transmission line either for splitting or combining power in a waveguide system.

There are basically 2 types of tees namely

1.) H- plane Tee junction
2.) E-plane Tee junction

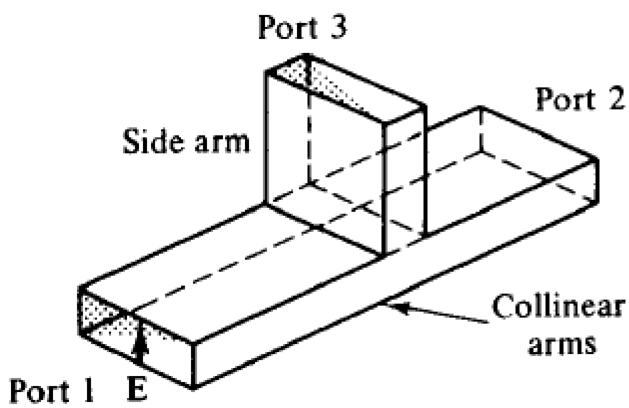
A combination of these two tee junctions is called a hybrid tee or " Magic Tee".

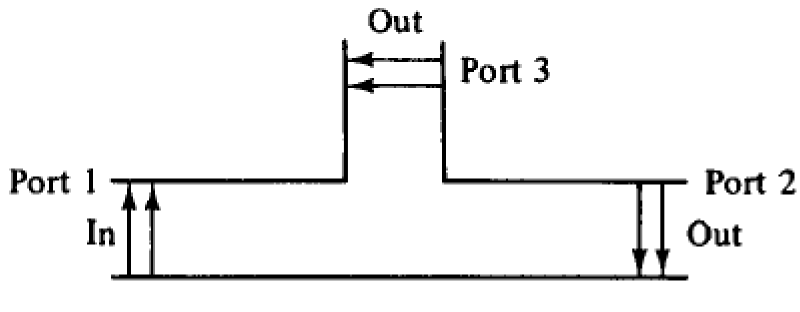
E-plane Tee(series tee):

An E-plane tee is a waveguide tee in which the axis of its side arm is parallel to the E field of the main guide . if the collinear arms are symmetric about the side arm.

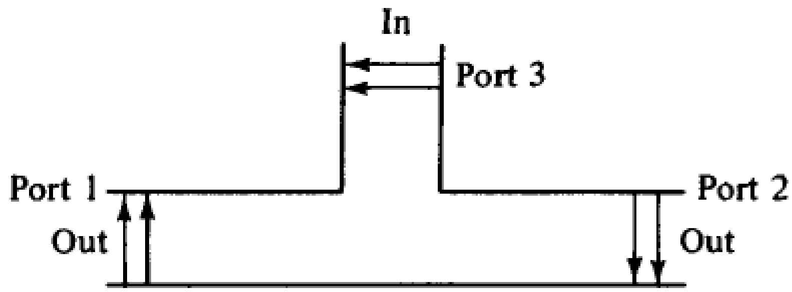
If the E-plane tee is perfectly matched with the aid of screw tuners at the junction , the diagonal components of the scattering matrix are zero because there will be no reflection.

When the waves are fed into side arm, the waves appearing at port 1 and port 2 of the collinear arm will be in opposite phase and in same magnitude.





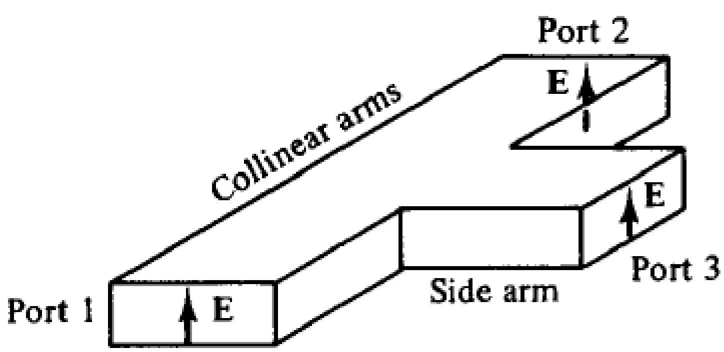
(a)



(b)

H-plane tee: (shunt tee)

An H-plane tee is a waveguide tee in which the axis of its side arm is shunting the E field or parallel to the H-field of the main guide.

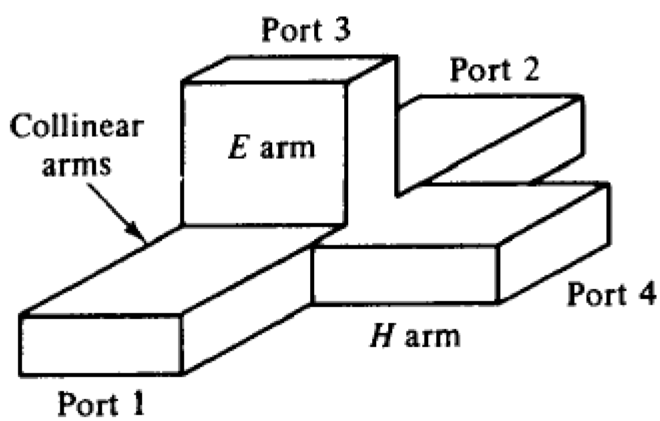


If two input waves are fed into port 1 and port 2 of the collinear arm, the output wave at port 3 will be in phase and additive .

If the input is fed into port 3, the wave will split equally into port 1 and port 2 in phase and in same magnitude .

Magic Tee (Hybrid Tees)

A magic tee is a combination of E-plane and H-plane tee. The characteristics of magic tee are:



1. If two waves of equal magnitude and same phase are fed into port 1 and port 2 the output will be zero at port 3 and additive at port 4.

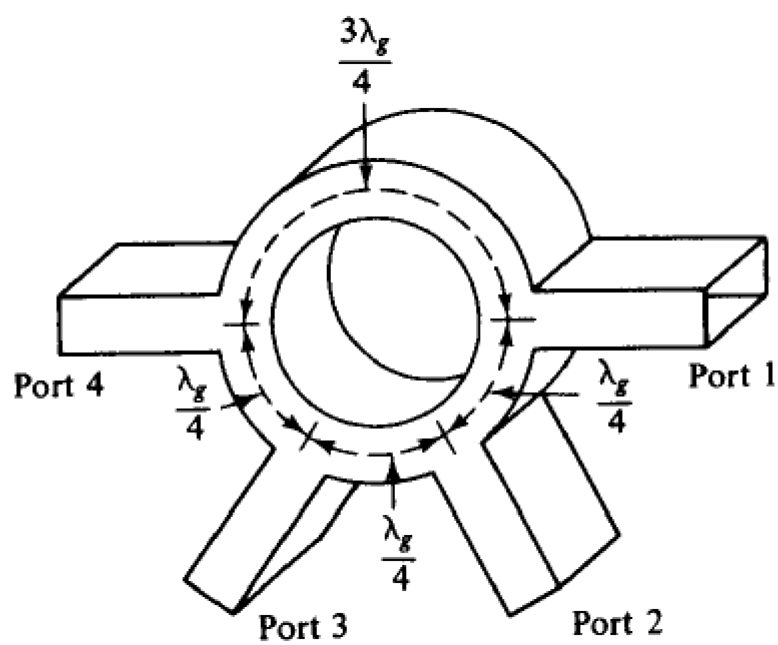
2. If a wave is fed into port 4 it will be divided equally between port 1 and port 2 of the collinear arms and will not appear at port 3.
3. If a wave is fed into port 3 , it will produce an output of equal magnitude and opposite phase at port 1 and port 2. the output at port 4 is zero.
4. if a wave is fed into one of the collinear arms at port 1 and port 2, it will not appear in the other collinear arm at port 2 or 1 because the E-arm causes a phase delay while H arm causes a phase advance.

Therefore the **S** matrix of a magic tee can be expressed as

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix}$$

Hybrid Rings(Rat Race circuits):

A hybrid ring consists of an annular line of proper electrical length to sustain standing waves, to which four arms are connected at proper intervals by means of series or parallel junctions.



The hybrid ring has characteristics similar to those of the hybrid tee. When a I wave is fed into port 1, it will not appear at port 3 because the difference of phase shifts for the waves traveling in the clockwise and counterclockwise direction is 180°. Thus the waves are canceled at port 3. For the same reason, the waves fed into port 2 will not emerge at port 4 and so on.

The S matrix for an ideal hybrid ring can be expressed as

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

It should be noted that the phase cancellation occurs only at a designated frequency for an ideal hybrid ring. In actual hybrid rings there are small leakage couplings and therefore the zero elements in the matrix are not equal to zero.

WAVE GUIDE CORNERS , BENDS AND TWISTS:

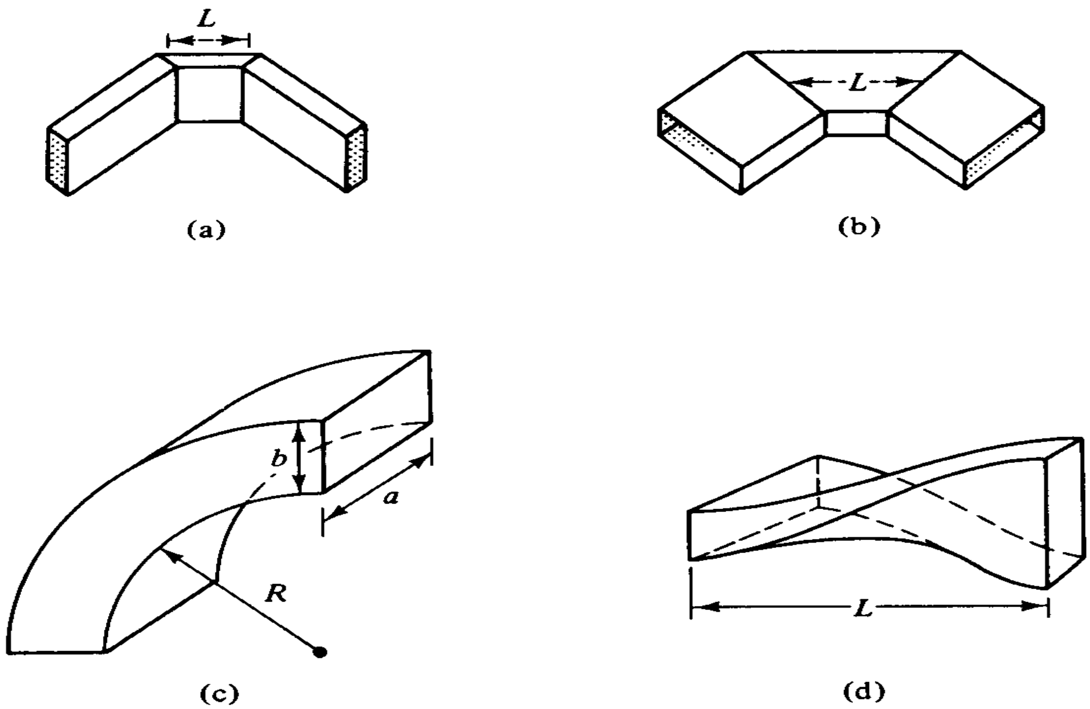
The waveguide corner, bend, and twist are shown in figure below, these waveguide components are normally used to change the direction of the guide through an arbitrary angle.

In order to minimize reflections from the discontinuities, it is desirable to have the mean length L between continuities equal to an odd number of quarter wave lengths. That is,

$$L = (2n + 1)\frac{\lambda_g}{4}$$

where n = 0, 1, 2, 3, ... , and Ag is the wavelength in the waveguide. If the mean length L is an odd number of quarter wavelengths, the reflected waves from both ends of the waveguide section are completely canceled. For the waveguide bend, the minimum radius of curvature for a small reflection is given by Southworth as

$R = 1.5b$ for an E bend $\quad R = 1.5a$ for an H bend

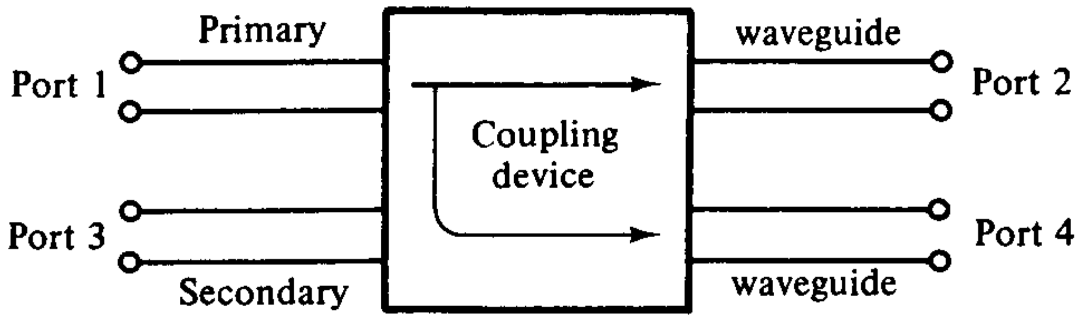


Waveguide corner, bend, and twist. (a) E-plane corner. (b) H-plane corner. (c) Bend. (d) Continuous twist.

DIRECTIONAL COUPLERS:

A directional coupler is a four-port waveguide junction as shown below. It Consists of a primary waveguide 1-2 and a secondary waveguide 3-4. When all Ports are terminated in their characteristic impedances, there is free transmission of the waves without reflection, between port 1 and port 2, and there is no transmission of power between port I and port 3 or between port 2 and port 4 because no coupling exists between these two pairs of ports. The degree of coupling between port 1 and port4 and between port 2 and port 3 depends on the structure of the coupler.

The characteristics of a directional coupler can be expressed in terms of its Coupling factor and its directivity. Assuming that the wave is propagating from port to port2 in the primary line, the coupling factor and the directivity are defined,



Directional coupler.

where PI = power input to port I

P3 = power output from port 3

P4 = power output from port 4

$$\text{Coupling factor (dB)} = 10 \log_{10} \frac{P_1}{P_4}$$

$$\text{Directivity (dB)} = 10 \log_{10} \frac{P_4}{P_3}$$

It should be noted that port 2, port 3, and port 4 are terminated in their characteristic impedances. The coupling factor is a measure of the ratio of power levels in the primary and secondary lines. Hence if the coupling factor is known, a fraction of power measured at port 4 may be used to determine the power input at port 1 .

This significance is desirable for microwave power measurements because no disturbance, which may be caused by the power measurements, occurs in the primary line. The directivity is a measure of how well the forward traveling wave in the primary waveguide couples only to a specific port of the secondary waveguide ideal directional coupler should have infinite directivity. In other words, the power at port 3 must be zero because port 2 and portA are perfectly matched. Actually well-designed directional couplers have a directivity of only 30 to 35 dB.

Several types of directional couplers exist, such as a two-hole direct couler, four-hole directional coupler, reverse-coupling directional coupler , and Bethe-hole directional coupler the very commonly used two-hole directional coupler is described here.

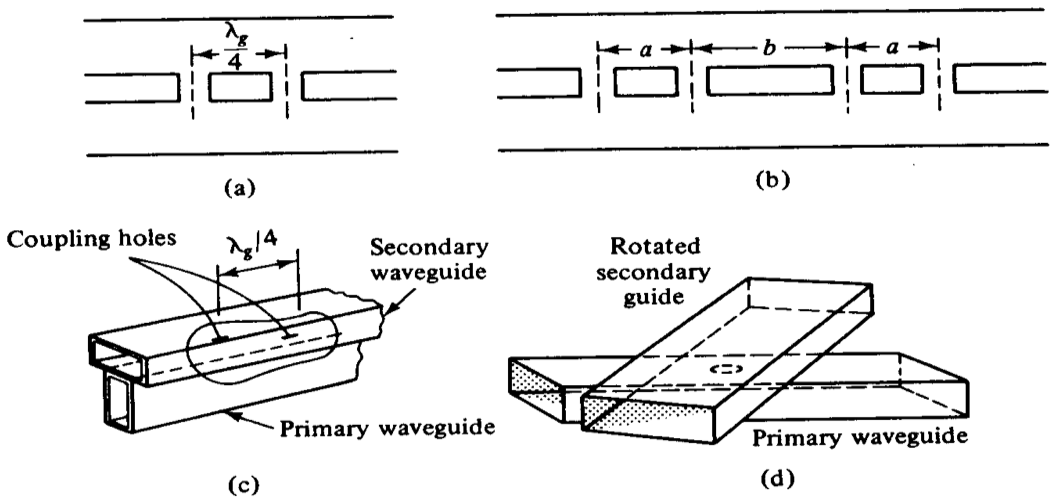


Figure 4-5-2 Different directional couplers. (a) Two-hole directional coupler. (b) Four-hole directional coupler. (c) Schwinger coupler. (d) Bethe-hole directional coupler.

TWO HOLE DIRECTIONAL COUPLERS:

A two hole directional coupler with traveling wave propagating in it is illustrated . the spacing between the centers of two holes is

$$L = (2n + 1)\frac{\lambda_g}{4}$$

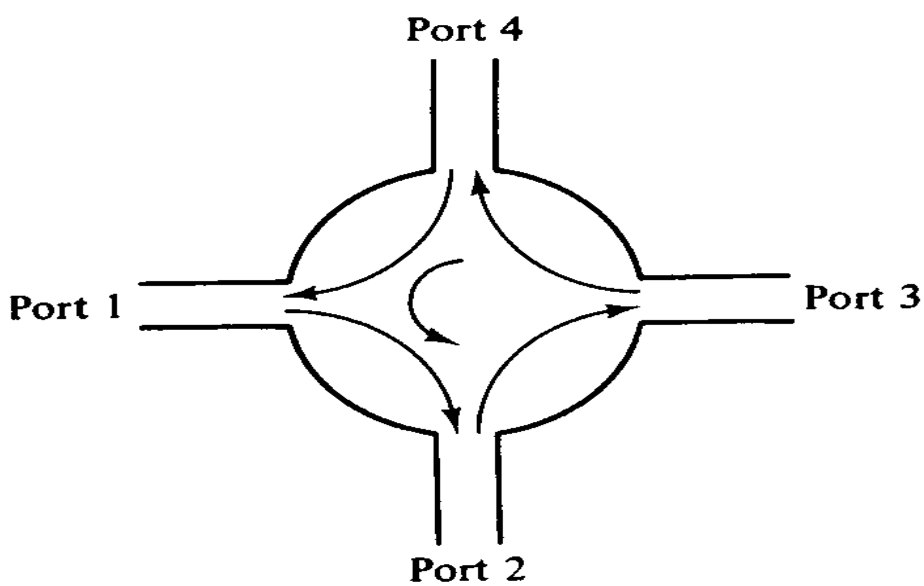
A fraction of the wave energy entered into port 1 passes through the holes and is radiated into the secondary guide as he holes act as slot antennas. The forward waves in the secondary guide are in same phase , regardless of the hole space and are added at port 4. the backward waves in the secondary guide are out of phase and are cancelled in port 3.

CIRCUALTORS AND ISOLATORS:

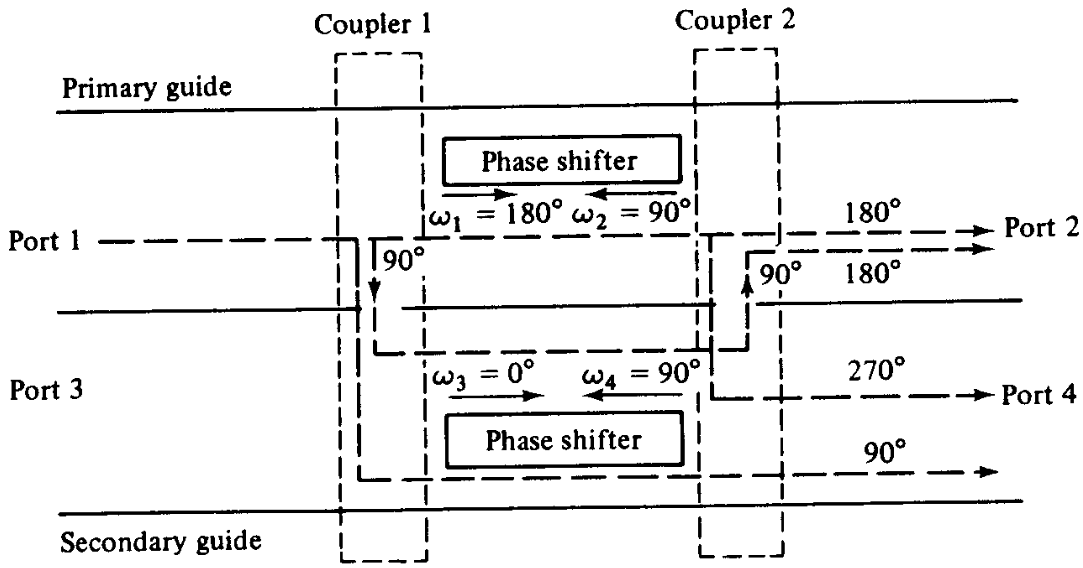
Both microwave circulators and isolators are non reciprocal transmission devices that use the property of Faraday rotation in the ferrite material. A non reciprocal phase shifter consists of thin slab of ferrite placed in a rectangular waveguide at a point where the dc magnetic field of the incident wave mode is circularly polarized. When a piece of ferrite is affected by a dc magnetic field the ferrite exhibits Faraday rotation. It does so because the ferrite is nonlinear material and its permeability is an asymmetric tensor.

MICROWAVE CIRCULATORS:

A *microwave circulator* is a multiport waveguide junction in which the wave can flow only from the *nth* port to the *(n* + I)th port in one direction Although there is no restriction on the number of ports, the four-port microwave circulator is the most common. One type of four-port microwave circulator is a combination of two 3-dB side hole directional couplers and a rectangular waveguide with two non reciprocal phase shifters.



The symbol of a circulator.



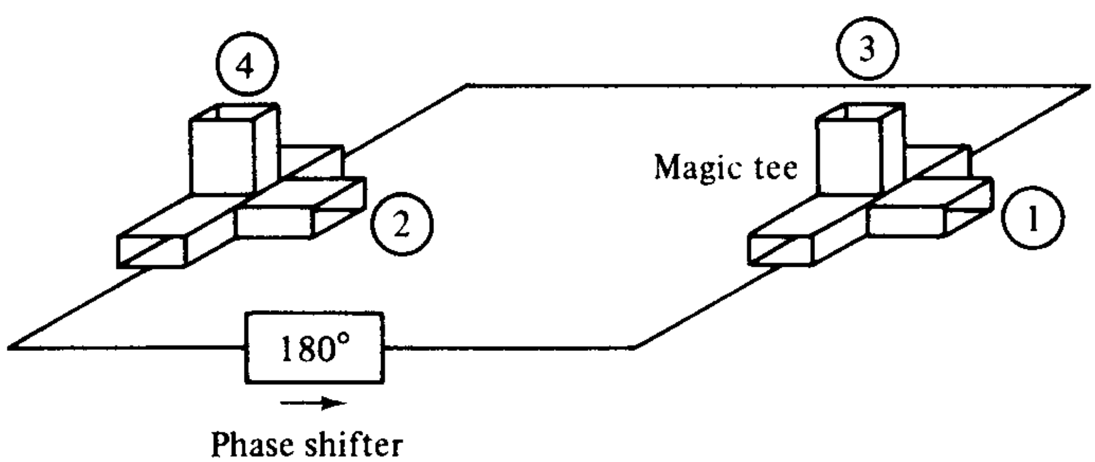
Schematic diagram of four-port circulator.

The operating principle of a typical microwave circulator can be analyzed with the aid of Fig shown above .Each of the two 3-dB couplers in the circulator introduces a phase shift of 90°, and each of the two phase shifters produces a certain amount of phase change in a certain direction as indicated. When a wave is incident to port 1,the wave is split into two components by coupler I. The wave in the primary guide arrives at port 2 with a relative phase' change of 180°. The second wave propagates through the two couplers and the secondary guide and arrives at port 2 with a relative phase shift of 180°. Since the two waves reaching port 2 are in phase, the power transmission is obtained from port 1 to port 2. However, the wave propagates through the primary guide, phase shifter, and coupler 2 and arrives at port 4 with a phase change of 270°. The wave travels through coupler 1 and the secondary guide, and it arrives at port 4 with a phase shift of 90°. Since the two waves reaching port 4 are out of phase by 180°, the power transmission from port 1 to port 4 is zero. In general, the differential propagation constants in the two directions of propagation in a waveguide containing ferrite phase shifters should be

$$\omega_1 - \omega_3 = (2m + 1)\pi \qquad \text{rad/s}$$
$$\omega_2 - \omega_4 = 2n\pi \qquad \text{rad/s}$$

where *m* and *n* are any integers, including zeros. A similar analysis shows that a wave incident to port 2 emerges at port 3 and so on. As a result, the sequence of power flow is designated as 1 ~ 2 ~ 3 ~ 4 ~ 1.

Many types of microwave circulators are in use today. However, their principles of operation remain the same. .A four-port circulator is constructed by the use of two magic tees and a phase shifter. The phase shifter produces a phase shift of 180°.



A four-port circulator.

A perfectly matched, lossless, and nonreciprocal four-port circulator has an S matrix of the form

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix}$$

Using the properties of S parameters the S-matrix is

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

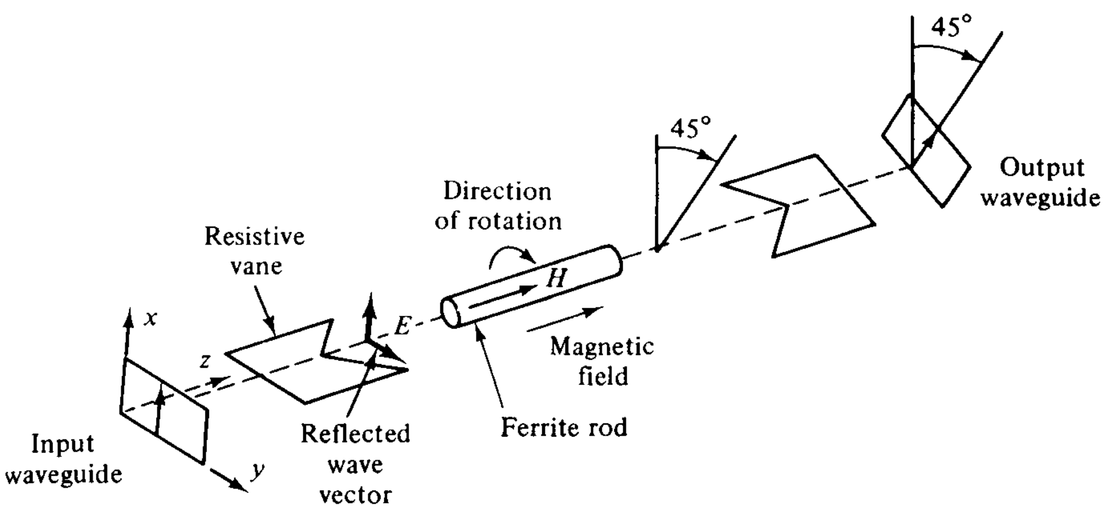
MICROWAVE ISOLATORS:

An *isolator* is a nonreciprocal transmission device that is used to isolate one component from reflections of other components in the transmission line. An ideal isolator completely absorbs the power for propagation in one direction and provides lossless transmission in the opposite direction. Thus the isolator is usually called *uniline.*

Isolators are generally used to improve the frequency stability of microwave generators, such as klystrons and magnetrons, in which the reflection from the load affects the generating frequency. In such cases, the isolator placed between the generator and load prevents the reflected power from the unmatched load from returning to the generator. As a result, the isolator maintains the frequency stability of the generator.

Isolators can be constructed in many ways. They can be made by terminating ports 3 and 4 of a four-port circulator with matched loads. On the other hand, isolators can be made by inserting a ferrite rod along the axis of a rectangular waveguide as shown below.

The isolator here is a Faraday-rotation isolator. Its operating principle can be explained as follows . The input resistive card is in the *y-z* plane, and the output resistive card is displaced 45° with respect to the input card. The dc magnetic field, which is applied longitudinally to the ferrite rod, rotates the wave plane of polarization by 45°. The degrees of rotation depend on the length and diameter of the rod and on the applied de magnetic field. An input TEIO dominant mode is incident to the left end of the isolator. Since the TEIO mode wave is perpendicular to the input resistive card, the wave passes through the ferrite rod without attenuation. The wave in the ferrite rod section is rotated clockwise by 45° and is normal to the output resistive card. As a result of rotation, the wave arrives at the output.



end without attenuation at all. On the contrary, a reflected wave from the output end is similarly rotated clockwise 45° by the ferrite rod. However, since the reflected wave is parallel to the input resistive card, the wave is thereby absorbed by the input card. The typical performance of these isolators is about 1-dB insertion loss in forward transmission and about 20- to 30-dB isolation in reverse attenuation.

RECOMMENDED QUESTIONS ON UNIT – 2

1. Discuss the various properties and characteristics of waveguides.
2. Show that waveguide acts as a high pass filter
3. Derive expressions for cutoff wavelength and cutoff frequency for TM waves propagating through rectangular waveguides.
4. Derive expressions for guide wavelength , phase and group velocity for TM waves in RWG
5. Draw the field patterns for the dominant TM and TE modes in rectangular waveguides.

6. Discuss the various types of loses occurring in rectangular waveguides.
7. Obtain an expression for attenuation in co-axial lines.
8. Derive an expression for frequency of oscillation for a rectangular and cylindrical resonator.
9. List the applications of cavity resonators.
10. Draw a neat diagram of H-plane Tee and explain its operation and derive the S matrix.
11. Draw a neat diagram of E-plane Tee and explain its operation and derive the S matrix.
12. Draw a neat diagram of MagicTee and explain its operation and derive the S matrix.
13. Explain the 2 hole directional coupler with sketch.
14. Explain the operation of a 3 port circulator
15. Explain the working of faraday rotation isolator.

UNIT - 3

MICROWAVE DIODES,

Transfer electron devices: Introduction, GUNN effect diodes – GaAs diode, RWH theory, Modes of operation, Avalanche transit time devices: READ diode, IMPATT diode, BARITT diode, Parametric amplifiers ,Other diodes: PIN diodes, Schottky barrier diodes.

7 Hours

TEXT BOOKS:

1. **Microwave Devices and circuits**- Liao / Pearson Education.

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2. **Microwave Engineering –** Annapurna Das, Sisir K Das TMH Publication, 2001.

REFERENCE BOOK:

1. **Microwave Engineering –** David M Pozar, John Wiley, 2e, 2004

Unit-3

MICROWAVE DIODES

TRANSFER ELECTRON DEVICES

INTRODUCTION:

The application of two-terminal semiconductor devices at microwave frequencies has been increased usage during the past decades. The CW, average, and peak power outputs of these devices at higher microwave frequencies are much larger than those obtainable with the best power transistor. The common characteristic of all active two-terminal solid-state devices is their negative resistance. The real part of their impedance is negative over a range of frequencies. In a positive resistance the current through the resistance and the voltage across it are in phase. The voltage drop across a positive resistance is positive and a power of *(12 R)* is dissipated in the resistance.

In a negative resistance, however, the current and voltage are out of phase by 180°. The voltage drop across a negative resistance is negative, and a power of *(-I!R)* is generated by the power supply associated with the negative resistance. In positive resistances absorb. power (passive devices), whereas negative resistances generate power (active devices). In this chapter the transferred electron devices(TEDs) are analyzed.

The differences between microwave transistors and transferred electron devices (TEDs) are fundamental. Transistors operate with either junctions or gates, but TEDs are bulk devices having no junctions or gates. The majority of transistors are fabricated from elemental semiconductors, such as silicon or germanium, whereas 1tDs are fabricated from compound semiconductors, such as

gallium arsenide (r.As),indium phosphide (lnP), or cadmium telluride (CdTe). Transistors operate
As "warm" electrons whose energy is not much greater than the thermal energy 0.026eVat room temperature) of electrons in the semiconductors.

GUNN EFFECT DIODES – GaAs diode

Gunn effect are named after J. B. Gunn who is 1963 discovered a periodic fluctuation of current passing through the n- type gallium arsenide . when the applied voltage exceeded a certain critical value.

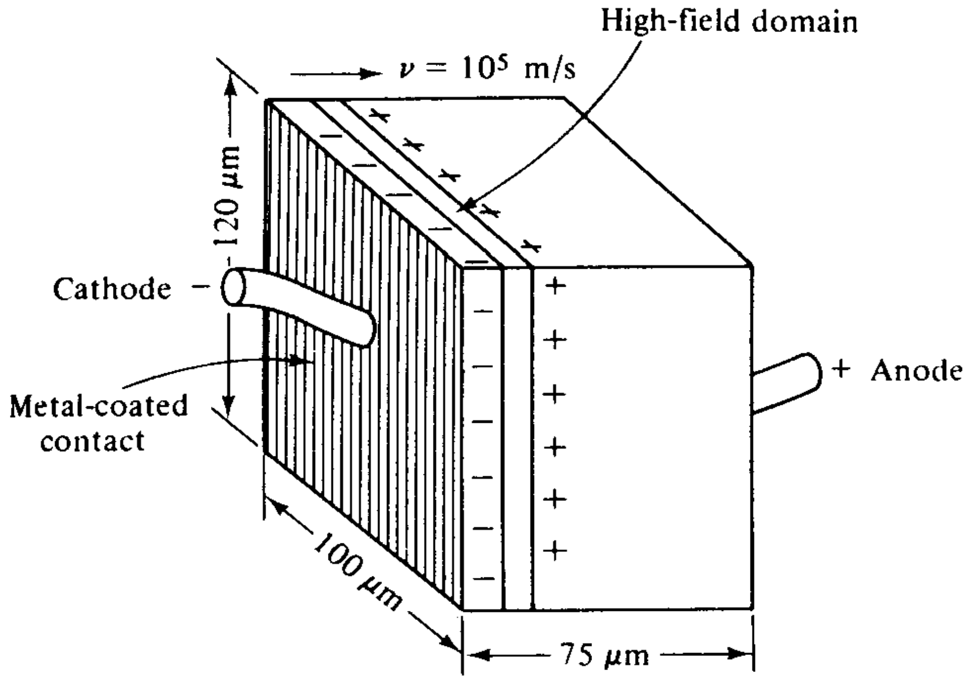
Shockley in 1954 suggested that the two terminal negative resistance devices using semiconductors had advantages over transistors at high frequencies.

In 1961 , Ridley and Watkins described a new method for obtaining negative differential mobility in semiconductors. The principle involved is to heat carriers in a light mass , low mobility , higher energy sub band when they have a high temperature.

Finally Kroemer stated that the origin of the negative differential mobility is Ridley Watkins Hilsum's mechanism of electron transfer into the valleys that occur in conduction bands.

Gunn effect:

The below figure shows the diagram of a uniform n-type GaAs diode with ohmic contacts at the end surfaces. Gunn stated that " Above some critical voltage , corresponding to an electric field of 2000 to 4000 Volts/cm, the current in every specimen became a fluctuating function of time.



Gunn Diodes

Single piece of GaAs or Inp and contains no junctions

Exhibits negative differential resistance

Applications:

low-noise local oscillators for mixers (2 to 140 GHz).

Low-power transmitters and wide band tunable sources

Continuous-wave (CW) power levels of up to several hundred mill watts can be obtained in the X-, Ku-, and Ka-bands. A power output of 30 mW can be achieved from commercially available devices at 94 GHz.

Higher power can be achieved by combining several devices in a power combiner.

Gunn oscillators exhibit very low dc-to-RF efficiency of 1 to 4%.

Gunn also discovered that the threshold electric field Eili varied with the length and type of material. He developed an elaborate capacitive probe for plotting the electric field distribution within a specimen of n-type GaAs of length *L* = 210 JLIll and cross-sectional area 3.5 x 10-3 cm2 with a low-field resistance of 16 n. Current instabilities occurred at specimen voltages above 59 V, which means that the threshold field is

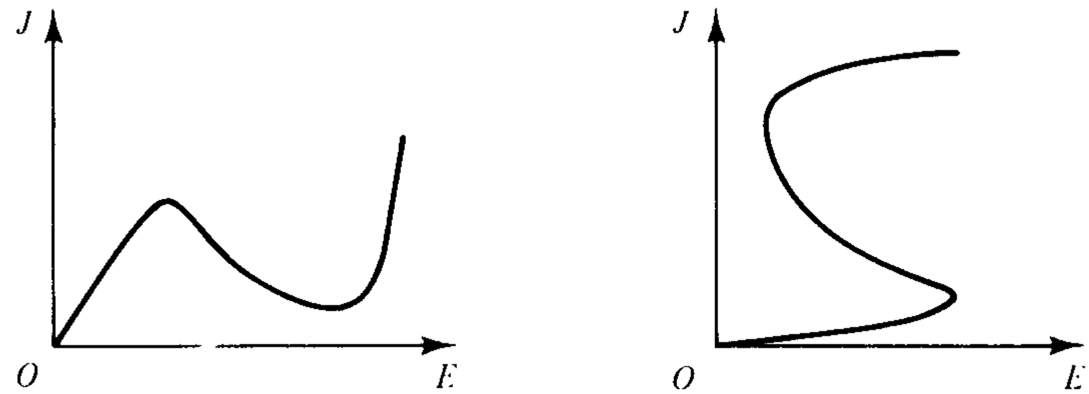
$$E_{th} = \frac{V}{L} = \frac{59}{210 \times 10^{-6} \times 10^2} = 2810 \text{ volts/cm}$$

RIDLEY WATKINS AND HILSUM THEORY:

Many explanations have been offered for the Gunn effect. In 1964 Kroemer [6] suggested that Gunn' s observations were in complete agreement with the Ridley-Watkins-Hilsum (RWH) theory.

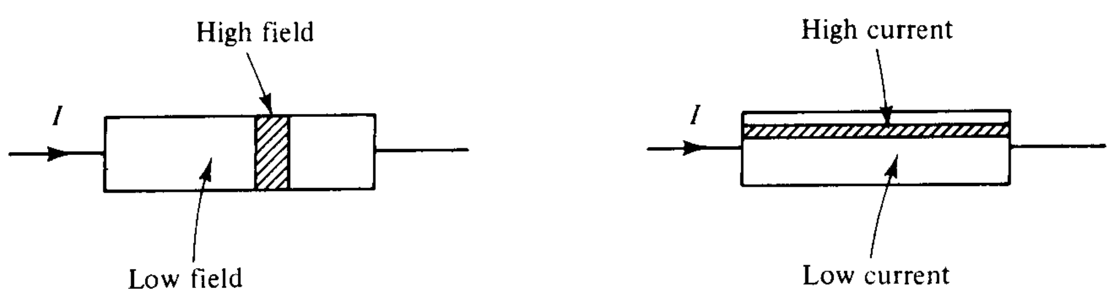
Differential Negative Resistance:

The fundamental concept of the Ridley-Watkins-Hilsum (RWH) theory is the differential negative resistance developed in a bulk solid-state III-V compound when either a voltage (or electric field) or a current is applied to the terminals of the sample. There are two modes of negative-resistance devices: voltage-controlled and current controlled Modes.



(a) Voltage-controlled mode (b) Current-controlled mode

In the voltage-controlled mode the current density can be multivalued, whereas in the current-controlled mode the voltage can be mu1tivalued. The major effect of the appearance of a differential negative-resistance region in the current density field curve is to render the sample electrically unstable. As a result, the initially homogeneous sample becomes electrically heterogeneous in an attempt to reach stability. In the voltage-controlled negative-resistance mode high-field domains are formed, separating two low-field regions. The interfaces separating low and high-field domains lie along equi potentials; thus they are in planes perpendicular to the current direction.

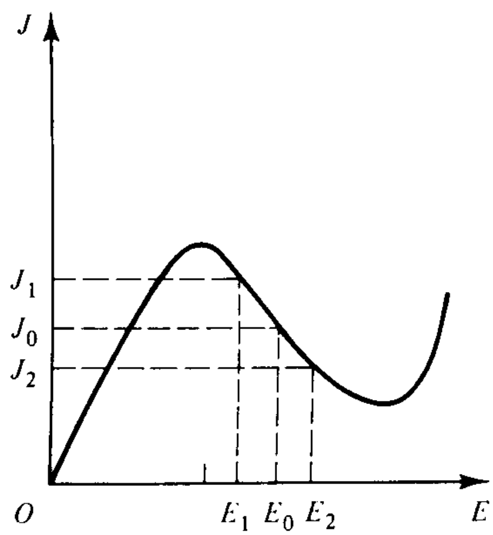


(a) High-field domain (b) High-current filament

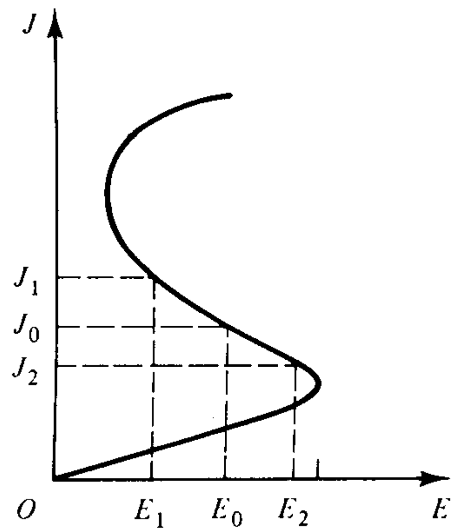
Expressed mathematically, the negative resistance of the sample at a particular region is

$$\frac{dI}{dV} = \frac{dJ}{dE} = \text{negative resistance}$$

If an electric field *Eo* (or voltage *Vo)* is applied to the sample, for example, the current density 10is generated. As the applied field (or voltage) is increased to *E1* (or *V2),* the current density is decreased to 12. When the field (or voltage) is decr~ to £. (or *VI),* the current density is increased to 1, . These phenomena of the voltage controlled negative resistance are shown in Fig. 7-2-3(a). Similarly, for the current controlled mode, the negative-resistance profile is as shown below.



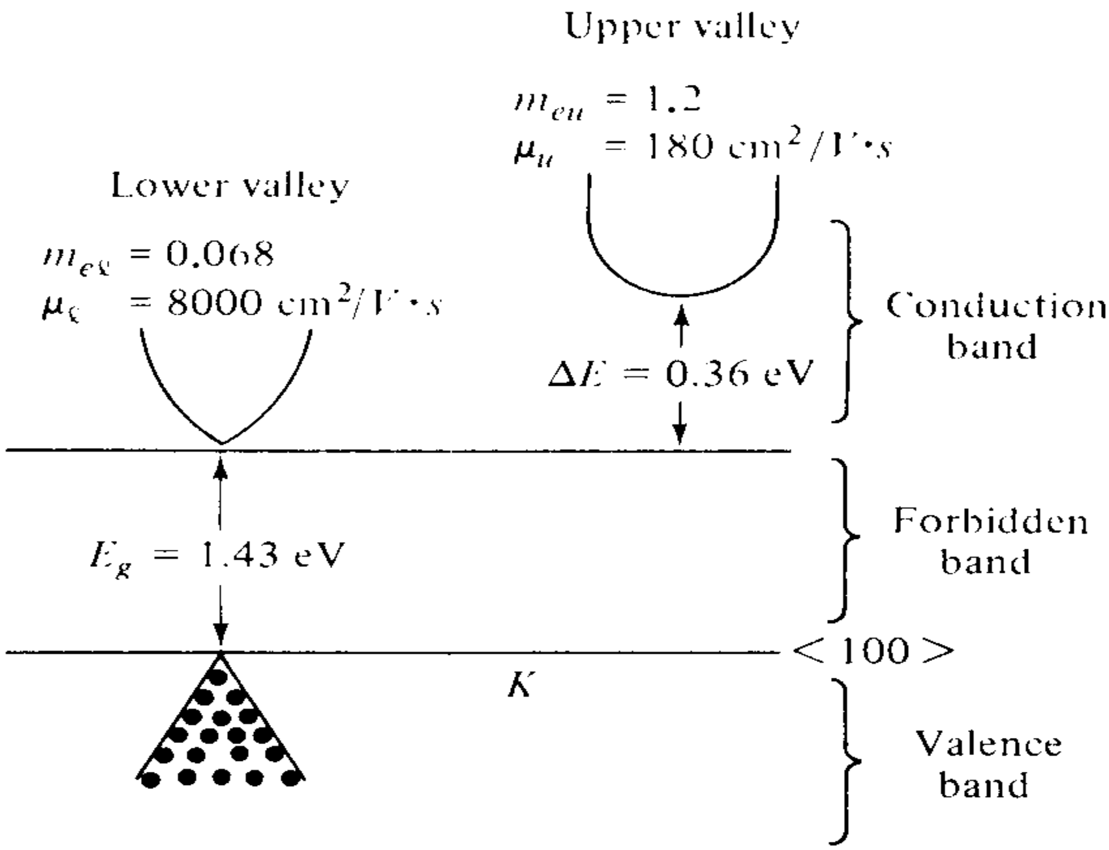
(a) Voltage-controlled mode



(b) Current-controlled mode

TWO VALLEY MODEL THEORY:

Kroemer proposed a negative mass microwave amplifier in 1958 [lO] and 1959 [II]. According *to* the energy band theory of the *n* -type GaAs, a high-mobility lower valley is separated by an energy of 0.36 eV from a low-mobility upper valley

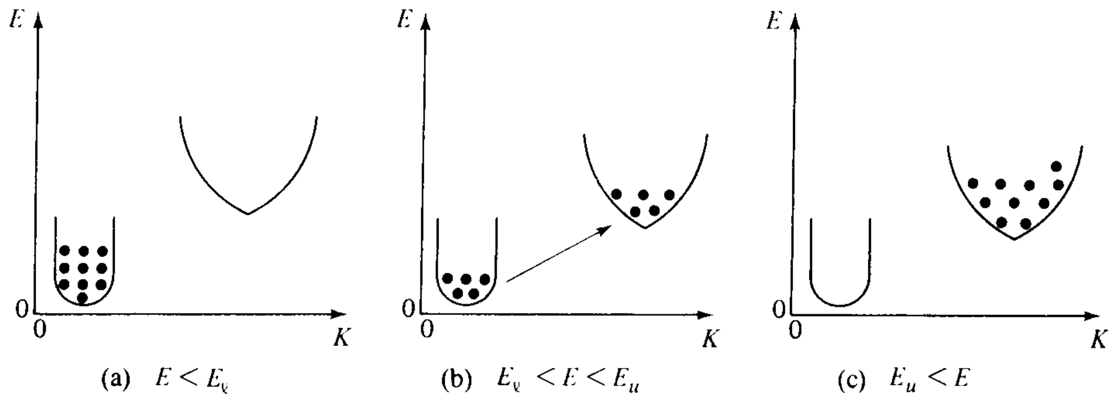


Electron densities in the lower and upper valleys remain the same under an Equilibrium condition. When the applied electric field is lower than the electric field of the lower valley *(E < Ee)*, no electrons will transfer to the upper valley.

When the applied electric field is higher than that of the lower valley and lower than that of the upper valley *(Ee < E < Eu))*, electrons will begin to transfer to the upper valley.

when the applied electric field is higher than that of the upper valley *(Eu < E)*, all electrons will transfer to the upper valley.

When a sufficiently high field *E* is applied to the specimen, electrons are accelerated and their effective temperature rises above the lattice temperature also increases. Thus electron density/I and are both functions of electric field *E*.



Transfer of electron densities.

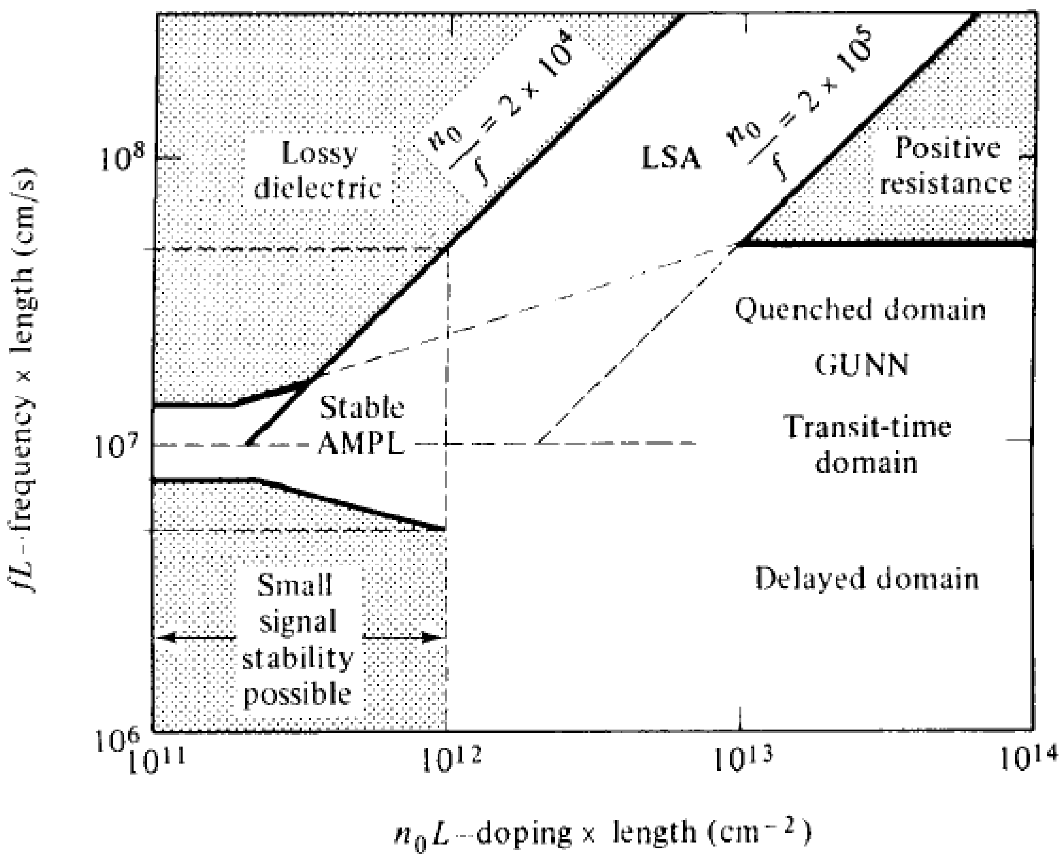
MODES OF OPERATION OF GUNN DIODE:

A gunn diode can operate in four modes:

1. Gunn oscillation mode
2. stable amplification mode
3. LSA oscillation mode
4. Bias circuit oscillation mode

Gunn oscillation mode: This mode is defined in the region where the product of frequency multiplied by length is about 107 cm/s and the product of doping multiplied by length is greater than 1012/cm2.In this region the device is unstable because of the cyclic formation of either the accumulation layer or the high field domain.

When the device is operated is a relatively high Q cavity and coupled properly to the load, the domain I quenched or delayed before nucleating.

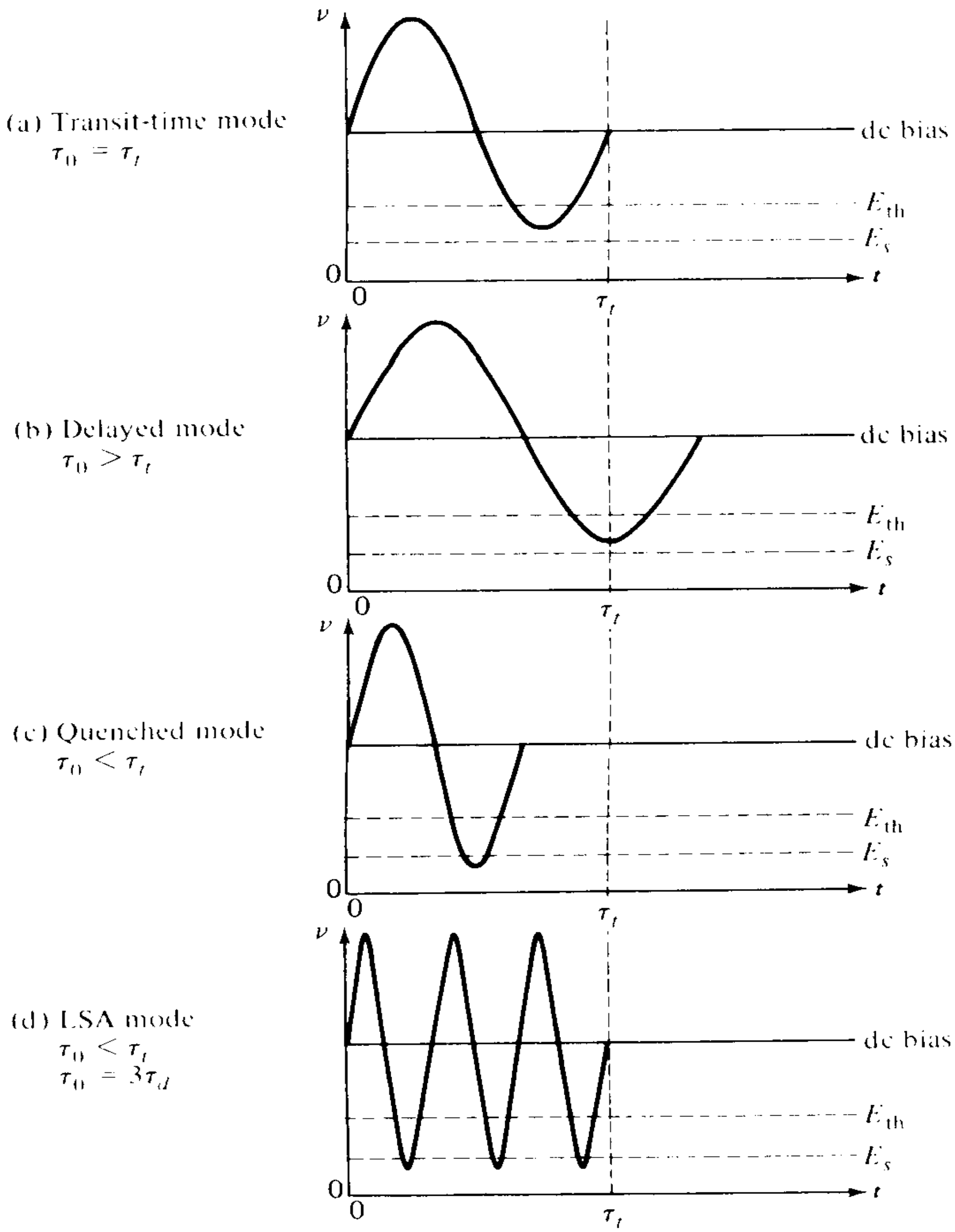


2.Stable amplification mode: This mode is defined in the region where the product of frequency times length is about 107 *cmls* and the product of doping times length is between l0^{11}and *1O^{12}/cm2*

.**3. LSA oscillation mode**: This mode is defined in the region where the product of frequency times length is above 10^{7} *cmls* and the quotient of doping divided by frequency is between 2 x 10^{4} and 2 x 10^{5}.

4. Bias-circuit oscillation mode: This mode occurs only when there is either Gunn or LSA oscillation. and it is usually at the region where the product of frequency times length is too small to appear in the figure. When a bulk diode is biased to threshold. the average current suddenly drops as Gunn oscillation begins.

The drop in current at the threshold can lead to oscillations in the bias circuit that are typically 1 kHz to 100 MHz .



Delayed domain mode (106 cm/s < *fL* < 107 cm/s). When the transit time is Chosen so that the domain is collected while *E* < Eth as shown in Fig. 7-3-4(b), a new domain cannot form until the field rises above threshold again. In this case, the oscillation period is greater than the transit time-that is, *To* > *T,*. This delayed mode is also called *inhibited mode.* The efficiency of this mode is about 20%.

Quenched domain mode (fL > 2 X 107 cm/s).

If the bias field drops below the sustaining field *Es* during the negative half-cycle as shown ,the domain collapses before it reaches the anode. When the bias field swings back above threshold ,a new domain is nucleated and the process repeats. Therefore the oscillations occur at the frequency of the resonant circuit rather than at the transit-time frequency, It has been found that the resonant frequency of the circuit is several times the transit-time frequency, since one dipole does not have enough time to readjust and absorb the voltage of the other dipoles . Theoretically, the efficiency of quenched domain oscillators can reach 13%

LSA MODE

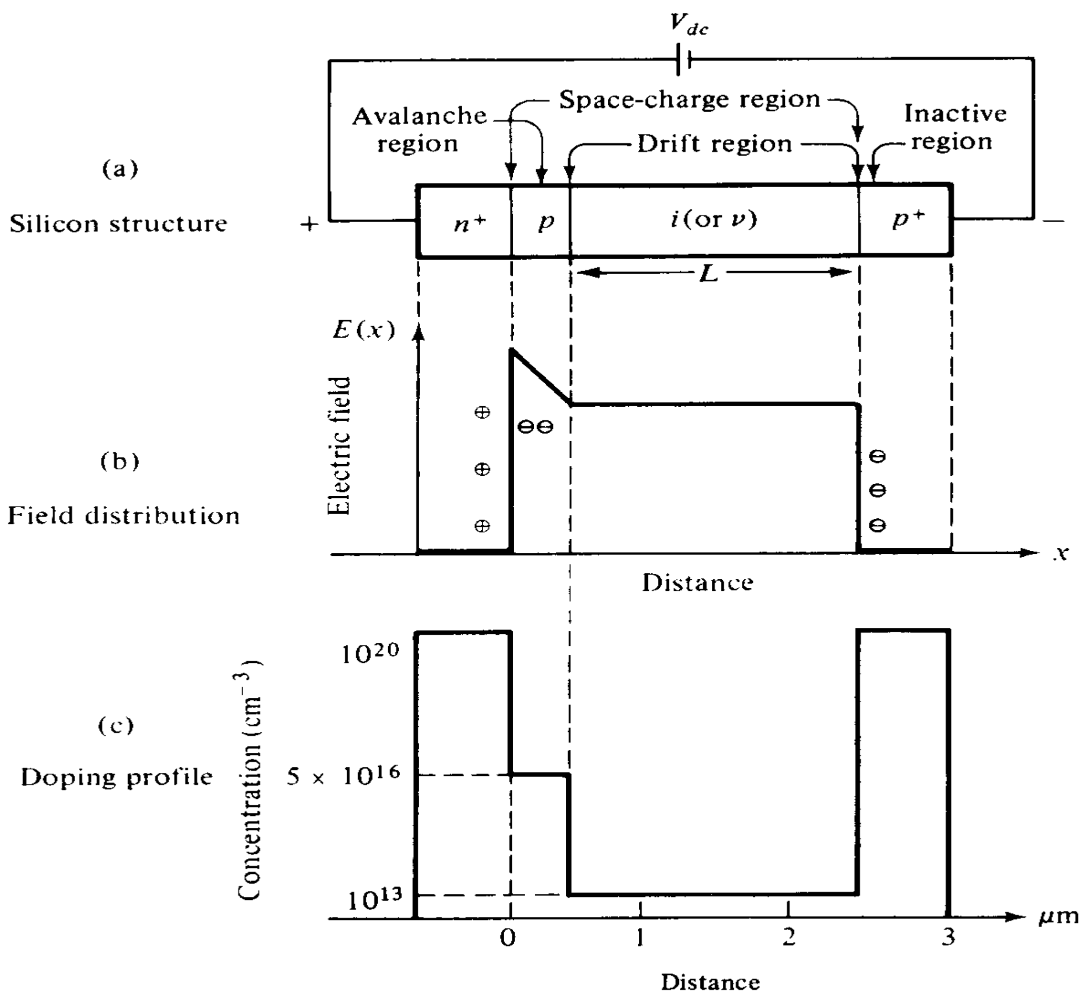
When the frequency is very high, the domains do not have sufficient time to form While the field is above threshold. As a result, most of the domains are maintained In the negative conductance state during a large fraction of the voltage cycle. Any Accumulation of electrons near the cathode has time to collapse while the signal is Below threshold. Thus the LSA mode *is* .the simplest mode of operation.

AVALANCHE TRANSIT TIEM DEVICES:

READ DIODE:

Read diode was the first proposed avalanche diode. The basic operating principles of IMPATT diode can be easily understood by first understanding the operation of read diode.

The basic read diode consists of four layers namely n+ p I p+ layers. The plus superscript refers to very high doping levels and ‘i’ denotes intrinsic layer.A large reverse bias is applied across diode . the avalanche multiplication occurs in the thin “p” region which is also called the high field region or avalanche region.



The holes generated during the avalanche process drift through the intrinsic region while moving towards p+ contact. The region between n+ p junction and the i-p+ junction is known as space charge region.

When this diode is reverse biased and placed inside an inductive microwave cavity microwave oscillations are produced due to the resonant action of the capacitive impedance of the diode and cavity inductance. The dc bias power is converted into microwave power by that read diode oscillator.

Avalanche multiplication occurs when the applied reverse bias voltage is greater then the breakdown voltage so that the space charge region extends from n+p junction through the p and I regions, to the i to p+ junction.

IMPATT DIODE:

lmpatt diodes are manufactured having different forms such as n+pip+, p+nin+, p+nn+ abrupt junction and p+ i n+ diode configuration. The material used for manufacture of these modes are either Germanium, Silicon, Gallium Arsenide (GaAs) or Indium Phosphide (In P).

Out of these materials, highest efficiency, higher operating frequency and lower noise is obtained with GaAs. But the disadvantage with GaAs is complex fabrication process and hence higher cost. The figure below shows a reverse biased n+ pi p+ diode with electric field variation, doping concentration versus distance plot, the microwave voltage swing and the current variation.

PRINICPLE OF OPERATION:

When a reverse bias voltage exceeding the breakdown voltage is applied, a high electric field appears across the n+ p junction. This high field intensity imparts sufficient energy to the valence electrons to raise themselves into the conduction band. This results avalanche multiplication of hole-electron pairs. With suitable doping profile design, it is possible to make electric field to have a very sharp peak in the close vicinity of the junction resulting in "impact avalanche multiplication". This is a cumulative process resulting in rapid increase of carrier density. To prevent the diode from burning, a constant bias source is used to maintain average current at safe limit 10, The diode current is contributed by the conduction electrons which move to the n+ region and the associated holes which drift through

the steady field and a.c. field.· The diode ~wings into and out of avalanche conditions under the influence of that reverse bias steady field and the a.c. field.

Due to the drift *time* of holes being' small, carriers drift to the end contacts before the a.c. voltage swings the diode out of the avalanche Due to building up of oscillations, the a.c. field takes energy from the applied bias lid the oscillations at microwave frequencies are sustained across the diode. Due to this a.c. field, the hole current grows exponentially to a maximum and again decays exponentially to Zero.

During this hole drifting process, a constant electron current is induced in the external Circuit which starts flowing when hole current reaches its peak and continues for half cycle Corresponding to negative swing of the a.c. voltage as shown in figure Thus a 180 degrees Phase shift between the external current and a.c. microwave voltage provides a negative Resistance for sustained oscillations.

The resonator is usually tuned to this frequency so that the IMPATI diodes provide a High power continuous wave (CW) and pulsed microwave signals.

$$\theta = \omega\tau = \omega\frac{L}{v_d}$$

$$\omega_r \equiv \left(\frac{2\alpha' v_d I_0}{\epsilon_s A}\right)^{1/2}$$

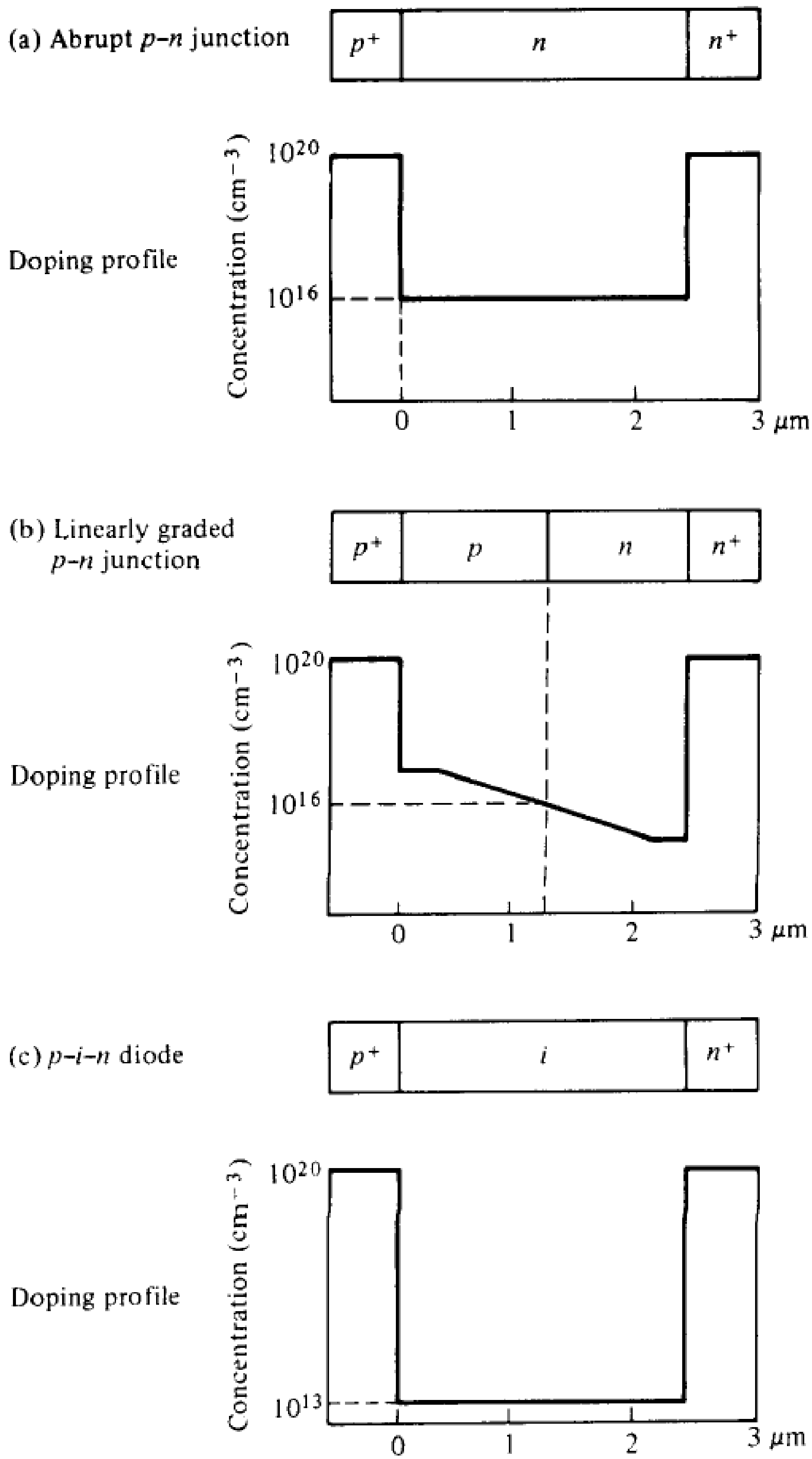
Applications of IMPATT Diodes

(i) Used in the final power stage of solid state microwave transmitters for communication purpose.

(ii) Used in the transmitter of TV system.

(iii) Used in FDM/TDM systems.

(iv) Used as a microwave source in laboratory for measurement purposes.


(a) Abrupt p–n junction
p+
n
n+
Doping profile
Concentration (cm−3)
10^20
10^16
0
1
2
3 μm
(b) Linearly graded p–n junction
p+
p
n
n+
Doping profile
Concentration (cm−3)
10^20
10^16
0
1
2
3 μm
(c) p–i–n diode
p+
i
n+
Doping profile
Concentration (cm−3)
10^20
10^13
0
1
2
3 μm

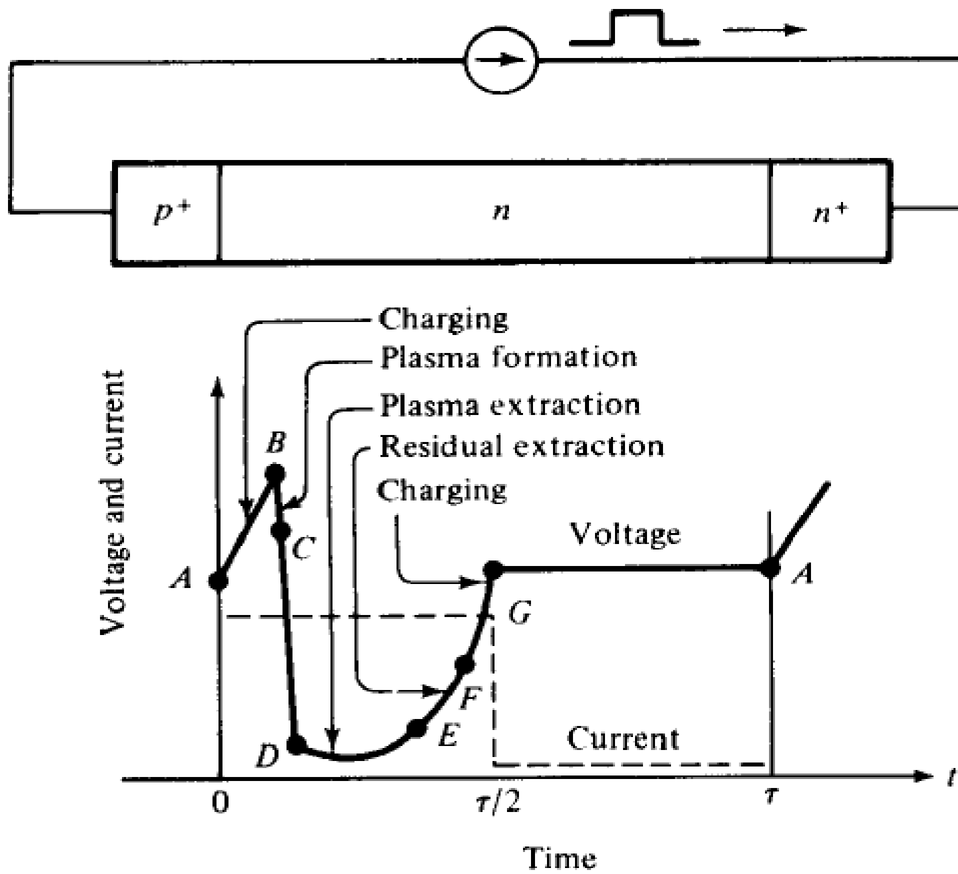
TRAPATT DIODE:

Silicon is usually used for the manufacture of TRAPATT diodes and have a configuration of p+ n n+ as shown .The p-N junction is reverse biased beyond the breakdown region, so that the current density is larger. This decreases the electric field in the space charge region and increases the carrier transit time. Due to this, the frequency of operation gets lowered to less than 10 GHz. But the efficiency gets increased due to low power dissipation.

Inside a co-axial resonator, the TRAPATT diode is normally mounted at a point where maximum RF voltage swing is obtained. When the combined dc bias and RF voltage exceeds breakdown voltage, avalanche occurs and a plasma of holes and electrons are generated which gets trapped. When the external circuit current flows, the voltage rises and the trapped plasma gets released producing current pulse across the drift space. The total transit time is the sum of the drift time and the delay introduced by the release of the trapped plasma. Due to this longer transit time, the operating frequency is limited to 10 GHz. Because the current pulse is associated with low voltage, the power dissipation is low resulting in higher efficiency.

The disadvantages of TRAPATT are high noise figure and generation of strong harmonics due to short duration of the current pulse.

TRAPATT diode finds application in S-band pulsed transmitters for pulsed array radar systems.



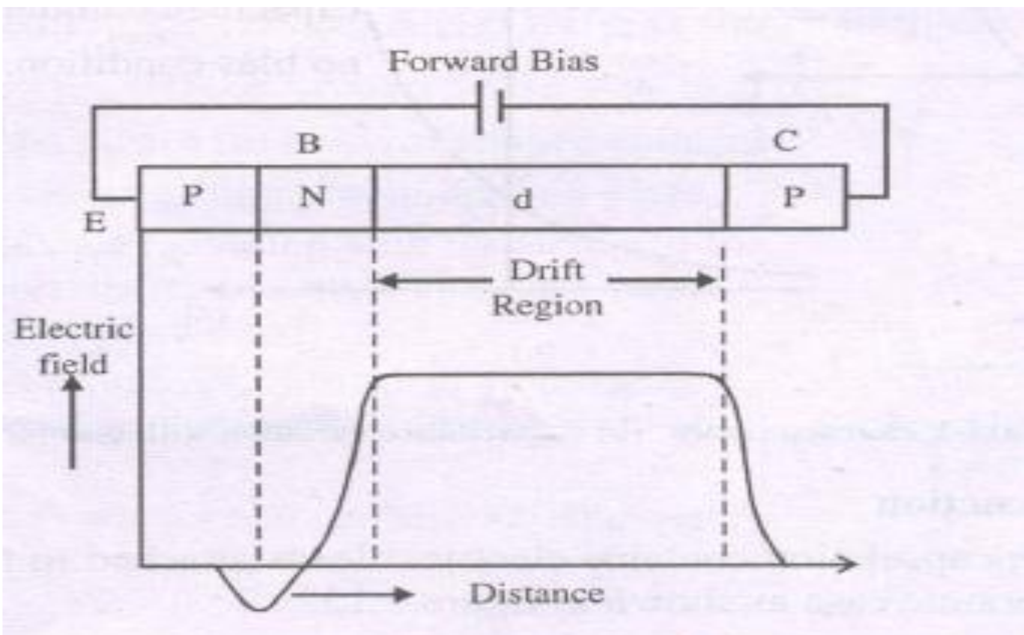
The electric field is expressed as

$$E(x,\ t) = E_m - \frac{qN_A}{\epsilon_s}x + \frac{Jt}{\epsilon_s}$$

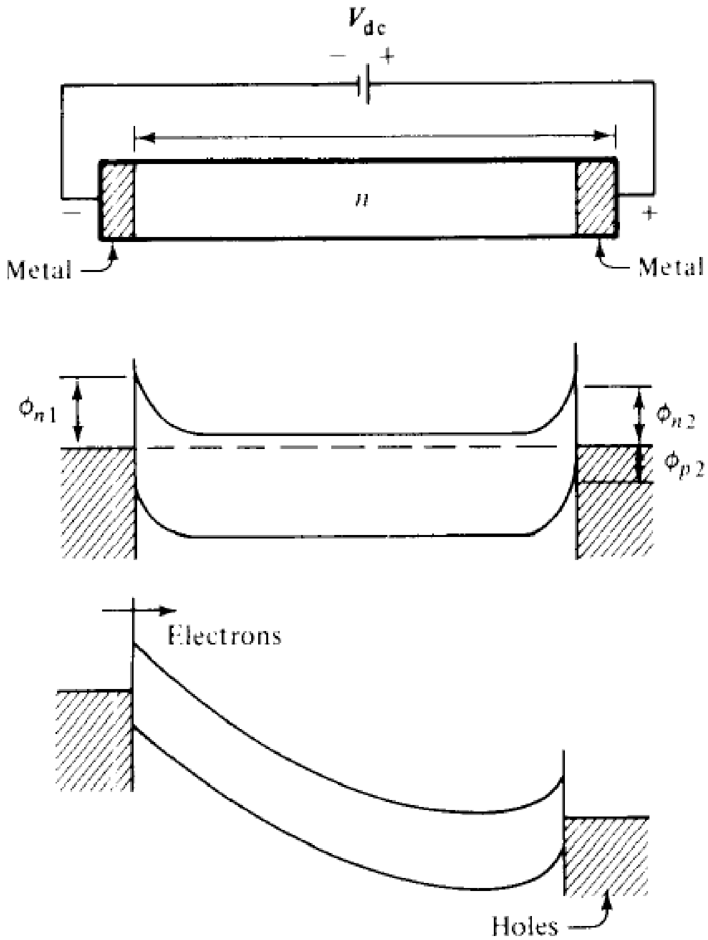
BARITT DIODE (Barrier injection transmit time devices):

BARITT devices are an improved version of IMPATT devices. IMPATT devices employ impact ionization techniques which is too noisy. Hence in order to achieve low noise figures, impact ionization is avoided in BARRITT devices. The minority injection is provided by punch-through of the intermediate region (depletion region). The process is basically of lower noise than impact ionization responsible for current injection in an IMPATT. The negative resistance is obtained on account of the drift of the injected holes to the collector end of the p-material.

The construction of a BARITT device consisting of emitter, base,
intermediate or drift or depleted region and collector. An essential requirement for the BARITT device is therefore that the intermediate drift region be entirely depleted to cause punch through to the emitter-base junction without causing avalanche breakdown of the base-collector junction.



The parasitic should be kept as low as possible. The equivalent circuit depends on the type of encapsulation and mounting make. For many applications, there should be a large capacitance variation, small value of minimum capacitance and series resistance Rs' Operation is normally limited to f/l0 [25 GHz for *Si* and 90 GHz for GaAs]. Frequency of operation beyond (f /10) leads to increase in R, decrease in efficiency and increase in noise.



(a) M–n–M diode

(b) Energy band diagram in thermal equilibrium

(c) Energy band under bias condition

PARAMETRIC AMPLIFIERS:

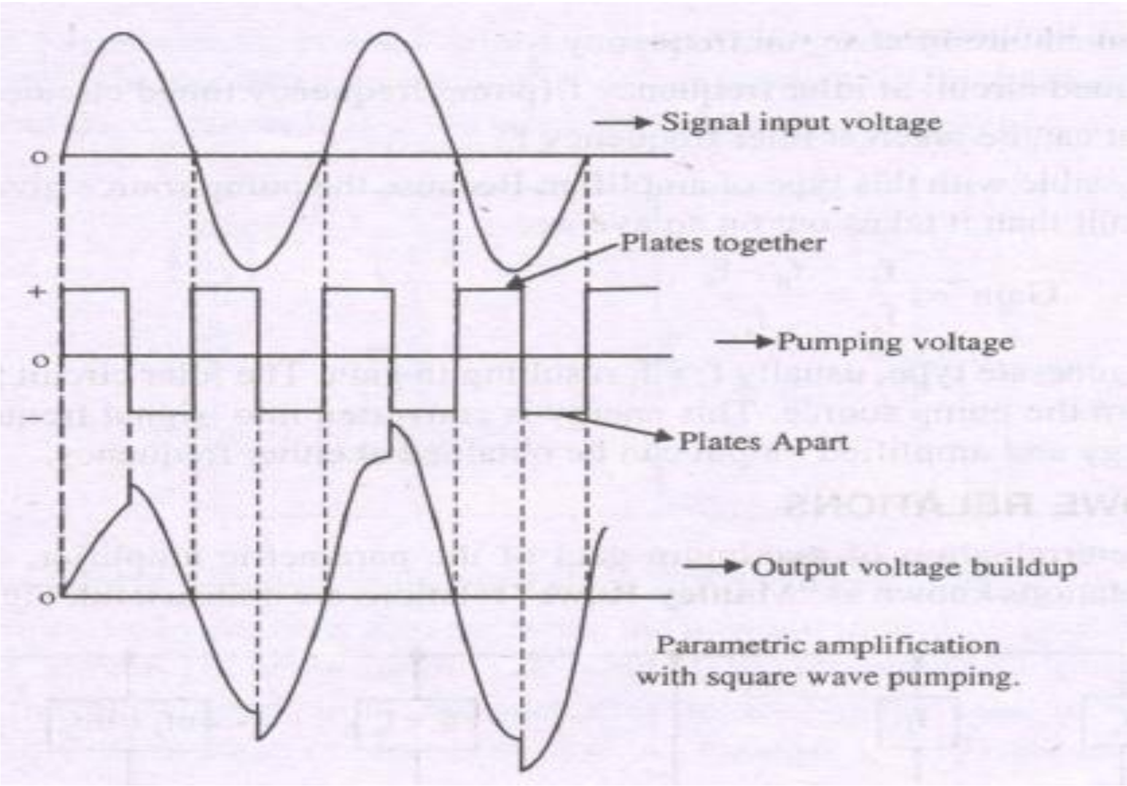
The parametric amplifier is an amplifier using a device whose reactance is varied to produce amplification. Varactor diode is the most widely used active element in a parametric amplifier. It is a low noise amplifier because no resistance is involved in the amplifying process. There will be no thermal noise, as the active element used involved is reactive (capacitive). Amplification is obtained if the reactance is varied electronically in some predetermined fashion.

Due to the advantage of low noise amplification, parametric amplifiers are extensively used in systems such as long range radars, satellite ground stations, radio telescopes, artificial satellites, microwave ground communication stations, radio astronomy etc.

Basic **Parametric Amplifier**

A conventional amplifier uses a variable resistance and a d.c. power supply. For a parametric amplifier, a variable reactance and an ac power supply are needed.

Pumping signal at frequency fp and a small amplitude signal at frequency fs are applied simultaneously to the device (varactor). The pump source supplies energy to the signal (at the signal frequency) resulting in amplification. This occurs at the active device where the capacitive reactance varies at the pump frequency.



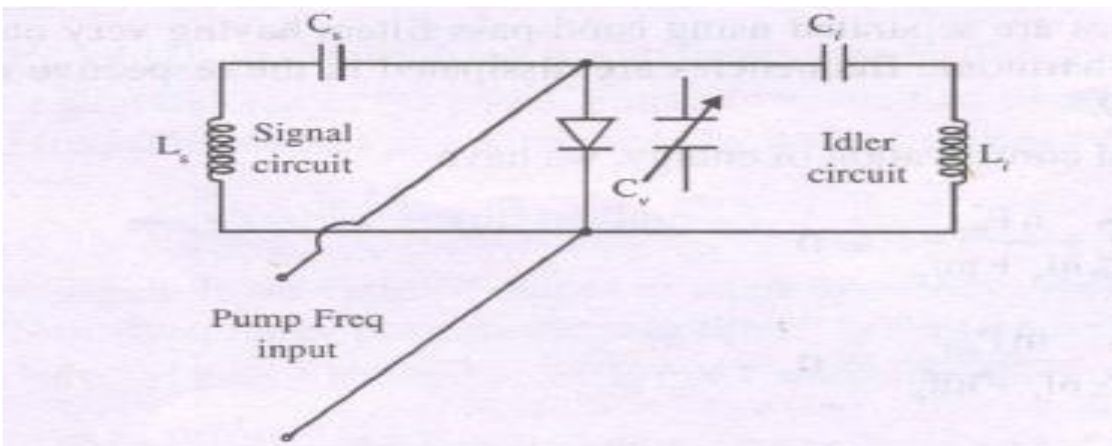
Parametric amplification with square wave pumping.

The voltage across the varactor is increased by the pumping signal at each signal voltage peak as shown above i.e., energy is taken from the pump source and added to the signal at the signal frequency. With an input circuit and load connected, amplification results.

One port non-degenerate amplifier is the most commonly used parametric amplifier. Only three frequencies are involved - the pump, the signal and the idler frequencies. If pump frequency is fp' the signal frequency is fs' then idler frequency is fj = fp - fs'

If fi = fs' then it is called Degenerate amplifier and

if fi is not equal to fs' then it is non-degenerate amplifier.



Ls Cs ~ tuned circuit at signal frequency fs

Lj Cj ~ tuned circuit at idler frequency fj (pump frequency tuned circuit is not shown),

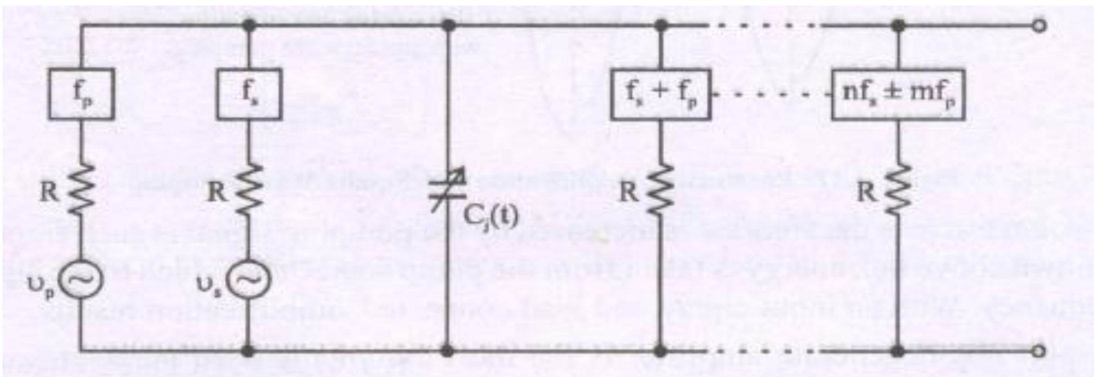
The output can be taken at idler frequency fr Gain is possible with this type of amplifier. Because the pump source gives more energy

$$\text{Gain} = \frac{f_i}{f_s} = \frac{f_p - f_s}{f_s}$$

In non-degenerate type, usually fj > fs resulting in gain. The idler circuit permits energy to be taken from the pump source. This energy is converted into signal frequency and idler frequency energy and amplified output can be obtained at either frequency.

MANLEY – ROWE RELATIONS:

For the determination of maximum gain of the parametric amplifier, a set of power conservation relations known as "Manley-Rowe" relations are quite useful.



two sinusoidal signals fp and fs applied across a lossless time varying non-linear capacitance Cj (t). At the output of this varying capacitance, harmonics of the two frequencies fp and fs are generated.

These harmonics are separated using band-pass filters having very narrow bandwidth.

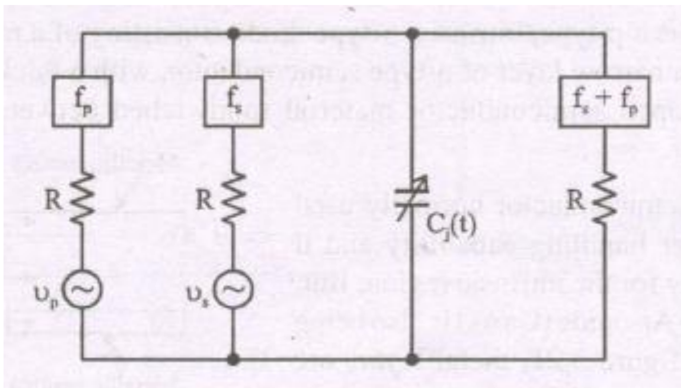
The power at these harmonic frequencies is dissipated in the respective resistive loads.

From the law of conservation of energy, we have

$$\sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \frac{n P_{mn}}{nf_s + mf_p} = 0$$

$$\sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \frac{m P_{mn}}{nf_s + mf_p} = 0$$

The above relations are called "Manley-Rowe" power conservation equations. When The power is supplied by the two generators, then Pmn is positive. In this case, power will flow into the non-linear capacitance. If it is the other way, then Pmn is negative.



As an example, let us consider the case when the power output flow is allowed at the sum frequency fp + fs only, with all the remaining harmonics being open circuited. With the above rest ructions, the quantities ‘m' and ‘n' can take on values -1,0 and respectively.

$$\frac{P_{01}}{f_s} + \frac{P_{11}}{f_s + f_p} = 0$$

$$\text{and } \frac{P_{10}}{f_p} + \frac{P_{11}}{f_s + f_p} = 0$$

The powers P01and P10 are considered positive, whereas P11 is considered negative. :. The power gain defined as the power output from the non-linear capacitor delivered to the load at sum frequency to that power received by it at a frequency fs is given by

$$G_p = \frac{P_{11}}{P_{01}} = \frac{f_s + f_p}{f_s} \text{ (for modulator)}$$

Thus the power gain is the ratio of output to input frequency. This type of parametric device is called "Sum-frequency parametric amplifier" or "up-converter".

On the other hand, if the signal frequency is fp + fs and output frequency is fs' then

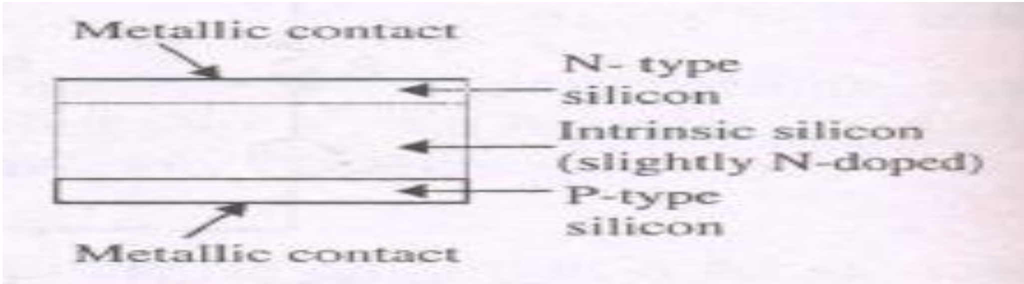
$$G_p = \frac{f_s}{f_p + f_s} \text{ (for demodulator)}$$

This type of parametric device will now be called "parametric down-converter" and the power gain becomes power attenuation.

PIN DIODE AND ITS APPLICATION:

The PIN diode is a p-type, intrinsic, n-type diode consisting of a narrow layer of p-type semiconductor and a narrow layer of n-type semiconductor, with a thicker region of intrinsic or very lightly n-doped semiconductor material sandwiched between them.

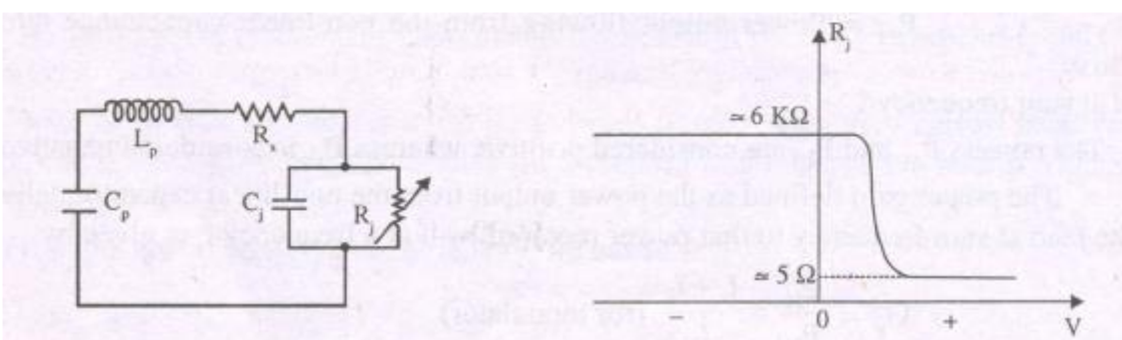
Silicon is the semiconductor normally used because of its power handling capability and it offers high resistively for the intrinsic region. But, now-a-days Gallium Arsenide (GaAs) is also being used. Metal layers are attached for contact purposes. Its main applications are in microwave switching and modulation.



PIN diode acts as a more or less ordinary diode at frequencies upto about 100 MHz. At high frequencies, it ceases to rectify and then acts as a variable resistance with an equivalent circuit and a resistance-voltage characteristics .In 'the equivalent circuit, Land C represent the package inductance and capacitance p p

respectively. R is the bulk semiconductor layer and contact resistance. R. and C. represent the respective junction resistance and capacitance of the intrinsic layer. When the bias is varied on the PIN diode, its microwave resistance R. changes from a typical value of 6 K under J

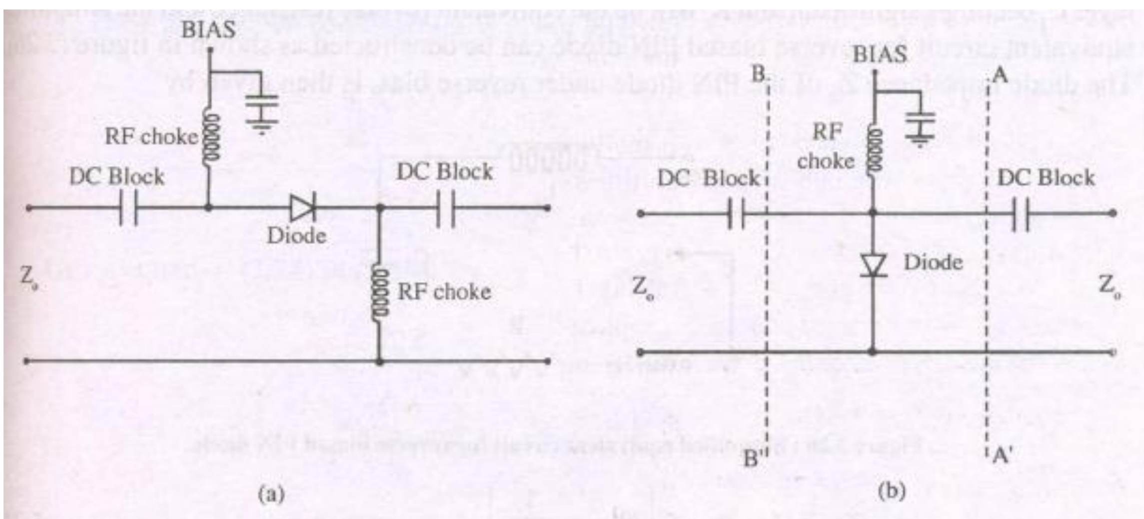
negative bias to perhaps 5 Q when the bias is positive .Thus, if the diode is mounted across a 50 Q co-axial line, it will not significantly load this line when it is back-biased, so that the power flow will not be interfered with. However, if the diode is now forward biased, its resistance drops significantly to 5Q, so that most of the power is reflected and hardly any is transmitted; the diode is acting as a switch.



APPLICATION OF PIN DIODE AS SINGLE POLE SWITCH:

A PIN diode can be used in either a series or a shunt configuration to form a single-pole, single-throw RF switch. These circuits are shown with bias networks below.

In the series configuration the switch is ON when thel diode is forward Biased and OFF when it is reverse biased. But, in shunt configuration of forward biasing the diode *"cuts-off'* the transmission and reverse biasing the diode ensures transmission from input to output. The DC blocks should have a very low impedance at RF operating frequency and RF choke inductors should have very high RF impedance.

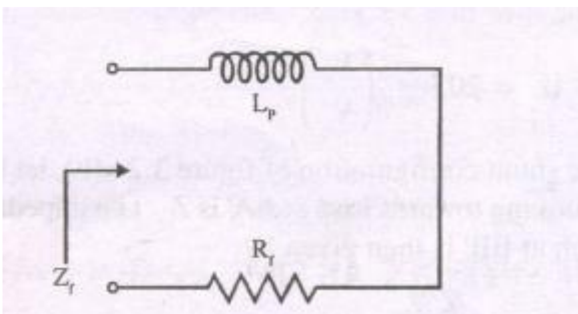


(a) Series configuration **(b) Shunt configuration.**

Ideally, a switch should have zero insertion loss in the ON state and infinite attenuation in the OFF state. Realistic switching elements, of course, result in some insertion loss for the ON state and finite attenuation for the OFF state due to non-zero forward bias resistance.

Similarly, for reverse bias shunt capacitor is not infinite & non-zero insertion loss results. Because of the large breakdown voltage (=500 volts) compared to an ordinary diode, PIN diode can be biased at high negative region so that large a.c. signal, superimposed on d.c. cannot make the device forward biased.

Forward Bias: When the PIN diode is forward biased, the capacitors C and C. almost behave as open circuits so that the equivalent circuit can now be simplified where Rf is the total forward resistance of the PIN diode given by



$$R_f = R_s + R_j$$

.. The diode impedance Zd of the PIN diode is given by

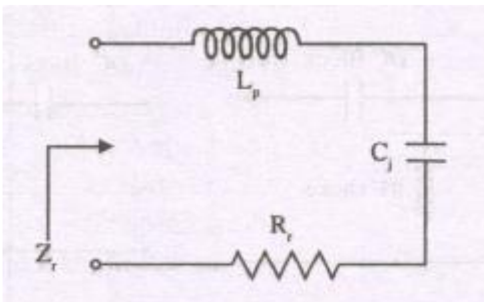
$$Z_d = Z_f = R_f + j\omega L_p$$

Reverse bias: When the PIN diode is reverse biased, the capacitance of the intrinsic layer C. becomes significant and Rr will be the equivalent reverse resistance and the simplified equivalent circuit for reverse biased PIN diode can be constructed as shown.

The diode impedance Zd of the PIN diode under reverse bias, is then given by

Reverse bias: When the PIN diode is reverse biased, the capacitance of the intrinsic layer C. becomes significant and Rr will be the equivalent reverse resistance and the simplified equivalent circuit for reverse biased PIN diode can be constructed ;

The diode impedance Zd of the PIN diode under reverse bias, is then given by

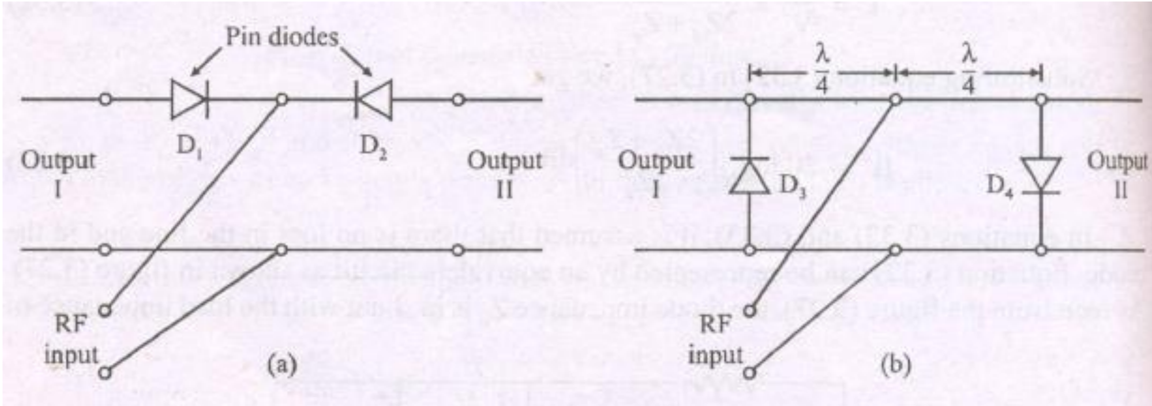


$$Z_d = Z_r = R_r + j\left(\omega L_p - \frac{1}{\omega C_j}\right)$$

PIN DIODE AS SPDT SWITCH:

Single-pole double throw (SPDT) action can be obtained by using a pair of PIN diodes either in series configuration or in shunt configuration as shown .In the series configuration of figure 3.29(a), when DI is forward biased and *Dz* reverse biased, connection is established between RF input and output I and no output at OUTPUT II.

When the biasing condition is reversed (D(reverse biased and *Dz* forward biased), connection is established between RF input and output II.



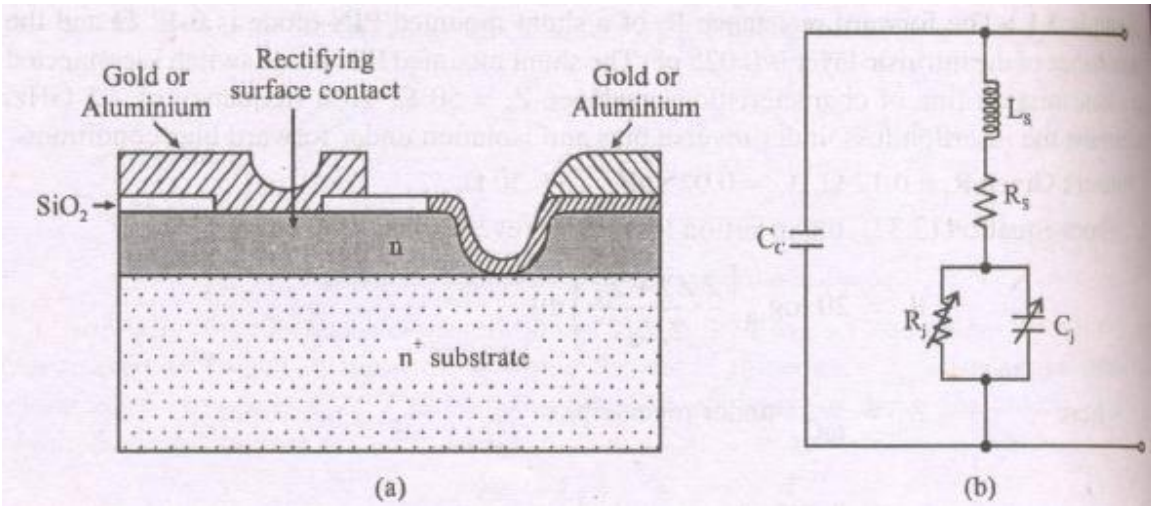
(a) Series configuration (b) Shunt configuration

In the shunt configuration when D3 is forward biased, it becomes short circuited throwing an open circuit at RF input line junction due to *(AI* 4) section. D4 is reverse biased so that it becomes open circuit (high impedance state) and connection is established between RF input and output II. When D3 is reverse biased and D4 forward biased, connection is established between RF input and output I.

SCHOTTKY BARRIER DIODE:

Schottky barrier diode is a sophisticated version of the point-contact silicon crystal diode, wherein the metal-semiconductor junction so formed is a surface rather than a point contact.

The advantage of schottky diode over point contact crystal diode is the elimination of minority carrier flow in the reverse-biased condition of the diode. Due to this elimination of holes, there is no delay due to hole-electron recombination (which is present in junction diodes) and hence the operation is faster. Because of larger contact area of rectifying contact compared to crystal diode, the forward resistance is lower as also noise. Noise figures as low as 3dB have been obtained with these diodes. Just like crystal diodes, the schottky diodes are also used in detection and mixing.



(a) Schottky barrier diode (b) Its equivalent circuit

The construction of schottky diode is illustrated in figure 3.30(a). The diode consists of n+ silicon substrate upon which a thin layer of silicon of 2 to 3 micron thickness is epitaxially grown. Then a thin insulating layer of silicon dioxide is grown thermally. After opening a window through masking process, a metal-semiconductor junction is formed by depositing metal over SiO_2. schottky diode which is almost identical with that of crystal diode.

RECOMMENDED QUESTIONS ON UNIT- 3

1. What is “ Gunn Effect”? with a neat diagram explain the constructional details of GUNN diode.
2. Explain the different modes of operation of Gunn diode oscillator.
3. Explain RWH theory for Transfer electron devices.
4. Explain the two valley theory model.
5. What are modes of operation of Gunn diode, explain.
6. With neat diagram explain the construction and operation of READ diode.
7. With neat diagram explain the construction and operation of IMPATT diode.
8. With neat diagram explain the construction and operation of TRAPATT diode.
9. With neat diagram explain the construction and operation of BARITT diode.
10. With neat diagram explain the construction and operation of SCHOTTKY barrier diode.
11. Explain the operation of a basic parametric amplifier with square wave pumping.
12. What are MANLEY –ROWE relations? How are they useful in understanding parametric amplifiers.

UNIT – 4

Microwave network theory and passive devices. Symmetrical Z and Y parameters, for reciprocal Networks, S matrix representation of multi port networks.

6 Hours

TEXT BOOKS:

1. **Microwave Devices and circuits**- Liao / Pearson Education.

2. **Microwave Engineering** – Annapurna Das, Sisir K Das TMH Publication, 2001.

REFERENCE BOOK:

1. **Microwave Engineering –** David M Pozar, John Wiley, 2e, 2004

UNIT – 4

MICROWAVE NETWORK THEORY AND PASSIVE DEVICES

INTRODUCTION

A microwave network consists of coupling of various microwave components and devices such as attenuators, phase shifters, amplifiers, resonators etc., to sources through transmission lines or waveguides. Connection of two or more microwave devices and components to a single point results in a *microwave junction.*

In a low frequency network, the input and output variables are voltage and current which can be related in terms of impedance Z-parameters, or admittance Y-parameters or hybrid h-parameters or ABCD parameters. These relationships for a two-port network of figure 4.1 can be represented by

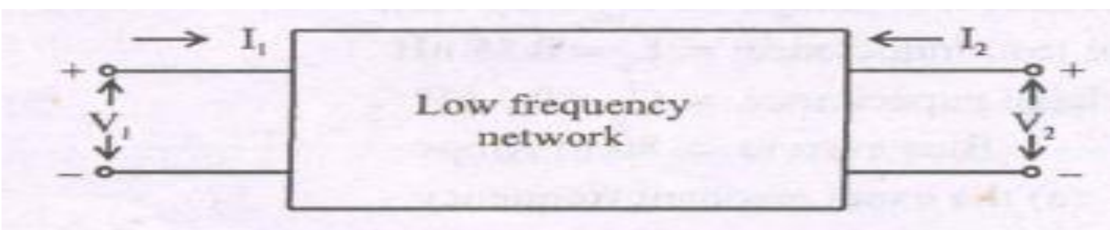


Figure 4.1: Low frequency two port network

$$\left.\begin{aligned} V_1 &= Z_{11}\, I_1 + Z_{12}\, I_2 \\ V_2 &= Z_{21}\, I_1 + Z_{22}\, I_2 \end{aligned}\right\} \qquad \text{..... (4.1)}$$

or
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \text{..... (4.2)}$$

$$\left.\begin{aligned} I_1 &= Y_{11}\, V_1 + Y_{12}\, V_2 \\ I_2 &= Y_{21}\, V_1 + Y_{22}\, V_2 \end{aligned}\right\} \qquad \text{..... (4.3)}$$

or
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \text{..... (4.4)}$$

$$\left.\begin{aligned} V_1 &= h_{11}\, I_1 + h_{12}\, V_2 \\ I_2 &= h_{21}\, I_1 + h_{22}\, V_2 \end{aligned}\right\} \qquad \text{..... (4.5)}$$

$$\text{or} \quad \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \qquad \text{..... (4.6)}$$

$$\left.\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}\right\} \qquad \text{..... (4.7)}$$

$$\text{or} \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad \text{..... (4.8)}$$

These parameters, Z, Y,h and ABeD parameters can be easily measured at low frequencies under short or open circuit conditions and can be used for analyzing the circuit.

The physical length of the device or the line at microwave frequencies, is comparable to or much larger than the wavelength. Due to this, the voltage and current are difficult to measure as also the above mentioned parameters. The reasons for this are listed as below.

(a) Equipment is not available to measure the total voltage and total current at any point.

(b) Over a wide range of frequencies, short and open circuits are difficult to realize.

(c) Active devices such as power transistors, tunnel diodes etc, will become unstable under short or open circuit conditions.

Therefore, a new representation is needed to overcome these problems at microwave frequencies. The logical variables are traveling waves rather than voltages and currents and these variables are labeled as "Scattering or S-parameters". These parameters for a two port network are represented as shown in figure 4.2 .

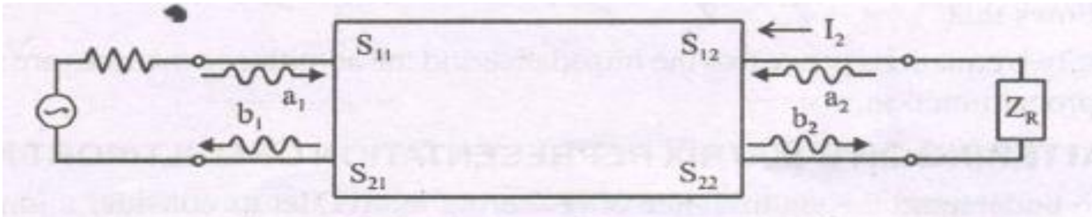


Figure 4.2 : S-parameters of a two-port network

These S-parameters can be represented in an equation form related to the traveling waves
a1,a2 and b1 b2 through

$$\left.\begin{aligned} b_1 &= S_{11}\,a_1 + S_{12}\,a_2 \\ b_2 &= S_{21}\,a_1 + S_{22}\,a_2 \end{aligned}\right\} \quad \text{..... (4.9)}$$

SYMMETRICAL Z AND Y MATRICES FOR RECIPROCAL NETWORK

In a reciprocal network, the junction media are characterized by scalar electrical parameters namely absolute permeability I..l and absolute permittivity E . In such a network, the impedance and the admittance matrices became symmetrical. This property can be proved by considering an N-port network. Let Ej and Hi be the respective electric and magnetic field intensities at the i^{th} port and let the total voltage Vo = 0 at all ports for n = 0, 1,2 .... except at i1hport.
Similarly if E. and H. are considered for the i^{th} port with V = 0 at other ports, then from reciprocity theorem.

S-MATRIX REPRESENTATION OF MULTIPORT NETWORK:

Let us now consider a junction of "n" number of rectangular waveguides as shown in figure 4.4. In this case, all "a' s" represent the incident waves at respective ports and all "b's" the reflected waves from the microwave junction coming out of the respective ports.

In this case also, equations (4.18) and (4.19) are still valid where S.. and S. have the following meanings:

Sii= Scattering coefficient corresponding to the input power applied at

IJ the i1hport and output power coming out of j^{th} port and

Sjj = Scattering coefficient corresponding to the power applied at the i^{th}port " . and output taken out of i1hport itself. This coefficient is a measure of amount of mismatch between the i^{th} port and the junction.

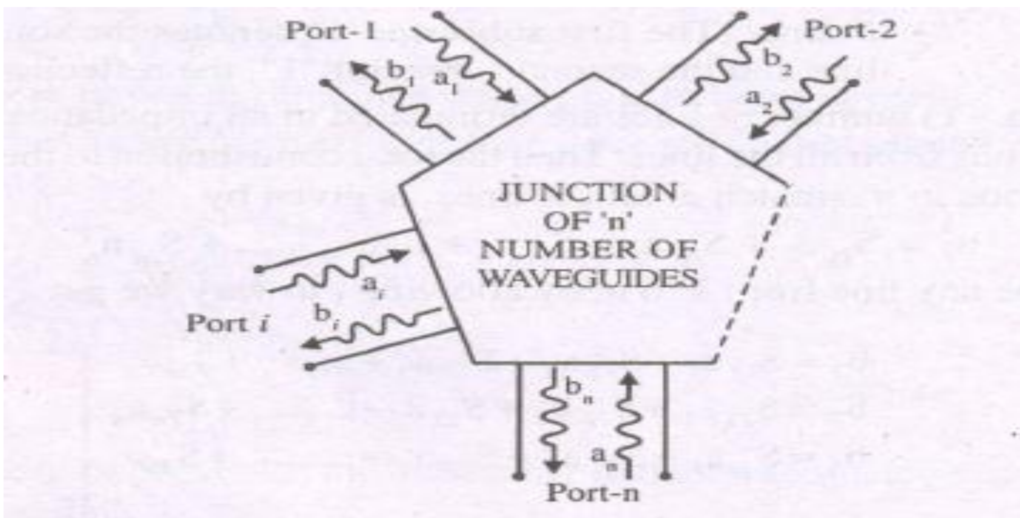


Figure 4.4 : Junction of "n" number of waveguides.

As an example let us consider a two-port network as shown in figure 4:5.

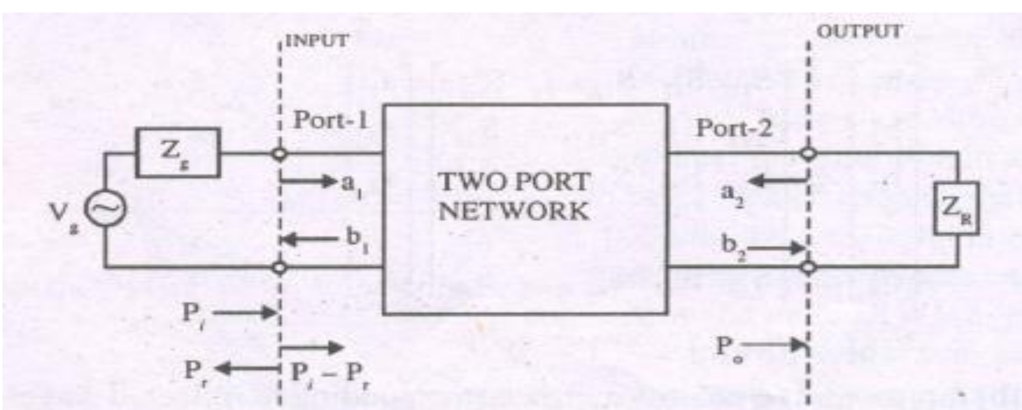


Figure 4.5 : Illustrating two-port network

The relationship. between the incident and reflected waves in terms of scattering coefficients can be written as

$$b_1 = S_{11} a_1 + S_{12} a_2 \quad \text{..... (4.20)}$$

$$b_2 = S_{21} a_1 + S_{22} a_2 \quad \text{..... (4.21)}$$

From these equations, the scattering coefficients are found as

$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$ = reflection coefficient at port-1 when port-2 is terminated with a matched load ($a_2 = 0$)

$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$ = reflection coefficient at port-2 when port-1 is terminated with a matched load ($a_1 = 0$)

$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$ = attenuation of the wave travelling from port-2 to port-1.

$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$ = attenuation of the wave travelling from port-1 to port-2.

In figure 4.5, we have

P_i = incident power at port-1

P_r = power reflected by the network coming out of port-1 itself.

P_o = output power at port-2.

The various losses can be expressed in terms of S-parameters as given below:

$$\text{Insertion loss in dB} = 10 \log_{10} \frac{P_i}{P_o} \quad \text{..... (4.22)}$$

But $P_i \propto |a_1|^2$

$P_o \propto |b_2|^2$

$$\therefore \quad \frac{P_i}{P_o} = \frac{|a_1|^2}{|b_2|^2} = \frac{1}{\left|b_2/a_1\right|^2} = \frac{1}{|S_{21}|^2} = \frac{1}{|S_{12}|^2} \quad \text{..... (4.23)}$$

$$\text{Insertion loss indB} = 10\log_{10}\frac{1}{|S_{21}|^2}$$

$$= 20\log_{10}\frac{1}{|S_{21}|} = 20\log_{10}\frac{1}{|S_{12}|} \quad \text{..... (4.24)}$$

2. **Transmission loss (or attenuation loss) in dB**

$$= 10\log_{10}\frac{P_i - P_r}{P_o}$$

$$= 10\log_{10}\frac{|a_1|^2 - |b_1|^2}{|b_2|^2}$$

$$= 10\log_{10}\frac{\dfrac{|a_1|^2 - |b_1|^2}{|a_1|^2}}{\dfrac{|b_2|^2}{|a_1|^2}} = 10\log_{10}\frac{1 - \dfrac{|b_1|^2}{|a_1|^2}}{\dfrac{|b_2|^2}{|a_1|^2}}$$

$$= 10\log_{10}\frac{1 - |S_{11}|^2}{|S_{21}|^2} \quad \text{..... (4.25)}$$

3. **Reflection loss in dB** $= 10\log_{10}\dfrac{P_i}{P_i - P_r}$

$$= 10\log_{10}\frac{|a_1|^2}{|a_1|^2 - |b_1|^2}$$

$$= 10\log_{10}\frac{1}{1 - \dfrac{|b_1|^2}{|a_1|^2}}$$

$$= 10\log_{10}\frac{1}{1 - |S_{11}|^2} \quad \text{..... (4.26)}$$

4. **Return loss in dB** $= 10\log_{10}\dfrac{P_i}{P_r}$

$$= 10\log_{10}\frac{|a_1|^2}{|b_1|^2}$$

$$= 10\log_{10}\frac{1}{|S_{11}|^2}$$

$$= 20\log_{10}\frac{1}{|S_{11}|} \quad \text{..... (4.27)}$$

PROPERTIES OF S-MATRIX

In general the scattering parameters are complex quantities having the following Properties:

Property (1) : When any Z1h port is perfectly matched to the junction, then there are no reflections from that port. Thus S ..= O. If all the ports are perfectly matched, then the leading diagonal II elements will all be zero.

Property (2) : Symmetric Property of S-matrix:- If a microwave junction satisfies reciprocity condition and if there are no active *devices*, then S parameters are equal to their corresponding transposes.

$$i.e., S_{ij} = S_{ji}$$

Proof : The steady-state voltage and current at the i^{th} port are given by

$$V_i = V_i^+ + V_i^- \qquad \text{..... (4.29)}$$

and

$$I_i = \frac{V_i^+}{Z_{oi}} - \frac{V_i^-}{Z_{oi}} \qquad \text{..... (4.30)}$$

where V_i^+ = incident voltage at the i^{th} port

V_i^- = reflected voltage from the i^{th} port.

Z_{oi} = characteristic impedance

From equations (4.29) and (4.30), V_i^+ and V_i^- can be found to be

$$V_i^+ = \frac{1}{2}(V_i + Z_{oi} I_i) \qquad \text{..... (4.31)}$$

$$V_i^- = \frac{1}{2}(V_i - Z_{oi} I_i) \qquad \text{..... (4.32)}$$

The average incident power (complex) at the i^{th} port is

$$\frac{1}{2} V_i I_i^* = \frac{|V_i^+|^2}{2Z_{oi}^*} \qquad \text{..... (4.33)}$$

The normalized incident and reflected voltages at the i^{th} port can be defined as

$$a_i = \frac{V_i^+}{\sqrt{Z_{oi}}} = \frac{1}{2}\left(\frac{V_i}{\sqrt{Z_{oi}}} + \sqrt{Z_{oi}}\, I_i\right) \qquad \text{..... (4.34)}$$

$$b_i = \frac{V_i^-}{\sqrt{Z_{oi}}} = \frac{1}{2}\left(\frac{V_i}{\sqrt{Z_{oi}}} - \sqrt{Z_{oi}}\, I_i\right) \qquad \text{..... (4.35)}$$

If the characteristic impedance is also normalized so that $\sqrt{Z_{oi}} = 1$, then

$$a_i = \frac{1}{2}(V_i + I_i) \qquad \text{..... (4.36)}$$

and

$$b_i = \frac{1}{2}(V_i - I_i) \qquad \text{..... (4.37)}$$

Property (3):- Unitary property for a lossless junction:-

.This property states that for any lossless network, the sum of the products of each term of anyone row or anyone column of the [SJ matrix with its complex conjugate is unity.

Proof:- From the principle of conservation of energy, if the junction is lossless, then the power input must be equal to power output. The incident and reflected waves are related to the incident and reflected voltages by

$$a = \frac{V^+}{\sqrt{Z_o}}$$

and $$b = \frac{V^-}{\sqrt{Z_o}}$$

$\therefore$ The incident power $= P^+ = \frac{1}{2}\, a\, a^* = \frac{1}{2}\, |a|^2$

and Reflected power $= P^- = \frac{1}{2}\, bb^* = \frac{1}{2}\, |b|^2$

When the junction is lossless, then no real power can be delivered to the network. Thus, if the characteristic impedances of all the ports are identical and assumed to be unity (perfectly normalized), the average power delivered to junction is *zero.*

$$\therefore \quad P_{av} = \frac{1}{2}\, \mathrm{Re} \sum_{i=1}^{n} V_i I_i^* = 0$$

where I_i^* = complex conjugate of I_i

Using equations (4.38) and (4.39) in (4.67), we get

$$\frac{1}{2}\, \mathrm{Re} \sum_{i=1}^{n} (a_i + b_i)\, (a_i - b_i)^* = 0$$

$$\therefore \quad \frac{1}{2}\, \mathrm{Re} \sum_{i=1}^{n} (a_i + b_i)\, (a_i^* - b_i^*) = 0$$

$$\therefore \quad \frac{1}{2}\, \mathrm{Re} \sum_{i=1}^{n} \left[(a_i a_i^* - b_i b_i^*) + (a_i^* b_i - a_i b_i^*) \right] = 0$$

$$\therefore \quad \sum_{i=1}^{n} \left[(a_i a_i^* - b_i b_i^*) \right] = 0$$

Equation (4.68) can be written in matrix form as

$$[a]\,[a]^* - [b]\,[b]^* = [0]$$

From equation (4.19), we have

$$[b] = [S]\,[a] \qquad \text{..... (4.70)}$$

Taking complex conjugate on both sides

$$[b]^* = [S]^*\,[a]^* \qquad \text{..... (4.71)}$$

Using equations (4.70) and (4.71) in (4.69), we get

$$[a]\,[a]^* - [S]\,[a]\,[S]^*\,[a]^* = 0$$

$$[a]\,[a]^* \left\{[U] - [S][S]^*\right\} = 0 \qquad \text{..... (4.72)}$$

Since $[a]\,[a]^*$ cannot be zero, we must have

$$[U] - [S]\,[S]^* = 0$$

$$\therefore \qquad \mathbf{[S]\,[S]^* = [U]} \qquad \text{..... (4.73)}$$

Where [U] is the unit matrix or identity matrix. Pre-multiplying both sides of equation (4.73) by $[S]^{-1}$, we get

$$[S]^{-1}\,[S]\,[S]^* = [S]^{-1}\,[U]$$

$$\therefore \qquad \mathbf{[S]^* = [S]^{-1}} \qquad \text{..... (4.74)}$$

Any matrix satisfying equation (4.74) is said to be "***UNITARY***". In other words, a matrix is said to satisfy "***unitary property***" when its complex conjugate is equal to its inverse.

From equation (4.62),

$$[S] = [S]_T$$

Using this in equation (4.74), we get

$$\mathbf{[S]^* = \{[S]_T\}^{-1}} \qquad \text{..... (4.75)}$$

A matrix that satisfies the condition of (4.75) is also termed as "***unitary matrix***".

The matrix equation of (4.75) can be written in summation form as

$$\sum_{k=1}^{n} S_{ki}\,S_{kj}^* = \delta_{ij} \text{ for all } i, j \qquad \text{..... (4.76)}$$

Where $\qquad \delta_{ij} = 1$ if $i = j$

and $\qquad \delta_{ij} = 0$ if $i \neq j$

(a) when $i = j$, equation (4.76) reduces to

$$\sum_{k=1}^{n} S_{ki}\,S_{ki}^* = 1 \qquad \text{..... (4.77)}$$

This property of equation (4.77) of [S] matrix is sometimes called "***UNITY PROPERTY***".

(b) When $i \neq j$, equation (4.76) reduces to

$$\sum_{k=1}^{n} S_{ki}\,S_{kj}^* = 0 \qquad \text{..... (4.78)}$$

This property of equation (4.78) on [S] matrix is sometimes called *"ZERO PROPERTY"*, In words, equation (4.77) states that the product of any column of [S] with the complex conjugate of that column gives unity, while equation (4.78) states that the product of any column of [S] with the complex conjugate of a different column gives zero.

Property (4) :- PHASE-SHIFT PROPERTY

Complex S-parameters of a network are defined with respect to the positions of the port or reference planes. For a two-port network with unprimed reference planes 1 and 2 as shown in figure 4.6, the S-parameters have definite complex values .

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

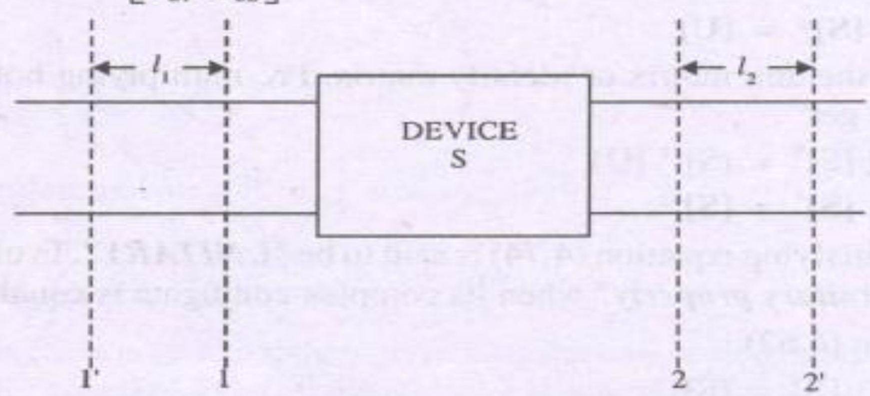


Figure 4.6 : Illustrating phase shift property of S

When the reference planes 1 and 2 are shifted outward to I' and 2' by electrical phase shifts,

$$[S'] = \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix} [S] \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix}$$

This property is valid for any number of ports.

.. For "n" number of ports,

$$[S'] = \begin{bmatrix} e^{-j\phi_1} & 0 & \ldots\ldots & 0 \\ 0 & e^{-j\phi_2} & \ldots\ldots & 0 \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots\ldots & e^{-j\phi_n} \end{bmatrix} [S] \begin{bmatrix} e^{-j\phi_1} & 0 & \ldots\ldots & 0 \\ 0 & e^{-j\phi_2} & \ldots\ldots & 0 \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots\ldots & e^{-j\phi_n} \end{bmatrix}.$$

The above property is called the *"PHASE SHIFT PROPERTY"* applicable to a shift of reference planes.

COMPARISON BETWEEN [S], [Z] AND [Y] MATRICES:

We know that impedance or admittance matrix for an N-port network represent all the circuit characteristics of the device at any given frequency. Like the impedance or admittance matrix for an N-port network, the *[S]* matrix also provides q complete description of the network as seen at its N ports. While the [Z] and [Y] matrices relate the total voltages and currents at the ports, the [S] matrix relates the voltage waves incident on the ports to those reflected from the ports .

From equation (4.52), the scattering matrix [S] is related to the impedance matrix [Z] by

$$[S] = \{[Z] - [U]\}\ \{[Z] + [U]\}^{-1}$$

In a similar way, the relationship between *[S]* and the admittance [Y] can also be expressed as

$$[S] = \{[U] - [Y]\}\ \{[U] + [Y]\}^{-1} \qquad \ldots\ldots (4.94)$$

The characteristics common between [S], [Z] and [Y] :

(i) the number of elements in all these matrices are same.

(ii) for reciprocal networks, all the 3 matrices [S], [Z] and [Y] are symmetric matrices.

(iii) The advantages of scattering matrix [S] over [Z] and [Y] can be listed as below:

(1) Using microwave measurement techniques, frequency, VSWR, power and phase of microwave signals can be easily measured. Measurement of VSWR is nothing but measurement of (b/a), power is measurement of lal2and measurement of phase is measurement of b2.Such a direct one-to-one relationship does not exist with [Z] or [Y] parameters.

(2)The power relations of lossless microwave circuits and devices can be readily Checked by using unitary property of [S] matrix. Such a quick check is not available with [Z] or [Y] matrices.

(3) The case of [Z] and [Y] matrices, the voltages and currents are functions of complex impedances and admittances respectively. When the reference planes are changed, there is change in both magnitude and phase of the impedances and admittances. But, in the case of [S] matrix, the change in reference plane changes only the phase of the scattering parameters.

RECOMMENDED QUESTIONS FOR UNIT – 4

1. **Explain the relation between incident and reflected waves in terms of scattering parameters for a two port network. Also explain physical significance of s-parameters.**
2. **Which properties are common in S, Z and Y matrices?**
3. **Two transmission lines of characteristic impedances Zj and Z2 are joined at plane PP'.Express s - parameters in terms of impedances**
4. **State and explain the properties of S-parameters**
5. **Explain S- matrix representation of muiltiport network.**
6. **What are the advantages of S parameters over Z and y parameters ?**

UNIT – 5

Microwave passive devices, Coaxial connectors and adapters, Phase shifters, Attenuators, Waveguide Tees, Magic tees.

4 Hours

TEXT BOOKS:

1. **Microwave Devices and circuits**- Liao / Pearson Education.
.
2. **Microwave Engineering –** Annapurna Das, Sisir K Das TMH Publication, 2001.

REFERENCE BOOK:

1. **Microwave Engineering –** David M Pozar, John Wiley, 2e, 2004

UNIT -5

MICROWAVE PASSIVE DEVICES

CO-AXIAL CABLES, CONNECTORS AND ADAPTERS

Coaxial Cables Microwave components and devices are interconnected using these co-axial cables of suitable length and operated at microwave frequencies. In this section let us consider some practical aspects of these co-axial cables. TEM mode is propagated through the co-axial line and the outer conductor guides these signals in the dielectric space between itself and inner conductor.

The outer conductor also acts as a shield to prevent the external signals to interfere with the internal signal. It also prevents the internal signal leakage. The co-axial cables usually possess characteristic impedance of either 50 ohms or 75 ohms Based on the structure of shielding, coaxial cables are classified into three basic types.

(i) Flexible co-axial Cable: Figure 5.1 shows the structure of flexible-type of co-axial cable consisting of low loss solid or foam type polyethylene dielectric. Electromagnetic shielding is provided for outer single braid or double braid of the flexible cable as shown, by using knitted metal wire mesh. The centre conductor usually consists of multi strand wire.

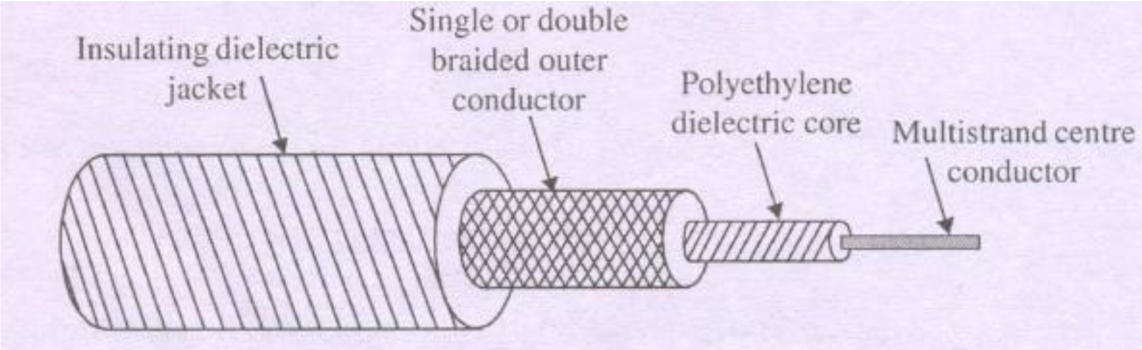


Figure 5.1 : Structure of flexible co-axial cable

(ii) Semi-rigid co-axial cable: Figure 5.2 shows the cross-sectional view of semi-rigid co-axial cable. Semi rigid co-axial cables make use of thin outer conductor made of copper and a strong inner conductor also made of copper. The region between the inner and outer conductor contains a solid dielectric. These cables can bent for convenient routing and are not as flexible as the first type.

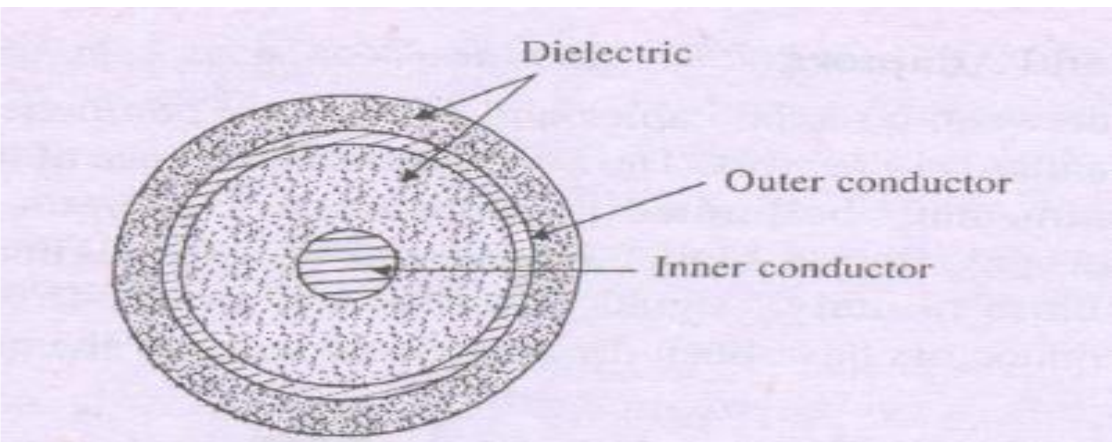


Figure 5.2 : Cross-section of semi-rigid co-axial cable

(ill) Rigid co-axial cable: Figure 5.3 shows the structure of a rigid co-axial cable consisting of inner and outer conductor with air as dielectric. To support the inner conductor at the centre dielectric spacers are introduced at regular intervals as shown. The thickness of these dielectric spacers is made small so that they do not produce significant discontinuities to the wave propagation.

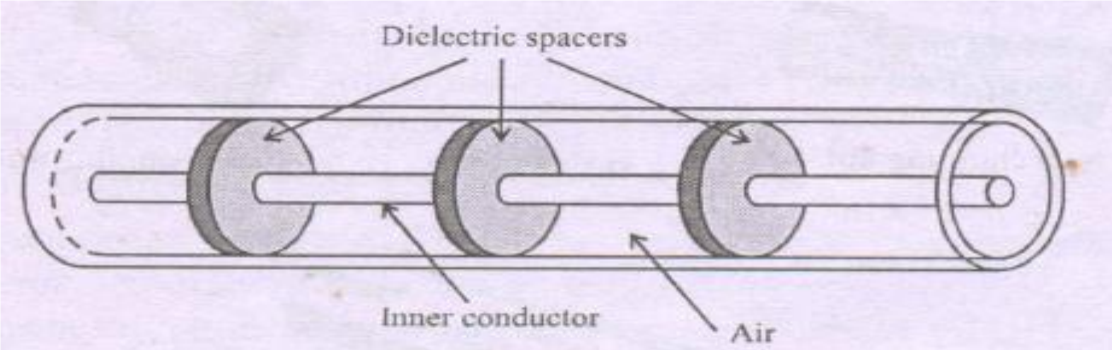


Figure 5.3 : Illustration of rigid co-axial cable

Co-axial cables can be used upto microwave -range of frequencies. Beyond these frequencies attenuation becomes very large (since attenuation increases with frequency) which makes co-axial cables unsuitable at higher frequencies. Some characteristics of standard coaxial cables with their radio guide (RG) and universal (U) numbers along with conductor (inner and outer) dimensions .

Coaxial Connectors and Adapters:

Interconnection between co-axial cables and microwave components is achieved with the help of shielded standard connectors. The average circumference of the co-axial cable, for mar high frequency operation must be limited to about one wavelength. This requirement is a VI necessary to reduce propagation at higher modes and also to eliminate erratic reflection coefficients (VSWR close to unity), signal distortion and power losses. Several types of co-axial connectors have been developed and some of them are described below.

(a) APC 3.5 (Amphenol Precision Connector - 3.5 mm)

HP (Hewlett - Packard) originally developed this connector, but it is now being manufactured by Amphenol. This connector can operate up to a frequency of 34 GHz and has a very low voltage standing wave ratio (VSWR). This connector provides repeatable connections and has 50 Q characteristic impedance. The male or female of SMA connector can be connected to the opposite type of APC 3.5 connector.

(b) APC -7 (Amphenol Precision connector -7 mm)

This connector was also developed by HP but improved later by Amphenol. This connector provides repeatable connections and used for very accurate 50 ohm measurement applications. This connector provides a coupling mechanism without male or female distinction (i.e., sexless) andits VSWR is extremely low, less than 1.02 in the frequency range upto 18 GHz.

(c) BNC (Bayonet Navy Connector)

This connector was developed during World War II and used for military applications. It has characteristic impedance 50 to 75 Q and is connected to flexible co-axial cable with diameters upto 0.635 cm. It is extensively used in almost all electronic measuring equipments upto 1 GHz of frequencies. BNC can be used even upto 4 GHz frequency and beyond that it
starts radiating electromagnetic energy.

(d) SMA (Sub-Miniature A type)

This type of connector is also called OSM connector as it is manufactured by Omni-Spectra Inc. SMA connectors are used on components for microwave systems. The disadvantage with these connectors is that at high frequencies greater than 24 GHz, it introduces higher ordermodes and hence not used above 24 GHz.

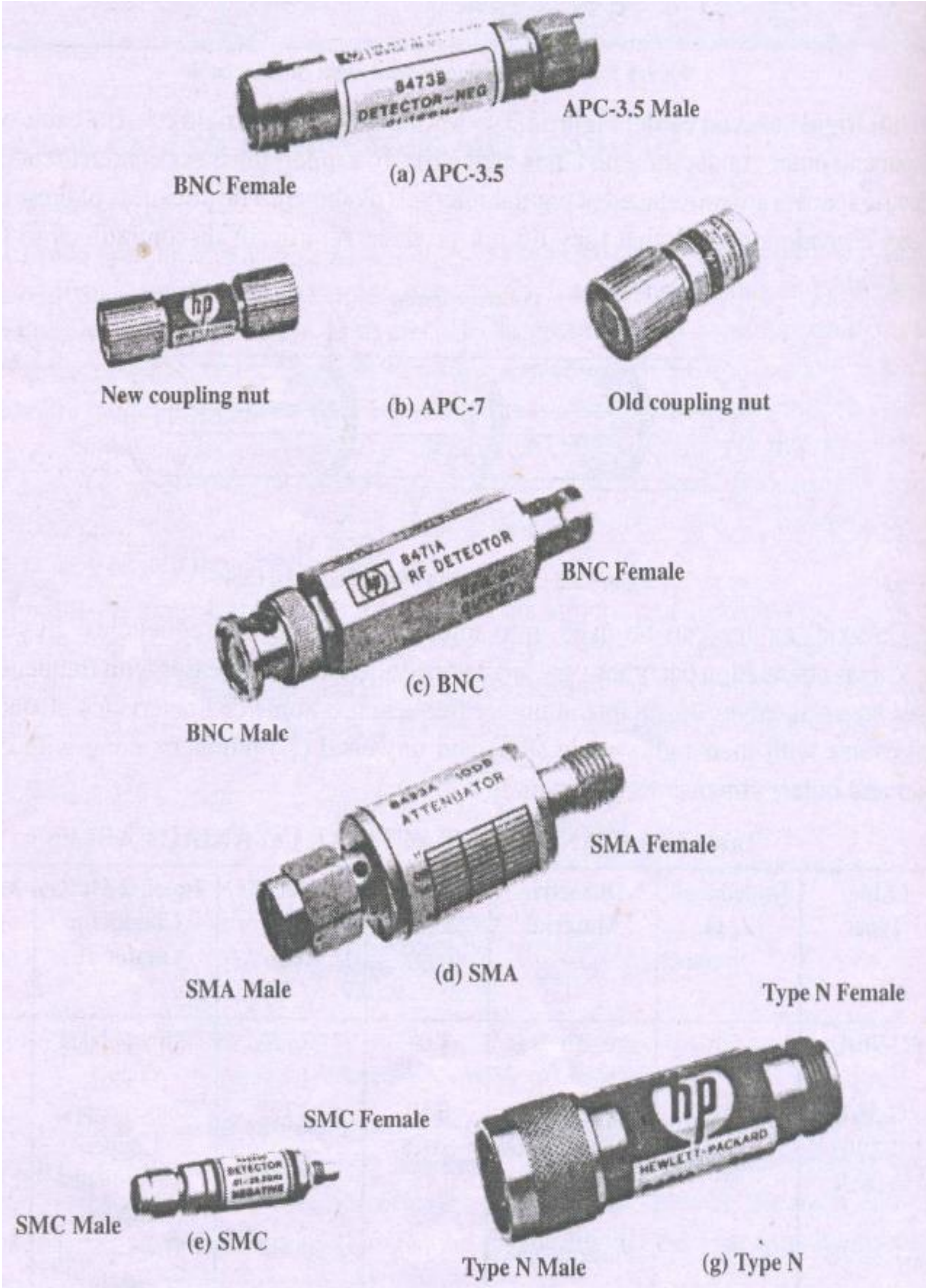


Figure 5.4 : Co-axial microwave connectors

(e) SMC (Sub-Miniature C-type)

This connector is manufactured by Sealectro Corporation and its size is smaller than SMA connector. It is a 50 Q connector that connects flexible cables upto a diameter of 0.317 cm and used upto a frequency of 7 GHz.

(0 TNC (Threaded Navy Connector)

This connector is an improved version of BNC in the sense that it is threaded. This threading prevents radiation at high frequencies so that it can be used upto about 12 GHz frequency.

(g) Type-N (Type-Navy) connector

It is a 50 Q or 75 Q connector having a very low value of VSWR less than 1.02. This was developed during World War II and extensively used as a microwave measurement connector up to a frequency of 18 GHz.

ATTENUATORS:

In order to control power levels in a microwave system by partially absorbing the transmitted microwave signal, attenuators are employed. Resistive films (dielectric glass slab coated with aquadag) are used in the design of both fixed and variable attenuators.

A co-axial fixed attenuator uses the dielectric lossy material inside the centre conductor of the co-axial line to absorb some of the centre conductor microwave power propagating through it dielectric rod decides the amount of attenuation introduced. The microwave power absorbed by the lossy material is dissipated as heat.

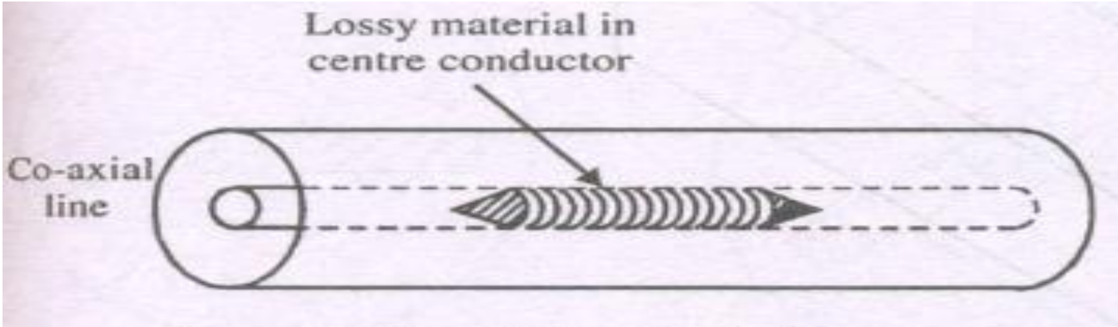


Figure 5.6 Co-axial line fixed attenuator

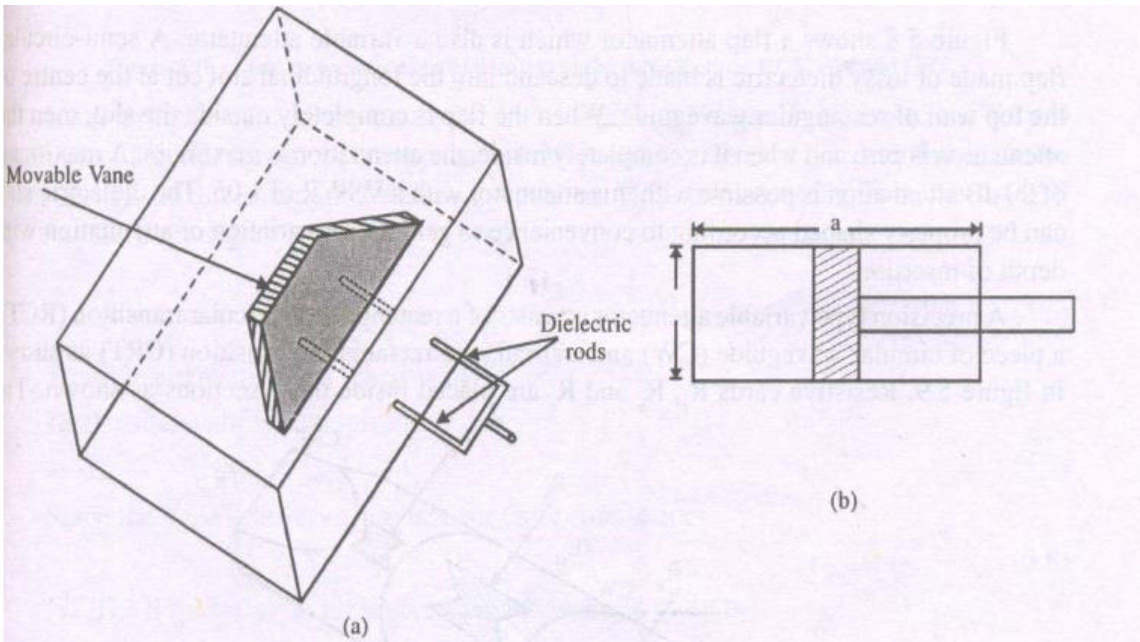


Figure 5.7 : Movable vane variable attenuator

In waveguides, the dielectric slab coated with aduadag is placed at the centre of the waveguide parallel to the maximum E-field for dominant TEIO mode. Induced current on the lossy material due to incoming microwave signal, results in power dissipation, leading to attenuation of the signal. The dielectric slab is tapered at both ends upto a length of more than half wavelength to reduce reflections as shown in figure 5.7. The dielectric slab may be made movable along the breadth of the waveguide by supporting it with two dielectric rods separated by an odd multiple of quarter guide wavelength and perpendicular to electric field. When the slab is at the centre, then the attenuation is maximum (since the electric field is concentrated at the centre for TEIO mode) and when it is moved towards one side-wall, the attenuation goes on decreasing thereby controlling the microwave power corning out of the other port.

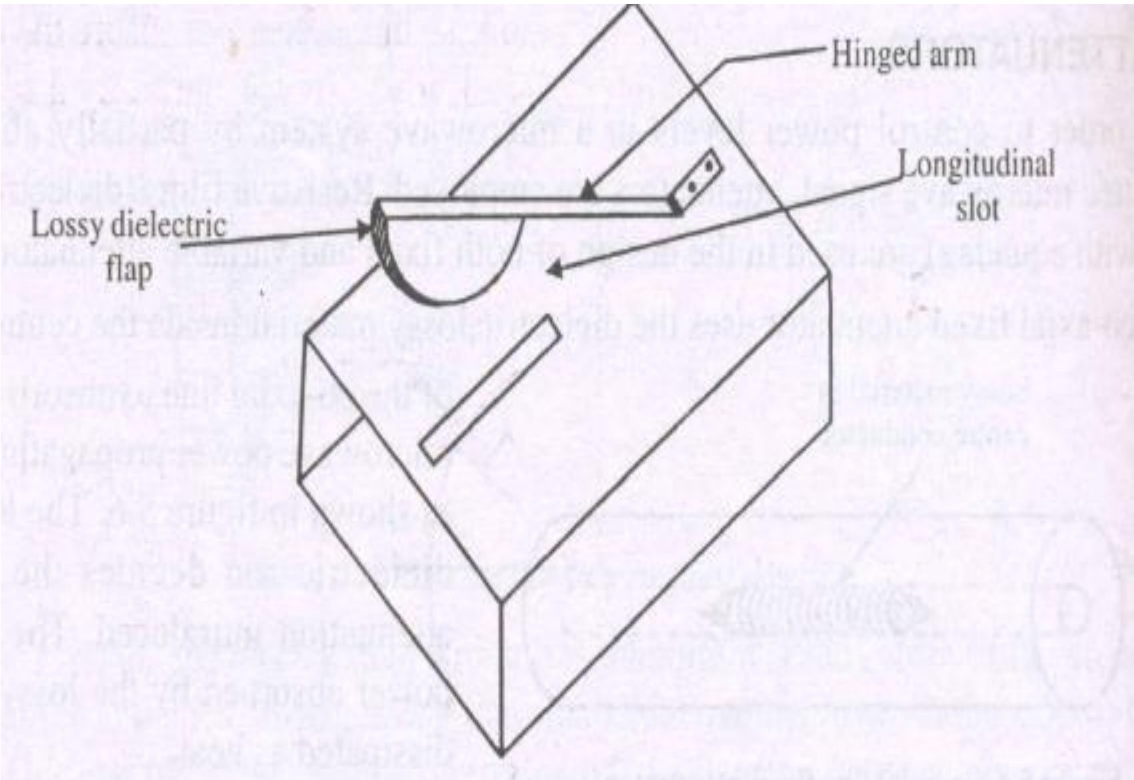


Figure 5.8 : Flap attenuator

Figure 5.8 shows a flap attenuator which is also a variable attenuator. A semi-circular flap made of lossy dielectric is made to descend into the longitudinal slot cut at the centre of the top wall of rectangular waveguide. When the flap is completely outside the slot, then the attenuation is zero and when it is completely inside, the attenuation is maximum. A maximum direction of 90 dB attenuation is possible with this attenuator with a VSWR of 1.05. The dielectric slab can be properly shaped according to convenience to get a linear variation of attenuation within the depth of insertion.

A precision type variable attenuator consists of a rectangular to circular transition (ReT), a piece of circular waveguide (CW) and a circular-to-rectangular transition

(CRT) as shown in figure 5.9. Resistive cards R , Rand R are placed inside these sections as shown. The centre circular section containing the resistive card Rb can be precisely rotated by 3600 with respect to the two fixed resistive cards. The induced current on the resistive card R due to the incident signal is dissipated as heat producing attenuation of the transmitted signal. TE mode in RCT is converted into TE in circular waveguide. The resistive cards R and R a kept perpendicular to the electric field of TEIO mode so that it does not absorb the energy. But any component parallel to its plane will be readily absorbed. Hence, pure TE mode is excited in circular waveguide section. II

If the resistive card in the centre section is kept at an angle 8 relative to the E-field direction of the TEll mode, the component E cos8 parallel to the card get absorbed while the component E sin 8 is transmitted without attenuation. This component finally comes out as E sin2θ as shown in figure 5.10.

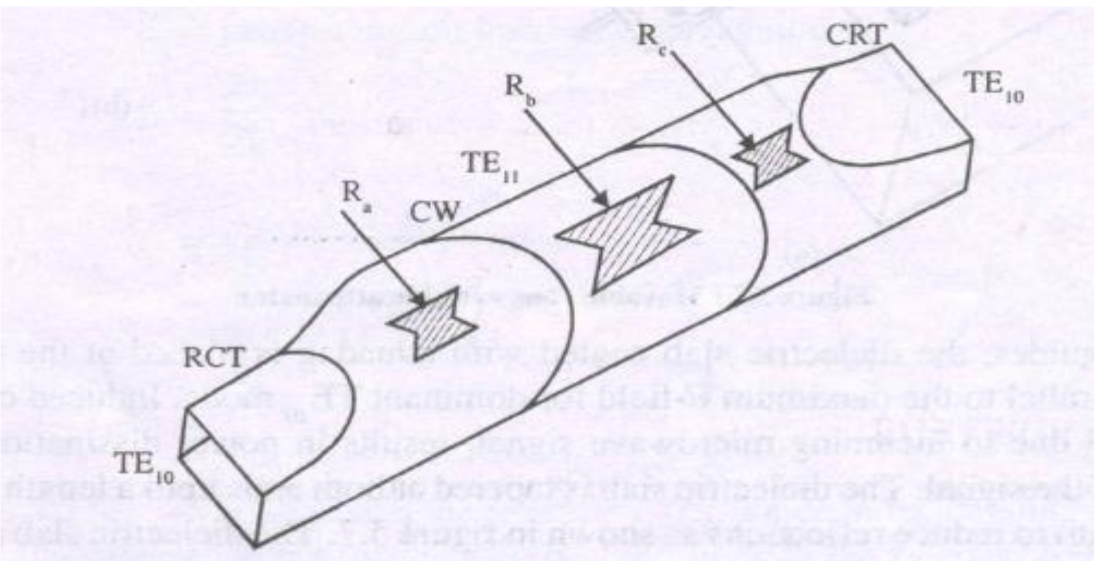


Figure 5.9 : Precision type variable attenuator

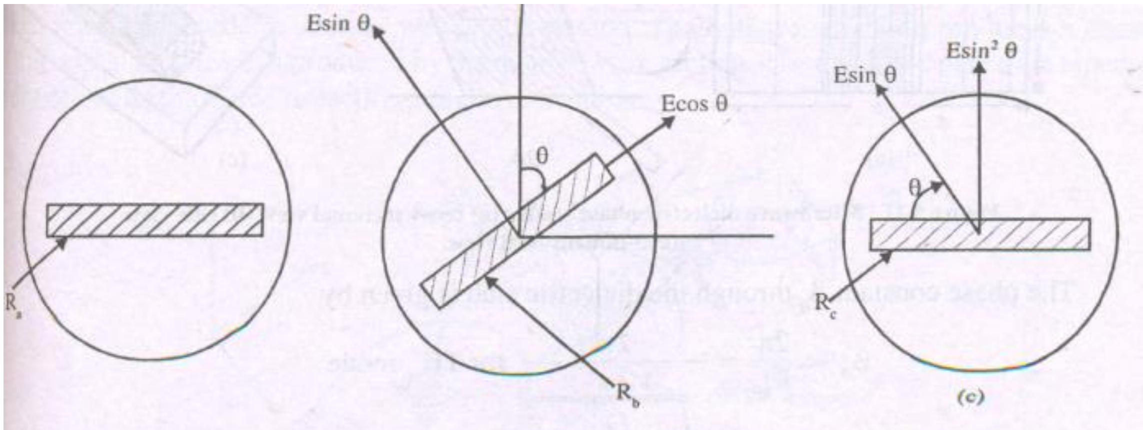


Figure 5.10 : Field components of electric field in the three sections RCT, CW and CRT

PHASE SHIFTERS:

A microwave phase shifter is a two port device which produces a variable shift in phase of the incoming microwave signal. A lossless dielectric slab when placed inside the rectangular waveguide produces a phase shift.

PRECISION PHASE SHIFTER

The rotary type of precision phase shifter is shown in figure 5.12 which consists of a circular waveguide containing a lossless dielectric plate of length 2l called "half-wave section", a section of rectangular-to-circular transition containing a lossless dielectric plate of length l, called "quarter-wave section", oriented at an angle of 45° to the broader wall of the rectangular waveguide and a circular-to-rectangular transition again containing a lossless dielectric plate of same length 1 (quarter wave section) oriented at an angle 45°. The incident TEIO mode becomes TEll mode in circular waveguide section. The half-wave section produces a phase shift equal to twice that produced by the quarter wave section. The dielectric plates are tapered at both ends to reduce reflections due to discontinuity.

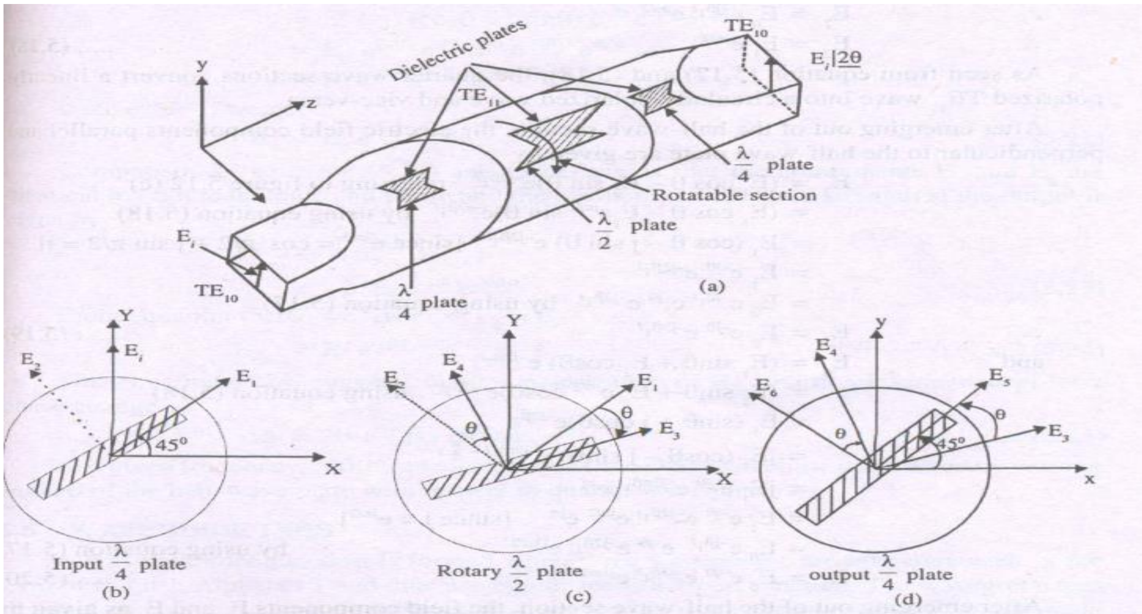


Figure 5.12 : Illustrating precision rotary phase shifter

When TEIO mode is propagated through the input rectangular waveguide of the rectangular to circular transition, then it is converted into TEll in the circular waveguide section. Let E; be the maximum electric field strength of this mode which is resolved into components, EI parallel to the plate and E2 perpendicular to El as shown in figure 5.12 (b). After propagation through the plate these components are given by

$$E_1 = (E_i \cos 45^0)\, e^{-j\beta_1 l} = E_0\, e^{-j\beta_1 l}$$

and $$E_2 = (E_i \sin 45^0)\, e^{-j\beta_2 l} = E_0\, e^{-j\beta_2 l}$$

Where $$E_0 = \frac{E_i}{\sqrt{2}}$$

The length I is adjusted such that these two components E1 and Ez have equal amplitude but differing in phase by = 90°.

$$E_1 = E_0\, e^{-j\beta_1 l}$$

$$E_2 = E_0\, e^{-j(\beta_1 l - 90^0)} = E_0\, e^{-j(\beta_1 l - \frac{\pi}{2})}$$

$$\therefore \quad E_2 = E_0\, e^{-j\beta_1 l}\, e^{j\pi/2}$$

$$\therefore \quad E_2 = E_1\, e^{j\pi/2}$$

The quarter wave sections convert a linearly polarized TEll wave into a circularly polarized wave and vice-versa. After emerging out of the half-wave section, the electric field components parallel and perpendicular to the half-wave plate are given by

$$E_3 = (E_1 \cos\theta - E_2 \sin\theta)e^{-j2\beta_1 l} \quad \text{referring to figure 5.12 (c)}$$
$$= (E_1 \cos\theta - E_1 e^{j\pi/2} \sin\theta)e^{-j2\beta_1 l} \quad \text{by using equation (5.18)}$$
$$= E_1 (\cos\theta - j\sin\theta)\, e^{-j2\beta_1 l} \quad [\text{since } e^{j\pi/2} = \cos \pi/2 + j \sin \pi/2 = j]$$
$$= E_1\, e^{-j\theta}\, e^{-j2\beta_1 l}$$
$$= E_0\, e^{-j\beta_1 l}\, e^{-j\theta}\, e^{-j2\beta_1 l} \quad \text{by using equation (5.17)}$$
$$\therefore \quad E_3 = E_0\, e^{-j\theta}\, e^{-j3\beta_1 l} \qquad \text{..... (5.19)}$$
and
$$E_4 = (E_1 \sin\theta + E_2 \cos\theta)\, e^{-j2\beta_2 l}$$
$$= (E_1 \sin\theta + E_1\, e^{j\pi/2} \cos\theta)e^{-j2\beta_2 l} \quad \text{using equation (5.18)}$$
$$= E_1 (\sin\theta + j\cos\theta)e^{-j2\beta_2 l}$$
$$= jE_1 (\cos\theta - j\sin\theta)\, e^{-j2(\beta_1 l - \frac{\pi}{2})}$$
$$= jE_1\, e^{-j\theta}\, e^{-j2\pi\beta_1 l}\, e^{j\pi}$$
$$= E_1\, e^{-j\theta}\, e^{-j2\beta_1 l}\, e^{j\pi/2}\, e^{j\pi} \quad [\text{since } j = e^{j\pi/2}]$$
$$= E_0\, e^{-j\beta_1 l}\, e^{-j\theta}\, e^{-j2\beta_1 l}\, e^{j3\pi/2} \quad \text{by using equation (5.17)}$$
$$\therefore \quad E_4 = E_0\, e^{-j\theta}\, e^{-j3\beta_1 l}\, e^{j3\pi/2} \qquad \text{..... (5.20)}$$

After emerging out of the half-wave section, the field components E3 and E4 as given by equations (5.19) and (5.20), may again be resolved into two TEll mQdes, polarized parallel and perpendicular to the output quarterwave plate. At the output end of this quarterwave plate, the field components parallel and perpendicular to the quarter wave plate, by referring to figure 5.12 (d), can be expressed as

$$E_5 = (E_3 \cos\theta + E_4 \sin\theta)\, e^{-j\beta_1 l}$$
$$= (E_0\, e^{-j\theta}\, e^{-j3\beta_1 l} \cos\theta + E_0\, e^{-j\theta}\, e^{-j3\beta_1 l}\, e^{j3\pi/2} \sin\theta)e^{-j\beta_1 l}$$

$$= E_0 (\cos\theta + e^{j3\pi/2} \sin\theta)\, e^{-j\theta}\, e^{-j3\beta_1 l}\, e^{-j\beta_1 l}$$

$$= E_0 (\cos\theta - j \sin\theta)\, e^{-j\theta} . e^{-j4\beta_1 l}$$

$$\therefore \quad E_5 = E_0\, e^{-j\theta}\, e^{-j\theta}\, e^{-j4\beta_1 l}$$

$$\therefore \quad E_5 = E_0\, e^{-j2\theta}\, e^{-j4\beta_1 l} \qquad \text{..... (5.21)}$$

$$\text{and} \quad E_6 = (E_4 \cos\theta - E_3 \sin\theta)\, e^{-j\beta_2 l}$$

$$\therefore \quad E_6 = (E_0\, e^{-j\theta}\, e^{-j3\beta_1 l}\, e^{j3\pi/2} \cos\theta - E_0\, e^{-j\theta}\, e^{-j3\beta_1 l} \sin\theta)\, e^{-j\beta_2 l}$$ by using equations (5.19) and (5.20)

$$\therefore \quad E_6 = E_0 (e^{j3\pi/2} \cos\theta - \sin\theta)\, e^{-j\theta}\, e^{-j3\beta_1 l}\, e^{-j(\beta_1 l - \frac{\pi}{2})}$$

$$= E_0 (-j \cos\theta - \sin\theta)\, e^{-j\theta}\, e^{-j3\beta_1 l}\, e^{-j\beta_1 l}\, e^{j\pi/2}$$

$$= E_0 (-j) (\cos\theta - j \sin\theta)\, e^{-j\theta}\, e^{-j4\beta_1 l}\, e^{j\pi/2}$$

$$= E_0\, e^{j3\pi/2}\, e^{-j\theta}\, e^{-j\theta}\, e^{-j4\beta_1 l}\, e^{j\pi/2}$$

$$= E_0\, e^{-j2\theta}\, e^{-j4\beta_1 l}\, e^{j2\pi}$$

since $e^{j2\pi} = 1$, we get

$$E_6 = E_0\, e^{-j2\theta}\, e^{-j4\beta_1 l} \qquad \text{..... (5.22)}$$

Comparison of equation (5.21) and (5.22) yields that the components Es and E6 are identical in both magnitude and phase and the resultant electric field strength at the output is given by

$$E_{out} = \sqrt{(E_5)^2 + (E_6)^2}$$

$$= \sqrt{2}\, E_0\, e^{-j2\theta}\, e^{-j4\beta_1 l}$$

WAVE GUIDE TEE JUNCTIONS:

A waveguide Tee is formed when three waveguides are interconnected in the form of English alphabet T and thus waveguide tee is 3-port junction. The waveguide tees are used to connects a branch or section of waveguide in series or parallel with the main waveguide transmission line either for splitting or combining power in a waveguide system.

There are basically 2 types of tees namely

1.H- plane Tee junction

2.E-plane Tee junction

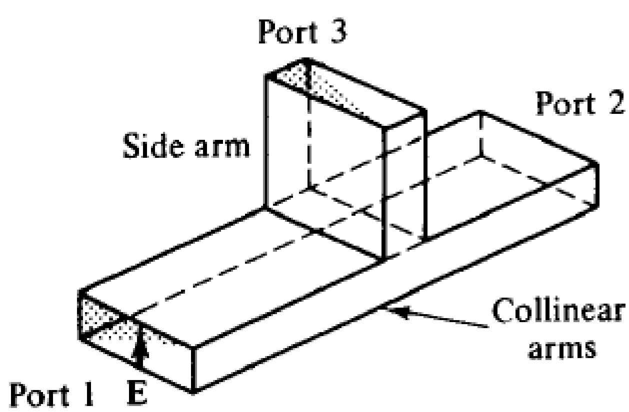
A combination of these two tee junctions is called a hybrid tee or " Magic Tee".

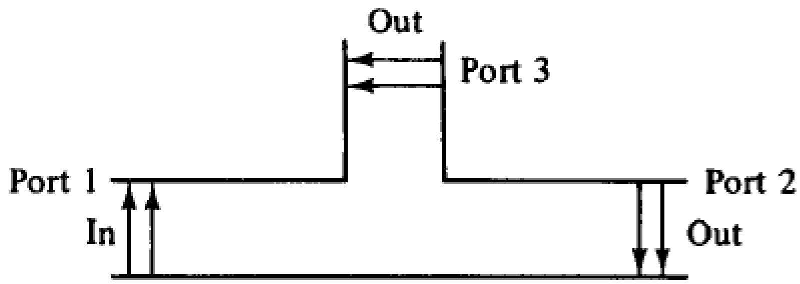
E-plane Tee(series tee):

An E-plane tee is a waveguide tee in which the axis of its side arm is parallel to the E field of the main guide . if the collinear arms are symmetric about the side arm.

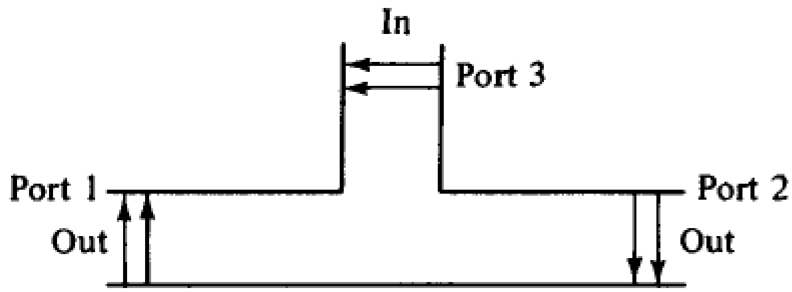
If the E-plane tee is perfectly matched with the aid of screw tuners at the junction , the diagonal components of the scattering matrix are zero because there will be no reflection.

When the waves are fed into side arm, the waves appearing at port 1 and port 2 of the collinear arm will be in opposite phase and in same magnitude.





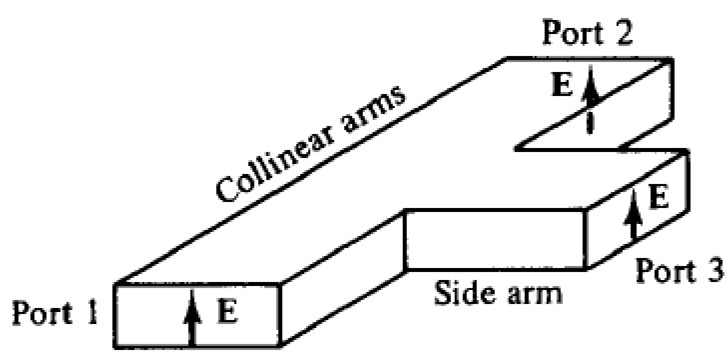
(a)



(b)

H-plane tee: (shunt tee)

An H-plane tee is a waveguide tee in which the axis of its side arm is shunting the E field or parallel to the H-field of the main guide.

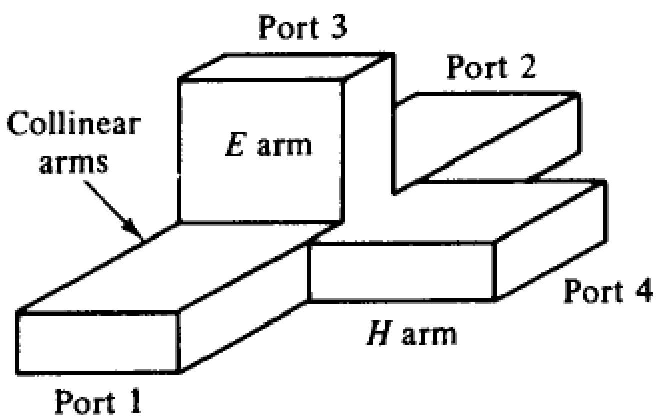


If two input waves are fed into port 1 and port 2 of the collinear arm, the output wave at port 3 will be in phase and additive .

If the input is fed into port 3, the wave will split equally into port 1 and port 2 in phase and in same magnitude .

Magic Tee (Hybrid Tees)

A magic tee is a combination of E-plane and H-plane tee. The characteristics of magic tee are:



1. If two waves of equal magnitude and same phase are fed into port 1 and port 2 the output will be zero at port 3 and additive at port 4.

3. If a wave is fed into port 4 it will be divided equally between port 1 and port 2 of the collinear arms and will not appear at port 3.

4. If a wave is fed into port 3 , it will produce an output of equal magnitude and opposite phase at port 1 and port 2. the output at port 4 is zero.

5. If a wave is fed into one of the collinear arms at port 1 and port 2, it will not appear in the other collinear arm at port 2 or 1 because the E-arm causes a phase delay while H arm causes a phase advance.

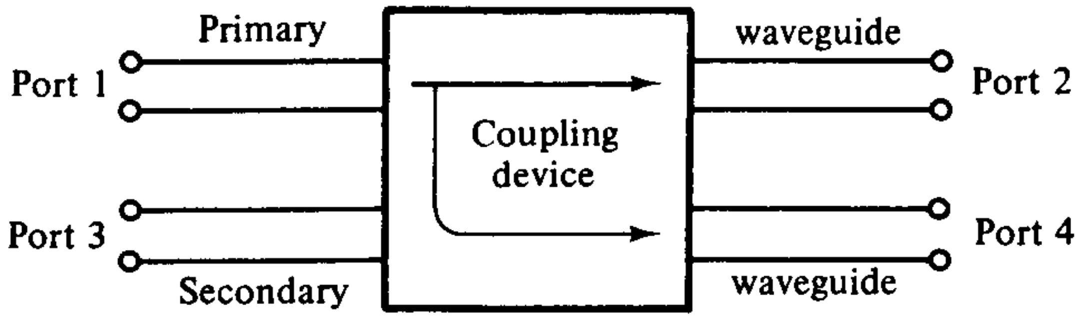
Therefore the **S** matrix of a magic tee can be expressed as

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix}$$

DIRECTIONAL COUPLERS:

A directional coupler is a four-port waveguide junction as shown below. It Consists of a primary waveguide 1-2 and a secondary waveguide 3-4. When all Ports are terminated in their characteristic impedances, there is free transmission of the waves without reflection, between port 1 and port 2, and there is no transmission of power between port I and port 3 or between port 2 and port 4 because no coupling exists between these two pairs of ports. The degree of coupling between port 1 and port4 and between port 2 and port 3 depends on the structure of the coupler.

The characteristics of a directional coupler can be expressed in terms of its Coupling factor and its directivity. Assuming that the wave is propagating from port to port2 in the primary line, the coupling factor and the directivity are defined,



Directional coupler.

where PI = power input to port I

P3 = power output from port 3

P4 = power output from port 4

$$\text{Coupling factor (dB)} = 10 \log_{10} \frac{P_1}{P_4}$$

$$\text{Directivity (dB)} = 10 \log_{10} \frac{P_4}{P_3}$$

It should be noted that port 2, port 3, and port 4 are terminated in their characteristic impedances. The coupling factor is a measure of the ratio of power levels in the primary and secondary lines. Hence if the coupling factor is known, a fraction of power measured at port 4 may be used to determine the power input at port 1 .

This significance is desirable for microwave power measurements because no disturbance, which may be caused by the power measurements, occurs in the primary line. The directivity is a measure of how well the forward traveling wave in the primary waveguide couples only to a specific port of the secondary waveguide ideal directional coupler should have infinite directivity. In other words, the power at port 3 must be zero because port 2 and portA are perfectly matched. Actually well-designed directional couplers have a directivity of only 30 to 35 dB.

Several types of directional couplers exist, such as a two-hole direct couler, four-hole directional coupler, reverse-coupling directional coupler , and Bethe-hole directional coupler the very commonly used two-hole directional coupler is described here.

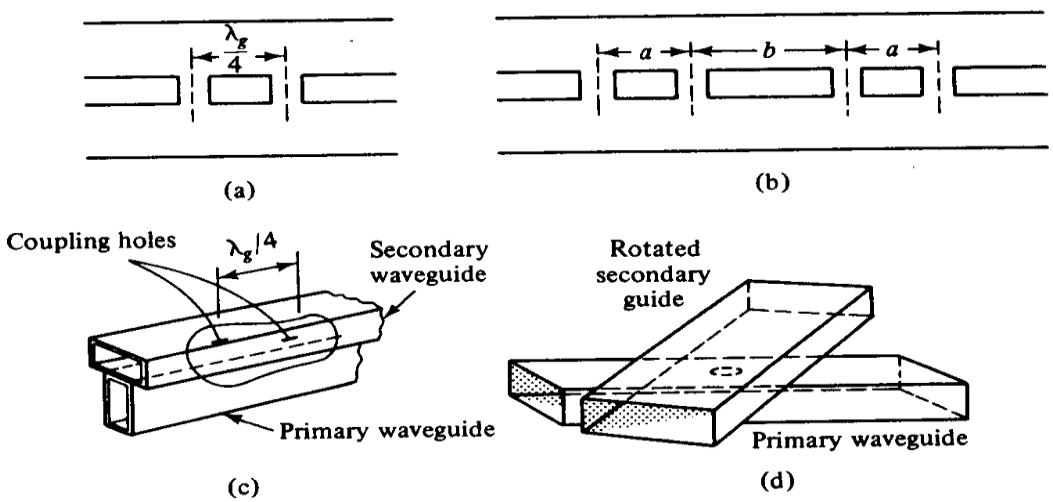


Figure 4-5-2 Different directional couplers. (a) Two-hole directional coupler. (b) Four-hole directional coupler. (c) Schwinger coupler. (d) Bethe-hole directional coupler.

TWO HOLE DIRECTIONAL COUPLERS:

A two hole directional coupler with traveling wave propagating in it is illustrated . the spacing between the centers of two holes is

$$L = (2n + 1)\frac{\lambda_g}{4}$$

A fraction of the wave energy entered into port 1 passes through the holes and is radiated into the secondary guide as he holes act as slot antennas. The forward waves in the secondary guide are in same phase , regardless of the hole space and are added at port 4. the backward waves in the secondary guide are out of phase and are cancelled in port 3.

S-matrix for Directional coupler:

The following characteristics arc observed in an ideal Directional Coupler:

1. Since the directional coupler is a 4-portjunction, the order or (S I matrix is 4 x 4 givcn by

$$[S]_{DC} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

2. Microwave power fed into port (I) cannot comc out of port (3) as port (3) is the back port. Therefore the scattering co-efficient S13 is zero...'

$$S_{13} = 0$$

3. Because of the symmetry of the junction, an input power at port (2) cannot couple to port (4) as port (4) is the back-port for port (2)

$$S_{24} = 0$$

4. Let us assume that port (3) and (4) are perfectly matched to the junction so that

$$S_{33} = S_{44} = 0$$

Then, the remaining two ports will be "automatically" matched to the junction

$$S_{11} = S_{22} = 0$$

From the symmetric property of ISI matrix, we have

$$S_{ij} = S_{ji}$$

With the above characteristic values for S-parameters, the matrix of (5.125) becomes

$$[S]_x = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

[Since $S_{21} = S_{12}$, $S_{31} = S_{13} = 0$, $S_{32} = S_{23}$, $S_{41} = S_{14}$, $S_{42} = S_{24} = 0$ and $S_{43} = S_{34}$]

From unitary property of equation we have

$$[S]\,[S]^* = [U]$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Considering 1st row and 1st column,

$$|S_{12}|^2 + |S_{14}|^2 = 1$$

Considering 2st row and 2nd column,

$$|S_{12}|^2 + |S_{23}|^2 = 1$$

Considering 3rd row and 3rd column,

$$|S_{23}|^2 + |S_{34}|^2 = 1$$

Considering 1st row and 3rd column,

$$S_{12}\,S_{23}^* + S_{14}\,S_{34}^* = 0$$

Comparison of equations (5.133) and (5.134) yields

$$S_{14} = S_{23}$$

Comparing equations (5.134) and (5.135), we get

$$S_{12} = S_{34}$$

Let S_{12} be "***real and positive*** "***equal*** to p

Then $S_{34} = p = S_{34}^* = S_{12}$

Using equations (5.137) and (5.139) in (5.136), we get

$$S_{12}\,S_{23}^* + S_{23}\,S_{12} = 0$$

$$\therefore \quad S_{12}\,(S_{23} + S_{23}^*) = 0$$

Since $S_{12} \neq 0$, we must have $S_{23} + S^*_{23} = 0$

Equation (5.140) will be satisfied only when S_{23} is purely imaginary.

Let $S_{23} = jq = S_{14}$

Using the above obtained values of S-parameters in the matrix of equation (5.131), we get

$$[S]_{DC} = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix} \quad \text{..... (5.142)}$$

The relationship between p and q can be obtained from equation (5.133) as

$$p^2 + q^2 = 1 \quad \text{...... (5.143)}$$

The quantity 'p' is called the "***transmission factor***" and 'q' is called the "***coupling factor***".

RECOMMENDED QUESTIONS FOR UNIT -5

1. **Explain the different co-axial connectors and adapters used for microwave applications.**
2. **Explain the different co-axial cables used for microwave applications.**
3. **Explain with a neat sketch a precision type variable attenuator**
4. **Explain with a neat sketch a flap type variable attenuator**
5. **Explain with a neat sketch a precision resistive type attenuator**
6. **With a neat sketch explain a precision rotary phase shifter**

7. Explain with neat sketch the construction and operation of H-plane Tee junction .
8. Explain with neat sketch the construction and operation of E-plane Tee junction .
9. Explain with neat sketch the construction and operation of Magic Tee
10. Explain the characteristics and S- matrix of H-plane Tee junction .
11. Explain the characteristics and S- matrix of E-plane Tee junction .
12. Explain the characteristics and S- matrix of Magic Tee junction .
13. Derive the scattering parameter of a directional coupler.

UNIT - 6

STRIP LINES: Introduction, Microstrip lines, Parallèle strip lines, Coplanar strip lines, Shielded strip Lines.

6 Hours

TEXT BOOKS:

1. **Microwave Devices and circuits**- Liao / Pearson Education.
2. **Microwave Engineering –** Annapurna Das, Sisir K Das TMH Publication, 2001.

REFERENCE BOOK:

1. **Microwave Engineering –** David M Pozar, John Wiley, 2e, 2004

UNIT – 6

STRIP LINES

Microstrip transmission line is a kind of "high grade" printed circuit construction, consisting of a track of copper or other conductor on an insulating substrate. There is a "backplane" on the other side of the insulating substrate, formed from similar conductor.

There is a "hot" conductor which is the track on the top, and a "return" conductor which is the backplane on the bottom. Microstrip is therefore a variant of 2-wire transmission line.

If one solves the electromagnetic equations to find the field distributions, one finds very nearly a completely TEM (transverse electromagnetic) pattern. This means that there are only a few regions in which there is a component of electric or magnetic field in the direction of wave propagation.

The field pattern is commonly referred to as a Quasi TEM pattern. Under some conditions one has to take account of the effects due to longitudinal fields. An example is geometrical dispersion, where different wave frequencies travel at different phase velocities, and the group and phase velocities are different.

The quasi TEM pattern arises because of the interface between the dielectric substrate and the surrounding air. The electric field lines have a discontinuity in direction at the interface. The boundary conditions for electric field are that the normal component (ie the component at right angles to the surface) of the electric field times the dielectric constant is continuous across the boundary; thus in the dielectric which may have dielectric constant 10, the electric field suddenly drops to 1/10 of its value in air. On the other hand, the tangential component (parallel to the interface) of the electric field is continuous across the boundary. In general then we observe a sudden change of direction of electric field lines at the interface, which gives rise to a longitudinal magnetic field component from the second Maxwell's equation, curl E = - dB/dt.

Since some of the electric energy is stored in the air and some in the dielectric, the effective dielectric constant for the waves on the transmission line will lie somewhere between that of the air and that of the dielectric. Typically the effective dielectric constant will be 50-85% of the substrate dielectric constant.

SUBSTRATE MATERIALS:

Important qualities of the dielectric substrate include

- The microwave dielectric constant
- The frequency dependence of this dielectric constant which gives rise to "material dispersion" in which the wave velocity is frequency-dependent
- The surface finish and flatness
- The dielectric loss tangent, or imaginary part of the dielectric constant, which sets the dielectric loss
- The cost
- The thermal expansion and conductivity
- The dimensional stability with time
- The surface adhesion properties for the conductor coatings
- The manufacturability (ease of cutting, shaping, and drilling)
- The porosity (for high vacuum applications we don't want a substrate which continually "out gasses" when pumped)

Types of substrate include plastics, sintered ceramics, glasses, and single crystal substrates (single crystals may have anisotropic dielectric constants; "anisotropic" means they are different along the different crystal directions with respect to the crystalline axes.)

Common substrate materials

- Plastics are cheap, easily manufacturability, have good surface adhesion, but have poor microwave dielectric properties when compared with other choices. They have poor dimensional stability, large thermal expansion coefficients, and poor thermal conductivity.
  - Dielectric constant: 2.2 (fast substrate) or 10.4 (slow substrate)
  - Loss tangent 1/1000 (fast substrate) 3/1000 (slow substrate)
  - Surface roughness about 6 microns (electroplated)
  - Low thermal conductivity, 3/1000 watts per cm sq per degree
- Ceramics are rigid and hard; they are difficult to shape, cut, and drill; they come in various purity grades and prices each having domains of application; they have low microwave loss and are reasonably non-dispersive; they have excellent thermal properties, including good dimensional stability and high thermal conductivity; they also have very high dielectric strength. They cost more than plastics. In principle the size is not limited.
  - Dielectric constant 8-10 (depending on purity) so slow substrate
  - Loss tangent 1/10,000 to 1/1,000 depending on purity
  - Surface roughness at best 1/20 micron
  - High thermal conductivity, 0.3 watts per sq cm per degree K
- Single crystal sapphire is used for demanding applications; it is very hard, needs orientation for the desired dielectric properties which are anisotropic;

is very expensive, can only be made in small sheets; has high dielectric constant so is used for very compact circuits at high frequencies; has low dielectric loss; has excellent thermal properties and surface polish.

- Dielectric constant 9.4 to 11.6 depending on crystal orientation (slow substrate)
- Loss tangent 5/100,000
- Surface roughness 1/100 micron
- High thermal conductivity 0.4 watts per sq cm per degree K

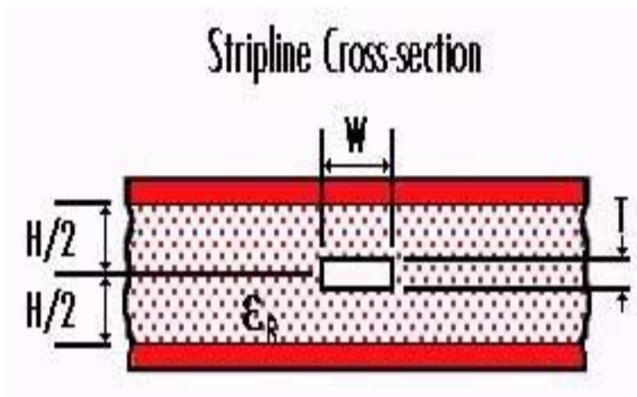
- Single crystal Gallium Arsenide (GaAs) and Silicon (Si) are both used for monolithic microwave integrated circuits (MMICs).
  - Dealing with GaAs first we have.....
    - Dielectric constant 13 (slow substrate)
    - Loss tangent 6/10,000 (high resistivity GaAs)
    - Surface roughness 1/40 micron
    - Thermal conductivity 0.3 watts per sq cm per degree K (high)

  GaAs is expensive and piezoelectric; acoustic modes can propagate in the substrate and can couple to the electromagnetic waves on the conductors.

The dielectric strength of ceramics and of single crystals far exceeds the strength of plastics, and so the power handling abilities are correspondingly higher, and the breakdown of high Q filter structures correspondingly less of a problem.

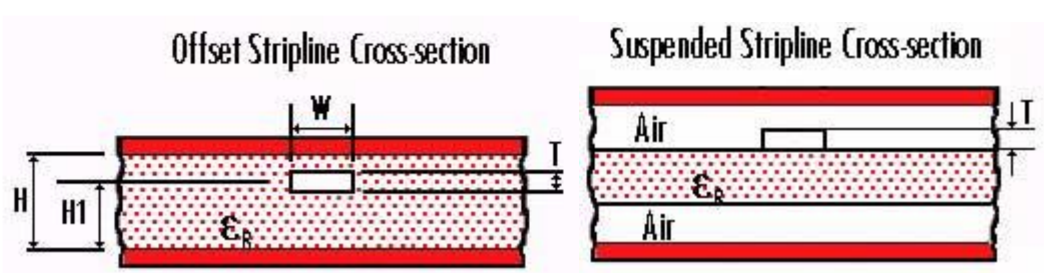
It is also a good idea to have a high dielectric constant substrate and a slow wave propagation velocity; this reduces the radiation loss from the circuits. However at the higher frequencies the circuits get impossible small, which restricts the power handling capability.

Stripline is a conductor sandwiched by dielectric between a pair of ground planes, much like a coax cable would look after you ran it over with your small-manhood indicating SUV (let's not go there...) In practice, strip line is usually made by etching circuitry on a substrate that has a ground plane on the opposite face, then adding a second substrate (which is metalized on only one surface) on top to achieve the second ground plane. Strip line is most often a "soft-board" technology, but using low-temperature co-fired ceramics (LTCC), ceramic stripline circuits are also possible.



Transmission lines on either of the interior metal layers behave very nearly like "classic" stripline, the slight asymmetry is not a problem. Excellent "broadside" couplers can be made by running transmission lines parallel to each other on the two surfaces.

Other variants of the stripline are offset strip line and suspended air stripline (SAS).



For stripline and offset stripline, because all of the fields are constrained to the same dielectric, the effective dielectric constant is equal to the relative dielectric constant of the chosen dielectric material. For suspended stripline, you will have to calculate the effective dielectric constant, but if it is "mostly air", the effective dielectric constant will be close to 1.

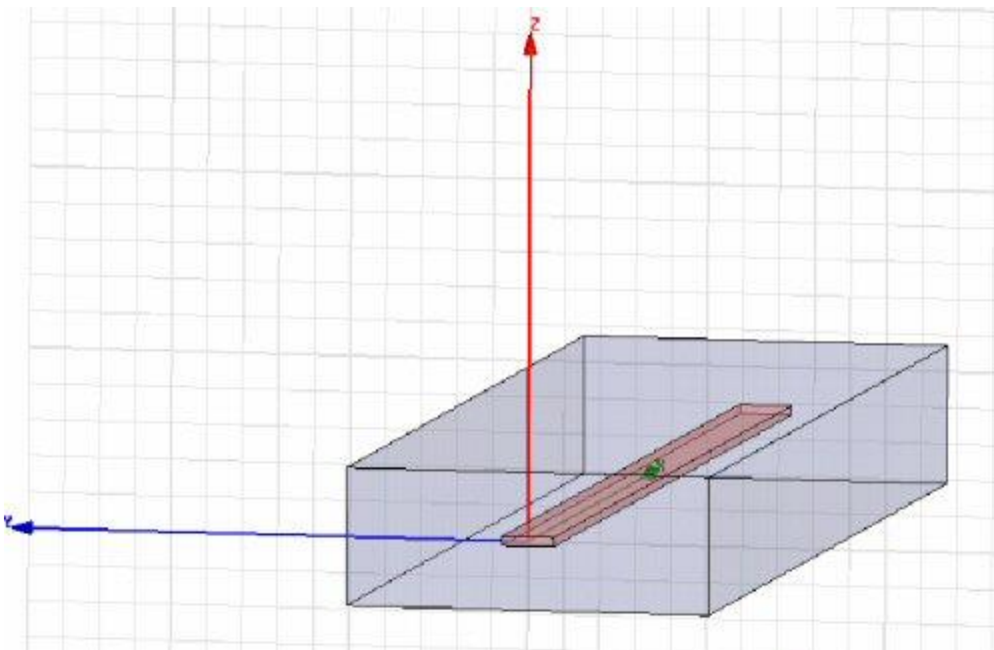
Advantages and disadvantages of stripline:

Stripline is a TEM (transverse electromagnetic) transmission line media, like coax. This means that it is non-dispersive, and has no cutoff frequency. Whatever circuits you can make on microstrip (which is quasi-TEM), you can do better using stripline, unless you run into fabrication or size constraints. Stripline filters and couplers always offer better bandwidth than their counterparts in microstrip.

Another advantage of stripline is that fantastic isolation between adjacent traces can be achieved (as opposed to microstrip). The best isolation results when a picket-fence of vias surrounds each transmission line, spaced at less than 1/4 wavelength. Stripline can be used to route RF signals across each other quite easily when offset stripline is used.

Disadvantages of stripline are two: first, it is much harder (and more expensive) to fabricate than microstrip. Lumped-element and active components either have to be buried between the ground planes (generally a tricky proposition), or transitions to microstrip must be employed as needed to get the components onto the top of the board.

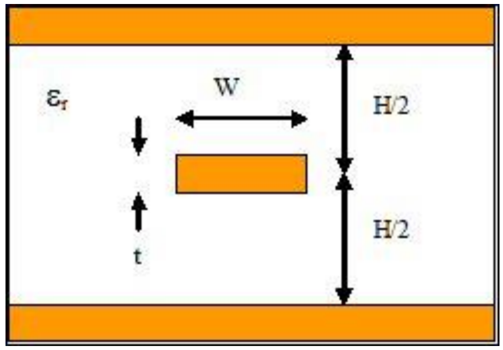
The second disadvantage of stripline is that because of the second ground plane, the strip widths are much narrower for a given impedance (such as 50 ohms) and board thickness than for microstrip. A common reaction to problems with microstrip circuits is to attempt to convert them to stripline. Chances are you'll end up with a board thickness that is four times that of your microstrip board to get equivalent transmission line loss. That means you'll need forty mils thick strip line to replace ten mil thick micro strip! This is one of the reasons that soft-board manufacturers offer so many thicknesses.



Stripline equations

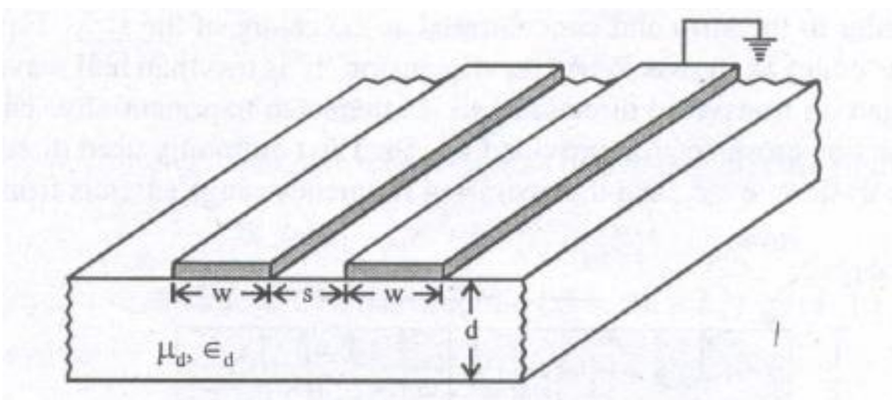
A simplified equation for characteristic impedance of stripline is given as:

$$Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \ln\left[\frac{4H}{0.67\pi W\left(0.8 + \frac{t}{D}\right)}\right]$$



COPLANAR STRIP LINES

A coplanar strip line consisting of two strip conductors each of width separated by a distance "s", mounted on a single dielectric substrate, with one conducting strip grounded. Since both the strips are on one side of the substrate unlike the parallel strip lines, connection of shunt elements is very easy. This is an added advantage in the manufacture of microwave integrated circuits (MICs). Because of this, reliability increases.



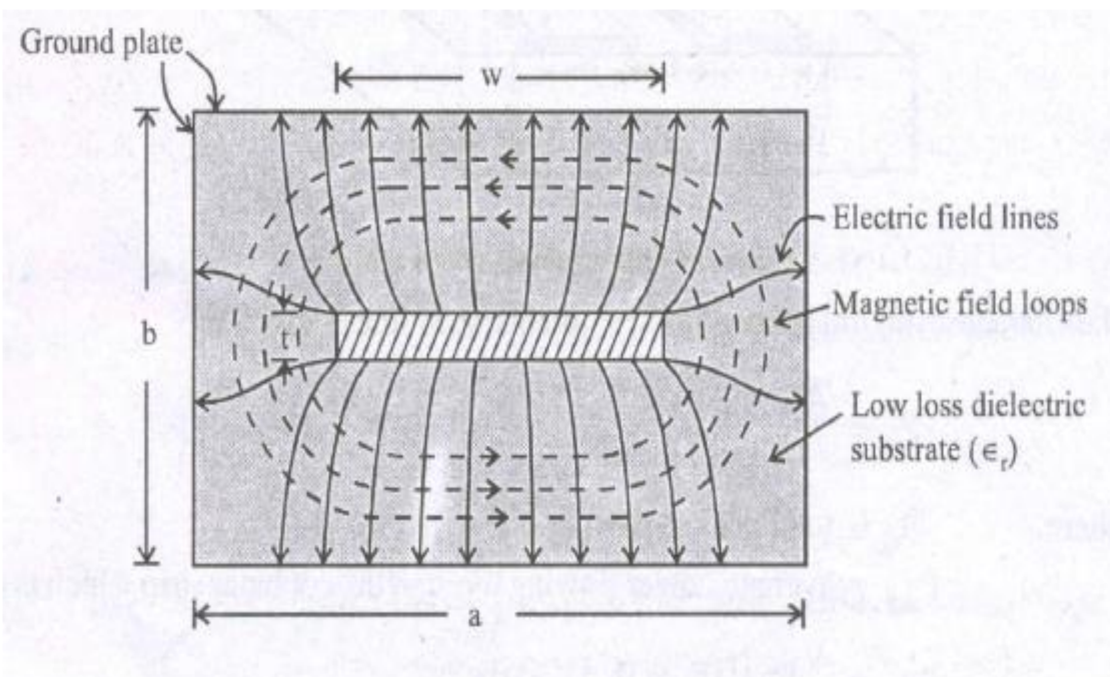
The characteristic impedance of the coplanar strip line is given by

$$Z_0 = \frac{2P_{avg}}{I_0^2}$$

P = average power flowing through the coplanar strip

SHIELDED STRIP LINES

The configuration of strip line consisting of a thin conducting strip of width "w" much greater than its thickness "t". This strip line is placed at the centre surrounded by a low-loss dielectric substrate of thickness "b", between two ground plates as shown. The mode of propagation is TEM (transverse electro-magnetic) wave where the electric field lines are perpendicular to the strip and concentrated at the centre of the strip. Fringing field lines also exist at the edges .When the dimension 'b' is less than half wavelength, the field cannot propagate in transverse direction and is attenuated exponentially. The energy will be confined to the line cross-section provided a> 5b. The commonly used dielectrics are teflon,polyolefine, polystyrene etc., and the operating frequency range extends from 100 MHz to 30 GHz.



The characteristic impedance for zero strip thickness is given by

$$Z_o = \frac{30}{\sqrt{\epsilon_R}} \ln\left[\frac{2(1+\sqrt{k})}{(1-\sqrt{k})}\right] \Omega \text{ for } \frac{w}{b} \le 0.5 \qquad \text{..... (6.45)}$$

$$Z_o = \frac{30\pi^2}{\sqrt{\epsilon_r}\ln\left[\frac{2(1+\sqrt{k'})}{(1-\sqrt{k'})}\right]} \Omega \text{ for } \frac{w}{b} > 0.5 \qquad \text{..... (6.46)}$$

Where $$k = \text{sech}\left(\frac{\pi w}{2b}\right) \qquad \text{..... (6.47)}$$

and $$k' = \sqrt{1-k^2} = \tanh\left(\frac{\pi w}{2b}\right) \qquad \text{..... (6.48)}$$

For non-zero strip thickness $\left(\frac{w}{b} >> 0.35\right)$, the characteristic impedance is given by

$$Z_o = \frac{94.15}{\sqrt{\epsilon_r}}\left[\frac{wK}{b} + \frac{C_f}{8.854\,\epsilon_r}\right]^{-1} \Omega \qquad \text{..... (6.49)}$$

Where $K = \frac{1}{1-t/b}$

t = thickness of the strip

C_f = fringing capacitance in pF/m due to fringing electric field at the edges.

$$C_f = \frac{8.854\,\epsilon_r}{\pi}\,[2K\,\ln(K+1) - (K-1)\,\ln(K^2-1)]\text{pF/m} \qquad \text{..... (6.50)}$$

In practice MICs use thickness 't' of the order of 1.5 to 3 mils [1 mil = 10^{-3} inch]. Since the mode of propagation is TEM, the wavelength in the line is $\lambda_o/\sqrt{\epsilon_r}$ where λ_o is the free-space wavelength.

LOSSES IN STRIP LINES:

For low-loss dielectric substrate, the attenuation factor in the strip line arises from conductor losses and is given by

$$\alpha_c = \frac{R_s}{Z_o b}\,\frac{(\pi w/b) + \ln\left(4b/\pi t\right)}{\ln 2 + (\pi W/2b)} \text{ nepers/unit length}$$

where $R_s = \sqrt{\pi f \mu/\sigma}$

The attenuation constant of a microstrip line depends on frequency of operation, electrical properties of substrate and the conductors and the geometry of mounting of strip on the dielectric.

When the dielectric substrate of dielectric constant is purely non-magnetic then three types of losses occur in microstrip lines . they are

1. Dielectric losses in substrate
1. Ohmic losses in strip conductor and ground plane
2. Radiation loss

Dielectric losses in substrate:

All dielectric materials possess some conductivity but it will be small , but when it is not negligible, then the displacement current density leads the conduction current density by 90 degrees, introducing loss tangent for a lossy dielectric .

1. Ohmic losses in strip conductor and ground plane

In a microstrip line the major contribution to losses at micro frequencies is from finite conductivity of microstrip conductor placed on a low loss dielectric substrate. Due to current flowing through the strip, there will be ohmic losses and hence attenuation of the microwave signal takes place. The current distribution in the transverse palne is fairly uniform with minimum value at the central axis and shooting up to maximum values at the edge of the strip.

2. Radiation losses:

At microwave frequencies , the microstrip line acts as an antenna radiating a small amount of power resulting in radiation losses. This loss depends on the thickness of the substrate, the characteristic impedance Z, effective dielectric constant and the frequency of operation.

For low-loss dielectric substrate, the attenuation factor in the strip line arises from conductor losses and is given by

$$\alpha_c = \frac{R_s}{Z_o b} \frac{(\pi w/b) + \ln\left(4b/\pi t\right)}{\ln 2 + (\pi W/2b)} \text{ nepers/unit length}$$

where $R_s = \sqrt{\pi f \mu / \sigma}$

Advantages and disadvantages of Planar Transmission Lines over Co-axial Lines:

Advantages:-

The advantages of planar transmission lines are

(a) very small size and hence low weight

(b) can be easily mounted on a metallic body including substrate.

(c) increased reliability

(d) cost is reduced due to small size

(e) series and shunt maintaining of components is possible

(f) the characteristic impedance Zo is easily controlled by defining the dimensions of the line in a single plane

(g) by changing the dimensions of the line in one plane only, it is possible to achieve accurate passive circuit design

Disadvantages:-

The disadvantages of planar transmission lines are

(a) low power handling capability due to small size

(b)The microstrip, slot and coplanar lines tend to radiate power resulting in radiation losses

(c) low Q-factor

RECOMMENDED QUESTIONS FOR UNIT – 6

1. With a neat sketch explain the different types of strip lines
2. What are the different losses taking place in microstrip line
3. Obtain the Characteristic equation for a parallel stripline.
4. Derive the equation for Z0 of shielded stripline.
5. Derive the equation for Z0 of coplanar stripline.
6. With equations explain the different losses in striplines.

UNIT – 7

AN INTRODUCTION TO RADAR: Basic Radar, The simple form of the Radar equation, Radar block diagram, Radar frequencies, application of Radar, the origins of Radar.

8 Hours

TEXT BOOKS:

1. **Microwave Devices and circuits**- Liao / Pearson Education.
2. **Introduction to Radar systems**-Merrill I Skolnik, 3rd Ed, TMH, 2001.
4. **Microwave Engineering** – Annapurna Das, Sisir K Das TMH Publication, 2001.

REFERENCE BOOK:

1. **Microwave Engineering –** David M Pozar, John Wiley, 2e, 2004

UNIT – 7

INTRODUCTION TO RADAR

Radar is an electromagnetic system for the detection and location of objects. It operates by transmitting a particular type of waveform, a pulse-modulated sine wave for example, and detects the nature of the echo signal.

Radar can be designed to see through those conditions impervious to normal human vision, such as darkness, haze, fog, rain, and snow. In addition, radar has the advantage of being able to measure the distance or range to the object.

An elementary form of radar consists of a transmitting antenna emitting electromagnetic radiation generated by an oscillator of some sort, a receiving antenna, and an energy-detecting device. or receiver.

A portion of the transmitted signal is intercepted by a reflecting object (target) and is reradiated in all directions. It is the energy reradiated in the back direction that is of prime interest to the radar.

The receiving antenna collects the returned energy and delivers it to a receiver, where it is processed to detect the presence of the target and to extract its location and relative velocity. The distance to the target is determined by measuring the time taken for the radar signal to travel to the target and back. The direction, or angular position, of the target may be determined from the direction of arrival of the reflected wave front.

The name radar reflects the emphasis placed by the early experimenters on a device to detect the presence of a target and measure its range. Radar is a contraction of the words radio detection and ranging. It was first developed as a detection device to warn of the approach of hostile aircraft and for directing antiaircraft weapons. Although a well-designed modern radar can usually extract more information from the target signal than merely range, the measurement of range is still one of radar's most important functions.

The most common radar waveform is a train of narrow, rectangular-shape pulses modulating a sine wave carrier. The distance, or range, to the target is determined by measuring the time ***TR*** taken by the pulse to travel to the target and return.

$$R = \frac{cT_R}{2} \tag{1.1}$$

The factor 2 appears in the denominator because of the two-way propagation of radar. With the range in kilometers or nautical miles, and ***TR*** in microseconds, ***Eq.*** (1.1) becomes

$$R(\text{km}) = 0.15 T_R(\mu\text{s}) \qquad \text{or} \qquad R(\text{nmi}) = 0.081\, T_R(\mu\text{s})$$

Once the transmitted pulse is emitted by the radar, a sufficient length of time must elapse to allow any echo signals to return and be detected before the next pulse may be transmitted.

Therefore the rate at which the pulses may be transmitted is determined by the longest range at which targets are expected. If the pulse repetition frequency is too high, echo signals from some targets might arrive after the transmission of the next pulse, and ambiguities in measuring range might result. Echoes that arrive after the transmission of the next pulse are called second-time-around (or multiple-time-around) echoes. Such an echo would appear to be at a much shorter range than the actual and could be misleading if it were not known to be a ***second-time-around echo***. The range beyond which targets appear as second-time-around echoes is called the ***maximum unambiguous range*** and is

$$R_{\text{unamb}} = \frac{c}{2f_p} \tag{1.2}$$

Where fp = pulse repetition frequency, in Hz.

1.2 THE SIMPLE FORM OF THE RADAR EQUATION

The radar equation relates the range of a radar to the characteristics of the transmitter,receiver, antenna, target, and environment. It is useful not just as a means for determining the maximum distance from the radar to the target, but it can serve both as a tool for understanding radar operation and as a basis for radar design.

If the power of the radar transmitter is denoted by P,, and if an isotropic antenna is used (one which radiates uniformly in all directions), the ***power*** density (watts per unit area) at a distance ***R*** from the radar is equal to the transmitter power divided by the surface area ***4∏R2*** of an imaginary sphere of radius R

$$\text{Power density from isotropic antenna} = \frac{P_t}{4\pi R^2} \tag{1.3}$$

Radars employ directive antennas to channel, or direct, the radiated power ***Pt*** into some
particular direction. The ***gain*** G of an antenna is a measure of the increased power radiated in the direction of the target as compared with the power that would have been radiated from an isotropic antenna. It may be defined as the ratio of the maximum radiation intensity from the subject antenna to the radiation intensity from a lossless, isotropic antenna with the same power input.

$$\text{Power density from directive antenna} = \frac{P_t G}{4\pi R^2} \tag{1.4}$$

The target intercepts a portion of the incident power and reradiates it in various directions The measure of the amount of incident power intercepted by the target and reradiated back in the direction of the radar is denoted as the radar cross section , and is defined by the relation

Power density of echo signal at radar $= \frac{P_t G}{4\pi R^2} \frac{\sigma}{4\pi R^2}$ (1.5)

The radar cross section a has units of area. It is a characteristic of the particular target and is a measure of its size as seen by the radar. The radar antenna captures a portion of the echo power. If the effective area of the receiving antenna is denoted ***A.,*** the power *P,* received by the radar is

$$P_r = \frac{P_t G}{4\pi R^2} \frac{\sigma}{4\pi R^2} A_e = \frac{P_t G A_e \sigma}{(4\pi)^2 R^4} \quad (1.6)$$

The maximum radar range ***Rmax*** is the distance beyond which the target cannot be detected. It occurs when the received echo signal power *P,* just equals the minimum detectable signal ***Smin***

$$R_{\max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 S_{\min}}\right]^{1/4} \quad (1.7)$$

This is the fundamental form of the radar equation.

Antenna theory gives the relationship between the transmitting gain and the receiving effective area of an antenna as

$$G = \frac{4\pi A_e}{\lambda^2} \quad (1.8)$$

Since radars generally use the same antenna for both transmission and reception, Eq. (1.8) can be substituted into Eq.

(1.7), first for ***A,*** then for G, to give two other forms of the radar equation

$$R_{max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right]^{1/4} \tag{1.9}$$

$$R_{max} = \left[\frac{P_t A_e^2 \sigma}{4\pi \lambda^2 S_{min}}\right]^{1/4} \tag{1.10}$$

These three forms (Eqs. 1.7, 1.9, and 1.10) illustrate the need to be careful in the interpretation of the radar equation.

1.3 RADAR BLOCK DIAGRAM AND OPERATION

The operation of a typical pulse radar may be described with the aid of the block diagram shown in Fig. 1.2. The transmitter may be an oscillator, such as a magnetron, that is "pulsed" (turned on and on) by the modulator to generate a repetitive train of pulses. The magnetron has probably been the most widely used of the various microwave generators for radar.

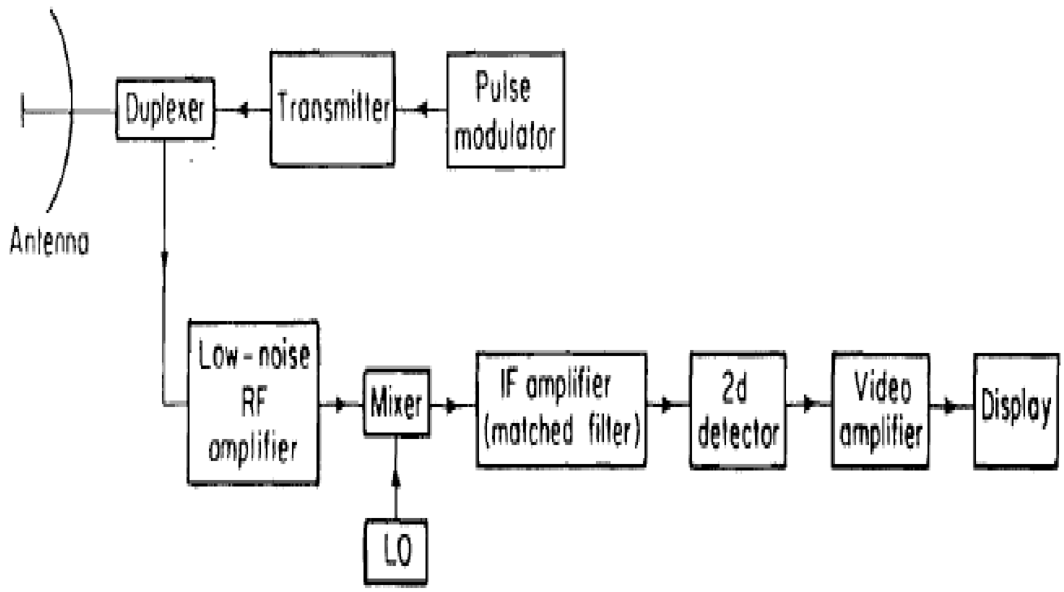


Figure 1.2 Block diagram of a pulse radar.

The waveform generated by the transmitter travels via a transmission line to the antenna, where it is radiated into space. A single antenna is generally used for both transmitting and receiving. The receiver must be protected from damage caused by the high power of the transmitter. This is the function of the duplexer. The duplexer also serves to channel the returned echo signals to the receiver and not to the transmitter. The duplexer might consist of two gas-discharge devices, one known as a TR (transmit-receive) and the other an ATR (anti-transmit-receive). The TR protects the receiver during transmission and the ATR directs the echo signal to the receiver during reception. Solid-state ferrite circulators and receiver protectors with gas-plasma TR devices and/or diode limiters are also employed as duplexers.

The receiver is usually of the super heterodyne type. The first stage might be a low-noise **RF** amplifier, such as a parametric amplifier or a low-noise transistor. However, it is not always desirable to employ a low-noise first stage in radar. The receiver input can simply be the mixer stage, especially in military radars that must operate in a noisy environment.

Although a receiver with a low-noise front-end will be more sensitive, the mixer input can have greater dynamic range, less susceptibility to overload, and less vulnerability to electronic interference.

The mixer and local oscillator (LO) convert the RF signal to an intermediate frequency **(IF).** A " typical" IF amplifier for an air-surveillance radar might have a center frequency of 30 or 60 MHz and a bandwidth of the order of one megahertz. After maximizing the signal-to-noise ratio in the IF amplifier, the pulse modulation is extracted by the second detector and amplified by the video amplifier to a level where it can be properly displayed, usually on a cathode-ray tube (CRT). Timing signals are also supplied to the indicator to provide the range zero. Angle information is obtained from the pointing direction of the antenna.

1.4 RADAR FREQUENCIES

Conventional radars generally have been operated at frequencies extending from about 220 **MHz** to 35 **GHz,** a spread of more than seven octaves. The place of radar frequencies in the electromagnetic spectrum is shown in Fig. 1.4. Some of the nomenclature employed to designate the various frequency regions.

Early in the development of radar, a letter code such as S, X, L, etc., was employed to designate radar frequency bands. Although its original purpose was to guard military secrecy, the designations were maintained, probably out of habit as well as the need for some convenient short nomenclature. This usage has continued and is now an accepted practice of radar engineers.

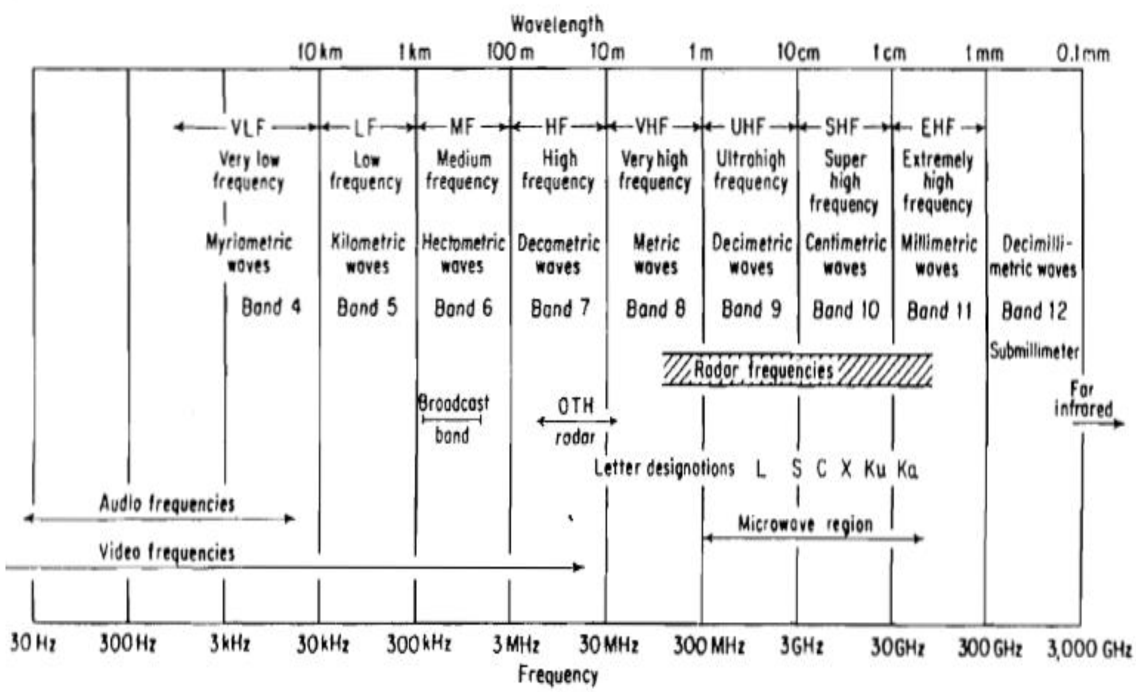


Figure 1.4 Radar frequencies and the electromagnetic spectrum.

Table 1.1 Standard radar-frequency letter-band nomenclature

Band designation	Nominal frequency range	Specific radiolocation (radar) bands based on ITU assignments for region 2
HF	3-30 MHz	
VHF	30-300 MHz	138-144 MHz 216-225
UHF	300-1000 MHz	420-450 MHz 890-942
L	1000-2000 MHz	1215-1400 MHz
S	2000-4000 MHz	2300-2500 MHz 2700-3700
C	4000-8000 MHz	5250-5925 MHz
X	8000-12,000 MHz	8500-10,680 MHz
K_u	12.0-18 GHz	13.4-14.0 GHz 15.7-17.7
K	18-27 GHz	24.05-24.25 GHz
K_a	27-40 GHz	33.4-36.0 GHz
mm	40-300 GHz	

1.5 RADAR DEVELOPMENT PRIOR TO WORLD WAR I1

Or

ORIGIN OF RADAR

Heinrich Hertz, in 1886, experimentally tested the theories of Maxwell and demonstrated the similarity between radio and light waves. Hertz showed that radio waves could be reflected by metallic and dielectric bodies.

In 1903 a German engineer by the name of Hulsmeyer experimented with the detection of radio waves reflected from ships. He obtained a patent in 1904 in several countries for an obstacle detector and ship navigational device.

Marconi recognized the potentialities of short waves for radio detection and strongly urged their use in 1922 for this application. In a speech delivered before the Institute of Radio Engineers.

In the autumn of 1922 A. H. I'aylor arid L. C. Young of tile Naval Research Laboratory detected a wooden ship using a CW wave-interference radar with separated receiver and transmitter. The wavelength was 5 m.

The first application of the pulse technique to the measurement of distance was in the basic scientific investigation by Breit and Tuve in 1925 for measuring the height of the ionosphere.

The first experimental radar systems operated with CW and depended for detection upon the interference produced between the direct signal received from the transmitter and the doppler-frequency-shifted signal reflected by a moving target. This effect is the same as the rhythmic flickering, or flutter, observed in an ordinary television receiver, especially on weak stations, when an aircraft passes overhead. This type of radar originally was called CW wave interference radar.

The first detection of aircraft using the wave-interference effect was made in June, 1930, by L. **A.** Hyland of the Naval Research Laboratory. The early CW wave-interference radars were useful only for detecting the ***presence*** of the target. The problem of extracting target-position information from such radars was a difficult one and could not be readily solved with the techniques existing at that time. **A** proposal was made by NRL in 1933 to employ a chain of transmitting and receiving stations along a line to be guarded. for the purpose of obtaining some knowledge of distance and velocity.

The United States Army Signal Corps also maintained an interest in radar during the early 1930s.In 1939 the Army developed the SCR-270, a long-range radar for early warning. The attack on Pearl Harbor in December, 1941, was detected by an SCR-270, one of six in Hawaii at the time.

By June, 1935, the British had demonstrated the pulse technique to measure range of an aircraft target. This was almost a year sooner than the successful NRL experiments with pulse radar.

1.6 APPLICATIONS OF RADAR

Radar has been employed on the ground, in the air, on the sea, and in space. Ground-based radar has been applied chiefly to the detection, location, and tracking of aircraft or space targets.

Shipboard radar is used as a navigation aid and safety device to locate buoys, shore lines, and other ships. as well as for observing aircraft. Airborne radar may be used to detect other aircraft, ships, or land vehicles or it may be used for mapping of land, storm avoidance, terrain avoidance, and navigation. In space, radar has assisted in the guidance of spacecraft and for the remote sensing of the land and sea.

The major user of radar, and contributor of the cost of almost all of its development, has been the military: although there have been increasingly important civil applications, chiefly for marine and air navigation.

1. *Air Traffic Control (A T C):* Radars are employed throughout the world for the purpose of
safely controlling air traffic en route and in the vicinity of airports. Aircraft and ground
vehicular traffic at large airports are monitored by means of high-resolution radar.

2.Aircraft Navigation: The weather-avoidance radar used on aircraft to outline regions of precipitation to the pilot is a classical form of radar. Radar is also used for terrain avoidance and terrain.

3. Ship Safety: Radar is used for enhancing the safety of ship travel by warning of potential collision with other ships, and for detecting navigation buoys, especially in poor visibility. Automatic detection and tracking equipments (also called plot extractors) are commercially available for use with such radars for the purpose of collision avoidance. Shore-based radar **of** moderately high resolution is also used for the surveillance of harbors as an aid to navigation.

4.Space: Space vehicles have used radar for rendezvous and docking, and for landing on the moon. Some of the largest ground-based radars are for the detection and tracking of
satellites. Satellite-borne radars have also been used for remote sensing.

5.Remote Sensing: Remote sensing with radar is also concerned with Earth resources, which includes the measurement and mapping of sea conditions, water resources, ice cover, agriculture, forestry conditions, geological formations, and environmental pollution. The platforms for such radars include satellites as well as aircraft.

6. Law Enforcement: The wide use of radar to measure the speed of automobile traffic by highway police, radar has also been employed as a means for the detection of intruders.

7.Military: The traditional role of radar for military application has been for surveillance, navigation, and for the control and guidance of weapons.

RECOMMENDED QUESITONS ON UNIT- 7

1. With the help of a block diagram, explain the operation of a radar system.

2. Derive radar range and equation.

3. Derive the radar equation. Discuss the effects of each parameter on the maximum detection range of the radar.

4.Write short notes of origin of radar

5.Wrtie any five important applications of Radar.

6. Write the frequency band designation of Radars

UNIT – 8

MTI AND PULSE DOPPLER RADAR: Introduction to Doppler and MTI Radar, delay line Cancellers, digital MTI processing, Moving target detector, pulse Doppler Radar.

7 Hours

TEXT BOOKS:

1. **Microwave Devices and circuits**- Liao / Pearson Education.
2. **Introduction to Radar systems**-Merrill I Skolnik, 3rd Ed, TMH, 2001.
3. **Microwave Engineering –** Annapurna Das, Sisir K Das TMH Publication, 2001.

REFERENCE BOOK:

1. **Microwave Engineering –** David M Pozar, John Wiley, 2e, 2004

UNIT - 8

MTI AND PULSE DOPPLAR RADAR

The ability of a radar receiver to detect a weak echo signal is limited by the noise energy that occupies the same portion of the frequency spectrum as does the signal energy. The weakest signal the receiver can detect is called the ***minimum detectable signal.***

8.1 THE DOPPLER EFFECT

A radar detects the presence of objects and locates their position in space by transmitting electromagnetic energy and observing the returned echo. A pulse radar transmits a relatively short burst of electromagnetic energy, after which the receiver is turned on to listen for the echo. The echo not only indicates that a target is present, but the time that elapses between the transmission of the pulse and the receipt of the echo is a measure of the distance to the target. Separation of the echo signal and the transmitted signal is made on the basis of differences in time.

The radar transmitter may be operated continuously rather than pulsed if the strong transmitted signal can be separated from the weak echo. The received-echo-signal power is considerably smaller than the transmitter power; it might be as little as 10^{18} that of the transmitted power-sometimes even less. Separate antennas for transmission and reception help segregate the weak echo from the strong leakage signal, but the isolation is usually not sufficient. **A** feasible technique for separating the received signal from the transmitted signal when there is relative motion between radar and target is based on recognizing the change in the echo-signal frequency caused by the doppler effect.

It is well known in the fields of optics and acoustics that if either the source of oscillation or the observer of the oscillation is in motion, an apparent shift in frequency will result. This is the *doppler effect.*

If R is the distance from the radar to target, the total number of wavelengths ***L*** contained in the two-way path between the radar and the target is $2R/\lambda$. The distance R and the wavelength ***L*** are assumed to be measured in the same units.

Since one wavelength corresponds to an angular excursion of **2**∏ radians, the total angular excursion made by the electromagnetic wave during its transit to and from the target is **4**∏**R /** λ radians. If the target is in motion, R and the phase **φ** are continually changing.

The doppler angular frequency ωd is given by

$$\omega_d = 2\pi f_d = \frac{d\phi}{dt} = \frac{4\pi}{\lambda}\frac{dR}{dt} = \frac{4\pi v_r}{\lambda}$$

where **fd** = doppler frequency shift and **L).** = relative (or radial) velocity of target with respect to radar. The doppler frequency shift

$$f_d = \frac{2v_r}{\lambda} = \frac{2v_r f_0}{c}$$

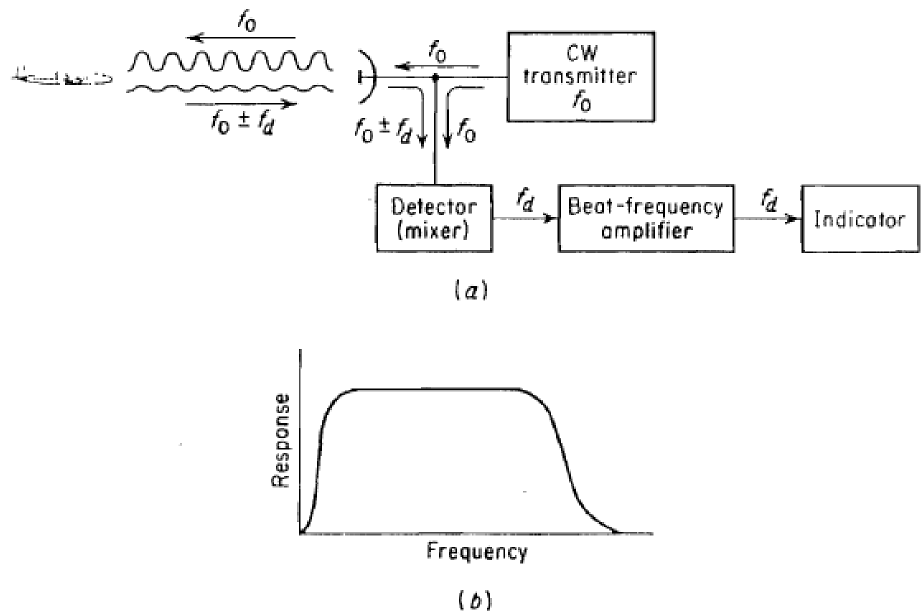
8.2 CW RADAR

Let us consider the simple CW radar as illustrated by the block diagram below. The transmitter generates a continuous (unmodulated) oscillation of frequency fo, which is radiated by **the** antenna. **A** portion of the radiated energy is intercepted by the target and is scattered, some of it in the direction of the radar, where it is collected by the receiving antenna.

If the target is in motion with a velocity ***v*****,** relative to the radar, the received signal will be shifted in frequency from the transmitted frequency fo by an amount + or – fd.The plus sign associated with the doppler frequency applies if the distance between target and radar is decreasing (closing target), that is, when the received signal frequency is greater than the transmitted signal frequency. The minus sign applies if the distance is increasing (receding target).

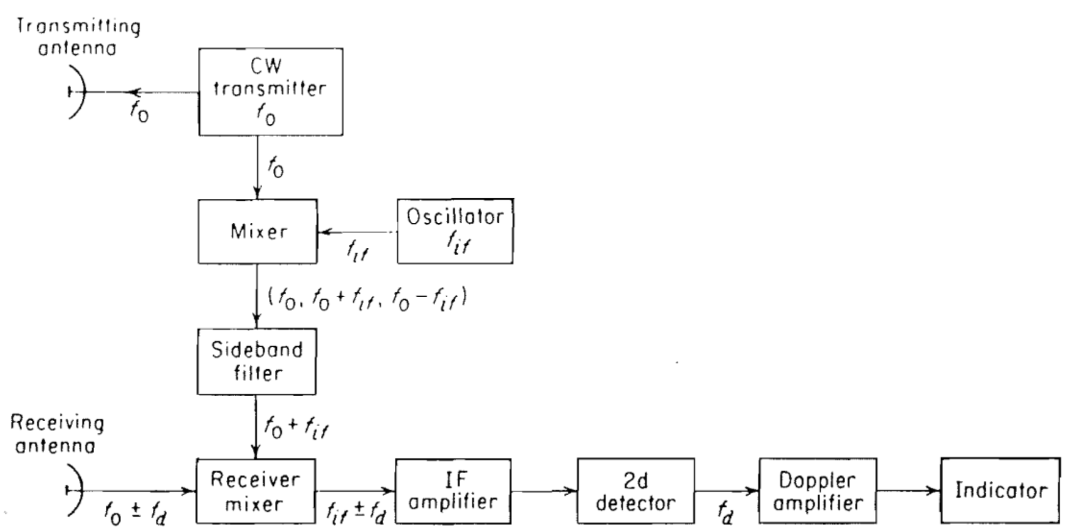
The received echo signal at a frequency enters the radar via the antenna and is heterodyned in the detector (mixer) with a portion of the transmitter signal fo to produce a doppler beat note of frequency fd. The sign of fd is lost in this process.

The purpose of the doppler amplifier is to eliminate echoes from stationary targets and to amplify the doppler echo signal to a level where it can operate an indicating device. The low-frequency cutoff must be high enough to reject tile d-c component caused by stationary targets, but yet it must be low enough to pass the smallest doppler frequency expected. Sometimes both conditions cannot he met simultaneously and a compromise is necessary. The upper cutoff frequency is selected to pass the lightest doppler frequency expected. The indicator might be a pair of earphones or a frequency meter.



(a) Simple CW radar block diagram; (b) response characteristic of beat-frequency amplifier.

Fig 8.1



Block diagram of CW doppler radar with nonzero IF receiver, sometimes called *sideband* ***Fig 8.2***

Intermediate-frequency receiver. The receiver of the simple CW radar of Fig 2 is in some respects analogous to a superheterodyne receiver. Receivers of this type are called homodyne receivers, or superheterodyne receivers with zero IF.

The function of the local oscillator is replaced by the leakage signal from the transmitter. Such a receiver is simpler than one with a more conventional intermediate frequency since no IF amplifier or local oscillator is required.

However, the simpler receiver is not as sensitive because of increased noise at the lower intermediate frequencies caused by flicker effect. Flicker-effect noise occurs in semiconductor devices such as diode detectors and cathodes of vacuum tubes.

For short-range, low-power, applications this decrease in sensitivity might be tolerated since it can be compensate by a modest increase in antenna aperture and/or additional transmitter power. But for 'maximum efficiency with CW radar, the reduction in sensitivity caused by the simple Doppler receiver with zero IF, cannot be tolerated.

The effects of flicker noise are overcome in the normal superheterodyne receiver by using an intermediate frequency which is high enough to render . the flicker noise small compared with the normal receiver noise. This results from the inverse, frequency dependence of flicker noise.

Separate antennas are shown for transmission and reception instead of the usual local oscillator found in the **convenient** receiver, the local oscillator (or reference signal) is derived in the receiver from a portion of the transmitted signal mixed with a locally generated signal of frequency equal to that of the receiver IF. Since the output of the mixer consists of two sidebands on either side of the carrier plus higher harmonics, a narrowband filter selects one of the sidebands as the reference signal. The improvement in receiver sensitivity with an intermediate-frequency super heterodyne might be as much as 30 dB over the simple receiver.

Applications of CW radar:

1. The chief use of the simple, unmodulated CW radar is for the measurement of the relative velocity of a moving target, as in the police speed monitor or in the previously mentioned rate-of-climb meter for vertical-take-off aircraft.

2. In support of automobile traffic, CW radar has been suggested for the control of traffic lights, regulation of toll booths, vehicle counting, as a replacement for the " fifth-wheel" speedometer in vehicle testing as a sensor in antilock braking systems, and for collision avoidance.

3. For railways, CW radar can be used as a speedometer to replace the conventional axle-driven tachometer.

4. It has been used for the measurement of **railroad-freight-car velocity** during humping operations in marshalling yards, and as

a detection device to give track maintenance personnel advance warning of approaching trains..

5. CW radar is also employed for monitoring the docking speed of large ships.

6. It has also seen application for intruder alarms and for the measurement of the velocity of missiles, ammunition, and baseballs.

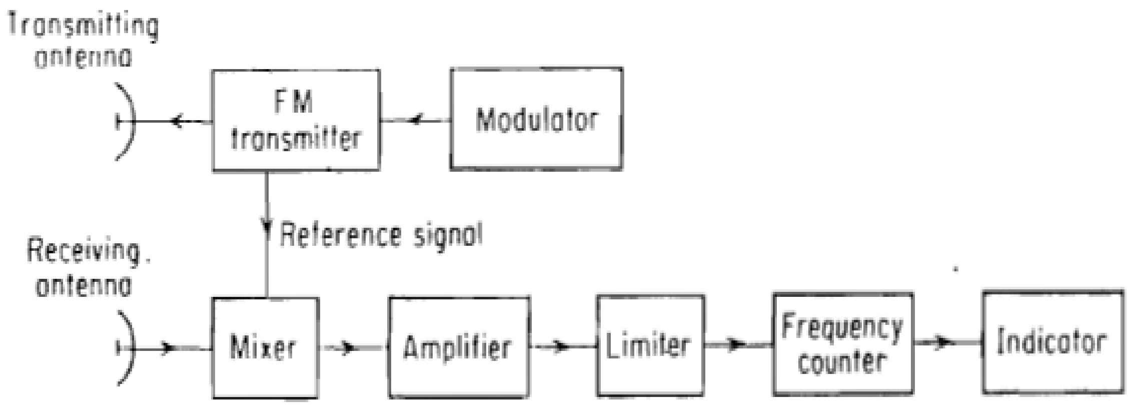
The principal advantage of a CW doppler radar over other (nonradar) methods of measuring speed is that there need not be any physical contact with the object whose speed is being measured. In industry this has been applied to the measurement of turbine-blade vibration, the peripheral speed of grinding wheels, and the monitoring of vibrations in the cables of suspension bridges.

8.3 FREQUENCY - MODULATED CW RADAR:

The inability of the simple CW radar to measure range is related to the relatively narrow spectrum (bandwidth) of its transmitted waveform. Some sort of timing mark must be applied to a CW carrier if range is to be measured. The timing mark permits the time of transmission and the time of return to be recognized. The sharper or more distinct the mark, the more
accurate the measurement of the transit time. But the more distinct the timing mark, the broader will be the transmitted spectrum. This follows from the properties of the Fourier transform.

The spectrum of a CW transmission can be broadened by the application of modulation, either amplitude. frequency, or phase. An example of an amplitude modulation is the pulse radar. The narrower the pulse, the more accurate the measurement of range and the broader the transmitted spectrum.

A block diagram illustrating the principle of the FM-CW radar is shown in above figure. A portion of the transmitter signal acts as the reference signal required to produce the beat frequency. It is introduced directly into the receiver via a cable or other direct connection.



Block diagram of FM-CW radar.

Fig 8.3

Ideally, the isolation between transmitting and receiving antennas is made sufficiently large so as to reduce to a negligible level the transmitter leakage signal which arrives at the receiver via the coupling between antennas. The beat frequency is amplified and limited to remove any amplitude fluctuations. The frequency of the amplitude-limited beat note is measured with a cycle-counting frequency meter calibrated in distance. The target was assumed to be stationary. If this assumption is not applicable, a doppler frequency shift will be superimposed on the FM range beat note and an erroneous range measurement results. The doppler frequency shift causes the frequency-time plot of the echo signal to be shifted up or down

$$f_b(\text{up}) = f_r - f_d$$
$$f_b(\text{down}) = f_r + f_d$$

When more than one target is present within the view of the radar, the mixer output will contain more than one difference frequency. If the system is linear, there will be a frequency component corresponding to each target. In principle, the range to each target may be determined by measuring the individual frequency components.

To measure the individual frequencies, they must be separated from one another. This might he accomplished with a bank of narrowband filters, or alternatively, a single frequency corresponding to a single target may be singled out and continuously observed with a narrow band tunable filter. If the FM-CW radar is used for single targets only, such as in the radio altimeter, it is not necessary to employ a linear modulation waveform.

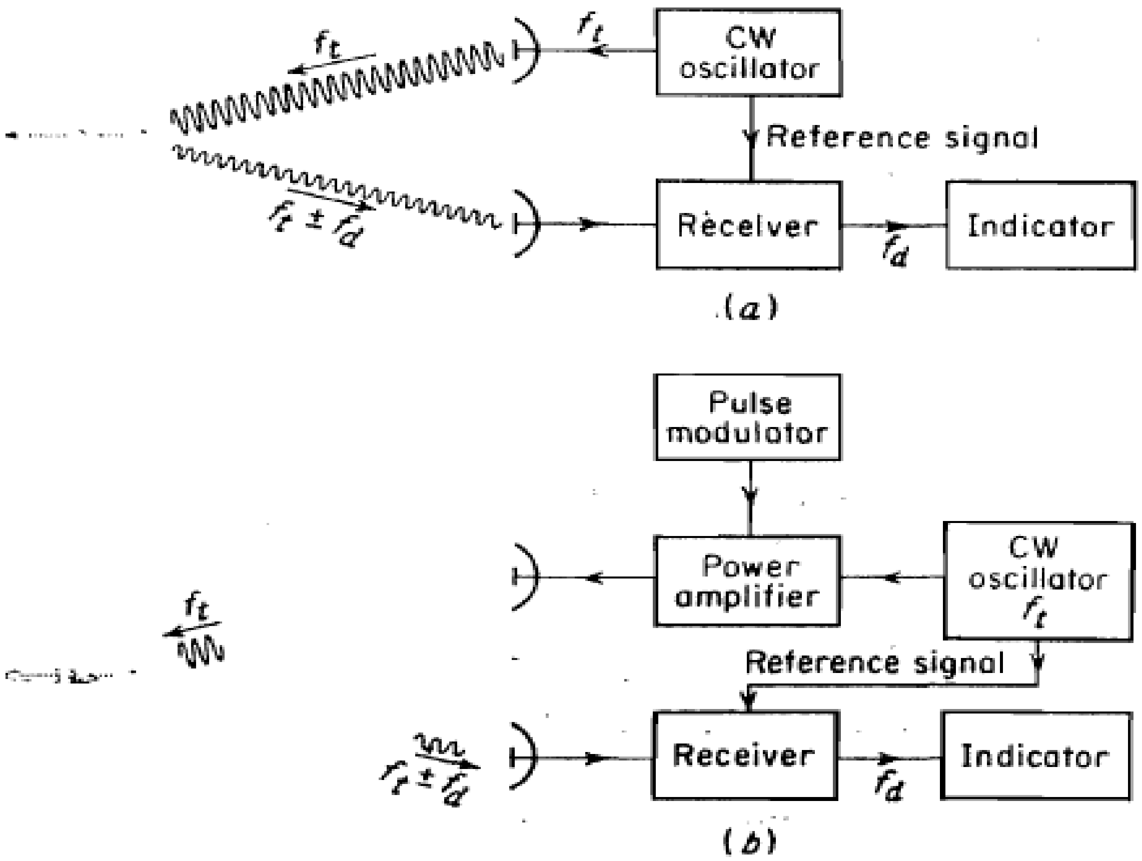
8.4 MTI RADARS

The doppler frequency shift produced by a moving target may be used in a pulse radar. just as in the CW radar to determine the relative velocity of a
target or to separate desired moving targets from undesired stationary objects (clutter). Although there are applications of pulse radar where a determination of the target's relative velocity is made from the doppler

frequency shift, the use of doppler to separate small moving targets in the presence of large clutter has probably been of far greater interest. Such a pulse radar that utilizes the doppler frequency shift as a means for discriminating moving from fixed targets is called an **M*TI*** (moving target indication) or a ***pulse doppler*** radar. The two are based on tile same physical principle, but in practice there are generally recognizable differences between them .

The MTI radar, for instance, usually operates with ambiguous doppler measurement but with unambiguous range measurement (no second-time'-around echoes). The opposite is generally the case for a pulse doppler radar. Its pulse repetition frequency is usually high enough to operate with unambiguous doppler (no blind speeds) but at the expense of range ambiguities. The discussion in this chapter, for the most part, is based on tile MTI radar, but much of what applies to MTI can be extended to **pulse** doppler radar as well.

MTI is a necessity in high-quality air-surveillance radars that operate in the presence of clutter. Its design is more challenging than that of a simple pulse radar or a simple CW radar. **An MTI** capability adds to a radar's cost and complex.

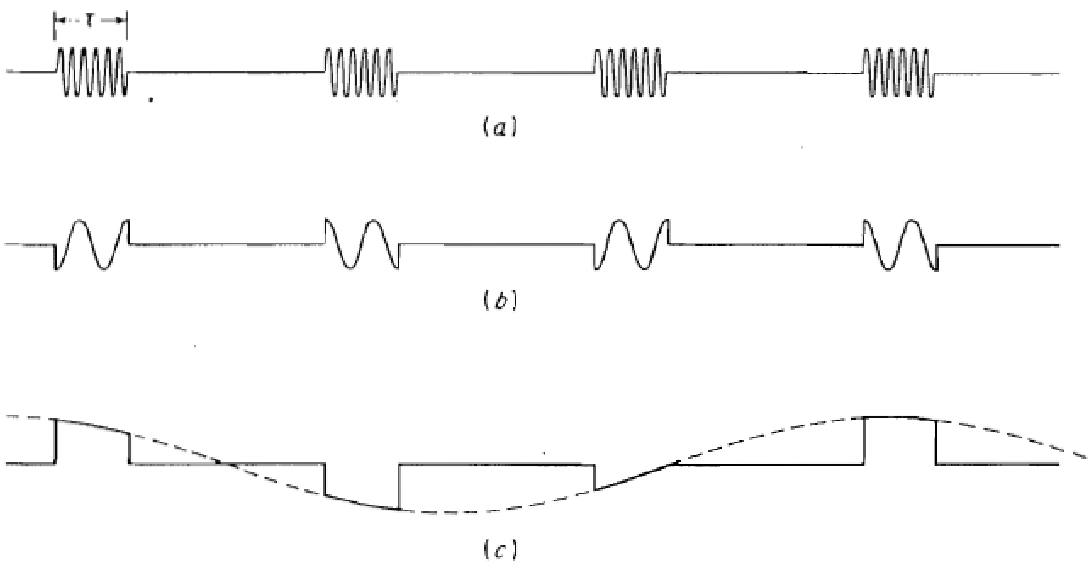


(a) Simple CW radar; pulse radar using doppler information.

Fig 8.4

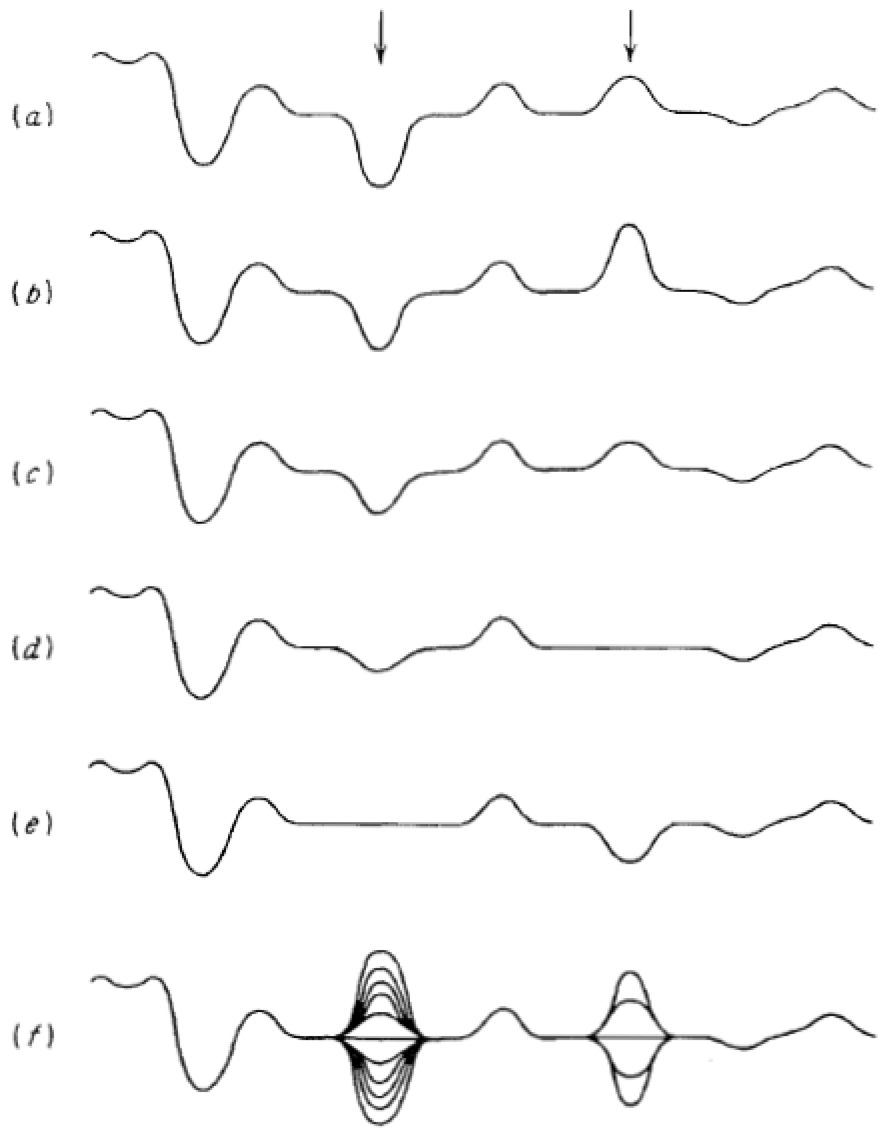
The doppler signal may be readily discerned from the information contained in a single pulse. If, on the other hand, f b is small compared with the reciprocal of the pulse duration, the pulses will be modulated with an amplitude.

Moving targets may be distinguished from stationary targets by observing the video output on an A-scope. O n the basis of a single sweep, moving targets cannot be distinguished from fixed targets. **I t** may be possible to distinguish extended ground targets from point targets by the stretching of the echo pulse. However, this is not a reliable means of discriminating moving from fixed targets since some fixed targets can look like point targets, e.g., a water tower. Also, some moving targets such as aircraft flying in formation can look like extended targets.)Successive A-scope sweeps (pulse-repetition intervals).



(a) RF echo pulse train; (b) video pulse train for doppler frequency $f_d > 1/\tau$; (c) video pulse train for doppler freuqncy $f_d < 1/\tau$.

Fig 8.5



(a–e) Successive sweeps of an MTI radar A-scope display (echo amplitude as a function of time); (f) superposition of many sweeps; arrows indicate position of moving targets.

Fig 8.5

Although the butterfly effect is suitable for recognizing moving targets on an A-scope, it is not appropriate for display on the PPI. One method commonly employed to extract Doppler information in a form

suitable for display on the PPI scope is with a delay-line canceller.. The delay-line canceller acts as a filter to eliminate the d-c component of fixed targets
and to pass the a-c components of moving targets. The video portion of the receiver is divided into two channels. One is a normal video channel. In the other, the video signal experiences a time delay equal to one pulse-repetition period (equal to the reciprocal of the pulse-repetition frequency). The outputs from the two channels are subtracted from one another. The fixed targets with unchanging amplitudes from pulse to pulse are canceled on subtraction. However, the amplitudes of the moving-target echoes are not constant from pulse **to** subtraction results in an uncancelled residue. The output of the subtraction circuit is bipolar video, just as was the input. Before bipolar video can intensity-modulate a PPI display, it must be converted to unipotential voltages (unipolar video) by a full-wave rectifier.

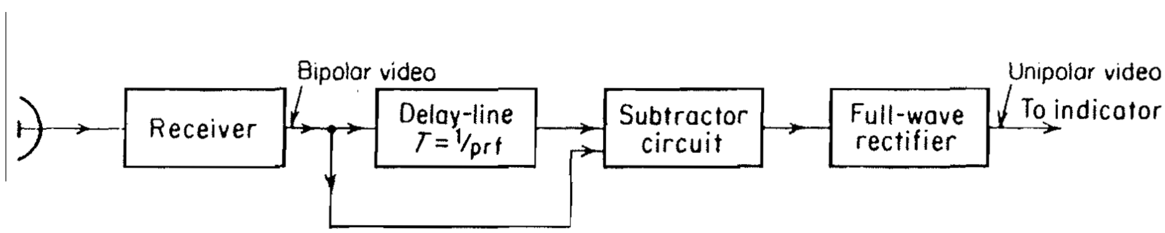
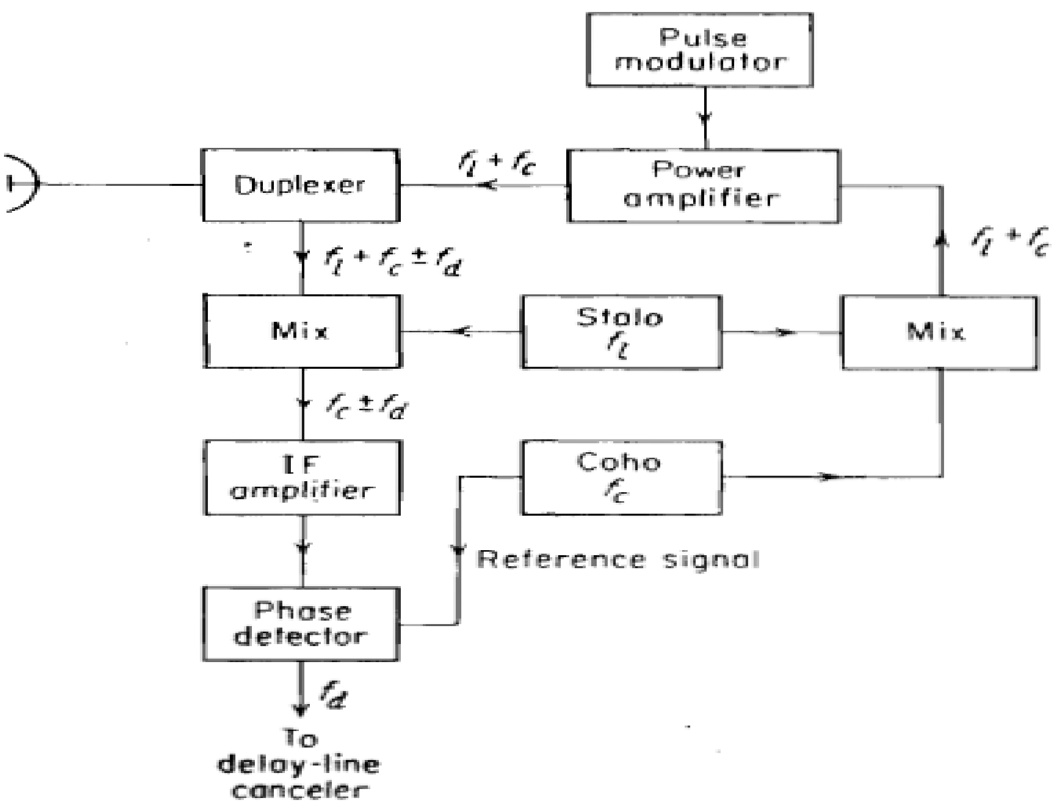


Fig : 8.6 MTI receiver with delay-line canceller

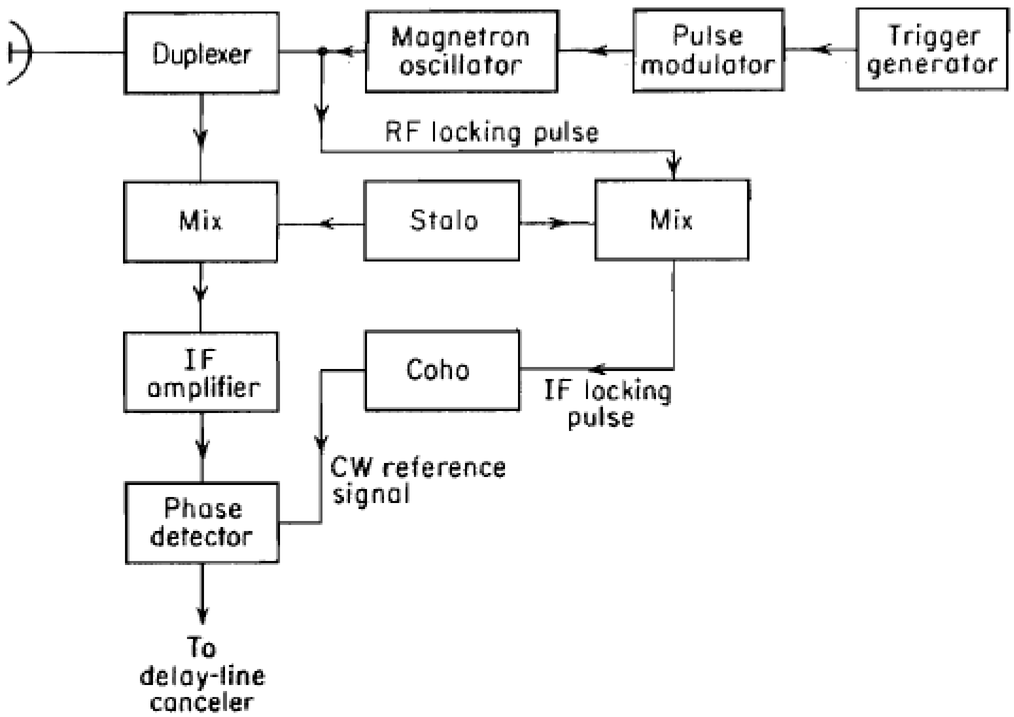
8.5 MTI RADAR WITH POWER AMPLIFIER TRANSMITTER:

The block diagram of a more common MTI radar employing a power amplifier is shown.The significant difference between this MTI configuration is the manlier in which the reference signal is generated. the coherent reference is supplied by the oscillator called the COHO, which stands for coherent oscillator. The coho is a stable oscillator whose frequency is the same as the intermediate frequency used in the receiver. In addition to providing the reference signal the output of the COHO fc; is also mixed with the local-oscillator frequency. The local oscillator- must also have a stable oscillator and is called STALO, for stable local oscillator. The RF echo signal is heterodyned with the stalo signal to produce the IF signal just as in conventional super heterodyne receiver.



Block diagram of MTI radar with power-amplifier transmitter.

Fig 8.7



Block diagram of MTI radar with power-oscillator transmitter.

Fig 8.8

Before the development of the klystron amplifier, the only high-power transmitter available at microwave frequencies for radar application was the magnetron oscillator.

A block diagram of an MTI radar (with a power oscillator) is shown **A** portion of the transmitted signal is mixed with the stalo output to produce an IF beat signal whose phase is directly related to the phase of the transmitter. This IF pulse is applied to the coho and causes the phase of the coho CW oscillation to "lock" in step with the phase of the

IF reference pulse. The phase of the coho is then related to the phase of the transmitted pulse and may be used as the reference signal for echoes received from that particular transmitted pulse.

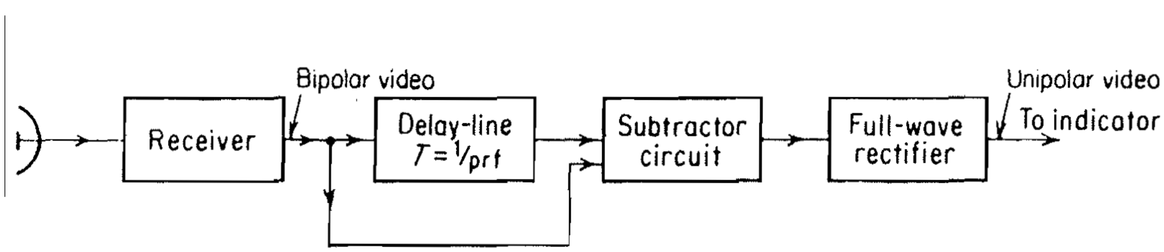
Upon the next transmission another IF locking pulse is generated to relock the phase of the CW coho until the next locking pulse comes along.

8.6 DELAY-LINE CANCELERS

The simple MTI delay-line canceller The simple MTI delay-line canceller The capability of this device depends on the quality of the medium used iis the delay line. The Pulse modulator delay line must introduce a time delay equal to the pulse repetition interval.

For typical ground-based air-surveillance radars this might be several milliseconds. Delay times of this magnitude cannot be achieved with practical electromagnetic transmission lines. By converting the electromagnetic signal to an 'acoustic signal it is possible to utilize delay lines of a reasonable physical length since the velocity of propagation of acoustic waves After the necessary delay is introduced by the acoustic line, the signal is converted back to an electromagnetic signal for further processing.

Fig 8.9



The early acoustic delay lines developed during World War 11 used liquid delay lines filled with either water or mercury.' Liquid delay lines were large and inconvenient to use. They were replaced in the mid-1950s by the solid fused-quartz delay line that used multiple internal reflections to obtain a compact device. These analog acoustic delay lines were, in turn supplanted in the early 1970s by storage devices based on digital computer technology. The use of digital delay lines requires that the output of the MTI receiver phase-detector be quantized into a sequence of digital words. The compactness and convenience of digital processing allows the implementation of more complex delay-line cancellers with filter characteristics not practical with analog methods.

One of the advantages of a time-domain delay-line canceller as compared to the more conventional frequency-domain filter is that a single network operates at all ranges and does not require a separate filter for each range resolution cell. Frequency-domain doppler filterbanks are of interest in some forms of MTI and pulse-doppler radar.

Filter characteristics of the delay-line canceller

The delay-line canceller acts as a filter which rejects the d-c component of clutter. Because of its periodic nature, the filter also rejects energy in the vicinity of the pulse repetition frequency and its harmonics.

$$V_1 = k \sin (2\pi f_d t - \phi_0)$$

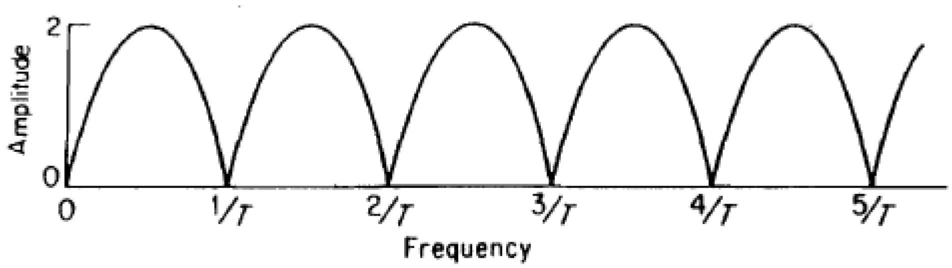
where Φ_0 = phase shift and k = amplitude of video signal. The signal from the previous transmission, which is delayed by a time T = pulse repetition interval, is

$$V_2 = k \sin [2\pi f_d(t - T) - \phi_0]$$

Everything else is assumed to remain essentially constant over the interval T so that k is the same for both pulses. The output from the subtractor is

$$V = V_1 - V_2 = 2k \sin \pi f_d T \cos \left[2\pi f_d\left(t - \frac{T}{2}\right) - \phi_0\right]$$

It is assumed that the gain through the delay-line canceller is unity. Thus the amplitude of the canceled video output is a function of the Doppler frequency shift and the pulse-repetition interval, or prf. The magnitude of the relative frequency-response of the delay-line canceler [ratio of the amplitude of the output from the delay-line canceler, to the amplitude of the normal radar video .



Frequency response of the single delay-line canceler; T = delay time = $1/f_p$.

Fig 8.9

Blind speeds: The response of the single-delay-line canceller will be zero whenever the argument Πfd T in the amplitude factor.

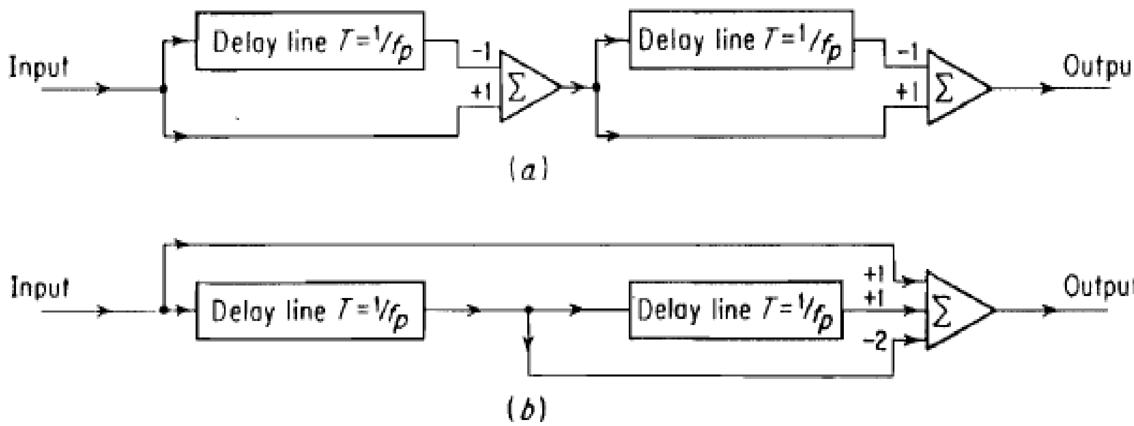
The blind speeds are one of the limitations of pulse MTI radar which do not occur with CW radar. They are present in pulse radar because doppler is measured by discrete samples (pulses) at the prf rather than continuously. If the first blind speed is to be greater than the maximum radial velocity expected from the target, the product ,If the first blind speed must be large. Thus the MTI radar must operate at long wavelengths (low frequencies) or with high pulse repetition frequencies, or both.

Double cancellation:

The frequency response of a single-delay-line canceller does not
always have as broad a clutter-rejection null as might be desired in the vicinity of d-c. The clutter-rejection notches may be widened by passing the output of the delay-line canceller through a second delay-line canceller. The output of the two single-delay line cancellers in cascade is the square of that from a single canceller.

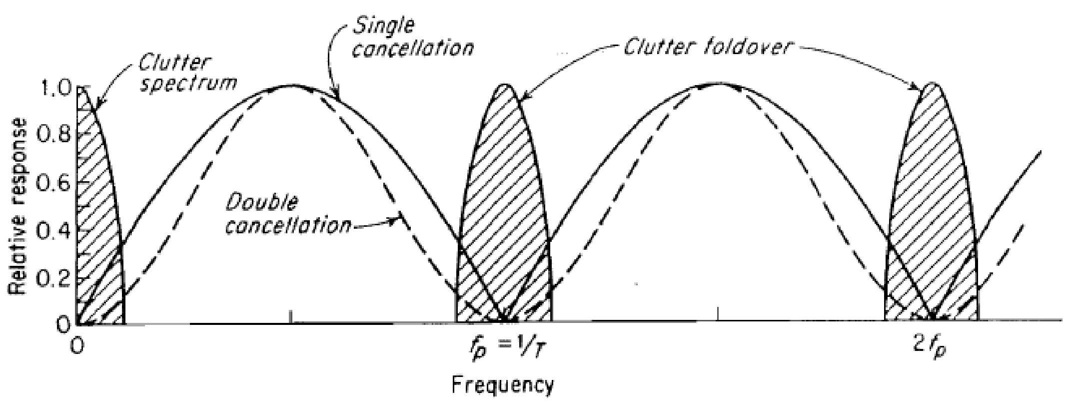
The two-delay-line configuration has the same frequency-response characteristic as the double-delay-line canceler. The operation of the device is as follows. **A** signal ***f (t)*** is inserted into the adder along with the signal from the preceding pulse period, with its amplitude weighted by the factor - 2, plus the signal from two pulse periods previous. The output of the adder is therefore

$$f(t) - 2f(t + T) + f(t + 2T)$$



(*a*) Double-delay-line canceler; (*b*) three-pulse canceler.

Fig 8.10



Relative frequency response of the single-delay-line canceler (solid curve) and the double-delay-line canceler (dashed curve). Shaded area represents clutter spectrum.

Fig 8.11

which is the same as the output from the double-delay-line canceller

$$f(t) - f(t+T) - f(t+T) + f(t+2T)$$

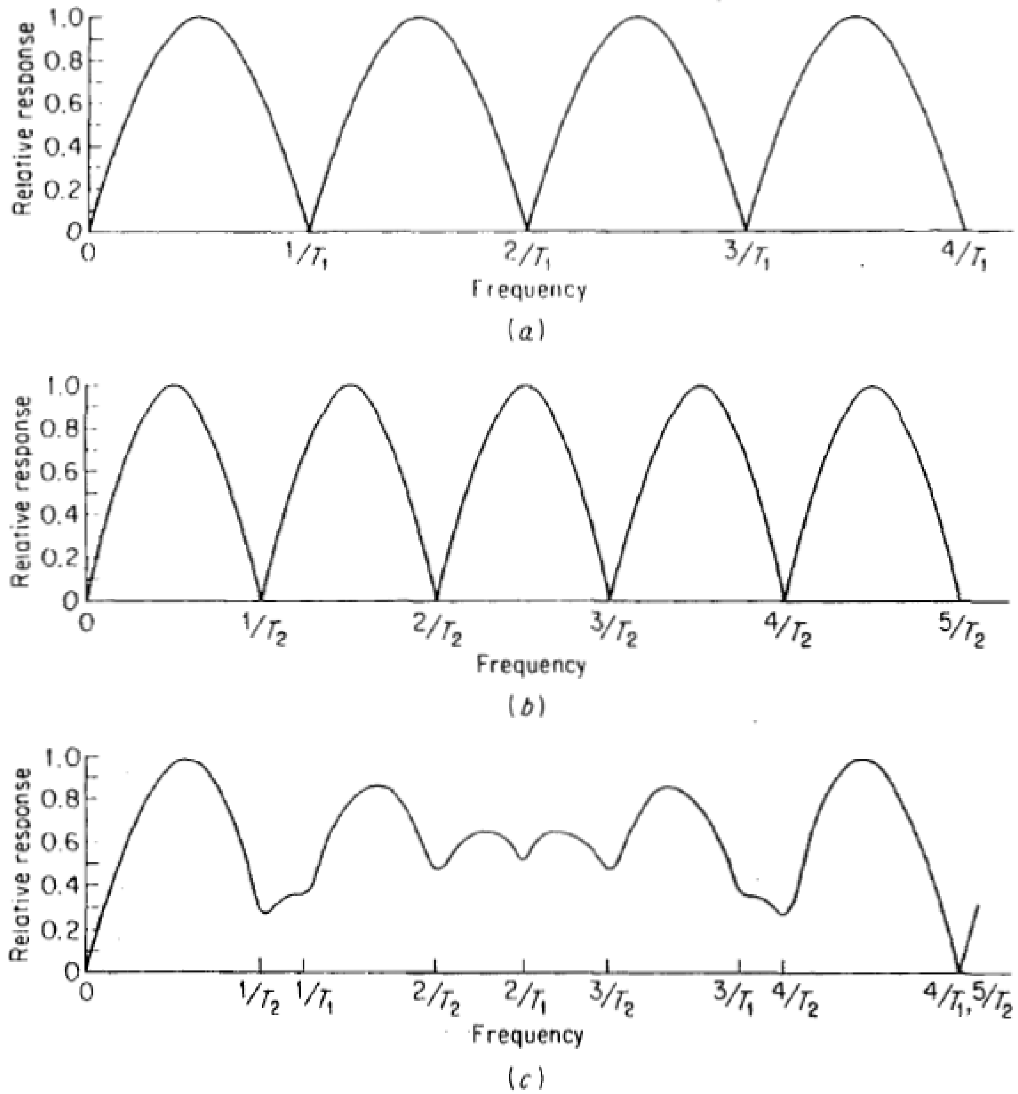
This configuration is commonly called the **three-pulse canceller**.

8.7 MULTIPLE, OR STAGGERED, PULSE REPETITION FREQUENCY

The use of more than one pulse repetition frequency offers additional flexibility in the design of MTI doppler filters. It not only reduces the effect of the blind speeds but it also allows a sharper low-frequency cutoff in the frequency response than might be obtained with a cascade of single-delay-line cancelers.

The blind speeds of two independent radars operating at the same frequency will be different if their pulse repetition frequencies are different. Therefore, if one radar were " blind " to moving targets, it would be unlikely that the other radar would be " blind" also. Instead of using two separate radars, the same result can be obtained with one radar which time-shares its pulse repetition frequency between two or more different values (multiple prf's). The pulse repetition frequency might be switched every other scan or every time the antenna is scanned a half beamwidth, or the period might be alternated on every other pulse. When the switching is pulse to pulse, it is known as a staggered prf.

An example of the composite (average) response of an MTI radar operating with two separate pulse repetition frequencies on a time-shared basis is shown below



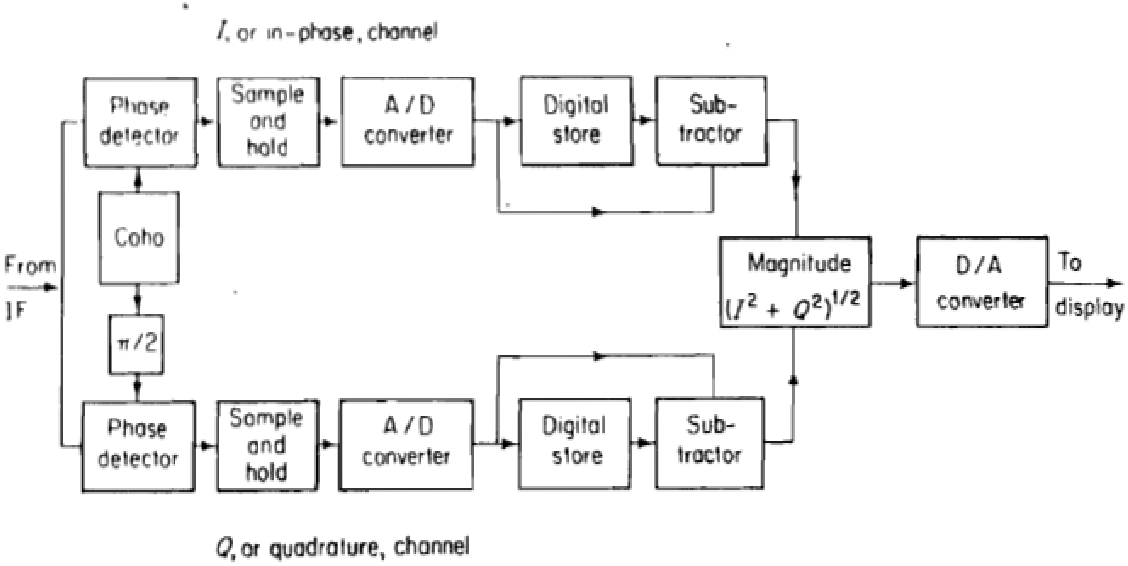
(a) Frequency-response of a single-delay-line canceler for $f_p = 1/T_1$; (b) same for $f_p = 1/T_2$; (c) composite response with $T_1/T_2 = \frac{4}{5}$.

Fig 8.12

8.8 DIGITAL MTI SIGNAL PROCESSOR:

A simple block diagram of a digital MTI processor is shown in Fig below. From the output of the IF amplifier the signal is split into two channels. One is denoted **I,** for ***in-phase channel.***The other is denoted **Q,** for ***quadrature*** channel, since a 90" phase change *(Π/2* radians) is introduces into the coherent reference signal at the phase detector. This causes the outputs of the two detectors to be 90 degrees out of phase.

The purpose of the quadrature channel is to eliminate the effects of blind phases.It is desirable to eliminate blind phases in any MTI processor, but it is seldom done with analog delay-line cancelers because of the complexity of the added analog delay lines of the second channel. The convenience of digital processing allows the quadrature channel to be added without significant burden so that it is often included in digital processing systems. It is for this reason it is shown in this block diagram, but was not included in the previous discussion of **MTI** delay-line cancellers.



Block diagram of a simple digital MTI signal processor. ***Fig 8.13***

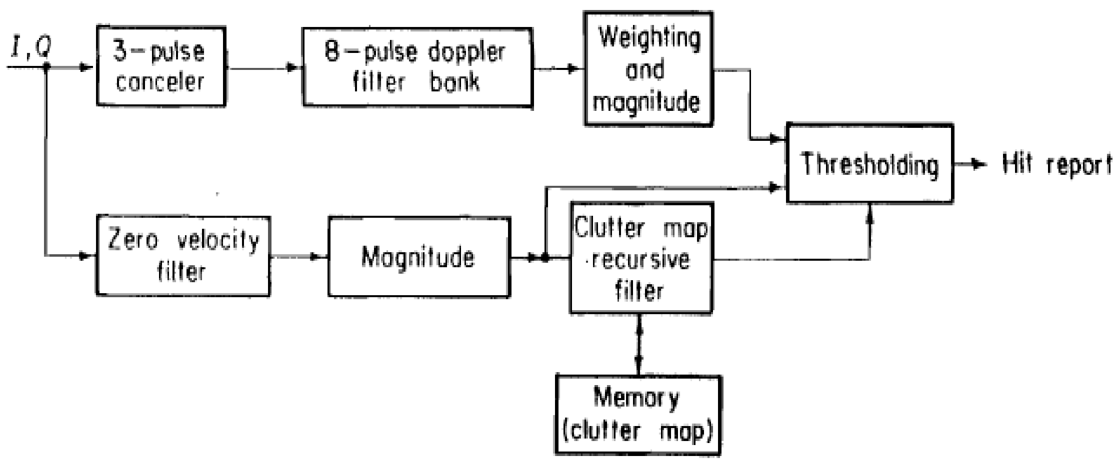
Following the phase detector the bipolar video signal is sampled at a rate sufficient to obtain one or more samples within each range resolution cell. These voltage samples are converted to a series of digital words by the analog-to-digital **(A/D)** converter.

The digital words are stored in a digital memory for one pulse repetition period and are then subtracted from the digital words of the next sweep. The digital outputs of the I and Q channels are combined by taking the square root of I^2 and Q^2 · The combined output is then converted to an analog signal by the digital-to-analog **(D/A)** converter. The unipolar video output is then ready to be displayed.

8.9 MOVING TARGET DETECTOR

A block diagram of the MTD processor is shown in Fig . The input on the left is from the output of the ***I*** and Q AID converters. The three-pulse canceler and the eight-pulse Doppler filter-bank eliminate zero-velocity clutter and generate eight overlapping filters covering the doppler interval, as described in the previous section. The use of a three-pulse canceler ahead of the fi1ter:bank eliminates stationary clutter and thereby reduces the dynamic range required of the doppler filter-bank.

The fast Fourier transform algorithm is listed to implement the doppler filter-bank. Since the first two pulses of a three-pulse canceler are meaningless only the last eight of the ten pulses output from the canceler are passed to the filter-bank. Following the filter-bank, weighting is applied in the frequency domain to reduce the filter sidelobes .



Simple block diagram of the Moving Target Detector (MTD) signal processor.

Fig 8.14

Separate thresholds are applied to each filter. The thresholds for the nonzero-velocity resolution cells are established by summing the detected outputs of the signals in the same velocity filter in 16 range cells, eight on either side of the cell of interest. Thus, each filter output is averaged over cne mile in range to establish the statistical mean level of nonzero-velocity clutter (such as rain) or noise. The filter thresholds are determined by multiplying the mean levels by an appropriate constant to obtain the desired false-alarm probability. This application of an adaptive threshold to each doppler filter at each range cell provides a constant false-alarm rate (CFAR) and results in ***Subweather visibility*** in that an aircraft with a radial velocity sufficiently different from the rain so as to fall into another filter can be seen even if the aircraft echo is substantially less than the weather echo.

A digital clutter map is generated which establishes the thresholds for the zero-velocity cells. The map is implemented with one word for each of the 365,000 range-azimuth cells. The original MTD stored the map on a magnetic disc memory. The purpose of the zero-velocity filter is to recover the clutter signal eliminated by the MTI delay-line canceler and to use this signal as a means for detecting targets on-crossing trajectories with zero velocities that would normally be lost in the usual MTI. Only targets larger than the clutter would be so detected.

8.10 LIMITATIONS TO MTI PERFORMANCE

The improvement in signal-to-clutter ratio of an MTI is affected by factors other than the design of the doppler signal processor. Instabilities of the transmitter and receiver, physical motions of the clutter, the finite time on target (or scanning modulation), and limiting in the receiver can all detract from the performance of an MTI radar.

MTI improvement f actor: The signal-to-clutter ratio at the output of the MTI system divided by the signal-to-clutter ratio at the input, averaged uniformly over all target radial velocities of interest.

Subclutter visibility : The ratio by which the target echo power may be weaker than the coincident clutter echo power and still be detected with specified detection and false alarm probabilities.

Clutter visibility factor : The signal-to-clutter ratio, after cancellation or doppler filtering, that provides stated probabilities of detection and false alarm.

Cltrtter attenuation: The ratio of clutter power at the canceller input to the clutter residue at the output, normalized to the attenuation of a single pulse passing through the unprocessed channel of the canceller.

Cancellation ratio: The ratio of canceller voltage amplification for the fixed-target echoes received with a fixed antenna, to the gain for a single pulse passing through the unprocessed channel of the canceller.

Equipment instabilities : Pulse-to-pulse changes in the amplitude, frequency, or phase of the transmitter signal, changes in the stalo or coho oscillators in the receiver, jitter in the timing of the pulse transmission, variations in the time delay through the delay lines, and changes in the pulse width can cause the apparent frequency spectrum from perfectly stationary clutter to broaden and thereby lower the improvement factor of an MTI radar.

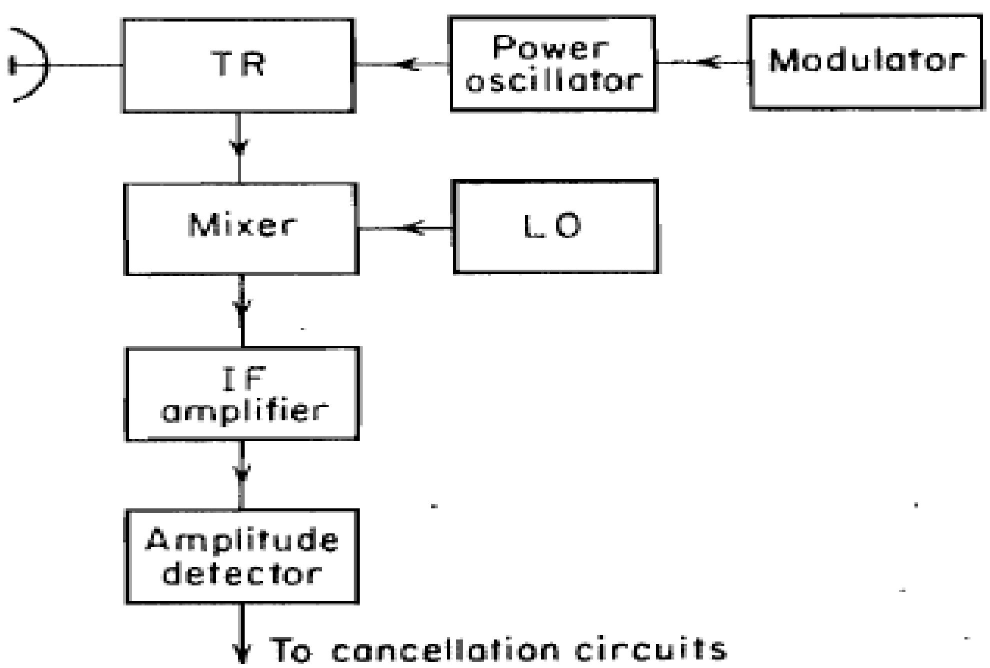
Internal fluctuation of clutter : Although clutter targets such as buildings, water towers, bare hills. or mountains produce echo signals that are constant in both phase and amplitude as a function of time, there are many types of clutter that cannot be considered as absolutely stationary. Echoes from trees, vegetation, sea, rain, and chaff fluctuate with time, and these fluctuations can limit the performance of MTI radar.

Antenna scanning modulation: As the antenna scans by a target, it observes the target for a finite time equal to , to = n where n, = number of hits received, fp = pulse repetition frequency, 0, = antenna beamwidth and antenna scanning rate. The received pulse train of finite duration to has a frequency spectrum (which can be found by taking the Fourier transform of the waveform) whose width is proportional to l/to. Therefore, even if the clutter were perfectly stationary, there will still be a finite width to the clutter spectrum because of the finite time on target. If the clutter spectrum is too wide because the observation time is too short, it will affect the improvement factor. This limitation has sometimes been called scanning fluctuations or scanning modulation.

8.11 NONCOHERENT MTI

The composite echo signal from a moving target and clutter fluctuates in both phase and amplitude. The coherent MTI and the pulse-doppler radar make use of the phase fluctuations in the echo signal to recognize the doppler component produced by a moving target. In these systems, amplitude fluctuations are removed by the phase detector. The operation of this type of radar, which may be called coherent MTI, depends upon a reference signal at the radar receiver that is coherent with the transmitter signal.

It is also possible to use the amplitude fluctuations to recognize the doppler component produced by a moving target. MTI radar which uses amplitude instead of phase fluctuations is called noncoherent.



Block diagram of a noncoherent MTI radar.

Fig 8.15

The noncoherent MTI radar does not require an internal coherent reference
signal or a phase detector as does the coherent form of MTI. Amplitude limiting cannot be employed in the non coherent MTI receiver, else the desired amplitude fluctuations would be lost. Therefore tile IF amplifier must be linear, or if a large dynamic range is required, it can be logarithmic. A logarithmic gain characteristic not only provides protection from saturation, but it also tends to make the clutter fluctuations at its output more uniform with variations in the clutter input amplitude.

The detector following the IF amplifier is a conventional amplitude detector. The phase detector is not used since phase information is of no interest to the non coherent radar. The local oscillator of the noncoherent radar does not have to he as frequency-stable as in the coherent MTI. The transmitter must be sufficiently stable over the pulse duration to prevent beats between overlapping ground clutter, but this is not as severe a requirement as in the case of coherent radar. The output of the amplitude detector is followed by an MTI processor such as a delay-line canceller.

The advantage of the noncoherent MTI is its simplicity; hence it is attractive for those applications where space and weight are limited. Its chief limitation is that the target must be in the presence of relatively large clutter signals if moving-target detection is to take place.

Clutter echoes may not always be present over the range at which detection is desired. The clutter serves the same function as does the reference signal in the coherent MTI. If clutter were not present, the desired targets would not be detected. It is possible, however, to provide a switch to disconnect the non coherent MTI operation and revert to normal radar whenever sufficient clutter echoes are not present. If the radar is stationary, a map of the clutter might be stored in a digital memory and used to determine when to switch in or out the non coherent **MTI .**

8.12 PULSE DOPPLER RADAR

A pulse radar that extracts the doppler frequency shift for the purpose of detecting moving targets in the presence of clutter is either an MTI radar or a ***pulse doppler radar.*** The distinction between them is based on the fact that in a sampled measurement system like a pulse radar, ambiguities can arise in both the doppler frequency (relative velocity) and the range (time delay) measurements. Range ambiguities are avoided with a ***low*** sampling rate (low pulse repetition frequency), and doppler frequency ambiguities are avoided with a high sampling rate. However, in most radar applications the sampling rate, or pulse repetition frequency, cannot be selected to avoid both types of measurement ambiguities.

Therefore a compromise must be made arid the nature of the compromise generally determines whether the radar is called an MTI or a pulse doppler. MTI usually refers to a radar in which the pulse repetition frequency is chosen low enough to avoid ambiguities in range (no multiple-time-around echoes). but with the consequence that the frequency measurement is ambiguous and results in blind speeds.

RECOMMENDED QUESTIONS ON UNIT- 8

1. Distinguish between MTI and pulse dopplar radar
2. With a neat block diagram explain the operation of CW radar.
3. With neat block diagram explain the operation of MTI radar.
4. What is blind speed? Obtain the expression for blind speed .
5. With a neat block diagram explain the operation of digital MTI processor
6. With a neat block diagram explain the operation of MTD processor
7. With a neat block diagram explain the operation of pulse Doppler radar.