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MOCK TEST

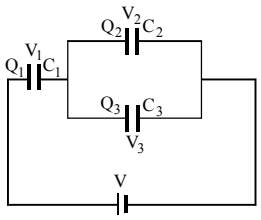
PART - I : PHYSICS

1. Force between two plates of a capacitor is

(a) $\frac{Q}{\varepsilon_0 A}$ (b) $\frac{Q^2}{2\varepsilon_0 A}$

(c) $\frac{Q^2}{\varepsilon_0 A}$ (d) none of these

2. Three capacitors C_1, C_2 and C_3 are connected to a battery as shown. With symbols having their usual meanings, the correct conditions are



(a) $Q_1 = Q_2 = Q_3$ and $V_1 = V_2 = V$
(b) $V_1 = V_2 = V_3 = V$
(c) $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_2$
(d) $Q_2 = Q_3$ and $V_2 = V_3$

3. A parallel plate condenser with oil between the plates (dielectric constant of oil K = 2) has a capacitance C. If the oil is removed, then capacitance of the capacitor becomes

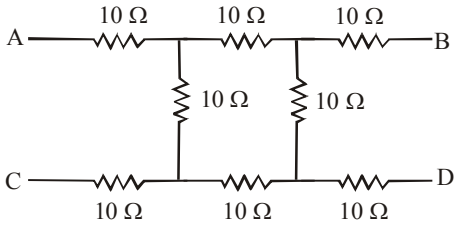
(a) $\sqrt{2}C$ (b) $2C$

(c) $\frac{C}{\sqrt{2}}$ (d) $\frac{C}{2}$

4. At the centre of a cubical box + Q charge is placed. The value of total flux that is coming out a wall is

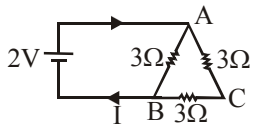
(a) Q/ε_o (b) $Q/3\varepsilon_o$
(c) $Q/4\varepsilon_o$ (d) $Q/6\varepsilon_o$

5. What will be the equivalent resistance of circuit shown in figure between two points A and D



(a) 10Ω (b) 20Ω (c) 30Ω (d) 40Ω

6. The current in the following circuit is



(a) 1A (b) $\frac{2}{3}$A (c) $\frac{2}{9}$A (d) $\frac{1}{8}$A

7. Kirchhoff's first law, i.e. $\Sigma i = 0$ at a junction, deals with the conservation of

(a) charge (b) energy
(c) momentum (d) angular momentum

8. The potential difference between the terminals of a cell in an open circuit is 2.2 V. When a resistor of 5Ω is connected across the terminals of the

cell, the potential difference between the terminals of the cell is found to be 1.8 V. The internal resistance of the cell is

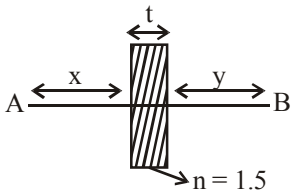
(a) $\frac{7}{12}\Omega$ (b) $\frac{10}{9}\Omega$ (c) $\frac{9}{10}\Omega$ (d) $\frac{12}{7}\Omega$

9. Direct current is passed through a copper sulphate solution using platinum electrodes. The elements liberated at the electrodes are
   (a) copper at anode and sulphur at cathode
   (b) sulphur at anode and copper at cathode
   (c) oxygen at anode and copper at cathode
   (d) copper at anode and oxygen at cathode
10. Two electric bulbs, one of 200 V, 40W and other of 200 V, 100W are connected in a domestic circuit. Then
   (a) they have equal resistance
   (b) the resistance of 40W bulb is more than 100W bulb
   (c) the resistance of 100W bulb is more than 40 W bulb
   (d) they have equal current through them
11. Two long parallel wires are at a distance of 1 metre. Both of them carry one ampere of current. The force of attraction per unit length between the two wires is
   (a) 2×10^{-7} N/m (b) 2×10^{-8} N/m
   (c) 5×10^{-8} N/m (d) 10^{-7} N/m
12. A charged particle of charge q and mass m enters perpendicularly in a magnetic field $\vec{B}$. Kinetic energy of the particle is E; then frequency of rotation is

   (a) $\frac{qB}{m\pi}$ (b) $\frac{qB}{2\pi m}$

   (c) $\frac{qBE}{2\pi m}$ (d) $\frac{qB}{2\pi E}$

13. If N is the number of turns in a coil, the value of self inductance varies as
   (a) N^0 (b) N
   (c) N^2 (d) N^{-2}
14. In an A.C. circuit with voltage V and current I the power dissipated is
   (a) dependent on the phase between V and I
   (b) $\frac{1}{\sqrt{2}}$ VI
   (c) $\frac{1}{2}$ VI
   (d) VI
15. A magnetic field of 2×10^{-2} T acts at right angles to a coil of area 100 cm^2, with 50 turns. The average e.m.f. induced in the coil is 0.1 V, when it is removed from the field in t sec. The value of t is
   (a) 10 s (b) 0.1 s
   (c) 0.01 s (d) 1 s
16. Two coils of self inductances 2 mH and 8 mH are placed so close together that the effective flux in one coil is completely linked with the other. The mutual inductance between these coils is
   (a) 6mH (b) 4mH
   (c) 16mH (d) 10mH
17. Which one of the following electromagnetic radiations has the smallest wavelength?
   (a) Ultraviolet waves (b) X-rays
   (c) γ-rays (d) Microwaves
18. Which one of the following phenomena is not explained by Huygen's construction of wavefront?
   (a) Refraction (b) Reflection
   (c) Diffraction (d) Orgin of spectra
19. Two light waves having their intensities in the ratio 16 : 9 interfere to produce interference pattern. What is the ratio of maximum intensity to minimum intensity in this pattern ?
   (a) 4 : 3 (c) 25 : 7
   (b) 625 : 49 (d) 49 : 1
20. A slab of thickness t and refractive index 1.5 is placed in between point A and B as shown in the figure given below. The optical path length between A and B is



(a) $\left(x+\frac{t}{2}+y\right)$ (b) $(x+t+y)$

(c) $\left(x+\frac{3}{2}t+y\right)$ (d) $\left(x+\frac{5}{2}t+y\right)$

21. If according to the Bohr model of the hydrogen atom, the ionization energy of the atom in its ground state is 13.6 eV, then the energy required to ionize the atom from its first excited state will be
(a) 6.8 eV (b) 3.4 eV
(c) 1.7 eV (d) 0.85 eV

22. Match List-I with List-II and select the correct answer using the codes given belowthe Lists

List-I	List-II
A. Balmer series	1. $\bar{v}=R_H\left(\frac{1}{3^2}-\frac{1}{n^2}\right)$
B. Brakett series	2. $\bar{v}=R_H\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$
C. Lyman series	3. $\bar{v}=R_H\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$
D. Paschen series	4. $\bar{v}=R_H\left(\frac{1}{4^2}-\frac{1}{n^2}\right)$

Codes :

	A	B	C	D
(a)	3	2	1	4
(b)	3	4	2	1
(c)	4	3	2	1
(d)	2	3	1	4

23. A ruby laser using a flash lamp is
1. A pulse laser
2. A three level laser
3. Cooled effectively

Which of the above statements are correct ?
(a) 1, 2 and 3 (b) 2 and 3
(c) 1 and 2 (d) 1 and 3

24. The population inversion necessary for laser action used in solid state lasers is
(a) electrical discharge
(b) inelastic atom-atom collision
(c) direct conversion
(d) optical pumping

25. The photo electric threshold of Tungsten is 2300Å. The energy of the electrons ejected from the surface by ultraviolet light of wavelength 1800Å is
(a) 0.15 eV (b) 1.5 eV
(c) 15 eV (d) 150 eV

26. Radiations of two photon's energy, twice and ten times the work function of metal are incident on the metal surface successsively. The ratio of maximum velocities of photoelectrons emitted in two cases is
(a) 1 : 2 (b) 1 : 3
(c) 1 : 4 (d) 1 : 1

27. If an electron and positron annihilate, then the energy released is
(a) 3.2×10^{-13} J (b) 1.6×10^{-13} J
(c) 4.8×10^{-13} J (d) 6.4×10^{-13} J

28. A is at rest while B moves at 0.8c. B measures an event to be 6s while A measures 3.6s. Who is correct?
(a) B is correct
(b) No conclusion is possible
(c) A is correct
(d) Both are correct

29. Which of the following are true regarding forces between nucleons inside the nucleus :
1. attractive in nature
2. electrical in nature
3. extremely short range
4. strongest forces in nature

Which of the above are correct ?
(a) 1, 2 and 4 (b) 2 and 3
(c) 1, 3 and 4 (d) 3 and 4

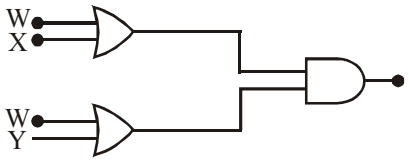
30. $^2_1H+{}^2_1H\rightarrow{}^3_2He+{}^1_0n+13$ MeV

In the above nuclear reaction, the binding energy of ^2_1H is 2.2 MeV. The binding energy of ^3_2He is
(a) 4.4 MeV (b) 8.6 MeV
(c) 13 MeV (d) 17.4 MeV

31. A radioactive nucleus has a half-life of 100 years. If the number of nuclei at t = 0 is N_0, what will be the number of nuclei which have decayed in 300 years ?
(a) $\frac{7N_0}{8}$ (b) $\frac{N_0}{2}$

(c) $\frac{3N_0}{4}$ (d) $\frac{N_0}{8}$

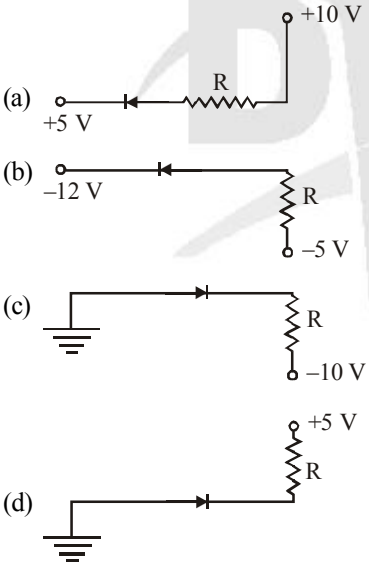
32. The diagram of a logic circuit is given below.



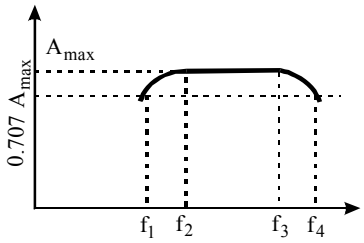
The output F of the circuit is represented by
(a) W + (X + Y) (b) W + (X.Y)
(c) W, (X + Y) (d) W, (X.Y)

33. NAND and NOR gates are called universal gates primarily because they
(a) are available universally
(b) can be combined to produce OR, AND and NOT gates
(c) are widely used in Integrated circuit packages
(d) are easiest to manufacture

34. Of the diodes shown in the following diagrams, which one is reverse biased ?



35. The frequency response curve of RC coupled amplifier is shown in figure. The band with of the amplifier will be



(a) $f_3 - f_2$ (b) $f_4 - f_2$ (c) $\frac{f_4 - f_2}{2}$ (d) $f_3 - f_1$

36. In common emitter amplifier the $\frac{I_c}{I_e}$ is 0.98. The current gain will be
(a) 4.9
(b) 7.8
(c) 49
(d) 78

37. An antenna is a device
(a) that converts electromagnetic energy into radio frequency signal
(b) that converts radio frequency signal into electromagnetic energy
(c) that converts guided electromagnetic waves into free space electromagnetic waves and vice-versa
(d) none of these

38. Consider telecommunication through optical fibres. Which of the following statements is not true ?
(a) Optical fibres may have homogeneous core with a suitable cladding
(b) Optical fibres can be of graded refractive index
(c) Optical fibres are subject to electromagnetic interference from outside
(d) Optical fibres have extremely low transmission loss

39. What should be the maximum acceptance angle at the aircore interface of an optical fibre if n_1 and n_2 are the refractive indices of the core and the cladding, respectively

(a) $\sin^{-1}(n_2/n_1)$ (b) $\sin^{-1}\sqrt{n_1^2 - n_2^2}$

(c) $\left[\tan^{-1}\frac{n_2}{n_1}\right]$ (d) $\left[\tan^{-1}\frac{n_1}{n_2}\right]$

40. What is a vidicon ?
(a) It is a type of camera
(b) It is a type of electric generator
(c) It is a type of heating device
(d) It is a type of optical device

PART - II : CHEMISTRY

41. Identify the correct statement for change of Gibbs energy for a system (ΔG_{system}) at constant temperature and pressure :
(a) If $\Delta G_{system} = 0$, the system has attained equilibrium
(b) If $\Delta G_{system} = 0$, the system is still moving in a particular direction
(c) If $\Delta G_{system} < 0$, the process is not spontaneous
(d) If $\Delta G_{system} > 0$, the process is not spontaneous

42. The factor of ΔG values is important in metallurgy. The ΔG values for the following reactions at 800°C are given as :

$S_{2(s)} + 2O_{2(g)} \longrightarrow 2SO_{2(g)}$; $\Delta G = -544$ kJ

$2Zn_{(s)} + S_{2(s)} \longrightarrow 2ZnS_{(s)}$; $\Delta G = -293$ kJ

$2Zn_{(s)} + O_{2(g)} \longrightarrow 2ZnO_{(s)}$; $\Delta G = -480$ kJ

Then ΔG for the reaction :

$2ZnS_{(s)} + 3O_{2(g)} \longrightarrow 2ZnO_{(s)} + 2SO_{2(g)}$

will be :
(a) –357 kJ (b) –731 kJ
(c) –773 kJ (d) –229 kJ

43. Considering entropy (S) as a thermodynamic parameter, the criterion for the spontaneity of any process is
(a) $\Delta S_{system} + \Delta S_{surroundings} > 0$
(b) $\Delta S_{system} - \Delta S_{surroundings} > 0$
(c) $\Delta S_{system} > 0$ only
(d) $\Delta S_{surroundings} > 0$ only

44. In the solid state, MgO has the same structure as that of sodium chloride. The number of oxygens surrounding each magnesium in MgO is
(a) 6 (b) 1
(c) 2 (d) 4

45. A solid with high electrical and thermal conductivity from the following is
(a) Si (b) Li
(c) NaCl (d) Ice

46. Four successive members of the first row transition elements are listed below with their atomic numbers. Which one of them is expected to have the highest third ionization enthalpy?
(a) Vanadium (Z = 23)
(b) Chromium (Z = 24)
(c) Manganese (Z = 25)
(d) Iron (Z = 26)

47. Which one of the following is expected to exhibit optical isomerism?
(en = ethylenediamine)
(a) cis-$[Pt(NH_3)_2 Cl_2]$
(b) trans-$[Pt(NH_3)_2Cl_2]$
(c) cis-$[Co(en)_2Cl_2]$
(d) trans-$[Co(en)_2Cl_2]$

48. For orthorhombic system axial ratios are $a \neq b \neq c$ and the axial angles are
(a) $\alpha = \beta = \gamma \neq 90°$
(b) $\alpha = \beta = \gamma = 90°$
(c) $\alpha = \beta = \gamma = 90°, \beta = 90°$
(d) $\alpha \neq \beta \neq \gamma = 90°$

49. Which one of the following will show paramagnetism corresponding to 2 unpaired electrons?(Atomic numbers : Ni = 28, Fe = 26)
(a) $[Fe F_6]^{3-}$ (b) $[Ni Cl_4]^{2-}$
(c) $[Fe (CN)_6]^{3-}$ (d) $[Ni (CN)_4]^{2-}$

50. The most durable metal plating on iron to protect against corrosion is
(a) Nickel plating (b) Tin plating
(c) Copper plating (d) Zinc plating

51. The most convenient method to protect the bottom of ship made of iron is
(a) Coating it with red lead oxide
(b) White tin plating
(c) Connecting it with Mg block
(d) Connecting it with Pb block

52. Which of the following coordination compounds would exhibit optical isomerism?
(a) pentamminenitrocobalt(III) iodide
(b) diamminedichloroplatinum(II)
(c) trans-dicyanobis (ethylenediamine) chromium (III) chloride
(d) tris-(ethylendiamine) cobalt (III) bromide

53. Lanthanoids are
(a) 14 elements in the sixth period (atomic no. = 90 to 103) that are filling 4f sublevel
(b) 14 elements in the seventh period (atomic no. = 90 to 103) that are filling 5f sublevel
(c) 14 elements in the sixth period (atomic no. = 58 to 71) that are filling 4f sublevel
(d) 14 elements in the seventh period (atomic no. = 58 to 71) that are filling 4f sublevel

54. The radioactive isotope $^{60}_{27}Co$ which is used in the treatment of cancer can be made by (n, p) reaction. For this reaction the target nucleus is
(a) $^{59}_{28}Ni$ (b) $^{59}_{27}Co$
(c) $^{60}_{28}Ni$ (d) $^{60}_{27}Co$

55. The age of most ancient geological formations is estimated by
(a) Potassium–argon method
(b) Carbon-14 dating method
(c) Radium-silicon method
(d) Uranium-lead method.

56. The activation energy for a simple chemical reaction $A \rightarrow B$ is E_a in forward direction. The activation energy for reverse reaction
(a) Is always double of E_a
(b) Is negative of E_a
(c) Is always less than E_a
(d) Can be less than or more than E_a

57. Select the rate law that corresponds to data shown for the following reaction

$A + B \longrightarrow$ products.

Exp.	[A]	[B]	Initial rate
1	0.012	0.035	0.1
2	0.024	0.070	0.8
3	0.024	0.035	0.1
4	0.012	0.070	0.8

(a) rate = $k[B]^3$
(b) rate = $k[B]^4$
(c) rate = $k[A][B]^3$
(d) rate = $k[A]^2[B]^2$

58. The plot of concentration of the reactant vs. time for a reaction is a straight line with a negative slope. The reaction follows a
(a) zero order rate equation
(b) first order rate equation
(c) second order rate equation
(d) third order rate equation

59. A substance 'A' decomposes by a first order reaction starting initially with [A] = 2.00 m and after 200 min, [A] becomes 0.15 m. For this reaction $t_{1/2}$ is
(a) 53.72 min (b) 50.49 min
(c) 48.45 min (d) 46.45 min

60. Which one of the following information can be obtained on the basis of Le Chatelier principle?
(a) Dissociation constant of a weak acid
(b) Entropy change in a reaction
(c) Equilibrium constant of a chemical reaction
(d) Shift in equilibrium position on changing value of a constraint.

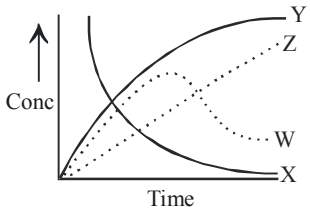
61. Equivalent conductances of NaCl, HCl and CH_3COONa at infinite dilution are 126.45, 426.16 and 91 $ohm^{-1}\ cm^2$ respectively. The equivalent conductance of CH_3COOH at infinite dilution would be
(a) 101.38 $ohm^{-1}\ cm^2$ (b) 253.62 $ohm^{-1}\ cm^2$
(c) 390.71 $ohm^{-1}\ cm^2$ (d) 678.90 $ohm^{-1}\ cm^2$

62. The standard e.m.f. of a galvanic cell involving cell reaction with n = 2 is found to be 0.295 V at 25°C. The equilibrium constant of the reaction would be (Given F = 96500 C mol^{-1}; R = 8.314 $JK^{-1}mol^{-1}$)
(a) 2.0×10^{11} (b) 4.0×10^{12}
(c) 1.0×10^{2} (d) 1.0×10^{10}

63. In the silver plating of copper, $K[Ag(CN)_2]$ is used instead of $AgNO_3$. The reason is
(a) A thin layer of Ag is formed on Cu
(b) More voltage is required
(c) Ag^+ ions are completely removed from solution
(d) Less availability of Ag^+ ions, as Cu cannot displace Ag from $[Ag(CN)_2]^-$ ion

64. If 0.5 amp current is passed through acidified silver nitrate solution for 100 minutes. The mass of silver deposited on cathode, is (eq.wt.of silver nitrate = 108)
(a) 2.3523 g (b) 3.3575 g
(c) 5.3578 g (d) 6.3575 g

65. For the reaction $A + B \longrightarrow C + D$. The variation of the concentration of the products is given by the curve



(a) Z (b) Y
(c) W (d) X

66. Acetamide and ethylamine can be distinguished by reacting with
(a) Aqueous HCl and heat
(b) Aqueous NaOH and heat
(c) Acidified $KMnO_4$
(d) Bromine water.

67. Aniline when diazotized in cold and when treated with dimethyl aniline gives a coloured product. Its structure would be

(a) $CH_3NH-C_6H_4-N=N-C_6H_4-NHCH_3$

(b) $CH_3-C_6H_4-N=N-C_6H_4-NH_2$

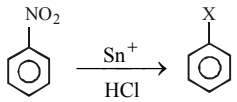
(c) $(CH_3)_2N-C_6H_4-N=N-C_6H_5$

(d) $(CH_3)_2N-C_6H_4-NH-C_6H_5$

68. Electrolytic reduction of nitrobenzene in weakly acidic medium gives
(a) N-Phenylhydroxylamine
(b) Nitrosobenzene
(c) Aniline
(d) p-Hydroxyaniline

69. If an organic compound shows a streching ($\rangle C = O$) frequency of 1720 cm^{-1}. The organic compound is _____.
(a) An aliphatic aldehyde
(b) An α, β unsaturated aldehyde
(c) A phenolic aldehyde
(d) An aromatic aldehyde

70. In the given reaction,

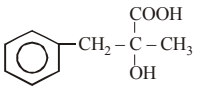
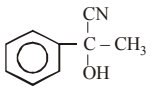


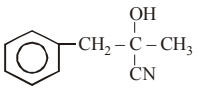
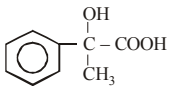
X- stands for
(a) $-NH_2$ (b) $SnCl_2$
(c) $-Cl$ (d) $-NH_3^+Cl^-$

71. In a set of reactions acetic acid yielded a product D.

$$CH_3COOH \xrightarrow{SOCl_2} A \xrightarrow[Anhy.\,AlCl_3]{Benzene} B \xrightarrow{HCN} C \xrightarrow{H.OH} D$$

The structure of D would be:

(a)  (b) 

(c)  (d) 

72. Ketones

[$R-\underset{\underset{O}{\|}}{C}-R_1$, where $R = R_1$ = alkyl groups]

can be obtained in one step by
(a) oxidation of primary alcohols
(b) hydrolysis of esters
(c) oxidation of tertiary alcohols
(d) reaction of acid halides with alcohols

73. Phenylmethyl ketone can be converted into ethylbenzene in one step by which of the following reagents?
(a) $LiAlH_4$ (b) Zn-Hg/HCl
(c) $NaBH_4$ (d) CH_3MgI

74. Ethanol and dimethyl ether form a pair of functional isomers. The boiling point of ethanol is higher than that of dimethyl ether, due to the presence of
(a) H-bonding in ethanol
(b) H-bonding in dimethyl ether
(c) CH_3 group in ethanol
(d) CH_3 group in dimethyl ether

75. Propan - 1- ol may be prepared by the reaction of propene with
(a) H_3BO_3
(b) H_2SO_4/H_2O
(c) B_2H_6,NaOH–H_2O_2
(d) $CH_3\overset{\overset{O}{\|}}{C}-O-O-H$

76. Sodium formate on heating yields
(a) Oxalic acid and H_2
(b) Sodium oxalate and H_2

(c) CO_2 and NaOH
(d) Sodium oxalate.

77. An ester (A) with molecular fomula, $C_9H_{10}O_2$ was treated with excess of CH_3MgBr and the complex so formed was treated with H_2SO_4 to give an olefin (B). Ozonolysis of (B) gave a ketone with molecular formula C_8H_8O which shows +ve iodoform test. The structure of (A) is
(a) $C_6H_5COOC_2H_5$
(b) $C_2H_5COOC_6H_5$
(c) $H_3COCH_2COC_6H_5$
(d) $p-H_3CO-C_6H_4-COCH_3$

78. Self condensation of two moles of ethyl acetate in presence of sodium ethoxide yields
(a) acetoacetic ester
(b) methyl acetoacetate
(c) ethyl propionate
(d) ethyl butyrate

79. Methanol is industrially prepared by
(a) Oxidation of CH_4 by steam at 900°C
(b) Reduction of HCHO using $LiAlH_4$
(c) Reaction HCHO with a solution of NaOH
(d) Reduction of CO using H_2 and $ZnO-Cr_2O_3$.

80. Indicate which of the nitrogen compounds amongst the following would undergo Hoffmann's reaction, i,e., reaction with Br_2 and strong KOH to furnish the primary amine ($R-NH_2$) ?
(a) $R-CO-NHCH_3$ (b) $R-CO-ONH_4$
(c) $R-CO-NH_2$ (d) $R-CO-NHOH$.

PART - III : MATHEMATICS

81. If $\sin(y+z-x), \sin(z+x-y)$ and $\sin(x+y-z)$ are in A.P. (with $x, y, z \neq \pi/2$), then tan x, tan y and tan z are in
(a) A.P. (b) GP.
(c) H.P. (d) none of these.

82. The number of roots of equation $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$ is
(a) 0 (b) 1
(c) 2 (d) infinite

83. In order that the equation $2ax(ax+nc)+(n^2-2)c^2=0$ may have real roots, n belongs to interval
(a) $[-2, 2]$ (b) $(-\infty, -2)$
(c) $(2, \infty)$ (d) $(-\infty, \infty)$

84. The portion of a tangent to a parabola $y^2 = 4ax$ cut off between the directrix and the curve subtends an angle θ at the focus, where θ =
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

85. There are 25 trees at equal distances of 5 meters in a line with a well, the distance of the well from the nearest tree being 10 metres. A gardener waters all the trees separately starting from the well and he returns to the well after watering each tree to get water for the next. The total distance the gardener will cover in order to water all the trees is
(a) 3550m (b) 3434m
(c) 3370m (d) 3200m

86. If $ax^2+by^2+cz^2-1=0$ and $lx+my+nz=0$ then
(a) $\frac{dy}{dx}=\frac{c\ell y-anx}{bnz-cmx}$ (b) $\frac{dy}{dx}=\frac{c\ell z-anx}{bny-cmz}$
(c) $\frac{dz}{dx}=\frac{amy-b\ell x}{bny-cmz}$ (d) $\frac{dz}{dx}=\frac{amz-b\ell x}{bny-cmz}$

87. If three positive number a, b, c, are in H.P. then
(a) $b^2 < ac$ (b) $a^n+c^n > 2b^n$
(c) Both correct (d) none correct

88. If $\int \sin^3 x \cos^5 dx = A\sin^4 x + B\sin^6 x + C\sin^8 x + D$.
Then
(a) $A=\frac{1}{4}, B=-\frac{1}{3}, C=\frac{1}{8}, D \in R$
(b) $A=\frac{1}{8}, B=\frac{1}{4}, C=\frac{1}{3}, D \in R$
(c) $A=0, B=-\frac{1}{6}, C=\frac{1}{8}, D \in R$
(d) none of these.

89. In a ΔABC, the area of the triangle is $b^2-(c-a)^2$, then tan B is equal to :

(a) $\frac{1}{4}$ (b) $\frac{8}{15}$

(c) $\frac{1}{2}$ (d) $\frac{15}{8}$

90. If $\varphi(x) = \log_5 \log_3 x$, then $\varphi'(e) =$

(a) 1 (b) $e \log_e 5$

(c) $\frac{1}{e}\log_5 e$ (d) $\log_e 5$

91. If $f(x) = x^\alpha \log x$ and $f(0) = 0$, then the value of α for which Rolle's theorem can be applied in [0, 1] is

(a) –2 (b) –1

(c) 0 (d) 1/2

92. The spheres $x^2 + y^2 + z^2 + x + y + z - 1 = 0$ and $x^2 + y^2 + z^2 + x + y + z - 5 = 0$

(a) intersect in a plane

(b) intersect in five points

(c) do not intersect

(d) none of these.

93. The value of integral $\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta \, d\theta}{\sin^4\theta + \cos^4\theta}$ is

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

94. Which of the following sums is correct?

(a) $\sum_{n=1}^{\infty} \frac{1}{(n+1)!} = e - 1$

(b) $\sum_{n=1}^{\infty} \frac{1}{(n+2)!} = e - 2$

(c) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)!} = \frac{e^2 - 1}{2e}$

(d) $\sum_{n=1}^{\infty} \frac{1}{(2n+1)!} = \frac{e^2 - 2e}{2e}$

95. The locus of the mid points of the focal chords of the parabola $y^2 = 4ax$ is another parabola whose vertex is given by

(a) (a, 0) (b) (0, a)

(c) (–a, 0) (d) None of these

96. The solution to the differential equation $(x+y)^2 \frac{dy}{dx} = a^2$ is

(a) $x + y = a \tan\left(\frac{y+c}{a}\right)$

(b) $y = a \tan^{-1}(x+y) + c$

(c) $y\left(1 - \frac{x}{4}\right) = \tan\frac{y}{a} + c$

(d) none of these

97. If $\sum_{k=1}^{m} (k^2 + 1)k! = 1999 \times 2000!$, then m is

(a) 1999 (b) 2000

(c) 2001 (d) None of these

98. If $\sin^{-1}(1-x) = 2\sin^{-1} x + \frac{\pi}{2}$, then :

(a) $x = 0, \frac{1}{2}$ (b) $x = \frac{1}{2}$

(c) $x = 0$ only (d) none of these

99. If a is a non-zero real number, then the vectors

$\vec{\alpha} = a\hat{i} + 2a\hat{j} - 3a\hat{k}$,

$\vec{\beta} = (2a+1)\hat{i} + (2a+3)\hat{j} + (a+1)\hat{k}$,

$\vec{\gamma} = (3a+5)\hat{i} + (a+5)\hat{j} + (a+2)\hat{k}$ are

(a) coplanar if $a < 0$

(b) coplanar if $a > 0$

(c) always coplanar

(d) never coplanar

100. If B is a non-singular matrix and A is a square matrix, then det $(B^{-1}AB)$ is equal to

(a) det (A^{-1}) (b) det (B^{-1})

(c) det (A) (d) det (B)

101. $\int \sqrt[3]{x}\sqrt[7]{1+\sqrt[3]{x^4}}\,dx$ is equal to

(a) $\frac{21}{32}\left(1+\sqrt[3]{x^4}\right)^{8/7}+C$ (b) $\frac{32}{21}\left(1+\sqrt[3]{x^4}\right)^{8/7}+C$

(c) $\frac{7}{32}\left(1+\sqrt[3]{x^4}\right)^{8/7}+C$ (d) None of these.

102. The sum of the sereis $1.2.3.4+2.3.4.5+3.4.5.6+.....$ upto n terms is equal to

(a) $\frac{1}{5}[(n+1)(n+2)(n+3)(n+4)-24]$

(b) $\frac{1}{5}(n+1)(n+2)(n+3)(n+4)(n+5)$

(c) $\frac{1}{5}n(n+1)(n+2)(n+3)(n+4)$

(d) none of these

103. If $x\cos\alpha+y\sin\alpha=p$, a variable chord of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{2a^2}=1$ subtends a right angle at the centre of the hyperbola, then the chords touch a fixed circle whose radius is equal to

(a) $\sqrt{2}a$ (b) $\sqrt{3}a$

(c) $2a$ (d) $\sqrt{5}a$

104. If $\int x^2\cos x\,dx = f(x)\sin x+g(x)\cos x+c$.

Then

(a) $f(x)=x^2,\ g(x)=x$

(b) $f(x)=x^2+2, g(x)=x$

(c) $f(x)=x^2,\ g(x)=2x-1$

(d) $f(x)=x^2-2,\ g(x)=2x$

105. If a chord which is normal to the parabola at one end and subtends a right angle at the vertex, then slope of the chord is

(a) 1 (b) –2

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

106. In each of a set of games it is 2 to 1 in favour of the winner of the previous game. The chance that the player who wins the first game shall win three at least of the next four is

(a) $\frac{8}{27}$ (b) $\frac{4}{81}$

(c) $\frac{4}{9}$ (d) None

107. The normal to the curve represented parametrically by $x=a(\cos\theta+\theta\sin\theta)$ and $y=a(\sin\theta-\theta\cos\theta)$ at any point θ, is such that it

(a) makes a constant angle with x-axis

(b) is at a contant distance from the origin

(c) touches a fixed circle

(d) satisfies (b) and (c)

108. If $x\in R$ and the roots of $ax^2+bx+c=0$ are non-real complex, then the sign of $a^2x^2+abx+ac$ is

(a) always positive

(b) always negative

(c) always non-negative

(d) none

109. If $y=e^{ax}\sin(bx+c)$, then $\frac{d^2y}{dx^2}-2a\frac{dy}{dx}+\left(a^2+b^2\right)y=$

(a) 0 (b) y

(c) a^2 (b) b^2

110. The value of $\int_0^{\frac{\pi}{2}}\log_e\sin x\,dx=\int_0^{\frac{\pi}{2}}\log_e\cos x\,dx$ is

(a) $\frac{\pi}{2}\log 2$ (b) $-\frac{\pi}{2}\log 2$

(c) $\pi\log\left(\frac{1}{2}\right)$ (d) none of these.

111. If A, B, C are the angles of a triangle then the value of determinant $\begin{vmatrix}\sin 2A & \sin C & \sin B\\ \sin C & \sin 2B & \sin A\\ \sin B & \sin A & \sin 2C\end{vmatrix}$ is

(a) π (b) 0

(c) 2π (d) None of these

112. The length of projection of the segment joining P(–1, 2, 0) and Q (1, –1, 2) on the plane $2x - y - 2z = 4$ is

(a) 3 (b) 4

(c) 5 (d) $3\sqrt{3}$

113. A square hole is punched out of a circular lamina of diameter a. The diagonal of the square is equal to the radius of the circle and one of the corners is at the centre of the circle. The distance of the C.G. of the remainder from the centre of the circle is :

(a) $\frac{a}{\pi+4}$ (b) $\frac{a}{8\pi-4}$

(c) $\frac{a}{5\pi-4}$ (d) $\frac{a}{5\pi+4}$

114. The quantity $(1000^{1000} - 1001^{999})$ is

(a) >0 (b) <0

(c) $=0$ (d) None

115. If a = 0 and b = 1, where a and b are two boolean variables, then a + b =

(a) 1 (b) 0

(c) 1 or 0 (d) none

116. Let P be a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with foci F_1 and F_2. If A is the area of the triangle PF_1F_2, then the maximum value of A is

(a) 2abe (b) abe

(c) $\frac{1}{2}$abe (d) None of these

117. Two cars started from a place, one moving due north and the other due west with equal speed v. The rate at which they were being separated from each other is :

(a) 2v (b) $\sqrt{2}$v

(c) $\frac{v}{\sqrt{2}}$ (d) v

118. Let A = {1,2,3,4,5,6,7,8} which of the following relations from A to **R** is NOT a function?

(a) $R_1 = \{(x, f(x)) : x \in A, f(x) = 6x + 7\}$

(b) $R_2 = \{(x, f(x)) : x \in A, f(x) = |x| + 9\}$

(c) $R_3 = \left\{(x, f(x)) : x \in A, f(x) = \frac{1}{x^2+7}\right\}$

(d) $R_4 = \{(x, f(x)) : x \in A, f(x) = \pm 4x\}$

119. Let $f : R \to R$ be such that f(1) = 3 and f'(1) = 6. Then $\lim_{x\to 0}\left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ equals

(a) 1 (b) $e^{1/2}$

(c) e^2 (d) e^3

120. The value of $(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11})$ is

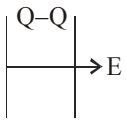
(a) 49 (b) 16

(c) 16ω (d) $49\omega^2$

HINTS & SOLUTION

1. (b) The magnitude of electric field by any one plate is

$$\frac{\sigma}{2\varepsilon_o} \text{ or } \frac{Q}{2A\varepsilon_o}$$



now force magnitude is $|Q||E|$ i.e.

$$|F| = \frac{Q^2}{2A\varepsilon_o}$$

2. (c) C_2 and C_3 are parallel so $V_2 = V_3$
C_1 and combination of C_2 & C_3 is in series.
So, $V = V_2 + V_1$ or $V = V_3 + V_1$
and also $Q_1 = Q_2 + Q_3$

3. (d) When oil is placed between space of plates

$$C = \frac{2A\varepsilon_0}{d} \quad \ldots(1)$$

$$\left[\because C = \frac{KA\varepsilon_0}{d}, \text{ if } K = 2\right]$$

When oil is removed $C' = \frac{A\varepsilon_0}{d}$ .....(2)

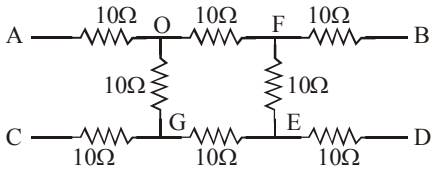
on comparison both equation, weget $C' = C/2$

4. (d) According to Gauss' Law

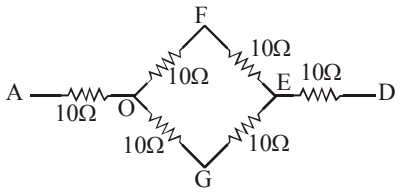
$$\oint E.ds = \frac{Q_{\text{enclosed by closed surface}}}{\varepsilon_o} = \text{flux}$$

so total flux $= Q/\varepsilon_o$
Since cube has six face so flux coming out through one wall or one face is $Q/6\varepsilon_o$.

5. (c)



Equivalent Circuit



Equivalent Resistance of circuit

$$= 10\Omega + \frac{20\times 20}{20+20} + 10 = 10 + 10 + 10 = 30\Omega$$

6. (a) Resistance of ACB, $R' = 3\Omega + 3\Omega = 6\Omega$.
For net resistance between A and B; $R' = 6\Omega$ and 3Ω are in parallel.

$$R_{AB} = \frac{3\times 6}{3+6} = \frac{18}{9} = 2\Omega$$

$$\text{Current in circuit } (I) = \frac{E}{R_{AB}} = \frac{2}{2} = 1A$$

7. (a) We know from the Kirchhoff's first law that the algebraic sum of the current meeting at any junction in the circuit is zero (i.e. $\Sigma i = 0$) or the total charge remains constant. Therefore, Kirchhoff's first law at a junction deals with the conservation of charge.

8. (b) $E = V + ir$

$$2.2 = 1.8 + \frac{1.8}{5}\times r \Rightarrow r = \frac{10}{9}\Omega$$

9. (c) In the electrolysis of $CuSO_4$, oxygen is liberated at anode and copper is deposited at cathode.

10. (b) $$R_1 = \frac{V^2}{P_1} = \frac{200\times 200}{40} = 1000\,\Omega$$

$$R_2 = \frac{V^2}{P_2} = \frac{200\times 200}{100} = 400\,\Omega$$

$\therefore R_1$ (for 40W) $> R_2$ (for 100 W)

11. (a) $$F = \frac{\mu_0}{4\pi}\times\frac{2i_1i_2l}{r} = 10^{-7}\times\frac{2\times 1\times 1\times 1}{1}$$
$= 2\times 10^{-7}$ N/m.
[This relates to the definition of ampere]

12. (b) For circular path in magnetic field,
$mr\omega^2 = qvB$

$$\Rightarrow \omega^2 = \frac{qvB}{mr} \qquad \text{As } v = r\omega$$

$$\therefore \omega^2 = \frac{q(r\omega)B}{mr} \Rightarrow \omega = \frac{qB}{m}$$

$\therefore$ If ν is frequency of roatation, then

$$\nu = \frac{\omega}{2\pi} \Rightarrow \nu = \frac{qB}{2\pi m}$$

13. (c) $$L = \frac{N\phi}{i};\ \phi = BA;\ B = \frac{\mu_0 Ni}{2R}$$

$$\Rightarrow L = \frac{N}{i}\left(\frac{\mu_0 Ni}{2R}\right)A = \frac{\mu_0 N^2}{2R}A \Rightarrow L \propto N^2$$

14. (a) Power dissipated = $E_{rms}.\ I_{rms} = (E_{rms})\ (I_{rms})\cos\theta$
Hence, power dissipated depends upon phase difference.

15. (b) $e = \frac{-(\phi_2 - \phi_1)}{t} = \frac{-(0 - NBA)}{t} = \frac{NBA}{t}$

$$t = \frac{NBA}{e} = \frac{50 \times 2 \times 10^{-2} \times 10^{-2}}{0.1} = 0.1\ s$$

16. (b) Mutual Inductance of two coils

$$M = \sqrt{M_1 M_2} = \sqrt{2mH \times 8mH} = 4mH$$

17. (c)

Rays	Wavelength [Range in m]
X-rays	1×10^{-11} to 3×10^{-8}
γ-rays	6×10^{-14} to 1×10^{-11}
Microwaves	10^{-3} to 0.3
Radiowaves	10 to 10^4

Wavelength of U.V. rays ranges from 6×10^{-8} to 4×10^{-7}.

18. (d) Huygen's construction of wavefront does not apply to origin of spectra which is explained by quantum theory.

19. (d) Let the amplitude of the waves be a_1, a_2 and the intensities I_1 and I_2. The ratio of this intensities is 16 : 9

Then $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{16}{9}$

$$\therefore \frac{a_1}{a_2} = \frac{4}{3} \quad \text{....(1)}$$

In interference, the maximum and minimum resultant amplitudes are $(a_1 + a_2)$ and $(a_1 - a_2)$ respectively

$$\therefore \frac{\text{Maximum intensity } I_{max}}{\text{Minimum intensity } I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

But from equation (1),

$$a_1 = \frac{4a_2}{3}$$

$$\therefore \frac{I_{max}}{I_{min}} = \frac{\left(\frac{4a_2}{3} + a_2\right)^2}{\left(\frac{4a_2}{3} - a_2\right)^2}$$

$$\therefore \frac{I_{max}}{I_{min}} = \frac{49}{1} = 49:1$$

20. (c) The optical path length between A and B is

$$x + (t \times \mu) + y = x + (1.5t) + y = x + \frac{3}{2}t + y$$

21. (b) Energy in first excited state

$$\frac{E_1}{2^2} = -\frac{13.6}{4} = -3.4\ eV$$

Hence, energy required to ionize atom from its first excited state will be 3.4 eV.

22. (b) For Balmer series, electron jumps from higher orbit to secondary orbit, thus for series $\bar{\nu} = R_H\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$

For Brackett series, the electron jumps from higher orbits to fourth stationary orbit ($n_f = 4$)

$$\bar{\nu} = R_H\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$$

For Lyman series, electron jumps from higher orbit to the first stationary orbit i.e., $n_f = 1$

$$\bar{\nu} = R_H\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

For Paschen series, electron jumps from higher orbits to third stationary orbit ($n_f = 3$)

$$\bar{\nu} = R_H\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$$

23. (a) All statements for ruby laser using flash lamp is correct because it is a pulse laser, three level laser as well as cooled effectively.

24. (c) A Febry Perot interferometer consists of two plane parallel glass plates. With this interferometer fringes of constant inclination are obtained by transmitted light after multiple reflection between the glass plate.

25. (b) $E_k = \frac{hc}{e}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)(\text{in eV})$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}}\left(\frac{10^{10}}{1800} - \frac{10^{10}}{2300}\right) = 1.5\,eV$$

26. (b) $\frac{1}{2}m v_1^2 = 2W_0 - W_0 = W_0$ and

$$\frac{1}{2}m v_2^2 = 10W_0 - W_0 = 9W_0$$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{W_0}{9W_0}} = \frac{1}{3}$$

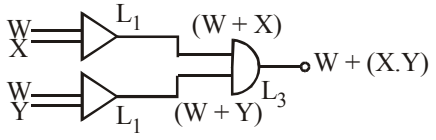
27. (b) Energy of electron $= m_e c^2$
Energy of positron $= m_p c^2$
$m_e = m_p$ c = speed of light.
Thus according to conservation of energy released
$= 2m_e c^2 = 2 \times 9.1 \times 10^{-31} \left(3 \times 10^8\right)^2$
$= 1.6 \times 10^{-13}$ Joules.

28. (d) Both are correct since the observers are inertial. If they move with respect to each other, they will always disagree.

29. (c) Forces between nucleons inside the nucleus are
(i) attractive in nature
(ii) extremely short range
(iii) strongest forces in nature

30. (d) ${}_1^2H + {}_1^2H \rightarrow {}_2^3He + {}_0^1n + 13\ MeV$
The binding energy of ${}_1^2H$ is 2.2
$(2.2) \times 2 = 13\ MeV +$ Binding energy
$(4.4) = -13\ MeV +$ Binding energy
Binding energy of He $= 13 + 4.4 = 17.4$

31. (a) Half live of given radioactive nucleus = 100 years
$\therefore$ Number of half lives is 300 years
$$= \frac{300}{100} = 3$$
Now the number of atoms left after n half-lives is
$$N = N_0 \left(\frac{1}{2}\right)^n = N_0 \left(\frac{1}{2}\right)^3 = \frac{N_0}{8}$$
Hence the number of nuclei which have decayed in 300 years is
$$N_0 - N_0/8 = \frac{7N_0}{8}$$

32. (b)


33. (b)

34. (d) Positive terminal is at lower potential (–10V) and negative terminal is at higher potential 0V.

35. (b)

36. (c) We know that for common base
$$\alpha = \frac{i_c}{i_e} = \frac{\text{collector current}}{\text{emmiter current}} = 0.98$$
& for common emitter
$$\beta = \frac{i_c}{i_b} = \frac{\text{collector current}}{\text{base current}}$$
$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = \frac{0.98}{0.02} = 49.$$

37. (c) An antenna is a metallic structure used to radiate or receive EM waves.

38. (c) Optical fibres are not subjected to electromagnetic interference from outside.

39. (b) Core of acceptance angle
$$\theta = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$

40. (a) A vidicon is a type of camera.

41. (a) If $\Delta G_{system} = 0$ the system has attained equilibrium is right choice.
In it alternative (d) is most confusing as when $\Delta G > 0$, the process may be spontaneous when it is coupled with a reaction which has $\Delta G < 0$ and total ΔG is negative, so right answer is (a).

42. (b) For the reaction
$2\ ZnS \rightarrow 2\ Zn + S_2\ ;\ \Delta G_1^\circ = 293\ kJ$ .....(1)
$2\ Zn + O_2 \rightarrow ZnO\ ;\ \Delta G_2^\circ = -480\ kJ$ .....(2)
$S_2 + 2\ O_2 \rightarrow 2\ SO_2\ ;\ \Delta G_3^\circ = -544\ kJ$ .....(3)
ΔG° for the reaction
$2\ ZnS + 3\ O_2 \rightarrow 2\ ZnO + 2\ SO_2$
can be obtained by adding equation (1), (2) and (3)
$\Rightarrow \Delta G^\circ = 293 - 480 - 544 = -731\ kJ$

43. (a) For a spontaneous process, ΔS_{total} is always positive

44. (a) Since MgO has a rock salt structure. In this structure each cation is surrounded by six anions and vice versa.

45. (b) Out of the given substances, only Li has high electrical and thermal conductivity.

46. (c) For third ionization enthalpy last configuration of
V $-4s^0\ 3d^3$

	3d				4s
	↑	↑	↑		

Cr $-4s^0\ 3d^4$

	↑	↑	↑	↑	

Mn $-4s^0\ 3d^5$

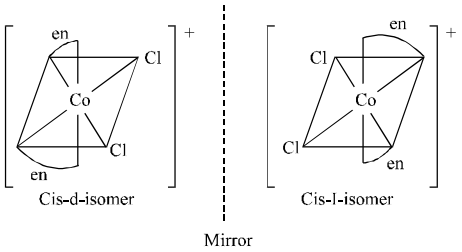
	↑	↑	↑	↑	↑

Fe $-4s^0\ 3d^6$

	↑↓	↑	↑	↑	↑

For third Ionization enthalpy Mn has stable configuration due to half filled d-orbital.

47. (c)



Transform of $[M(AA)_2\ a_2]^{n\pm}$ does not shows optical isomerism.

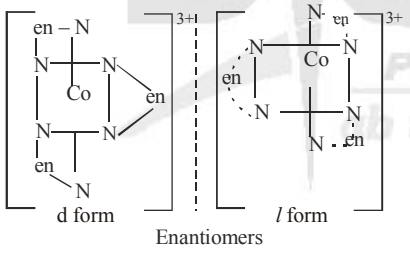
48. (b) For orthorhombic system $\alpha = \beta = \gamma = 90°$

49. (b) As in $[NiCl_4]^{-2}$ Chloride ion being a weak ligand is not able to paired the electron in d orbital.

50. (d) Galvanisation is the process of deposition of zinc metal on the surface of Fe to prevent it from rusting etc. Zinc forms a protective layer of basic carbonate ($ZnCO_3$. $Zn(OH)_2$) on it.

51. (c) For bottom of ship to be protected it is connected with more reactive metal than iron like magnesium. This technique is called cathodic protection.

52. (d) The optical isomers are pair of molecules which are non super imposable mirror images of each other



The two optically active isomers are collectivity called enantiomers.

53. (c) Lanthanides are 4 *f*-series elements starting from cerium (Z= 58) to lutetium (Z = 71). These are placed in the sixth period and in third group.

54. (c) $[X] + {}^{1}_{0}n \rightarrow {}^{60}_{27}Co + {}^{1}_{1}H$

Balancing the mass and atomic numbers on both sides ${}^{60}_{28}X + {}^{1}_{0}n \rightarrow {}^{60}_{27}Co + {}^{1}_{1}H$

Thus X should be ${}^{60}_{28}Ni$

55. (d) Age of geological formations (i.e. predicting the age of the earth and rocks) is estimated by U– Pb method, also known as helium dating.

Note : C-14 dating method is used to predict the age of fossils or dead animals or a fallen tree.

56. (d) The activation energy of reverse reaction will depend upon whether the forward reaction is exothermic or endothermic.

As $\Delta H = E_{a\ (forward\ reaction)} - E_{a(backward\ reaction)}$

For exothermic reaction

ΔH = –ve

$\therefore -\Delta H = E_{a(f)} - E_{a(b)}$

or $E_{a(f)} = E_{a(b)} - \Delta H$

$\therefore \quad E_{a(f)} < E_{a(b)}$

for endothermic reaction

ΔH = + ve

$\therefore \Delta H = E_{a(f)} + E_{a(b)}$ or $E_{a(f)} = \Delta H + E_{a(b)}$

$\therefore \quad E_{a(f)} > E_{a(b)}$.

57. (a) From data 1 and 3, it is clear that keeping (B) const, [A] is doubled, rate remains unaffected. Hence rate is independent of [A]. from 1 and 4, keeping [A] constant, [B] is doubled, rate become 8 times. Hence rate $\propto [B]^3$.

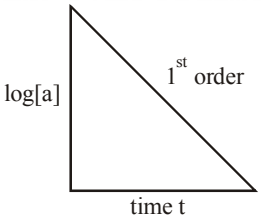
58. (b) For a first reaction $\frac{dx}{dt} = k(a-x)$

on intergration , $\int \frac{dx}{(a-x)} = \int k\,dt$

i.e $-\ell n\,(a-x) = kt + c$ or $kt = \ell n\,a - \ell n\,(a-x)$

or $kt = 2.303\,[\log a - \log (a-x)]$

Thus if we plot a graph between log a & t we get



59. (a) Given initial concentration (a) = 2.00 m; Time taken (t) = 200 min and final concentration (a – x) = 0.15 m. For a first order reation rate constant,

60. (d) According to Le-chatelier's principle" whenever a constraint is applied to a system in equilibrium, the system tends to readjust so as to nullify the effect of the constraint.

61. (c) By Kohlraush's law, $\wedge^0_{eq}\ NaCl = 126.45$

$$\lambda^0_{Na^+} + \lambda^0_{Cl^-} = 126.45 \qquad ....(1)$$

$$\lambda^0_{H^+} + \lambda^0_{Cl^-} = 426.16 \quad ...(2)$$

$$\lambda^0_{CH_3OO^-} + \lambda^0_{Na^+} = 91 \quad ....(3)$$

on adding (2) and (3) then subtract (1) from it

$$\lambda^0_{CH_3COO^-} + \lambda^0_{Na^+} = 517.16 - 126.45$$

$$\wedge^0_{(CH_3COOH)} = 390.71 \text{ ohm}^{-1}\text{ cm}^2$$

62. (d) $E° = \frac{0.0591}{n}\log_{10} K$

Here, $n = 2, E° = 0.295$

$$\therefore \log_{10} K = \frac{2 \times 0.295}{0.0591} = 10 \text{ or } K = 10^{10}$$

63. (d) In the silver plating of copper, $K[Ag(CN)_2]$ is used instead of $AgNO_3$. Copper being more electropositive readily precipitate silver from their salt solution

$$Cu + 2AgNO_3 \longrightarrow Cu(NO_3)_2 + Ag$$

whereas in $K[Ag(CN)_2]$ solution a complex anion $[Ag(CN)_2]^-$ is formed and hence Ag^+ are less available in the solution and therefore copper cannot displace Ag from its complex ion.

64. (b) Given current (i) = 0.5 amp;
Time (t) = 100 minutes × 60 = 6000 sec
Equivalent weight of silver nitrate (E) = 108.
According to Faraday's first law of electrolysis

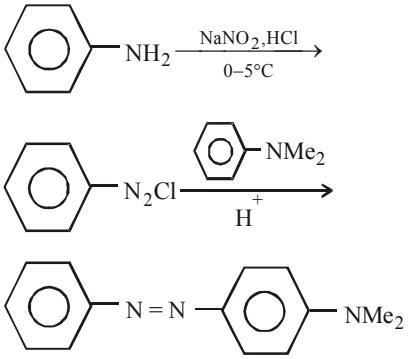
$$W = \frac{Eit}{96500} = \frac{108 \times 0.5 \times 6000}{96500} = 3.3575 \text{ g.}$$

65. (b) Order of reaction is independent of given factors.

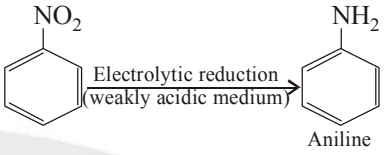
66. (b) Acetamide and ethylamine can be distinguished by heating with NaOH solution.Acetamide evolves NH_3 but ethylamine does not.

$$CH_3CONH_2 + NaOH \xrightarrow{\Delta} CH_3COONa + NH_3$$

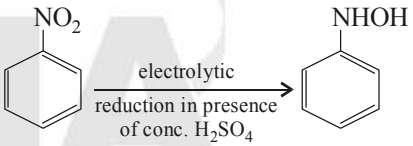
$$CH_3CH_2NH_2 + NaOH \longrightarrow \text{No reaction.}$$

67. (c) 

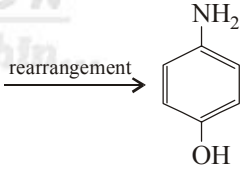
68. (c) **Electrolytic reduction of Nitroalkane in weakly acidic medium give aniline**



Whereas in strongly acidic medium it gives p-hydroxyaniline



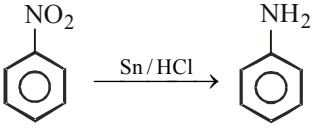
(Strongly acidic medium)



p-Hydroxyaniline

69. (a) An aliphatic aldehyde.
Generally H – C = O streching frequency is about 2830 – 2695 cm^{-1} but for aliphatic aldehyde is 1740 – 1720 cm^{-1}.

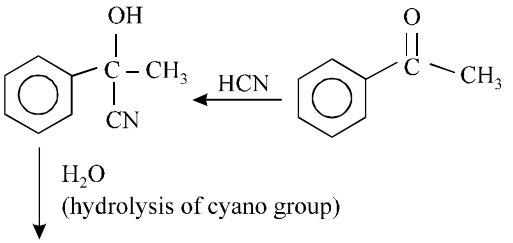
70. (a) Nitrobenzene is reduced to aniline in the presence of Sn/HCl.



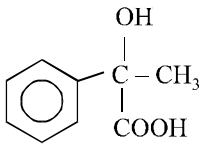
Nitrobenzene Aniline
Hence, option (a) is correct.

71. (d) $CH_3COOH \xrightarrow{SOCl_2} CH_3-COCl$

Friedel craft's reaction (with $AlCl_3$)



H_2O (hydrolysis of cyano group)

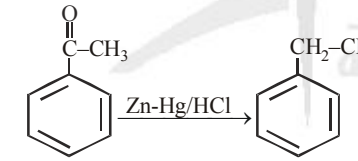


72. (c) By oxidation of tertiary alcohol with stronger oxidising agents ketones may be formed along with carboxylic acid.

$(CH_3)_3COH \xrightarrow{4[O]} CH_3COCH_3+CO_2+2H_2O$

$(CH_3)_3COH \xrightarrow{8[O]} CH_3COOH+2CO_2+3H_2O$

73. (b)



Phenyl methyl ketone — ethyl benzene

This reaction is known as clammensen's reduction.

74. (a) Due to H-bonding, the boiling point of ethanol is much higher than that of the isomeric diethyl ether.

75. (c) $CH_3-CH=CH_2 \xrightarrow[NaOH/H_2O_2]{B_2H_6} CH_3-CH_2-CH_2-OH$

Propanol

76. (b) $2HCOONa \xrightarrow[360°]{\Delta} \begin{matrix}COONa\\|\\COONa\end{matrix} + H_2$

sod.formate

77. (a)

$C_6H_5COOC_2H_5 \xrightarrow{CH_3MgBr} C_6H_5-C(OMgBr)(CH_3)-OC_2H_5$

$\xrightarrow{-Mg(OC_2H_5)Br} C_6H_5-C(=O)-CH_3 \xrightarrow[CH_3MgBr]{Excess}$

$C_6H_5-C(OMgBr)(CH_3)_2 \xrightarrow{H_2O} C_6H_5-C(CH_3)_2-OH$

$\xrightarrow[\Delta]{Con.\ H_2SO_4} C_6H_5-C(CH_3)=CH_2 \xrightarrow{Ozonolysis}$

'B'

$C_6H_5COCH_3+HCHO$

$C_6H_5COCH_3 \xrightarrow{3I_2+4NaOH} CHI_3$

78. (a) It is an example of Claisen condensation. The product is acetoacetic ester.

$CH_3C(=O)-\boxed{OC_2H_5+H}-CH_2-C(=O)-OC_2H_5 \longrightarrow$

$CH_3-C(=O)-CH_2-C(=O)-OC_2H_5$

(ethyl acetoacetate)

79. (d) Water gas is mixed with half its volume of hydrogen. The mixture is compressed to approximately 200 – 300 atmospheres. It is then passed over a catalyst $|ZnO + Cr_2O_3|$ at 300°C. Methyl alcohol vapours are formed which are condensed

$CO+2H_2 \xrightarrow[300°C]{ZnO+Cr_2O_3} CH_3OH$

Compressed gas — Methyl alcohol

80. (c) $R-CONH_2 \xrightarrow{Br_2/KOH} RNH_2$

This is known as Hoffmann's Bromamide reaction.

81. (a) For the given A.P., we have

$\sin(z+x-y)-\sin(y+z-x)=\sin(x+y-z)-\sin(z+x-y)$

$\Rightarrow 2\cos z\sin(x-y)=2\cos x\sin(y-z)$

$\Rightarrow \dfrac{\sin x\cos y-\cos x\sin y}{\cos x\cos y}$

$=\dfrac{\sin y\cos z-\cos y\sin z}{\cos y\cos z}$

$\Rightarrow \tan x-\tan y=\tan y-\tan z$, showing that tan x, tan y and tan z are in A.P.

82. (b) $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

$\Rightarrow \tan^{-1} \frac{2x+3x}{1-2x.3x} = \tan^{-1} 1$

$\Rightarrow \frac{5x}{1-6x^2} = 1 \Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow (6x-1)(x+1) = 0 \Rightarrow x = \frac{1}{6}$ or -1

Now for $x = -1$, LHS of equation becomes negative, so $x = \frac{1}{6}$.

83. (a) The given equation is

$2a^2x^2 + 2ancx + (n^2-2)c^2 = 0$

It has real roots, So

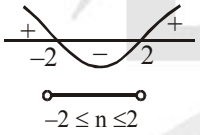
$D \geq 0 \Rightarrow (2anc)^2 - 4 \cdot 2a^2 \cdot (n^2-2)c^2 \geq 0$

$\Rightarrow n^2 - 2(n^2 - 2) \geq 0$

$[\because 4a^2c^2$ is positive$]$

$\Rightarrow n^2 - 4 \leq 0 \Rightarrow (n+2)(n-2) \leq 0$

$\therefore -2 \leq n \leq 2 \Rightarrow n \in [-2, 2]$

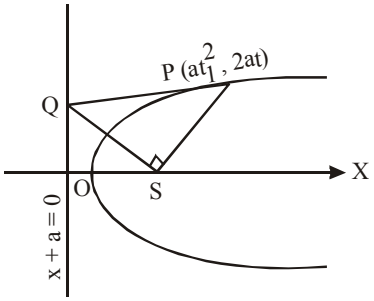


$-2 \leq n \leq 2$

84. (c) The equation of the tangent at P $(at^2, 2at)$ to $y^2 = 4ax$ is $ty = x + at^2$ ...(1)

It meets the directrix $x = -a$

$\therefore \quad ty = -a + at^2 \Rightarrow y = \frac{a(t^2-1)}{t}$



Thus, (1) meets the directrix at $Q\left(-a, \frac{a(t^2-1)}{t}\right)$

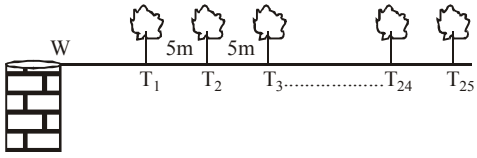
Now, slope of PS is $m_1 = \frac{2at-0}{at^2-a} = \frac{2t}{t^2-1}$

and slope of QS is m_2

$$= \frac{\frac{a(t^2-1)}{t} - 0}{-a-a} = -\frac{(t^2-1)}{2t}$$

Since $m_1 m_2 = -1$, therefore PQ subtends a right angle at the focus.

85. (c) Obviously the well (W) must be on one side of the trees $T_1, T_2, \ldots\ldots, T_{25}$,



The total distance covered by the gardener

$= WT_1 + (2WT_1 + T_1T_2) + (2WT_2 + T_2T_3) + \ldots\ldots + (2WT_{24} + T_{24}T_{25})$

$= 10 + (2\times10+5) + (2\times15+5) + \ldots\ldots\ldots$ + to 25 terms

$= 10 + (25 + 35 + 45 + \ldots\ldots\ldots$to 24 terms$)$

$= 10 + \frac{24}{2}[2\times25 + (24-1)\times10] = 10 + 12[50+230] = 3370$

86. (b) Let $f(x, y, z) = ax^2 + by^2 + cz^2 - 1 = 0$ and $\phi(x, y, z) = \ell x + my + nz = 0$

$\therefore \quad \frac{df}{dx} = 0$ and $\frac{d\phi}{dx} = 0$

But $\frac{df}{dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}.\frac{dy}{dx} + \frac{\partial f}{\partial z}.\frac{dz}{dx}$

or $\quad 0 = 2ax + 2by\frac{dy}{dx} + 2cz\frac{dz}{dx}$

or $\quad ax + by\frac{dy}{dx} + cz\frac{dz}{dx} = 0$ .....(1)

Similarly from $\frac{d\phi}{dx} = 0$,

we obtain, $\ell + m\frac{dy}{dx} + n\frac{dz}{dx} = 0$ .....(2)

Solve (1) and (2) by the method of cross-multiplication,

$$\frac{1}{bny - cmz} = \frac{\frac{dy}{dx}}{c\ell z - anx} = \frac{\frac{dz}{dx}}{amx - b\ell y}$$

$$\therefore \frac{dy}{dx} = \frac{c\ell z - anx}{bny - cmz} \text{ and } \frac{dz}{dx} = \frac{amx - b\ell y}{bny - cmz}$$

87. (c) Clearly b is H.M. of a and c

Also the G.M. of a and c is $\sqrt{ac}$

Since

$G.M. > H.M \Rightarrow \sqrt{ac} > b \Rightarrow b^2 < ac$ .....(i)

Again, for the positive numbers a^n, c^n

$$G.M. = \sqrt{a^n c^n} \text{ and } A.M. = \frac{a^n + c^n}{2}$$

$$\text{Since A.M.} > \text{G.M.} \Rightarrow \frac{a^n + c^n}{2} > \sqrt{a^n c^n}$$

$$\Rightarrow \frac{a^n + c^n}{2} > \left(\sqrt{ac}\right)^n > b^n \quad \text{(Using (i))}$$

$$\Rightarrow a^n + c^n > 2b^n$$

88. (a) $I = \int \sin^3 x . \cos^5 x \, dx$

Put $\sin x = t \Rightarrow \cos x \, dx = dt$

$$I = \int \sin^3 x . \cos^4 x . \cos x \, dx = \int t^3 (1 - t^2)^2 dt$$

$$= \int \left(t^3 - 2t^5 + t^7\right) dt = \frac{1}{4}t^4 - \frac{2}{6}t^6 + \frac{1}{8}t^8 + c$$

$$= \frac{1}{4}\sin^4 x - \frac{1}{3}\sin^6 x + \frac{1}{8}\sin^8 x + c$$

ALTERNATE:

$$I = \int \sin^3 x . \cos^5 x \, dx = \int \sin^2 x . \cos^5 x . \sin x \, dx$$

Put $\cos x = t \Rightarrow -\sin x \, dx = dt$

$$I = \int (1 - t^2) t^5 (-dt) = \int \left(t^7 - t^5\right) dt = \frac{t^8}{8} - \frac{t^6}{6} + c$$

$$= \frac{1}{8}\cos^8 x - \frac{1}{6}\cos^6 x + c$$

[Not that this result is in terms of powers of cos x]

89. (b) $\sqrt{s(s-a)(s-b)(s-c)} = 4(s-a)(s-c)$

$[b^2 - (c-a)^2 = (b+c-a)(b-c+a)$

$= 2(s-a)2(s-c)]$

Squaring and simplifying we get

$s(s-b) = 16(s-a)(s-c)$

$$\Rightarrow \frac{(s-a)(s-c)}{s(s-b)} = \frac{1}{16} \Rightarrow \tan^2 \frac{B}{2} = \frac{1}{16}$$

$$\Rightarrow \tan \frac{B}{2} = \frac{1}{4}$$

$$\therefore \tan B = \frac{2\tan \frac{B}{2}}{1 - \tan^2 \frac{B}{2}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15}$$

90. (c) If $\varphi(x) = \log_5 \log_3 x = \log_5 \left(\frac{\log_e x}{\log_e 3}\right)$

$$= \log_5 (\log_e x) - \log_5 (\log_e 3)$$

$$= \frac{\log_e (\log_e x)}{\log_e 5} - \log_5 (\log_e 3)$$

$$\therefore \varphi'(x) = \frac{1}{\log_e 5} \cdot \frac{1}{\log_e x} \cdot \frac{1}{x} - 0$$

$$\Rightarrow \varphi'(e) = \frac{1}{\log_e 5} \cdot \frac{1}{\log_e e} \cdot \frac{1}{e} = \frac{1}{e \log_e 5}$$

91. (d) For Rolle's theorem in [a, b], f(a) = f(b), In [0, 1] $\Rightarrow$ f(0) = f(1) = 0

$\because$ the function has to be continuous in [0, 1]

$$\Rightarrow f(0) = \lim_{x \to 0^+} f(x) = 0 \Rightarrow \lim_{x \to 0} x^\alpha \log x = 0$$

$$\Rightarrow \lim_{x \to 0} \frac{\log x}{x^{-\alpha}} = 0$$

Applying L.H. Rule $\lim_{x \to 0} \frac{1/x}{-\alpha x^{-\alpha - 1}} = 0$

$$\Rightarrow \lim_{x \to 0} \frac{-x^\alpha}{\alpha} = 0 \Rightarrow \alpha > 0$$

92. (c)

93. (c) Substitute $\sin^2 \theta = t$

$\Rightarrow 2 \sin \theta \cos \theta \, d\theta = dt$

$\Rightarrow \sin 2\theta \, d\theta = dt$

Also when $x = 0, t = 0$ and when $x = \frac{\pi}{2}, t = 1$

$\therefore$ The integral becomes

$$I = \int_0^1 \frac{dt}{t^2 + (1-t)^2} = \int_0^1 \frac{dt}{2t^2 - 2t + 1}$$

$$= \frac{1}{2} \int_0^1 \frac{dt}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \left[\frac{1}{2}\cdot\frac{1}{\frac{1}{2}}\tan^{-1}\frac{t-\frac{1}{2}}{\frac{1}{2}}\right]_0^1 = \left[\tan^{-1}\frac{2t-1}{1}\right]_0^1$$

$$= \tan^{-1}(1) - \tan^{-1}(-1)$$

$$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

94. (c) We have, $T_n = \frac{1}{(2n-1)!}$

$$\therefore \sum_{n=1}^{\infty} T_n = 1 + \frac{1}{3!} + \frac{1}{5!} + \ldots\ldots = \frac{1}{2}(e - e^{-1})$$

$$= \frac{e^2 - 1}{2e}$$

95. (a) The equation of the parabola is $y^2 = 4ax$...(1)
Let (x_1, y_1) be the mid point of a focal chord of the parabola.
Equation of chord having (x_1, y_1) as its mid point is

$T = S_1$ i.e. $yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$

or $yy_1 - 2ax = y_1^2 - 2ax_1$

Since it passes through the focus (a, 0)

$\therefore \quad 0 - 2a^2 = y_1^2 - 2ax_1$ or

$y_1^2 - 2a(x_1 - a) = 0$

Thus, the locus of the mid point (x_1, y_1) is $y^2 = 2a(x - a)$, which is a parabola whose vertex is (a, 0).

96. (a) Put $x + y = z \Rightarrow 1 + \frac{dy}{dx} = \frac{dz}{dx}$

The equation then becomes

$$z^2\left(\frac{dz}{dx} - 1\right) = a^2 \Rightarrow \frac{dz}{dx} = 1 + \frac{a^2}{z^2} = \frac{z^2 + a^2}{z^2}$$

$$\Rightarrow \frac{z^2}{z^2 + a^2}dz = dx$$

Integrating we get

$$\Rightarrow \frac{z^2}{z^2 + a^2}dz = \int dx + D$$

$$\Rightarrow \int\left[1 - \frac{a^2}{z^2 + a^2}\right]dz = x + D$$

$$\Rightarrow z - \frac{a^2}{a}\tan^{-1}\frac{z}{a} = x + D$$

$$\Rightarrow a\tan^{-1}\frac{z}{a} = z - x - D$$

$$\Rightarrow \frac{z}{a} = \tan\left(\frac{z - x - D}{a}\right)$$

$$\Rightarrow x + y = a\tan\left(\frac{y + c}{a}\right), \quad \text{where } c = -D$$

97. (a) The general term

$T_k = (k^2 + 1)k! = (k^2 - 1 + 2)(k)!$

$= (k-1)(k+1)k! + 2k!$

$T_k = (k-1)(k+1)! + 2(k)!$

$= (k + 2 - 3)(k+1)! + 2(k)!$

$= (k+2)! - 3(k+1)! + 2(k)!$

$\therefore \; T_1 = (3)! - 3(2)! + 2(1)!;$

$T_2 = (4)! - 3(3)! + 2(2)!;$

$T_3 = (5)! - 3(4)! + 2(3)!$

....

....

$T_m = (m+2)! - 3(m+1)! + 2(m)!$

$\therefore \sum_{k=1}^{m} T_k = T_1 + T_2 + T_3 + \ldots\ldots\ldots\ldots + T_m$

$= -3(2)! + 2(1)! + 2(2)! + (m+1)! + (m+2)! - 3(m+1)!$

$= (m+2)! - 2(m+1)! = (m+1)!(m+2-2)$

$= m(m+1)!$

Now, $m(m+1)! = 1999 \times 2000! \Rightarrow m = 1999$

98. (c) $2\sin^{-1}x = \sin^{-1}(1-x) - \frac{\pi}{2} \Rightarrow 2\alpha = \beta - \frac{\pi}{2}$

where, $\alpha = \sin^{-1}x \Rightarrow x = \sin\alpha$

and $\beta = \sin^{-1}(1-x) \Rightarrow \sin\beta = 1 - x$

Now, $\cos 2\alpha = \cos\left(\beta - \frac{\pi}{2}\right) = \sin\beta$

$\Rightarrow 1 - 2\sin^2\alpha = \sin\beta \Rightarrow 1 - 2x^2 = 1 - x$

$\Rightarrow x(2x-1) = 0 \Rightarrow x = 0$ or $x = \frac{1}{2}$

Only $x = 0$ satisfies the given equation.

99. (d) We have

$$[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$$

$$= a\{(2a+3)(a+2)-(a+5)(a+1)\} - 2a\{(2a+1)(a+2)-(3a+5)(a+1)\} - 3a\{(2a+1)(a+5)-(3a+5)(2a+3)\}$$

$$= a(15a^2+31a+37), \text{ on simplification}$$

$$= 15a\left\{\left(a+\frac{31}{30}\right)^2+\frac{1259}{900}\right\} \neq 0$$

[for all non-zero a]

Hence, the given vectors are non-coplanar, for all $a \neq 0$.

100. (c) $\det(B^{-1}AB) = \det(B^{-1})\det A \det B$

$= \det(B^{-1}).\det B.\ \det A = \det(B^{-1}B).\det A$

$= \det(1).\ \det A = 1.\ \det A = \det A.$

101. (a) Put $1+x^{4/3} = x^7 \Rightarrow x^{1/3}dx = \frac{21}{4}z^6dz$

$$\therefore \int \sqrt[3]{x}\sqrt[7]{1+\sqrt[3]{x^4}}\,dx = \int z\left(\frac{21}{4}z^6\right)dz = \frac{21}{4}\cdot\frac{z^8}{8}+C$$

$$= \frac{21}{32}\left(1+\sqrt[3]{x^4}\right)^{8/7}+C$$

102. (c) The general term $x_k = k(k+1)(k+2)(k+3)$ or

$$(k+4)x_k = k(k+1)(k+2)(k+3)(k+4)$$

$$= kx_{k+1}$$

$$\therefore 5x_k = kx_{k+1}-(k-1)x_k$$

$$= f(k+1)-f(k)$$

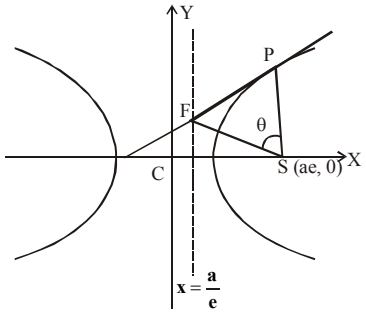
where $f(k) = (k-1)x_k$

$$\text{So, } S_n = \sum_{k=1}^{n} x_k = \frac{1}{5}\sum_{k=1}^{n}[f(k+1)-f(k)]$$

$$= \frac{1}{5}[f(n+1)-f(1)] = \frac{1}{5}[nx_{n+1}-0.x_1]$$

$$= \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$$

103. (a) Equation of the pair of straight lines joining the centre (origin) and the intersection points of the variable chord and the hyperbola is



$$\frac{x^2}{a^2}-\frac{y^2}{2a^2}-\left[\frac{x\cos\alpha+y\sin\alpha}{p}\right]^2 = 0 \quad \text{....(i)}$$

[By homogenising the equation of hyperbola with the help of the equation of the straight lines. See the pair of straight lines].

The equation (i) represents a pair of perpendicular lines, if coefficient of x^2 + coefficient of $y^2 = 0$

i.e.,

$$\left[\frac{1}{a^2}-\frac{\cos^2\alpha}{p^2}\right]+\left[-\frac{1}{2a^2}-\frac{\sin^2\alpha}{p^2}\right] = 0$$

$$\Rightarrow \frac{1}{a^2}-\frac{1}{2a^2} = \frac{1}{p^2}$$

$\therefore p^2 = 2a^2 \Rightarrow p = \pm\sqrt{2}a$

$\therefore$ The equation of the chord is $x\cos\alpha + y\sin\alpha = \pm\sqrt{2}a$

Which is the equation of straight line always touching the circle $x^2+y^2 = (\sqrt{2}a)^2$, a fixed circle of radius $\sqrt{2}a$.

104. (d) $I = \int x^2\cos x\,dx$

$$= x^2\int\cos x\,dx - \int\left(\frac{d}{dx}x^2\int\cos x\,dx\right)dx$$

[Integrating by parts taking x^2 as first function]

$$= x^2\sin x - \int 2x\sin x\,dx$$

$$= x^2\sin x - 2\left[x\int\sin x\,dx - \int\left(\frac{d}{dx}x\int\sin x\,dx\right)dx\right]$$

[Integrating by parts again taking x as first function]

$= x^2 \sin x - 2\left[x(-\cos x) + \int \cos x\, dx\right]$

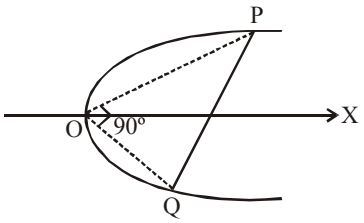
$= x^2 \sin x + 2x\cos x - 2\sin x + c$

$= (x^2 - 2)\sin x + 2x\cos x + c$

$\Rightarrow f(x) = x^2 - 2,\ g(x) = 2x$

105. (c) Chord PQ is normal to the parabola at Q and is equation is

$y = mx - 2am - am^3$ ...(1)



The slope of normal PQ is m = tan θ. The joint equation of OP and OQ is obtained by making the equation of parabola $y^2 = 4ax$ homogeneous with the help of (1). Thus joint equation of lines OP, OQ is

$y^2 = 4ax\left\{\frac{mx - y}{2am + am^3}\right\}$

or $m(2 + m^2)y^2 + 4xy - 4mx^2 = 0$ ...(2)

Since $\angle POQ = \frac{\pi}{2}$, the sum of the coefficients of x^2 and y^2 in (2) must be zero.

$m(2 + m^2) - 4m = 0$ or $m(m^2 - 2) = 0$

As $m \neq 0$, we get $m^2 = 2$ or $m = \pm\sqrt{2}$.

Thus, $m = \sqrt{2}$.

106. (c) Let W stand for the winning of a game and L for losing it. Then there are 4 mutually exclusive possibilities

(i) W, W, W (ii) W, W, L, W

(iii) W, L, W, W (iv) L, W, W, W.

[Note that case (i) includes both the cases whether he losses or wins the fourth game.]

By the given conditions of the question, the probabilities for (i), (ii), (iii) and (iv) respectively are

$\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3};\ \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3};\ \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{2}{3}$

and $\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}$.

Hence the required probability

$= \frac{8}{27} + \frac{4}{81} + \frac{4}{81} + \frac{4}{81} = \frac{36}{81} = \frac{4}{9}$.

[$\because$ The probability of winning the game if previous game was also won is $\frac{2}{1+2} = \frac{2}{3}$ and the probability of winning the game if previous game was a loss is $\frac{1}{1+2} = \frac{1}{3}$].

107. (d) We have

$\frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta) = a\theta\cos\theta$

and

$\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta) = a\theta\sin\theta,$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \tan\theta$

So, the slope of the normal is $-\frac{1}{dy/dx} = -\cot\theta$, which varies as θ.

The equation of the normal at any point is

$y - a(\sin\theta - \theta\cos\theta)$

$= -\cot\theta[x - a(\cos\theta + \theta\sin\theta)]$

$\Rightarrow x\cos\theta + y\sin\theta = a.$

Clearly, it is a line at a constant distance | a | from the origin.

Further, the equation $x\cos\theta + y\sin\theta = a$ represents the equation of a tangent to the circle $x^2 + y^2 = a^2$ for any value of θ, which is a fixed circle.

108. (a) The roots of $ax^2 + bx + c = 0$ are non-real complex.

So, discriminant $D = b^2 - 4ac < 0$

$\Rightarrow a(ax^2 + bx + c) > 0\ \forall\ x$

$\Rightarrow a^2x^2 + abx + ac > 0\ \forall\ x \in R$

109. (a) If $y = e^{ax}\sin(bx + c)$ ....(i)

Then,

$\frac{dy}{dx} = ae^{ax}\sin(bx + c) + b\cos(bx + c)e^{ax}$

$= ay + be^{ax}\cos(bx + c)$ ....(ii)

Again $\frac{d^2y}{dx^2} = a\frac{dy}{dx} + b.ae^{ax}\cos(bx+c)$

$$-b^2e^{ax}\sin(bx+c)$$

$$= a\frac{dy}{dx} + a\left(\frac{dy}{dx} - ay\right) - b^2y \quad [\text{from (i) \& (ii)}]$$

$$= 2a\frac{dy}{dx} - (a^2+b^2)y$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2+b^2)y = 0$$

110. (b) $I = \int_0^{\frac{\pi}{2}} \log\sin x\,dx$ ...(i)

$$= \int_0^{\frac{\pi}{2}} \log\sin\left(\frac{\pi}{2} - x\right)dx = \int_0^{\frac{\pi}{2}} \log\cos x\,dx \quad ...(ii)$$

From (i) and (ii)

$$2I = \int_0^{\frac{\pi}{2}} \log\sin x\,dx + \int_0^{\frac{\pi}{2}} \log\cos x\,dx$$

$$= \int_0^{\pi/2} \log\frac{\sin 2x}{2}dx$$

$$= \int_0^{\frac{\pi}{2}} \log\sin 2x\,dx - \int_0^{\frac{\pi}{2}} \log 2\,dx \;;$$

$$= \int_0^{\frac{\pi}{2}} \log\sin 2x\,dx - \frac{\pi}{2}\log 2$$

Put $2x = t \Rightarrow 2dx = dt$

$$2I = \frac{1}{2}\int_0^{\pi} \log\sin t\,dt - \frac{\pi}{2}\log 2$$

$$= \frac{1}{2}\cdot 2\int_0^{\frac{\pi}{2}} \log\sin t\,dt - \frac{\pi}{2}\log 2$$

[Since $\sin(\pi - t) = \sin t$, see property No. 8]

$$2I = I - \frac{\pi}{2}\log 2 \Rightarrow I = -\frac{\pi}{2}\log_e 2$$

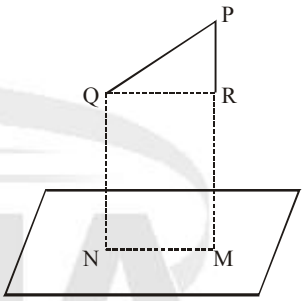
111. (b) Let $\Delta = \begin{vmatrix} \sin 2A & \sin C & \sin B \\ \sin C & \sin 2B & \sin A \\ \sin B & \sin A & \sin 2C \end{vmatrix}$

$$= \begin{vmatrix} 2\sin A\cos A & \sin C & \sin B \\ \sin C & 2\sin B\cos B & \sin A \\ \sin B & \sin A & 2\sin C\cos C \end{vmatrix}$$

The above determinant is the product of two determinants,

$$\Delta = \begin{vmatrix} \sin A & \cos A & 0 \\ \sin B & \cos B & 0 \\ \sin C & \cos C & 0 \end{vmatrix} \times \begin{vmatrix} \cos A & \sin A & 0 \\ \cos B & \sin B & 0 \\ \cos C & \sin C & 0 \end{vmatrix} = 0$$

112. (b) Given plane is $2x - y - 2z - 4 = 0$.



From points P and Q draw PM and QN perpendiculars on the given plane and $QR \perp MP$.

$$|MP| = \frac{|2(-1) - 2 - 0 - 4|}{\sqrt{2^2 + (-1)^2 + (-2)^2}} = \frac{8}{3},$$

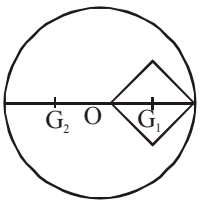
$$|NQ| = \frac{|2.1 - (-1) - 2.2 - 4|}{\sqrt{2^2 + (-1)^2 + (-2)^2}} = \frac{5}{3}$$

Also

$$|PQ| = \sqrt{(1+1)^2 + (-1-2)^2 + (2-0)^2} = \sqrt{17}$$

$$\therefore |NM| = |QR| = \sqrt{PQ^2 - RP^2} = \sqrt{17-1} = 4.$$

113. (b) The area of the circle is $\pi\left(\frac{1}{2}a\right)^2 = \frac{1}{4}\pi a^2$



If x be side of square, then diagonal is $\sqrt{(x^2+x^2)} = \frac{1}{2}a$ given.

$\therefore\ x^2 = \frac{1}{8}a^2 = \text{area of square}$

Also $OG_1 = \frac{1}{2}\left(\frac{1}{2}a\right) = \left(\frac{1}{4}a\right)$.

Area of remainder $= \pi\left(\frac{1}{4}a^2\right) - \left(\frac{1}{8}a^2\right)$.

Taking moments about O, we get

$$\left(\pi\frac{a^2}{4} - \frac{a^2}{8}\right).OG_2 = x^2.\frac{a}{4} = \frac{a^2}{8}\cdot\frac{a}{4}$$

$\therefore\ OG_2 = \frac{a}{8\pi - 4}$.

114. (a) The given quantity is of the form $n^n - (n+1)^{n-1}$, where $n = 1000$

Now, $n^n - (n+1)^{n-1} = n^n - \frac{(n+1)^n}{n+1}$

$$= n^n - \frac{n^n}{n+1}\left\{1+\frac{1}{n}\right\}^n = n^n\left\{1 - \frac{k}{n+1}\right\}$$

Where $k = \left(1+\frac{1}{n}\right)^n$, which lies between 2 and 3

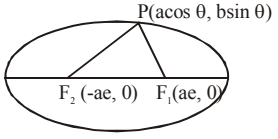
$\therefore\ \frac{k}{n+1} < 1$ if $n \geq 2$

$\therefore\ n^n - (n+1)^{n-1} > 0$ if $n \geq 2$

115. (a) $a = 0, b = 1$

$a + b = \text{Max}(0, 1) = 1$

116. (b) Let $P \equiv (a\cos\theta, b\sin\theta)$



Then, A = area of ΔPF_1F_2

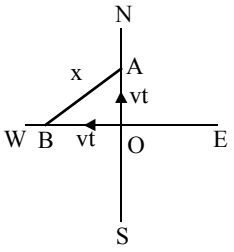
$$= \frac{1}{2}\begin{vmatrix} a\cos\theta & b\sin\theta & 1 \\ ae & 0 & 1 \\ -ae & 0 & 1 \end{vmatrix} = \left|\frac{1}{2}\cdot 2ae\cdot b\sin\theta\right|$$

$= abe\,|\sin\theta|$

Clearly, A is maximum when $|\sin\theta| = 1$

$\therefore$ Maximum value of A = abe

117. (b) Let the two cars move from O, one towards north and the other due west with equal speed v.



At time t they reach at A and B respectively.
Then from the formula,
distance = velocity × time,
we have O A = v t and OB = v.t.
Join A and B. Let AB = x

Then we have to find $\frac{dx}{dt}$.

From the right angled ΔAOB we have

$AB^2 = OA^2 + OB^2$

or $x^2 = (v.t)^2 + (v.t)^2 = 2v^2t^2$;

or $x = \sqrt{2v^2t^2} = \sqrt{2}.v.t.$

$\therefore \frac{dx}{dt} = \sqrt{2}v.$ which is constant.

Hence the two cars separated from each other with a constant speed $\sqrt{2}v$.

118. (d) In case of $R_1, f(x) = 6x + 7$

clearly every element of A has a unique image hence, R_1 represents a function.
Similarly, R_2 and R_3 also represent functions.
In case of R_4, $f(x) = \pm 4x$
$\therefore$ every element of A has two unequal images for example $f(1) = \pm 4$, $f(2) = \pm 8$, etc.
$\therefore$ R_4 is not a function

119. (c) $\lim\limits_{x\to 0}\left(\frac{f(1+x)}{f(1)}\right)^{1/x} = e^{\lim\limits_{x\to 0}\frac{1}{x}\left(\frac{f(1+x)-f(1)}{f(1)}\right)}$

$$= e^{\frac{f'(1)}{f(1)}} = e^2$$

120. (a) $\omega^{10} = \omega^9.\omega = \omega$, $\omega^{11} = \omega^9.\omega^2 = \omega^2$
$\therefore (2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11})$
$= (2-\omega)^2(2-\omega^2)^2$
Now $x^3 - 1 = (x-1)(x-\omega)(x-\omega^2)$,
dividing by $x - 1$
$\therefore x^2 + x + 1 = (x-\omega)(x-\omega^2)$
or $(x^2+x+1)^2 = (x-\omega)^2(x-\omega^2)^2$.
Put $x = 2$, we get $(2-\omega)^2(2-\omega^2)^2 = (7)^2 = 49$