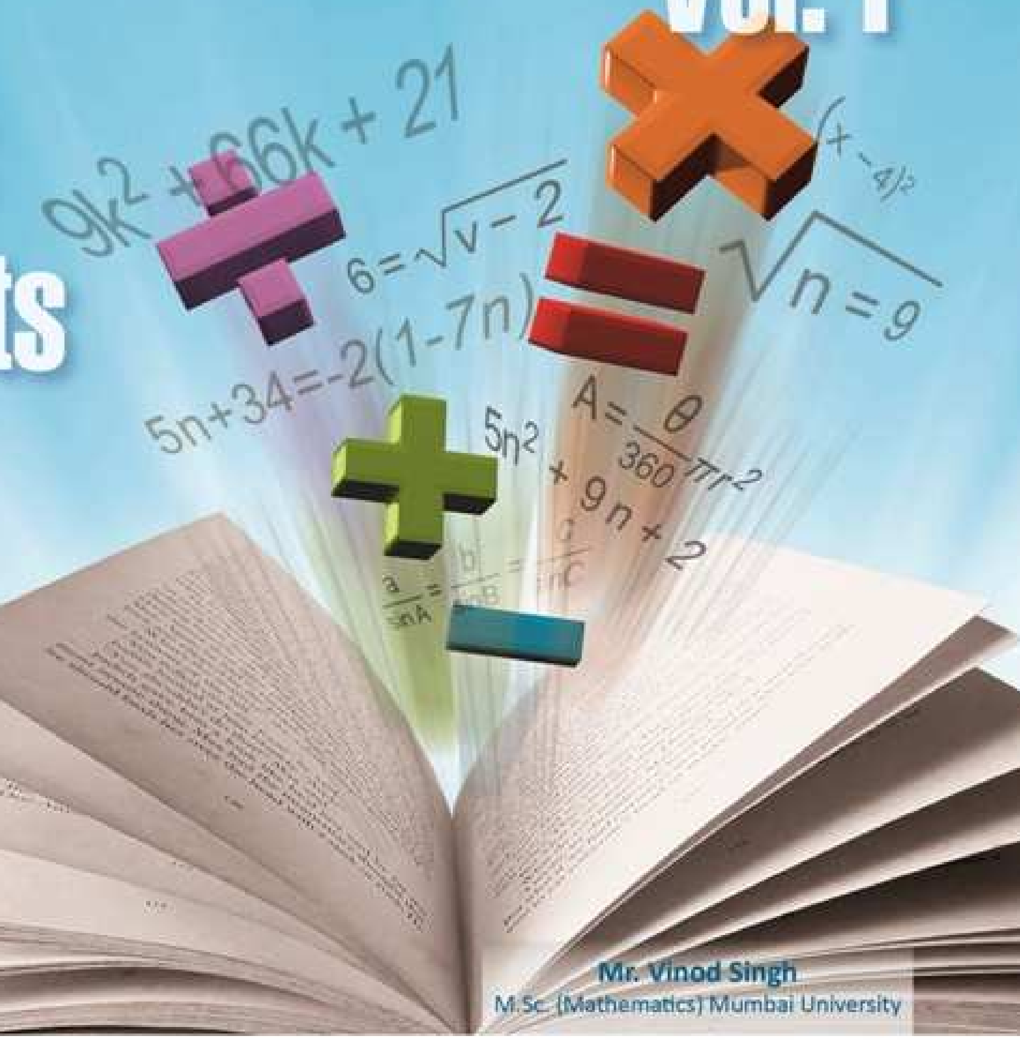
JEE - Main
Mathematics

For all Engineering Entrance Examinations held across India.

2946 MCQ's with Hints

Vol. I



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JEE – Main

Mathematics

Vol. I

Salient Features

- Exhaustive coverage of MCQs subtopic wise.
- '2946' MCQs including questions from various competitive exams.
- Precise theory for every topic.
- Neat, Labelled and authentic diagrams.
- Hints provided wherever relevant.
- Additional information relevant to the concepts.
- Simple and lucid language.
- Self evaluative in nature.

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PREFACE

Mathematics is the study of quantity, structure, space and change. It is one of the oldest academic discipline that has led towards human progress. Its root lies in man's fascination with numbers.

Maths not only adds great value towards a progressive society but also contributes immensely towards other sciences like Physics and Chemistry. Interdisciplinary research in the above mentioned fields has led to monumental contributions towards progress in technology.

Target's "Maths Vol. I" has been compiled according to the notified syllabus for JEE (Main), which in turn has been framed after reviewing various national syllabus.

Target's "Maths Vol. I" comprises of a comprehensive coverage of theoretical concepts and multiple choice questions. In the development of each chapter we have ensured the inclusion of shortcuts and unique points represented as an 'Important Note' for the benefit of students.

The flow of content and MCQ's has been planned keeping in mind the weightage given to a topic as per the JEE (Main).

MCQ's in each chapter are a mix of questions based on theory and numerical and their level of difficulty is at par with that of various engineering competitive examinations.

This edition of "Maths Vol. I" has been conceptualized with a complete focus on the kind of assistance students would require to answer tricky questions, which would give them an edge over the competition.

Lastly, I am grateful to the publishers of this book for their persistent efforts, commitment to quality and their unending support to bring out this book, without which it would have been difficult for me to partner with students on this journey towards their success.

All the best to all Aspirants!

Yours faithfully,
Author

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01 Sets, Relations and Functions

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1.1 Sets

1. Definition:

Any collection of well defined and distinct objects is called a set.

By **"Well-defined collection"** we mean that given a set and an object, it must be possible to decide whether or not the object belongs to the set. The objects in a set are called its members or elements.

Sets are usually denoted by capital letters A, B, C, X, Y, Z etc.

Elements of the sets:

The elements of the set are denoted by small letters i.e., a, b, c, x, y, z etc.

If x is an element of a set A, we write $x \in A$ and if x is not an element of A, we write $x \notin A$.

Eg.

If A = {1, 2, 3, 4, 5}, then $3 \in A$ but $6 \notin A$.

Important Note

❖ Every set is a collection of objects but every collection of objects is not a set.

Examples of well defined collections:

i. The collection of vowels in English alphabet is a set containing five elements a, e, i, o, u.
ii. The collection of first five prime nos. is a set containing the elements 2, 3, 5, 7, 11.
iii. The collection of rivers of India.
iv. The collection of all states of India.
v. The collection of the solutions of the equation $x^2 - 5x + 6 = 0$.
vi. The set of all lines in a particular plane.

Examples of not well defined collections, hence not sets:

i. The collection of **good cricket players** of India.
ii. The collection of **bright students** in class XI of a school.
iii. The collection of **beautiful girls** of the world.
iv. The collection of **rich persons** in India.
v. The collection of **most talented writers** of India.
vi. The collection of **most dangerous animals** of the world.

2. Symbols:

Symbol	Meaning
$\Rightarrow$	Implies
$\in$	Belongs to
$A \subset B$	A is a subset of B
$\Leftrightarrow$	Implies and is implied by
$\notin$	Does not belong to
s.t (: or |)	Such that
$\forall$	For all or for every
$\exists$	There exists
iff	if and only if
&	And
a/b	a is a divisor of b
N	Set of natural nos.
I or Z	Set of integers
R	Set of real nos.
C	Set of complex nos.
Q	Set of rational nos.

3. Representation of a set:

There are two methods for representing a set:

i. Tabulation or Roster or Enumeration or Listing method:

In this method, we list all the members of the set, separating them by commas and enclosing them in curly brackets {}.

Egs.

a. If A is the set of all prime nos. less than 10, then A = {2, 3, 5, 7}.
b. If A is the set of all even nos. lying between 2 and 20, then A = {4, 6, 8, 10, 12, 14, 16, 18}.

ii. Set builder or Rule or Property method:

In this method, we write the set by some special property and write it as

$A = \{x : P(x)\}$
$\quad = \{x/x \text{ has the property } P(x)\}$

and read it as "A is the set of all elements x such that x has the property P".

Egs.

a. If A = {1, 2, 3, 4}, then we can write A = $\{x \in N : x < 5\}$.

b. If A is the set of all odd integers lying between 2 and 51, then A = $\{x : 2 < x < 51, x \text{ is odd}\}$.

Important Notes

❖ The order of writing the elements of a set is immaterial.
Eg.
{1, 2, 3}, {2, 3, 1}, {3, 2, 1}, {1, 3, 2} all denote the same set.

❖ An element of a set is not written more than once. Thus the set {1, 2, 3, 4, 3, 3, 2, 1, 2, 1, 4} can be written as {1, 2, 3, 4}.

4. Null or Empty or Void set:

A set having no element is called a null set. It is denoted by ϕ or { }.

i. ϕ is unique.

ii. ϕ is a subset of every set.

iii. ϕ is never written within brackets i.e., $\{\phi\}$ is not a null set

Egs.

a. $\{x : x \in N, 4 < x < 5\} = \phi$

b. $\{x : x \in R, x^2 + 1 = 0\} = \phi$

c. $\{x : x^2 = 25, x \text{ is an even no.}\} = \phi$

5. Singleton set or Unit set:

A set having one and only one element is called singleton or unit set.

Egs.

i. $\{x : x - 3 = 4\} = \{7\}$ is a singleton set.

ii. $\{x : x + 4 = 0, x \in Z\} = \{-4\}$

iii. $\{x : |x| = 7, x \in N\} = \{7\}$

6. Finite and Infinite sets:

A set is called a **finite set** if it is either void set or its elements can be listed (counted, labelled) by natural numbers 1, 2, 3, ……. and the process of listing or counting of elements surely comes to an end.

And a set which is not finite is called an **infinite set.**

Egs.

i. A = {a, e, i, o, u} is a finite set.

ii. B = {1, 2, 3, 4, …..} is an infinite set.

7. Cardinal number of a finite set:

Number of elements in a finite set A is called cardinal number of a finite set and is denoted by n(A) or o(A). It is also called order of a finite set.

Eg.

If A = {1, 2, 3, 4, 5, 6}, then o(A) = 6

8. Equal sets:

Two sets A and B are said to be equal if every element of A is an element of B and every element of B is an element of A.

Symbolically: A = B if $x \in A \Leftrightarrow x \in B$

Eg.

If A = {4, 8, 10} and B = {8, 4, 10}, then A = B.

9. Equivalent sets:

Two finite sets A and B are equivalent if o(A) = o(B).

Eg.

Sets A = {1, 3, 5, 7},
B = {10, 12, 14, 16} are equivalent
[$\because$ o(A) = 4 = o(B)]

Important Note

❖ Equal sets are always equivalent but equivalent sets may not be equal. In above e.g. A $\neq$ B although they are equivalent.

10. Subsets:

If every element of A is also an element of a set B, then A is called a subset of B.

We write A $\subseteq$ B, which is read as "A is a subset of B" or "A is contained in B".

Thus, $A \subseteq B \Leftrightarrow \{x \in A \Rightarrow x \in B\}$

i. Every set is a subset of itself i.e., $A \subseteq A$.

ii. ϕ is a subset of every set.

iii. If $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$

iv. A = B iff $A \subseteq B$ and $B \subseteq A$

a. Proper subsets: If A is a subset of B and A $\neq$ B, then A is a proper subset of B. If a set A is non-empty, then the null set is a proper subset of A.
We write this as $A \subset B$.

Important Note

❖ If $A \subseteq B$, we may have $B \subseteq A$ but if $A \subset B$, we cannot have $B \subset A$.

b. **Improper subsets:** The null set ϕ is subset of every set and every set is subset of itself, i.e., $\phi \subset A$ and $A \subseteq A$ for every set A.
They are called improper subsets of A.

Thus, every non-empty set has two improper subsets.

It should be noted that ϕ has only one subset ϕ, which is improper.

Eg.

Let A = {1, 2}. Then A has ϕ, {1}, {2}, {1, 2} as its subsets out of which ϕ and {1, 2} are improper and {1} and {2} are proper subsets.

11. Universal set:

Superset of all the sets, i.e., all sets are contained in this set. This is usually denoted by Ω or S or U or X.

12. Power set:

The set of all the subsets of a given set A is said to be the power set A and is denoted by P(A).

Important Note

❖ If A has n elements i.e., o(A) = n, then $o(P(A)) = 2^n$

Eg.

Let A = {a, b, c}, then

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

Here, o(A) = 3

$\therefore$ $o(P(A)) = 2^3 = 8$

13. Operations on sets:

i. Union of sets:

The union of two sets A and B is the set of all those elements which are either in A or in B or in both.

This set is denoted by $A \cup B$ or A + B

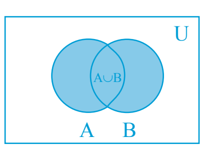
[read as 'A union B' or 'A join B']

Symbolically,

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Eg.

If A = {1, 2, 3} and B = {1, 3, 5, 7}, then $A \cup B$ = {1, 2, 3, 5, 7}



Important Note

❖ $A_1 \cup A_2 \cup \ldots\ldots \cup A_n = \bigcup_{i=1}^{n} A_i$

ii. Intersection of sets:

The intersection of two sets A and B is the set of all elements which are common in A and B.

This set is denoted by $A \cap B$ or AB

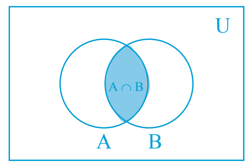
[read as 'A intersection B' or 'A meet B']

Symbolically,

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

Eg.

If A = {1, 2, 3, 4} and B = {2, 4, 6}, then $A \cap B$ = {2, 4}.

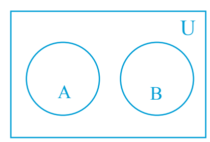


iii. Disjoint sets:

If two sets A and B have no common element i.e., $A \cap B = \phi$, then the two sets A and B are called disjoint or mutually exclusive events.

Eg.

If A = {1, 2, 3} and B = {a, b, c}, then $A \cap B = \phi$.



iv. Difference of sets:

Let A and B be two sets. The difference of A and B written as A – B, is the set of all those elements of A which do not belong to B.

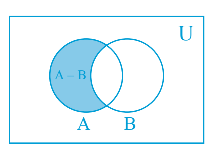
Thus, $A - B = \{x : x \in A \text{ and } x \notin B\}$

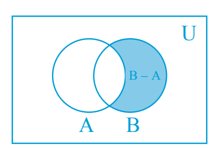
Similarly, the difference B – A is the set of all those elements of B that do not belong to A.

i.e., $B - A = \{x \in B \text{ and } x \notin A\}$

Eg.

If A ={1,3,5,7,9} and B ={2,3,5,7,11}, then A – B = {1, 9} and B – A = {2, 11}.





Important Notes

- $A - B = \phi$ if $A \subset B$
- $A - B \neq B - A$
- The sets $A - B$, $B - A$ and $A \cap B$ are disjoint sets
- $A - B \subseteq A$ and $B - A \subseteq B$
- $A - \phi = A$ and $A - A = \phi$

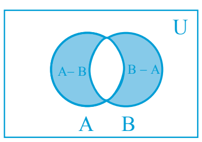
v. Symmetric difference of two sets:

Let A and B be two sets. Then symmetric difference of two sets A and B is the set $(A - B) \cup (B - A)$ or $(A \cup B) - (A \cap B)$ and is denoted by $A \Delta B$ or $A \oplus B$.

i.e., $A \Delta B$ or $A \oplus B = (A - B) \cup (B - A)$
$= (A \cup B) - (A \cap B)$

Eg.

If $A = \{1, 3, 5, 7, 9\}$ and
$B = \{2, 3, 5, 7, 11\}$, then
$A \Delta B = (A - B) \cup (B - A)$
$= \{1, 9\} \cup \{2, 11\}$
$= \{1, 2, 9, 11\}$



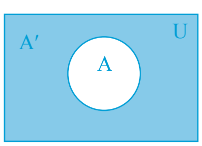
vi. Complement of a set:

Let U be the universal set and A be a set such that $A \subset U$, then the complement of A, denoted by A' or A^c or $U - A$ is defined as

A' or $A^c = \{x : x \in U \text{ and } x \notin A\}$

Eg.

Let $U = \{x : x$ is a letter in English alphabet$\}$
and $A = \{x : x$ is a vowel$\}$, then
$A' = \{x : x$ is a consonant$\}$



Important Notes

- $U' = \phi$
- $\phi' = U$
- $A \cup A' = U$
- $A \cap A' = \phi$
- $(A')' = A$

14. Laws or properties of algebra of sets:

i. Idempotent laws:

For any set A, we have

a. $A \cup A = A$
b. $A \cap A = A$

ii. Identity laws:

For any set A, we have

a. $A \cup \phi = A$
b. $A \cap \phi = \phi$
c. $A \cup U = U$
d. $A \cap U = A$

iii. Commutative laws:

For any two sets A and B, we have

a. $A \cup B = B \cup A$
b. $A \cap B = B \cap A$
c. $A \Delta B = B \Delta A$

i.e., union, intersection and symmetric difference of two sets are commutative.

But difference and cartesian product of two sets are not commutative.

iv. Associative laws:

If A, B and C are any three sets, then

a. $(A \cup B) \cup C = A \cup (B \cup C)$
b. $A \cap (B \cap C) = (A \cap B) \cap C$
c. $(A \Delta B) \Delta C = A \Delta (B \Delta C)$

i.e. union, intersection and symmetric difference of three sets are associative.

But difference and cartesian product of three sets are not associative.

v. Distributive laws:

If A, B and C are any three sets, then

a. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
b. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

vi. De-Morgan's law:

If A, B and C are any three sets then

a. $(A \cup B)' = A' \cap B'$
b. $(A \cap B)' = A' \cup B'$
c. $A - (B \cup C) = (A - B) \cap (A - C)$
d. $A - (B \cap C) = (A - B) \cup (A - C)$

vii. For any two sets A and B:

a. $P(A) \cap P(B) = P(A \cap B)$
b. $P(A) \cup P(B) \subseteq P(A \cup B)$
c. if $P(A) = P(B) \Rightarrow A = B$

where P(A) is the power set of A

15. More results on operations on sets:

For any sets A and B, we have

i. $A \subseteq A \cup B$, $B \subseteq A \cup B$,
$A \cap B \subseteq A$, $A \cap B \subseteq B$

ii. $A - B = A \cap B'$, $B - A = B \cap A'$

iii. $(A - B) \cap B = \phi$

iv. $(A - B) \cup B = A \cup B$

v. $A \subseteq B \Leftrightarrow B' \subseteq A'$

vi. $A - B = B' - A'$

vii. $(A \cup B) \cap (A \cup B') = A$

viii. $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

ix. $A - (A - B) = A \cap B$

x. $A - B = B - A \Leftrightarrow A = B$ and $A \cup B = A \cap B \Rightarrow A = B$

16. Results on cardinal number of some sets:

If A, B and C are finite sets and U be the universal set, then

i. $n(A \cup B) = n(A) + n(B)$ if A and B are disjoint sets.

ii. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

iii. $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

iv. $n(A) = n(A - B) + n(A \cap B)$
$n(B) = n(B - A) + n(A \cap B)$
Here, $n(A - B) = n(A) - n(A \cap B)$
and $n(A - B) = n(A \cup B) - n(B)$

v. $n(A') = n(U) - n(A)$

vi. $n(A' \cap B') = n(A \cup B)'$
$= n(U) - n(A \cup B)$

vii. $n(A' \cup B') = n(A \cap B)'$
$= n(U) - n(A \cap B)$

viii. $n(A \cap B') = n(A) - n(A \cap B)$

ix. $n(A \cap B)$
$= n(A \cup B) - n(A \cap B') - n(A' \cap B)$

x. $n(A \cup B \cup C)$
$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

xi. If $A_1, A_2, A_3, \ldots, A_n$ are disjoint sets, then $n(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n)$
$= n(A_1) + n(A_2) + n(A_3) + \ldots + n(A_n)$

xii. $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

xiii. $n(A \cap B' \cap C')$
$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
$n(B \cap A' \cap C')$
$= n(B) - n(B \cap C) - n(B \cap A) + n(A \cap B \cap C)$
$n(C \cap A' \cap B')$
$= n(C) - n(C \cap A) - n(C \cap B) + n(A \cap B \cap C)$

xiv. $n(A' \cap B' \cap C')$
$= n[(A \cup B \cup C)']$
$= n(U) - n(A \cup B \cup C)$

17. Ordered pair:

If A be a set and a, b $\in$ A, then the ordered pair of elements a and b in A are denoted by (a, b), where a is called the first co-ordinate and b is called the second co-ordinate.

Important Notes

❖ Ordered pairs (a, b) and (b, a) are different,
i.e., $(a, b) \neq (b, a)$

❖ Ordered pairs (a, b) and (c, d) are equal iff a = c and b = d
i.e., (a, b) = (c, d) iff a = c, b = d.

18. Cartesian product of two sets:

i. Let A and B be two non-empty sets. The cartesian product of A and B denoted by $A \times B$ is defined as the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$
Symbolically,
$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$
Similarly,
$B \times A = \{(b, a) : b \in B \text{ and } a \in A\}$
Eg.
If $A = \{1, 2, 3\}$ and $B = \{x, y\}$, then
$A \times B = \{(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)\}$
and
$B \times A = \{(x, 1), (y, 1), (x, 2), (y, 2), (x, 3), (y, 3)\}$

Important Note

❖ If $A \neq B$, then $A \times B \neq B \times A$

ii. If there are three sets A, B, C and $a \in A$, $b \in B$, $c \in C$, then we form an ordered triplet (a, b, c). The set of all ordered triplets (a, b, c) is called the cartesian product of these sets A, B and C.
i.e., $A \times B \times C = \{(a, b, c): a \in A, b \in B, c \in C\}$

19. Order of A × B:

i. If $o(A) = m$ and $o(B) = n$, then $o(A \times B) = mn$

ii. If $A = \phi$, $B = \phi$, then $A \times B = \phi$

iii. If $A = \phi$, $B = \{a, b, c\}$, then $A \times B = \phi$
Similarly,
If $A = \{a, b, c\}$, $B = \phi$, then $A \times B = \phi$

20. Some results on cartesian products of sets:

i. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

ii. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

iii. $A \times (B - C) = (A \times B) - (A \times C)$

iv. $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

v. If $A \subseteq B$ and $C \subseteq D$, then
$(A \times C) \subseteq (B \times D)$

vi. If $A \subseteq B$, then
$A \times A \subseteq (A \times B) \cap (B \times A)$

vii. If A and B are non-empty subsets, then
$A \times B = B \times A \Leftrightarrow A = B$.

viii. If $A \subseteq B$, then $(A \times C) \subseteq (B \times C)$

1.2 Relations

1. Relations from a set A to a set B:

A relation (or binary relation) R, from a non-empty set A to another non-empty set B, is a subset of $A \times B$.
i.e., $R \subseteq A \times B$ or $R \subseteq \{ (a, b): a \in A, b \in B\}$
Now, if (a, b) be an element of the relation R, then we write a R b (read as'a' is related to 'b')
i.e., $(a, b) \in R \Leftrightarrow a\,R\,b$
In particular, if B = A, then the subsets of $A \times A$ are called relations from the set A to A.
i.e., any subset of $A \times A$ is said to be a relation on A.

Egs.

i. Let $A=\{1, 3, 5, 7\}$ and $B=\{6, 8\}$, then R be the relation 'is less than' from A to B is
1R6, 1R8, 3R6, 3R8, 5R6, 5R8, 7R8

$\therefore$ $R = \{(1, 6), (1, 8), (3, 6), (3, 8), (5, 6), (5, 8), (7, 8)\}$

ii. Let $A = \{1, 2, 3, \ldots.., 34\}$, then R be the relation 'is one fourth of' on A is
1R4, 2R8, 3R12, 4R16, 5R20, 6R24, 7R28, 8R32

$\therefore$ $R = \{(1, 4), (2, 8), (3, 12), (4, 16), (5, 20), (6, 24), (7, 28), (8, 32)\}$

2. Number of possible relations from A to B:

If A has m elements and B has n elements, then $A \times B$ has $m \times n$ elements and total number of possible relations from A to B is 2^{mn}.

3. Domain and Range of a relation:

i. Domain of $R = \{a : (a, b) \in R\}$
i.e., if R is a relation from A to B, then the set of first elements of ordered pairs in R is called the domain of R.

ii. Range of $R = \{b : (a, b) \in R\}$
i.e., if R is a relation from A to B, then the set of second elements of ordered pairs in R is called the range of R.

Eg.

If $R = \{(4, 7), (5, 8), (6, 10)\}$ is a relation from the set $A = \{1, 2, 3, 4, 5, 6\}$ to the set $B = \{6, 7, 8, 9, 10\}$, then
domain of $R = \{4, 5, 6\}$ and
range of $R = \{7, 8, 10\}$.

Important Notes

- If $R = A \times B$, then domain of $R \subseteq A$ and range of $R \subseteq B$.
- The domain as well as range of the empty set ϕ is ϕ.
- If R is a relation from the set A to the set B, then the set B is called the co-domain of the relation R.
i.e., Range $\subseteq$ Co-domain.

4. Inverse relation:

If R is a relation from a set A to a set B, then the inverse relation of R, to be denoted by R^{-1}, is a relation from B to A.
Symbolically,
$R^{-1} = \{(b, a) : (a, b) \in R\}$
Thus, $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}\ \forall\ a \in A, b \in B$

i. Domain (R^{-1}) = Range (R) and
Range (R^{-1}) = Domain (R)

ii. $(R^{-1})^{-1} = R$

Eg.

If $R = \{(1, 2), (3, 4), (5, 6)\}$, then
$R^{-1} = \{(2, 1), (4, 3), (6, 5)\}$

$\therefore$ $(R^{-1})^{-1} = \{(1, 2), (3, 4), (5, 6)\} = R$

Here, domain (R) = {1, 3, 5},
range (R) = {2, 4, 6} and
domain (R^{-1}) = {2, 4, 6},
range (R^{-1}) = {1, 3, 5}

Clearly, dom (R^{-1}) = range (R) and
range (R^{-1}) = dom (R)

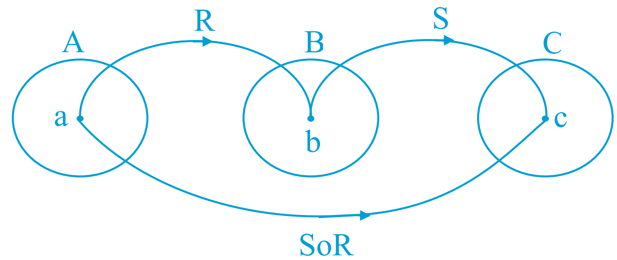
5. **Universal relation:**
A relation R in a set A is called the universal relation in A if $R = A \times A$.
Eg.
If $A = \{a, b, c\}$, then
$A \times A = \{(a, a), (a, b), (a, c), (b, a), (b, c),$
$(b, b) (c, a), (c, b), (c, c)\}$
is the universal relation in A.

6. **Identity relation:**
A relation R in a set A is called identity relation in A, if
$R = \{(a, a) : a \in A\} = I_A$
Eg.
If $A = \{a, b, c\}$, then $I_A = \{(a, a), (b, b), (c, c)\}$

7. **Void relation:**
A relation R in a set A is called void relation if $R = \phi$.

8. **Various types of relation:**
Let A be a non-empty set, then a relation R on A is said to be
i. **Reflexive:**
If aRa $\forall$ a $\in$ A i.e., $(a, a) \in R \ \forall \ a \in A$
Eg.
If $A = \{2, 4, 7\}$, then relation
$R = \{(2, 2), (4, 4), (7, 7)\}$ is reflexive.
ii. **Symmetric:**
If aRb $\Rightarrow$ bRa $\forall$ a, b $\in$ A
i.e., if $(a, b) \in R \Rightarrow (b, a) \in R \ \forall \ a, b \in A$
Eg.
If $A = \{2, 4, 7\}$, then
$R = \{(2, 4), (4, 2), (7, 7)\}$ is symmetric.
iii. **Transitive:**
If aRb and bRc $\Rightarrow$ aRc $\forall$ a, b, c $\in$ A
i.e., if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \ \forall \ a, b, c \in A$.
Eg.
If $A = \{2, 4, 7\}$, then relation
$R = \{(2, 4), (4, 7), (2, 7), (4, 4)\}$ is transitive.
iv. **Anti-symmetric:**
If aRb and bRa $\Rightarrow$ a = b $\forall$ a, b $\in$ A
v. **Equivalence relation:**
A relation R on a set A is said to be an equivalence relation on A iff R is
i. Reflexive
ii. Symmetric and
iii. Transitive
i.e., for equivalence relation R in A
i. aRa $\forall$ a$\in$A
ii. aRb $\Rightarrow$ bRa $\forall$ a, b $\in$ A
iii. aRb and bRc $\Rightarrow$ aRc $\forall$ a, b, c $\in$ A

9. **Composition of two relations:**
If A, B and C are three sets such that $R \subseteq A \times B$ and $S \subseteq B \times C$, then $(SoR)^{-1} = R^{-1}oS^{-1}$. It is clear that aRb, bSc $\Rightarrow$ aSoRc.

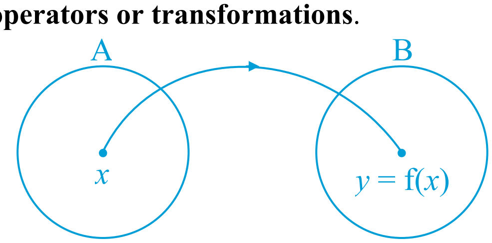


This relation is called the composition of R and S.
Eg.
If $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$, $C = \{p, q, r, s\}$ be three sets such that $R = \{(1, a), (2, b), (1, c), (2, d)\}$ is a relation from A to B and $S = \{(a, s), (b, r), (c, r)\}$ is a relation from B to C, then SoR is a relation from A to C given by
$SoR = \{(1, s), (2, r), (1, r)\}$
In this case, RoS does not exist.
In general, RoS $\neq$ SoR.

10. If R is a relation on a set A, then
i. R is reflexive $\Rightarrow R^{-1}$ is reflexive
ii. R is symmetric $\Rightarrow R^{-1}$ is symmetric
iii. R is transitive $\Rightarrow R^{-1}$ is transitive

1.3 Functions or Mappings

1. **Definition:**
Let A and B be any two non-empty sets. If to each element $x \in A \ \exists$ a unique element $y \in B$ under a rule f, then this relation is called function from A into B and is written as $f : A \to B$ or $A \xrightarrow{f} B$.
The other terms used for functions are **operators or transformations**.



Important Notes

- If $x \in A$, $y = f(x) \in B$, then $(x, y) \in f$
- If $(x_1, y_1) \in f$ and $(x_2, y_2) \in f$, then $y_1 = y_2$

Real valued function:

Let f : A → B and A ⊆ R & B ⊆ R be defined by y = f(x), where $x \in$ A, $y \in$ B, then f is called a real valued function of a real variable.

2. Domain, Co-domain and Range:

i. Domain: The set of A is called the domain of f i.e., all possible values of x for which f(x) exists (denoted by D_f).

ii. Co-domain: The set of B is called the co-domain of f (denoted by C_f).

iii. Range: The set of all f - images of the elements of A is called the range of function f.

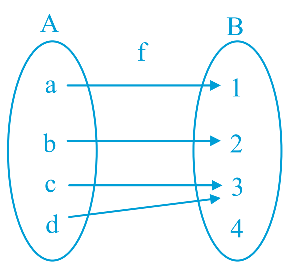
i.e., all possible values of f(x), for all values of x (denoted by R_f)

∴ Range of f = {f(x) : $x \in$ A}

Important Note

❖ The range of f is always a subset of co-domain B. i.e., $R_f \subseteq C_f$

Eg.



From figure: Domain = {a, b, c, d} = A

Co-domain = {1, 2, 3, 4} = B

Range = {1, 2, 3}

So, $R_f \subseteq C_f$

3. Algebra of functions:

Let f and g be two real valued functions with domains D_f and D_g, then

i. Sum function is defined by

(f + g) (x) = f(x) + g (x)

and **domain** of f(x) + g(x) is $D_f \cap D_g$.

ii. Difference function is defined by (f − g)(x) = f(x) − g(x)

and **domain** of f(x) − g(x) is $D_f \cap D_g$.

iii. Multiplication by scalar is defined by (α f)(x) = α f(x)

iv. Product function is defined by (fg) (x) = f(x). g(x) and **domain** of f(x) g(x) is $D_f \cap D_g$.

v. Quotient function is defined by

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, g($x$) ≠ 0 and

domain of $\frac{f(x)}{g(x)}$ is $D_f \cap D_g - \{g(x) = 0\}$

vi. Domain of $\sqrt{f(x)}$ is $D_f \cap \{x : f(x) \geq 0\}$

4. One-one function:

A function f : A → B is said to be one-one if different elements of A have different images in B i.e., no two different elements of A have the same image in B. Such a mapping is also known as **injective mapping** or an **injection** or **monomorphism**.

Method to test one-one: If $x_1, x_2 \in$ A, then $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ and

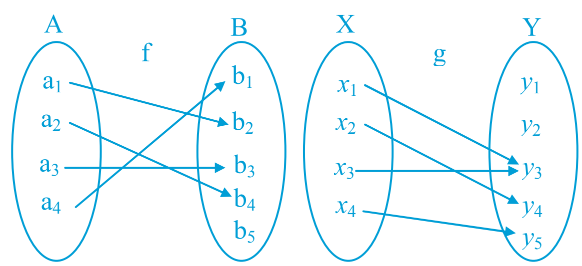
$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

Important Note

❖ A function is one-one, if it is increasing or decreasing.

Eg.

Let f : A → B and g : X → Y be two functions represented by the following diagrams.



Clearly, f : A → B is a one-one function. But g : X → Y is not one-one function because two distinct elements x_1 & x_3 have the same image under function g.

5. Onto function:

Let f : A → B, if every element in B has at least one pre-image in A, then f is said to be **onto function** or **surjective mapping** or **surjection**.

Important Note

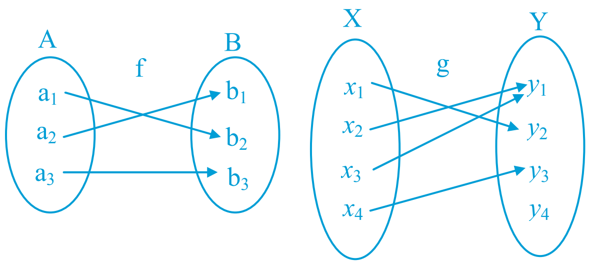
❖ If f^{-1} (y) $\in$ A, ∀ $y \in$ B, then function is onto.

In other words,

Range of f = Co-domain of f

Eg.

In the following diagrams:



f : A → B is onto function. But g : X → Y is not onto funtion because Range ≠ Co-domain.

6. Into function:

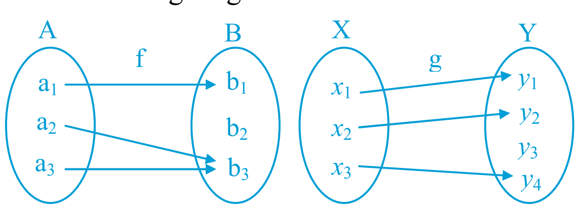
A funtion f : A → B is an into function, if there exists an element in B having no pre-image in A.

Important Note

- ❖ If f(A) ⊂ B i.e., Range ⊂ Co-domain, then the function is into or f : A → B is an into function, if it is not an onto function.

Eg.

The following diagrams show into functions:



Because in both the diagrams $R_f \subset C_f$.

7. Bijection (one-one onto function):

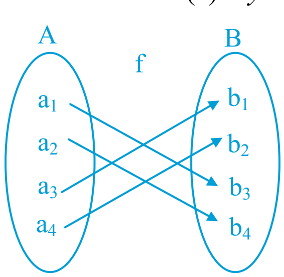
A function f : A → B is a **bijection** or **bijective** , if it is one-one as well as onto.

In other words, a function f : A → B is a bijection if

i. it is one-one i.e., $f(x) = f(y) \Rightarrow x = y$ $\forall\ x, y \in A$

ii. it is onto i.e., $\forall\ y \in B$, there exists $x \in A$ such that $f(x) = y$

Eg.



Clearly, f is a bijection, since it is both injective as well as surjective.

8. Many-one function:

A function f : A → B is said to be a many-one function, if two or more elements of set A have the same image in B.

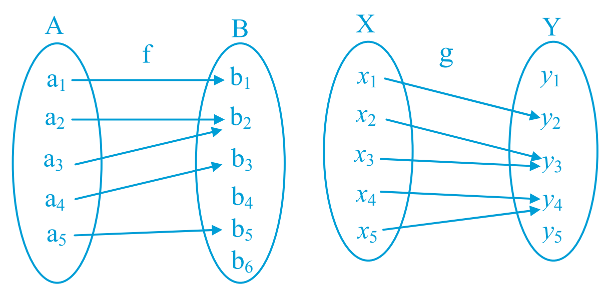
In other words, f : A → B is a many-one function, if it is not a one-one function.

Important Notes

- ❖ f : A → B is a many-one function, if there exists $x_1, x_2 \in A$ such that $x_1 \neq x_2$ but $f(x_1) = f(x_2)$
- ❖ It can also be defined as a function is many-one, if it has local maximum or local minimum.

Eg.

The following diagrams show many-one functions:



9. Inverse of a function:

If f : A → B be one-one and onto function, then the mapping $f^{-1}(B) \to A$ such that $f^{-1}(b) = a$ (where $a \in A$ & $b \in B$) is called inverse function of the function f : A → B.

or

Let f : A → B be a one-one and onto function, then there exists a unique function, g : B → A such that $f(x) = y \Leftrightarrow g(y) = x$, $\forall\ x \in A$ and $y \in B$. Then g is said to be inverse of f.

Thus, $g = f^{-1}: B \to A = \{(f(x), x) \mid (x, f(x)) \in f\}$

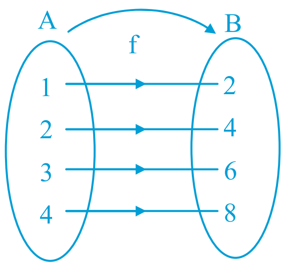
Eg.

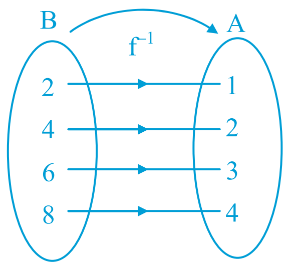
Let us consider one-one function with domain A and range B,

where A = {1, 2, 3, 4} and B = {2, 4, 6, 8} and f : A → B is given by $f(x) = 2x$, then write f and f^{-1} as a set of ordered pairs.

So, f = {(1, 2), (2, 4), (3, 6), (4, 8)}

and f^{-1} = {(2, 1), (4, 2), (6, 3), (8, 4)}





Important Notes

In above function,
- Domain of f = {1, 2, 3, 4} = range of f^{-1}
- Range of f = {2, 4, 6, 8} = domain of f^{-1}

Which represents for a function to have its inverse, it must be **one-one onto** or **bijective**.

10. Graph of a function:

If f : A → B be a function defined by $y = f(x)$, then graph of f is defined as a subset of A × B given by

$G(f) = \{(x, f(x)) : x \in A\}$

11. Some particular functions with their graphs:

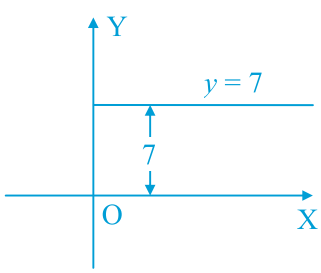
i. Constant function: A function f : X → Y is said to be constant function, if its range is a singleton set i.e., $f(x) = c \ \forall \ x \in X$, where c is some constant.

Eg.

f : R → R defined by $y = f(x) = 7$ is a constant function

[∵ f(1) = 7, f(2) = 7, f(3) = 7, …..]

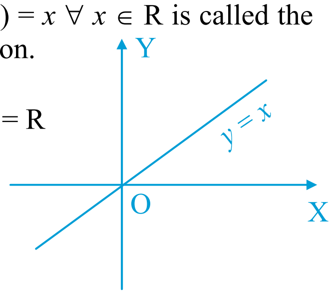
Here, $D_f = R$ and $R_f = 7 = c$



ii. Identity function: The function f defined by $f(x) = x \ \forall \ x \in R$ is called the identity function.

Here,

$D_f = R$ and $R_f = R$



iii. Polynomial function: A function f defined by

$f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$;

where $a_0, a_1, a_2, \dots, a_n$ are real constants and n is non-negative integer, is called a polynomial function.

iv. Rational function: A function f(x) which can be expressed as $\dfrac{g(x)}{h(x)}$, where g(x) and h(x) are polynomials and $h(x) \neq 0$ is called a rational function.

v. Modulus function or Absolute value or Numerical function:

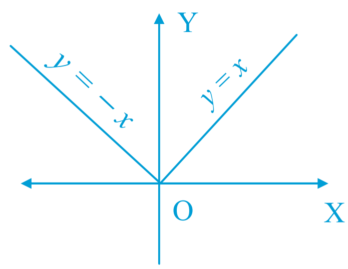
A function f : R → R defined by

$f(x) = |x|$ or $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

is called the absolute value or modulus function.

Here, $D_f = R$ and

$R_f = R^+ = [0, \infty)$



Properties of Modulus of a real number:

$\forall \ x, y \in R$, we have

a. $|x| = \max(x, -x)$
b. $|x|^2 = |-x|^2 = x^2$
c. $|xy| = |x|\,|y|$
d. $\left|\dfrac{x}{y}\right| = \dfrac{|x|}{|y|}$, $[y \neq 0]$
e. $|x + y| \leq |x| + |y|$
f. $|x - y| \leq |x| + |y|$
g. $|x - y| \geq \big||x| - |y|\big|$
h. $|x + y| \geq \big||x| - |y|\big|$
i. $|x| \leq k \Rightarrow -k \leq x \leq k, (k > 0)$
j. $|x| \geq k \Rightarrow -k \geq x$ or $x \geq k, (k > 0)$

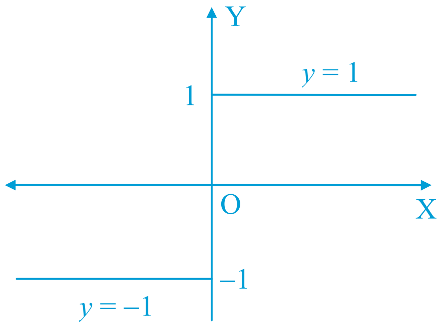
vi. Signum function:

The function f defined by

$f(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ or $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$

is called the signum function.

Here, $D_f = R$ and $R_f = \{-1, 0, 1\}$



vii. Greatest Integer function or Step function or Floor function:

The function f defined by $f(x) = [x] \le x$, $\forall\ x \in R$ is called greatest integer function.

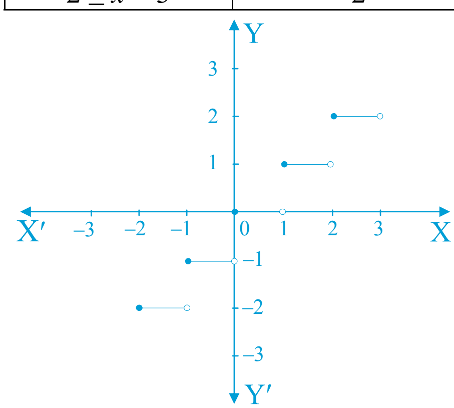
$[x]$ indicates the integral part of x which is nearest and smaller integer is x.

Thus, $[x]$ = x (if x is an integer)
= an integer immediately on the left of x (if x is not an integer)

Here, $D_f = R$ and $R_f = I$

For graph of f, we construct the table of values

x	$y = [x]$
$-2 \le x < -1$	-2
$-1 \le x < 0$	-1
$0 \le x < 1$	0
$1 \le x < 2$	1
$2 \le x < 3$	2



Some facts about the function $f(x) = [x]$:

a. $[x] = x$ iff $x \in I$
b. $[x] < x$ iff $x \notin I$
c. $[x] = k$, $(k \in I)$ iff $k \le x < k + 1$
d. $[x + I] = [x] + I$, if I is an integer and $x \in R$
e. $[-x] = -[x]$, if $x \in I$
f. $[-x] = -[x] - 1$, if $x \notin I$

viii. Fractional part function:

The function defined by the rule $f(x) = x - [x]$, where $[x]$ indicates the integral part of x is called the fractional part function.

Here, $D_f = R\ \forall\ x \in R$

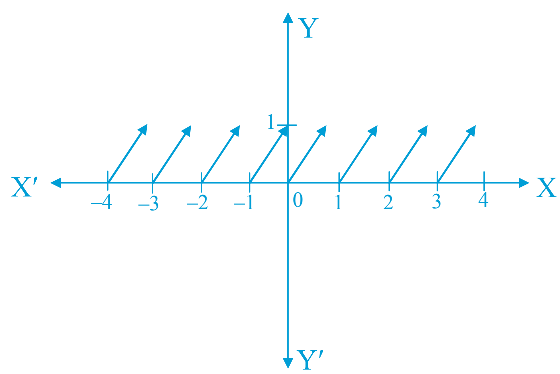
and $R_f = [0, 1)$ $\left[\begin{array}{l} \because\ [x] \le x < [x] + 1 \\ \Rightarrow 0 \le x - [x] < 1 \\ \Rightarrow 0 \le f(x) < 1 \end{array}\right]$

Some facts about the function $f(x) = x - [x]$:

a. $f(x) = 0$ iff x is an integer.
b. $f(x) = x$ iff $0 \le x < 1$.
c. $0 < f(x) < 1$ iff x is not an integer.
d. $f(1 + x) = f(x)\ \forall\ x \in R$
i.e., f is a periodic function with period 1.

For graph of f, construct the table of values:

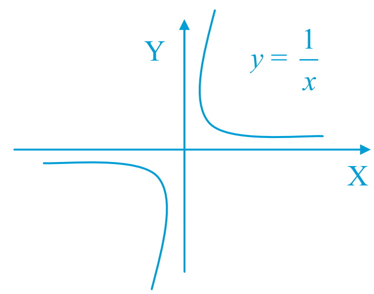
x	$[x]$	$y = x - [x]$
$-2 \le x < -1$	-2	$x + 2$
$-1 \le x < 0$	-1	$x + 1$
$0 \le x < 1$	0	x
$1 \le x < 2$	1	$x - 1$
$2 \le x < 3$	2	$x - 2$



ix. Reciprocal function:

The function f defined by $f(x) = \dfrac{1}{x}$ is called reciprocal function.

Here, $D_f = R - \{0\}$ and $R_f = R - \{0\}$



x. Exponential function:

If $a > 0$, then the function defined by $f(x) = a^x \ \forall \ x \in R$, is called the general exponential function with base a.

Here,

$$D_f = R \text{ and } R_f = \begin{cases} \{1\} & \text{if } a = 1 \\ (0, \infty) & \text{if } a > 0,\ a \neq 1 \end{cases}$$

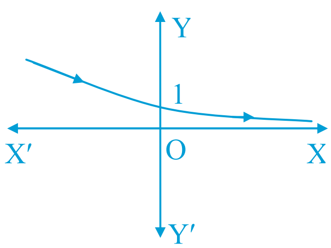
In particular, $f(x) = e^x$, $x \in R$ is called the **natural exponential function**.

Here, $D_f = R$ and $R_f = (0, \infty)$

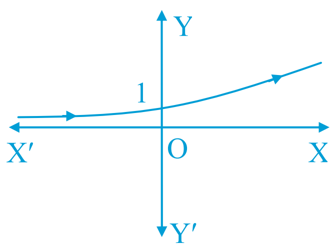
Important Note

❖ a^x increases if $a > 0$ and a^x decreases if $0 < a < 1$.

If $0 < a < 1$



If $a > 1$



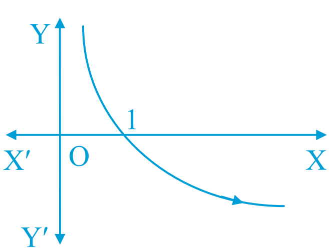
xi. Logarithmic function:

The function defined by $f(x) = y = \log_a x$ iff $x = a^y$ $(a > 0,\ a \neq 1)$, $x > 0$ is called logarithmic function.

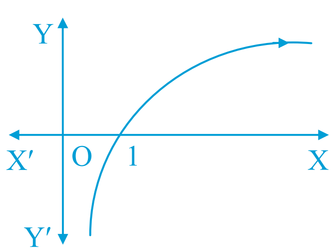
Here, $D_f = (0, \infty)$ and $R_f = R$.

In particular, the function $f(x) = \log_e x$ is called **natural logarithmic function** and $f(x) = \log_{10} x$ is called **common logarithmic function**.

If $0 < a < 1$



If $a > 1$



Some properties of logarithmic function:

a. $y = \log_a x$ iff $x = a^y$, $x > 0$, $y \in R$

b. $\log_a 1 = 0$ and $\log_a a = 1$

c. $a^{\log_a x} = x$, for $x > 0$

d. $\log_a (xy) = \log_a x + \log_a y$, $x > 0$, $y > 0$.

e. $\log_a \left(\dfrac{x}{y}\right) = \log_a x - \log_a y$

f. $\log_a (x^n) = n \log_a x$

g. $\log_{a^n} x = \dfrac{1}{n} \log_a x$

h. $\log_a x = \dfrac{\log x}{\log a}$

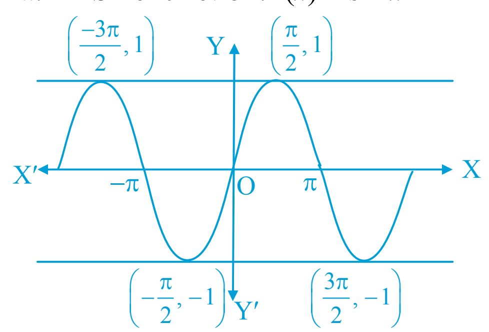
i. For $x \leq 0$, $\log_a x$ is not defined.

j. $\log_a x$ decreases if $0 < a < 1$ and increases if $a > 1$.

xii. Power function: A function $f : R \to R$ defined by $f(x) = x^\alpha$, $\alpha \in R$ is called a power function.

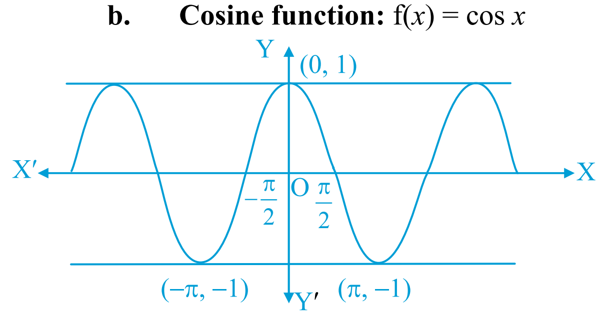
xiii. Trigonometric functions:

a. Sine function: $f(x) = \sin x$



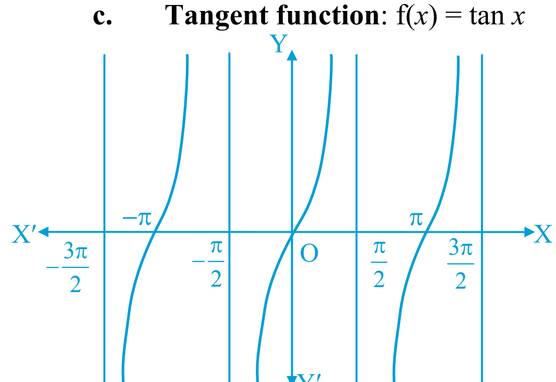
The domain of sine function is R and the range is $[-1, 1]$.

b. Cosine function: $f(x) = \cos x$



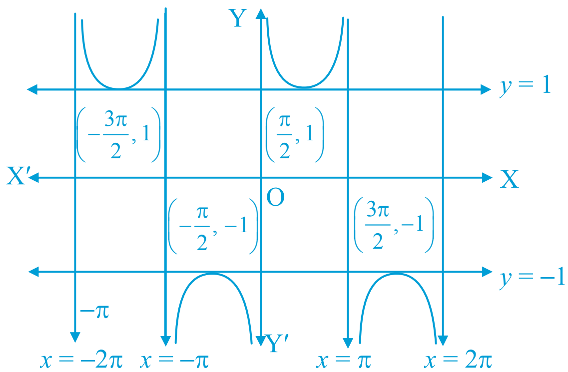
The domain of cosine function is R and the range is $[-1, 1]$.

c. Tangent function: $f(x) = \tan x$



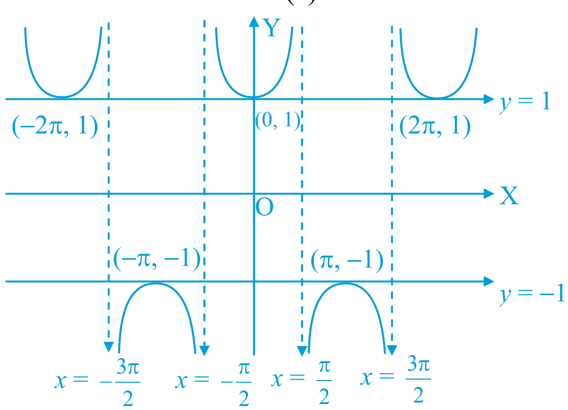
Here, domain of tangent function is $R - \left\{\frac{(2n+1)\pi}{2},\ n \in I\right\}$ and range is R.

d. Cosecant function: $f(x) = \text{cosec } x$



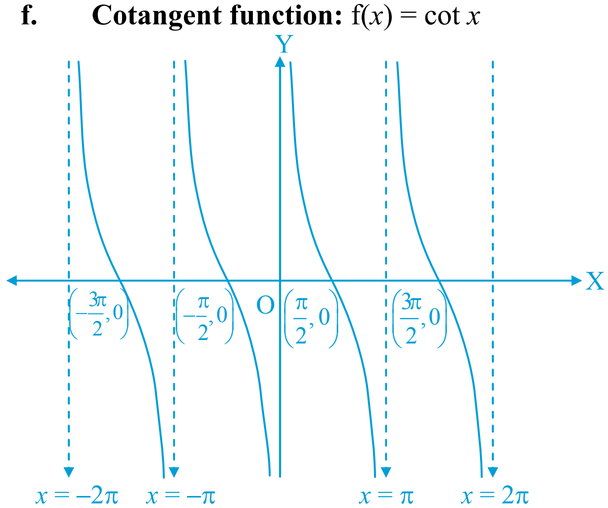
Here, domain $\in R - \left\{(2n\pi + 1)\frac{\pi}{2}, n \in I\right\}$ and range $\in R - (-1, 1)$.

e. Secant function: $f(x) = \sec x$



Here, domain $\in R - \left\{(2n + 1)\frac{\pi}{2}\right\}$ and range $\in R - (-1, 1)$.

f. Cotangent function: $f(x) = \cot x$



Here, domain $\in R - \{n\pi \,/\, n \in I\}$ and range $\in R$.

12. Domain and range of some standard functions:

Function	Domain	Range
Polynomial function	R	R
Identity function x	R	R
Constant function K	R	{K}
Reciprocal function $\frac{1}{x}$	R – {0}	R – {0}
x^2, $\lvert x\rvert$	R	$[0, \infty)$
x^3, $x\lvert x\rvert$	R	R
Signum function	R	{– 1, 0, 1}
$x + \lvert x\rvert$	R	$[0, \infty)$
$x - \lvert x\rvert$	R	$R^- \cup \{0\}$
$[x]$	R	I
$x - [x]$	R	$[0, 1)$
$\sqrt{x}$	$[0, \infty)$	$[0, \infty)$
a^x	R	R^+
$\log x$	R^+	R
$\sin x$	R	$[-1, 1]$
$\cos x$	R	$[-1, 1]$
$\tan x$	$R - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots\right\}$	R
$\cot x$	$R - \{0, \pm\pi, \pm 2\pi, \ldots\}$	R
$\sec x$	$R - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots\right\}$	$(-\infty, -1] \cup [1, \infty)$
$\text{cosec } x$	$R - \{0, \pm\pi, \pm 2\pi, \ldots\}$	$(-\infty, -1] \cup [1, \infty)$
$\sin^{-1} x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	R	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1} x$	R	$(0, \pi)$
$\sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\text{cosec}^{-1} x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

13. Even and odd functions:

A function $y = f(x)$ is said to be

i. **Even** if $f(-x) = f(x)$

ii. **Odd** if $f(-x) = -f(x)$

iii. **Neither even nor odd** if $f(-x) \neq \pm f(x)$

Egs.

i. $f(x) = e^x + e^{-x}$, $f(x) = x^2$, $f(x) = x \sin x$, $f(x) = \cos x$, $f(x) = x^2\cos x$ all are **even functions**.

ii. $f(x) = e^x - e^{-x}$, $f(x) = \sin x$, $f(x) = x^3$, $f(x) = x \cos x$, $f(x) = x^2 \sin x$ all are **odd functions**.

Properties of even and odd functions:

i. The product of two even or two odd functions is an even function.

ii. The product of an even function by an odd function or vice versa is an odd function.

iii. The sum of even and odd function is neither even nor odd function.

iv. Zero function $f(x) = 0$ is the only function which is even and odd both.

v. Every function $f(x)$ can be expressed as the sum of even and odd function. i.e.,

$$f(x) = \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(x) - f(-x)]$$

$$= F(x) + G(x)$$

Here, $F(x)$ is even and $G(x)$ is odd.

[$\because F(-x) = F(x)$ and $G(-x) = -G(x)$]

14. Periodic function:

A function is said to be periodic function, if there exists a constant $T > 0$ such that $f(x + T) = f(x - T) = f(x)\ \forall\ x \in$ domain. Here, the least positive value of T is called the period of the function.

i. **Periodic functions**

Functions	Period
$\sin^n x$, $\cos^n x$; (if n = even)	π
$\sec^n x$, $\operatorname{cosec}^n x$; (if n is odd and fraction)	2π
$\lvert\sin x\rvert$, $\lvert\cos x\rvert$, $\lvert\tan x\rvert$, $\lvert\cot x\rvert$, $\lvert\operatorname{cosec} x\rvert$, $\lvert\sec x\rvert$	π
$x - [x]$, $\sin(x - [x])$, $\sin(x - [-x])$, $x - [-x]$	1
$\sin^{-1}(\sin x)$, $\cos^{-1}(\cos x)$	2π
$\left(\frac{1}{2}\right)^{\sin x}, \left(\frac{1}{2}\right)^{\cos x}, \left(\frac{1}{2}\right)^{\sin x} + \left(\frac{1}{2}\right)^{\cos x}$	2π
$\sqrt{\cos x}$, $\sqrt{\frac{1+\cos x}{2}}$	2π
$(\lvert\sin x\rvert + \lvert\cos x\rvert)$, $\sin^4 x + \cos^4 x$	$\frac{\pi}{2}$
$\cos x + \cos\frac{x}{2} + \cos\left(\frac{x}{2^2}\right) + \cos\left(\frac{x}{2^3}\right) + \ldots + \cos\left(\frac{x}{2^{n-1}}\right) + \cos\left(\frac{x}{2^n}\right)$	$2^n\pi$
$\cos(\cos x) + \cos(\sin x)$	$\frac{\pi}{2}$
$\sin(\sin x) + \sin(\cos x)$	2π
$\frac{\lvert\sin x + \cos x\rvert}{\lvert\sin x\rvert + \lvert\cos x\rvert} = \frac{\left\lvert\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)\right\rvert}{\lvert\sin x\rvert + \lvert\cos x\rvert}$	π
$2^{\sin x} + 2^{\cos x}$	2π

ii. **Some non-periodic functions:**

$\sin\sqrt{x}$, $\cos\sqrt{x}$, $\cos x^2$, $\sin x^2$, $x^2 \pm \cos x$, $x^2 \pm \sin x$, $\sin\frac{1}{x}$, $x \cos x$, $(\cos\sqrt{3}x + \cos 3x)$, $(\sin x + \{x\})$, $(\sin x + x - [x])$, $\left(\frac{1}{x}\right)$

[where $\{x\}$ is fractional part function & $[x]$ is a greatest integer function]

Properties of periodic function:

i. If $f(x)$ is periodic with period T, then

a. $a.f(x)$ is periodic with period T.

b. $f(x + a)$ is periodic with period T.

c. $f(x) \pm a$ is periodic with period T.

where a is any constant.

We know $\sin x$ has period 2π.

Then $f(x) = 5(\sin x) + 7$ is also periodic with period 2π.

i.e., "If constant is added, subtracted, multiplied or divided in periodic function, period remains same."

ii. If $f(x)$ is periodic with period T, then

$kf(ax + b)$ has period $\frac{T}{|a|}$.

i.e., period is only affected by coefficient of x, where k, a, b $\in$ constant.

We know $f(x) = \left\{5\sin\left(2x + \frac{\pi}{7}\right)\right\} - 12$ has the period $\frac{2\pi}{|2|} = \pi$, as $\sin x$ is periodic with period 2π.

iii. If $f_1(x)$, $f_2(x)$ are periodic functions with periods T_1, T_2 respectively, then we have, $h(x) = f_1(x) + f_2(x)$ has period

$$= \begin{cases} \frac{1}{2}\text{ L.C.M. of } \{T_1, T_2\}; & \text{If } f_1(x) \text{ and } f_2(x) \text{ are complementary} \\ & \text{pair wise comparable even functions} \\ \text{L.C.M. of } \{T_1, T_2\}; & \text{otherwise} \end{cases}$$

While taking L.C.M. we should always remember.

a. $\text{L.C.M. of } \left(\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\right) = \frac{\text{L.C.M. of (a, c, e)}}{\text{H.C.F. of (b, d, f)}}$

Eg.

$\text{L.C.M. of } \left(\frac{2\pi}{3}, \frac{\pi}{6}, \frac{\pi}{12}\right)$

$= \frac{\text{L.C.M. of } (2\pi, \pi, \pi)}{\text{H.C.F. of } (3, 6, 12)} = \frac{2\pi}{3}$

$\therefore$ $\text{L.C.M. of } \left(\frac{2\pi}{3}, \frac{\pi}{6}, \frac{\pi}{12}\right) = \frac{2\pi}{3}$

b. L.C.M. of rational with rational is possible.
L.C.M. of irrational with irrational is possible.
But L.C.M. of rational and irrational is not possible.
Eg. L.C.M. of $(2\pi, 1, 6\pi)$ is not possible as $2\pi, 6\pi \in$ irrational and $1 \in$ rational.

15. Some special functions:

i. If $f(x + y) = f(x) + f(y)$, then $f(x) = kx$.

ii. If $f(xy) = f(x) + f(y)$, then $f(x) = \log x$.

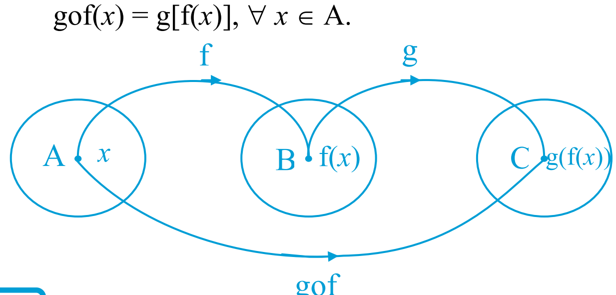
iii. If $f(x + y) = f(x).\ f(y)$, then $f(x) = e^x$.

iv. If $f(x)\ .f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$, then

$f(x) = x^n \pm 1$.

16. Composite function:

Let $f : A \to B$ be defined by $b = f(a)$ and
$g : B \to C$ be defined by $c = f(b)$, then
$h : A \to C$ be defined by $h(a) = g[f(a)]$
is called composite function.
We write $h = gof$
Thus, $gof : A \to C$ will be defined as $gof(x) = g[f(x)]$, $\forall\ x \in A$.



i. gof is defined, if $R_f \subseteq D_g$

ii. gof is one-one $\Rightarrow$ f is one-one.

iii. gof is onto $\Rightarrow$ f is onto.

iv. if f, g are one-one onto, then gof is also one-one onto.

v. f is even, g is even $\Rightarrow$ fog is even function.

vi. f is odd, g is odd $\Rightarrow$ fog is odd function.

vii. f is even, g is odd $\Rightarrow$ fog is even function.

viii. f is odd, g is even $\Rightarrow$ fog is even function.

ix. fog $\neq$ gof i.e., composite of functions is not commutative.

x. (fog)oh = fo(goh) i.e., composite of functions is associative.

xi. $(gof)^{-1} = (f^{-1}og^{-1})$

Important Note

❖ All functions are relation but all relations may not be a function.

Formulae

1.1 Sets

If A, B and C are finite sets and U be the universal set, then

1. $A \subseteq B$ iff $\{x \in A \Rightarrow x \in B\}$
2. $A = B$ iff $A \subseteq B$ and $B \subseteq A$
3. i. $P(A) = \{B : B \text{ is a subset of } A\}$
   ii. If A has n elements i.e., $o(A) = n$, then $o(P(A)) = 2^n$.
4. $A \cup B = \{x : x \in A \text{ or } x \in B\}$
5. $A \cap B = \{x : x \in A \text{ and } x \in B\}$
6. A and B are disjoint iff $A \cap B = \phi$.
7. $A - B = \{x : x \in A \text{ and } x \notin B\}$
8. $A - B = B - A$ iff $A = B$
9. $A - B = A$ iff $A \cap B = \phi$
10. $A \Delta B$ or $A \oplus B = (A - B) \cup (B - A)$ or $(A \cup B) - (A \cap B)$
11. $A'(\text{or } A^c) = \{x : x \in U \text{ and } x \notin A\}$
12. $A \cup A = A$ and $A \cap A = A$
    These are called **Idempotent** laws.
13. $A \cup B = A$ iff $B \subset A$ and
    $A \cap B = A$ iff $A \subset B$
14. $A \cup \phi = A$, $A \cap \phi = \phi$, $A \cup U = U$ and $A \cap U = A$
    These are called **Identity** laws.
15. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
    These are called **Distributive** laws.

16. $(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$,
$A - (B \cup C) = (A - B) \cap (A - C)$ and
$A - (B \cap C) = (A - B) \cup (A - C)$
These are called **De-Morgan's** law.
17. i. $P(A) \cap P(B) = P(A \cap B)$
ii. $P(A) \cup P(B) \subseteq P(A \cup B)$
iii. if $P(A) = P(B) \Rightarrow A = B$
where P(A) is the power set of A
18. $A - B = A \cap B' = A - (A \cap B)$
19. $n(A \cup B) = n(A) + n(B)$,
if A and B are disjoint sets.
20. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
21. $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$
22. $n(A - B) = n(A) - n(A \cap B)$
23. $n(A \cup B \cup C)$
$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$
24. $n(A \cap B' \cap C')$
$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
25. $A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$
26. $A \times B = B \times A$ iff $A = B$
27. $n(A \times B) = n(A).n(B)$
28. If $A \subseteq B$, then $A \times A \subseteq (A \times B) \cap (B \times A)$
29. $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

1.2 Relations

If A and B are finite sets and R be the relation, then
1. $R \subseteq A \times B$ i.e., $R \subseteq \{(a, b) : a \in A, b \in B\}$
2. i. If $n(A) = m$ and $n(B) = n$, then total number of possible relations from A to B is $= 2^{mn}$.
ii. The number of relations on finite set A having n elements is 2^{n^2}.
3. Domain of $R = \{a : (a, b) \in R\}$
Range of $R = \{b : (a, b) \in R\}$
4. $R^{-1} = \{(b, a) : (a, b) \in R\}$ is called **Inverse** relation.
5. $R = A \times A$ is called **Universal** relation.
6. $R = \{(a, a) : a \in A\} = I_A$ is called **Identity** relation.
7. $R = \phi$ is called **Void** relation.
8. If A be a non-empty set, then a relation R on A is said to be
i. Reflexive :
If $(a, a) \in R \ \forall a \in A$
ii. Symmetric :
If $(a, b) \in R \Rightarrow (b, a) \in R \ \forall a, b \in A$
iii. Transitive :
If $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow (a, c) \in R \ \forall a, b, c \in A$
iv. Anti-symmetric :
If $(a, b) \in R$ and $(b, a) \in R$
$\Rightarrow a = b \ \forall a, b \in A$
v. Equivalence :
iff it is reflexive, symmetric and transitive.

1.3 Functions

1. If $f : A \to B$ is a function, then
$x = y \Rightarrow f(x) = f(y) \ \forall x, y \in A$
2. A function $f : A \to B$ is a **one-one function** or an **injection**,
if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \ \forall x_1, x_2 \in A$
or
$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2) \ \forall x_1, x_2 \in A$
3. A function $f : A \to B$ is an **onto function** or a **surjection** if range (f) = co-domain (f).
4. A function $f : A \to B$ is an **into function,** if range (f) $\subset$ co-domain (f).
5. A function $f : A \to B$ is a **bijection** or **bijective**, if it is one-one as well as onto.
6. A function $f : A \to B$ is **many-one function**, if $x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2) \ \forall x_1, x_2 \in A$
7. For domain and range, if function is in the form
i. $\sqrt{f(x)}$, take $f(x) \geq 0$
ii. $\dfrac{1}{\sqrt{f(x)}}$, take $f(x) > 0$
iii. $\dfrac{1}{f(x)}$, take $f(x) \neq 0$

Shortcuts

1.1 Sets

1. The total number of subsets of a finite set containing n elements is 2^n.
2. Number of proper subsets of A containing n elements is $2^n - 1$.
3. Number of non-empty subsets of A containing n elements is $2^n - 1$.
4. Let A, B, C be any three sets, then
i. n (A only)
$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
ii. n (B only)
$= n(B) - n(B \cap C) - n(A \cap B) + n(A \cap B \cap C)$
iii. $n(C \text{ only}) = n(C) - n(C \cap A) - n(B \cap C) + n(A \cap B \cap C)$

5. Number of elements in exactly two of the sets A, B and C
$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$
6. Number of elements in exactly one of the sets A, B and C
$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$
7. Number of elements which belong to exactly one of A or B.
i.e., $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$
8. If $o(A \cap B) = n$, then
$o[(A \times B) \cap (B \times A)] = n^2$
9. If $N_a = \{an : n \in N\}$, then
$N_b \cap N_c = N_{(\text{L.C.M. of b and c})}$ where $a, b, c \in N$

1.2 Relations

1. The identity relation on a set A is an anti-symmetric relation.
2. The relation 'congruent to' on the set T of all triangles in a plane is a transitive relation.
3. If R and S are two equivalence relations on a set A, then $R \cap S$ is also an equivalence relation on A.
4. The union of two equivalence relations on a set is not necessarily an equivalence relation on the set.
5. The inverse of an equivalence relation is an equivalence relation.
6. If a set A has n elements, then the number of binary relations on $A = n^{n^2}$.
7. Empty relation is always symmetric and transitive.
8. A relation R on a non-empty set A is symmetric iff $R^{-1} = R$.
9. Total number of reflexive relations in a set with n elements $= 2^n$.

1.3 Functions

1. The number of functions from a finite set A into a finite set $B = [n(B)]^{n(A)}$
2. i. The domain of $\sqrt{a^2 - x^2}$ is $[-a, a]$.
ii. The domain of $\dfrac{1}{\sqrt{a^2 - x^2}}$ is $(-a, a)$.
iii. The domain of $\sqrt{x^2 - a^2}$ is $(-\infty, -a] \cup [a, \infty)$.
iv. The domain of $\dfrac{1}{\sqrt{x^2 - a^2}}$ is $(-\infty, -a) \cup (a, \infty)$.
3. i. The domain of $\sqrt{(x-a)(b-x)}$ when $a < b$ is $[a, b]$.
ii. The domain of $\dfrac{1}{\sqrt{(x-a)(b-x)}}$ when $a < b$ is (a, b).
iii. The domain of $\sqrt{(x-a)(x-b)}$ when $a < b$ is $(-\infty, a] \cup [b, \infty)$.
iv. The domain of $\dfrac{1}{\sqrt{(x-a)(x-b)}}$ when $a < b$ is $(-\infty, a) \cup (b, \infty)$.
4. i. The domain of $\sqrt{\dfrac{x-a}{x-b}}$ when $a < b$ is $(-\infty, a] \cup (b, \infty)$.
ii. The domain of $\sqrt{\dfrac{x-a}{x-b}}$ when $a > b$ is $(-\infty, b) \cup [a, \infty)$.
iii. The domain of $\sqrt{\dfrac{x-a}{b-x}}$ when $a < b$ is $[a, b)$.
iv. The domain of $\sqrt{\dfrac{x-a}{b-x}}$ when $a > b$ is $(b, a]$.
5. i. The domain of $\log(a^2 - x^2)$ is $(-a, a)$.
ii. The domain of $\log(x^2 - a^2)$ is $(-\infty, -a) \cup (a, \infty)$.
iii. The domain of $\log[(x - a)(b - x)]$ when $a < b$ is (a, b).
6. i. Range of $f(x) = \sqrt{a^2 - x^2}$ is $[0, a]$.
ii. Range of $f(x) = a\cos x + b\sin x + c$ is $[c - \sqrt{a^2 + b^2}, c + \sqrt{a^2 + b^2}]$.
7. The domain of the function
$f(x) = \dfrac{|x + c|}{x + c}$ is $R - \{-c\}$ and range $= \{-1, 1\}$.

8. If $y = f(x) = \dfrac{ax+b}{(x-a)}$, then $fof(x) = x$.

9. Any polynomial function $f : R \to R$ is onto if degree of f is odd and into if degree of f is even.

10. If $f(x)$ is periodic with period a, then $\dfrac{1}{f(x)}$ is also periodic with same period a.

11. If $f(x)$ is periodic with period a, $\sqrt{f(x)}$ is also periodic with same period a.

12. Period of algebraic functions $\sqrt{x}$, x^2, $x^3 + 5$ etc. doesn't exist.

13. i. If A and B have n and m distinct elements respectively, then the number of mappings from A to B = m^n.
 ii. If A = B, then the number of mapping = n^n.

14. The number of one-one functions that can be defined from a set A into a finite set B is
 ${}^{n(B)}P_{n(A)}$; if $n(B) \ge n(A)$
 0 ; otherwise

15. The number of onto functions that can be defined from a finite set A containing n elements onto a finite set B containing 2 elements = $2^n - 2$.

16. The number of onto functions from A to B, where o(A) = m, o(B) = n and $m \ge n$ is $\sum_{r=1}^{n} (-1)^{n-r}\, {}^nC_r (r)^m$.

17. The number of bijections from a finite set A onto a finite set B is
 n(A)! ; if n(A) = n (B)
 0 ; otherwise

18. If any line parallel to X-axis, cuts the graph of the function atmost one point, then the function is one-one.

19. If any vertical line does not meet the graph of the function $f(x)$, then the function is onto.

Multiple Choice Questions

1.1 Sets

1.1.1 Sets and their representation, Power set

1. The set of intelligent students in a class is [AMU 1998]
 (A) a null set
 (B) a singleton set
 (C) a finite set
 (D) not a well defined collection

2. The set B = $\{x : x$ is a positive prime $< 10\}$ in the tabular form is
 (A) {2, 3, 5, 7} (B) {3, 5, 7, 9}
 (C) {2, 3, 5, 6} (D) {2, 3, 7, 8}

3. If A is the set of numbers obtained by adding 1 to each of the even numbers, then its set builder notation is **[DCE 2002]**
 (A) $A = \{x : x$ is odd and $x > 1\}$
 (B) $A = \{x : x$ is odd and $x \in I\}$
 (C) $A = \{x : x$ is even$\}$
 (D) $A = \{x : x$ is an integer$\}$

4. In rule method the null set is represented by **[Karnataka CET 1998]**
 (A) { } (B) ϕ
 (C) $\{x : x = x\}$ (D) $\{x : x \neq x\}$

5. The set of all prime numbers is
 (A) a finite set (B) a singleton set
 (C) an infinite set (D) a null set

6. Which set is the subset of all given sets?
 (A) {1, 2, 3, 4,….} (B) {1}
 (C) {0} (D) { }

7. Which of the following is a true statement? **[UPSEAT 2005]**
 (A) $a \in \{a, b, c\}$ (B) $a \subseteq \{a, b, c\}$
 (C) $\phi \in \{a, b, c\}$ (D) none of these

8. Which of the following is a singleton set?
 (A) $\{x : |x| = 8, x \in Z\}$
 (B) $\{x : |x| = 4, x \in N\}$
 (C) $\{x : x^2 = 7, x \in N\}$
 (D) $\{x : x^2 + 2x + 1 = 0, x \in N\}$

9. If a set A has n elements, then the total number of subsets of A is **[Roorkee 1991; Karnataka CET 1992,2000]**
 (A) n (B) n^2
 (C) 2n (D) 2^n

10. If A = $\{x, y\}$, then the power set of A is **[Pb.CET 2004, UPSEAT 2000]**
(A) $\{x^x, y^y\}$
(B) $\{\phi, x, y\}$
(C) $\{\phi, \{x\}, \{2y\}\}$
(D) $\{\phi, \{x\}, \{y\}, \{x, y\}\}$

11. The number of proper subsets of the set $\{1, 2, 3\}$ is **[JMIEE 2000]**
(A) 5 (B) 6
(C) 7 (D) 8

12. The number of non-empty subsets of the set $\{1, 2, 3, 4\}$ is **[Karnataka CET 1997; AMU 1998]**
(A) 14 (B) 16
(C) 15 (D) 17

13. Which of the following is the empty set? **[Karnataka CET 1990]**
(A) $\{x : x$ is a real number and $x^2 - 1 = 0\}$
(B) $\{x : x$ is a real number and $x^2 + 1 = 0\}$
(C) $\{x : x$ is a real number and $x^2 - 9 = 0\}$
(D) $\{x : x$ is a real number and $x^2 = x + 2\}$

14. If A = $\{x : x$ is a multiple of 4$\}$ and B = $\{x : x$ is a multiple of 6$\}$, then A $\subset$ B consists of all multiples of **[UPSEAT 2000]**
(A) 16 (B) 12
(C) 8 (D) 4

15. If X = $\{64n : n \in N\}$ and Y = $\{3^{2n+2} - 8n - 9 : n \in N\}$, then
(A) X $\subset$ Y (B) Y $\subset$ X
(C) X = Y (D) none of these

16. Which of the following is not true?
(A) $0 \in \{0, \{0\}\}$ (B) $\{0\} \in \{0, \{0\}\}$
(C) $\{0\} \subset \{0, \{0\}\}$ (D) $0 \subset \{0, \{0\}\}$

17. Power set of the set A = $\{\phi, \{\phi\}\}$ is
(A) $\{\phi, \{\phi\}, \{\{\phi\}\}\}$
(B) $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$
(C) $\{\phi, \{\phi\}, A\}$
(D) none of these

18. Two finite sets have m and n elements. The total number of subsets of the first set is 48 more than the total number of subsets of the second set. The values of m and n are **[M.N.R.E.C. Allahabad 1988,91; Kerala P.E.T. 2003]**
(A) 7, 6 (B) 6, 3
(C) 6, 4 (D) 7, 4

19. The set A = $\{x : x \in R, x^2 = 16$ and $2x = 6\}$ equals **[Karnataka CET 1995]**
(A) ϕ (B) $\{14, 3, 4\}$
(C) $\{14, 4\}$ (D) $\{4\}$

20. If a set contains (2n + 1) elements, then the number of subsets of this set containing more than n elements is equal to **[UPSEAT 2001,04]**
(A) 2^{n-1} (B) 2^n
(C) 2^{n+1} (D) 2^{2n}

1.1.2 Union, Intersection and Complement of sets and their algebraic properties

21. If A $\cup$ B = B, then
(A) A $\subset$ B (B) B $\subset$ A
(C) A = B (D) A $\cap$ B = ϕ

22. $(A \cup B)^c$ =
(A) $A^c \cup B^c$ (B) $A^c \cap B^c$
(C) $A^c - B^c$ (D) None of these

23. If A and B are disjoint, then n(A $\cup$ B) is equal to
(A) n(A) (B) n(B)
(C) n(A) + n(B) (D) n(A).n(B)

24. If A, B and C are any three sets, then A – (B $\cap$ C) is equal to
(A) (A – B) $\cup$ (A – C)
(B) (A – B) $\cap$ (A – C)
(C) (A – B) $\cup$ C
(D) (A – B) $\cap$ C

25. If A, B and C are any three sets, then A $\cap$ (B $\cup$ C) is equal to
(A) (A $\cup$ B) $\cap$ (A $\cup$ C)
(B) (A $\cap$ B) $\cup$ (A $\cap$ C)
(C) (A $\cup$ B) $\cup$ (A $\cup$ C)
(D) none of these

26. If A is any set and U be the universal set, then
(A) A $\cup$ A′ = ϕ (B) A $\cup$ A′ = U
(C) A $\cap$ A′ = U (D) none of these

27. If A = $\{1, 2, 4, 5, 6\}$ and B = $\{2, 3, 4, 5, 6\}$, then A $\cap$ B is equal to
(A) $\{2, 3, 4\}$ (B) $\{1, 2, 3\}$
(C) $\{2, 4, 5, 6\}$ (D) $\{2, 3, 5, 6\}$

28. If A = $\{2, 3, 5, 8, 10\}$, B = $\{3, 4, 5, 10, 12\}$ and C = $\{4, 5, 6, 12, 14\}$, then (A $\cap$ B) $\cup$ (A $\cap$ C) is equal to
(A) $\{3, 5, 10\}$ (B) $\{2, 7, 10\}$
(C) $\{4, 5, 6\}$ (D) $\{3, 5, 12\}$

29. If A = { a, b, c}, B = {b, c, d}, C = {a, b, d, e}, then $A \cap (B \cup C)$ is **[Kurukshetra CEE 1997]**
(A) {a, b, c} (B) {b, c, d}
(C) {a, b, d, e} (D) {e}

30. If A = {1, 2, 4}, B = {2, 4, 5}, C = {2, 5}, then $(A - B) \times (B - C)$ is
(A) {1, 2, 3} (B) {1, 2, 5}
(C) {(1, 5)} (D) {(1, 4)}

31. If n(A) = 10, n(B) = 7 and n(C) = 6 for three disjoint sets A, B and C, then $n(A \cup B \cup C) =$
(A) 7 (B) 9
(C) 21 (D) 23

32. $A - B = \phi$ if
(A) $A \subset B$ (B) $B \subset A$
(C) A = B (D) $A \cap B = \phi$

33. If Q is a set of rational numbers and P is a set of irrational numbers, then
(A) $P \cap Q = \phi$ (B) $P \subset Q$
(C) $Q \subset P$ (D) $P - Q = \phi$

34. If the sets A and B are defined as $A = \{(x, y) : y = e^x, x \in R\}$; $B = \{(x, y) : y = x, x \in R\}$, then **[UPSEAT 1994,99,2002]**
(A) $B \subseteq A$ (B) $A \subseteq B$
(C) $A \cup B = A$ (D) $A \cap B = \phi$

35. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is **[Kerala (Engg.) 2002]**
(A) 40 percent (B) 60 percent
(C) 80 percent (D) 70 percent

36. If A = {1, 2, 3} and B = {3, 8}, then $(A \cup B) \times (A \cap B)$ is
(A) {(3, 1), (2, 2), (3, 3), (3, 8)}
(B) {(1, 3), (2, 3), (3, 3), (8, 3)}
(C) {(1, 2), (2, 2), (3, 3), (8, 8)}
(D) {(8, 3), (8, 2), (8, 1), (8, 8)}

37. In a class of 100 students, 55 students have passed in Mathematics and 67 students have passed in Physics, then the number of students who have passed in Physics only is **[DCE 1993; ISM Dhanbad 1994]**
(A) 22 (B) 45
(C) 33 (D) 65

38. Of the members of three athletic teams in a school 21 are in the cricket team, 26 are in hockey team and 29 are in the football team. Among them, 14 play hockey and cricket, 15 play hockey and football and 12 play football and cricket. Eight play all the three games. The total number of members in the three athletic teams is
(A) 43 (B) 76
(C) 49 (D) 78

39. If A and B are any two sets, then A – B is equal to
(A) $(A \cup B) - (A \cap B)$
(B) $A \cap B$
(C) $A \cap B'$
(D) B – A

40. If A and B are any two sets, then $A \cup (A \cap B)$ is equal to **[Karnataka CET 1996]**
(A) A (B) A^c
(C) B (D) B^c

41. If A and B are any two sets, then $(A \cup B) - (A \cap B)$ is equal to
(A) A – B
(B) B – A
(C) $(A - B) \cup (B - A)$
(D) none of these

42. If the set A has p elements, B has q elements, then the number of elements in $A \times B$ is **[Karnataka CET 1999]**
(A) p^2 (B) p + q
(C) pq (D) p + q + 1

43. If A and B are two sets, then $A \cap (A \cup B)'$ is equal to
(A) ϕ (B) A
(C) B (D) none of these

44. If A and B are two sets, then
(A) $A \cup B \subseteq A \cap B$ (B) $A \cap B \subseteq A \cup B$
(C) $A \cap B = A \cup B$ (D) none of these

45. If $N_a = \{an : n \in N\}$, then $N_5 \cap N_7 =$ **[Kerala (Engg.) 2005]**
(A) N_5 (B) N_7
(C) N_{12} (D) N_{35}

46. If A, B, C are three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then **[Roorkee 1991]**
(A) A = B (B) B = C
(C) A = C (D) A = B = C

47. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 5} and B = {6, 7}, then $A \cap B'$ is
(A) B' (B) B
(C) A' (D) A

48. If n(A) = 3 and n(B) = 6, then the minimum number of elements in $A \cup B$ is
[MNR 1987; Karnataka CET 1996]
(A) 3 (B) 9
(C) 6 (D) 12

49. If n(U) = 700, n(A) = 200, n(B) = 300 and $n(A \cap B) = 100$, then $n(A^c \cap B^c) =$
[Kurukshetra CEE 1999]
(A) 200 (B) 600
(C) 300 (D) 400

50. If A and B are two sets, then $A \cap (A \cap B)^c$ is equal to
[AMU 1998, K.U.K.C.E.E.T. 1999]
(A) A (B) B
(C) ϕ (D) $A \cap B^c$

51. If A = {a, b}, B = {c, d}, C ={d, e}, then {(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)} is equal to [AMU 1999, Him. CET 2002]
(A) $A \cap (B \cup C)$ (B) $A \times (B \cup C)$
(C) $A \cup (B \cap C)$ (D) $A \times (B \cap C)$

52. If n(A) = 4, n(B) = 3, $n(A \times B \times C) = 24$, then n(C) = [Kerala (Engg.) 2005]
(A) 1 (B) 2
(C) 12 (D) 17

53. If A = $\{x : x^2 - 5x + 6 = 0\}$, B = {2, 4}, C = {4, 5}, then $A \times (B \cap C)$ is
[Kerala PET 2002]
(A) {(2, 4), (3, 4)}
(B) {(4, 2), (4, 3)}
(C) {(2, 4), (3, 4), (4, 4)}
(D) {(2, 2), (3, 3), (4, 4)}

54. A – B = B – A if
(A) $A \subset B$ (B) $B \subset A$
(C) A = B (D) $A \cap B = \phi$

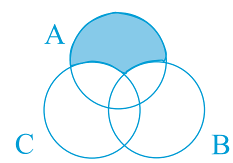
55. If $A \cap B = A$ and $A \cup B = A$, then
(A) $A \subset B$ (B) $B \subset A$
(C) A = B (D) none of these

56. If A and B are non-empty sets and $A \times B = B \times A$, then
(A) A is a proper subset of B
(B) B is a proper subset of A
(C) A = B
(D) none of these

57. If A = $\{x \in C : x^2 = 1\}$ and B = $\{x \in C : x^4 = 1\}$, then $A \Delta B$ is
(A) {–1, 1} (B) {–1, 1, i, –i}
(C) {–i, i} (D) none of these

58. If P, Q and R are subsets of a set A, then $R \times (P^c \cup Q^c)^c =$
[Karnataka CET 1993]
(A) $(R \times P) \cap (R \times Q)$
(B) $(R \times Q)^c \cap (R \times P)^c$
(C) $(R \times P) \cup (R \times Q)$
(D) none of these

59. The shaded region in the given figure is
[NDA 2000]
(A) $A \cap (B \cup C)$
(B) $A \cup (B \cap C)$
(C) $A \cap (B - C)$
(D) $A - (B \cup C)$



60. If n(U) = 20, n(A) = 12, n(B) = 9, $n(A \cap B) = 4$, where U is the universal set, A and B are subsets of U, then $n((A \cup B)^c) =$
[Kerala CET 2004, Him. CET 2007]
(A) 3 (B) 6
(C) 9 (D) 12

61. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9},
A = $\{x \in N : 30 < x^2 < 70\}$,
B = {x : x is a prime number less than 10},
then which of the following is false?
(A) $A \cup B = \{2, 3, 5, 6, 7, 8\}$
(B) $A \cap B = \{7, 8\}$
(C) $A - B = \{6, 8\}$
(D) $A \Delta B = \{2, 3, 5, 6, 8\}$

62. If A = $\{(x, y) : x^2 + y^2 = 25\}$ and B = $\{(x, y) : x^2 + 9y^2 = 144\}$, then $A \cap B$ contains [AMU 1996; Pb. CET 2002]
(A) one point (B) two points
(C) three points (D) four points

63. If two sets A and B are having 99 elements in common, then the number of elements common to each of the sets $A \times B$ and $B \times A$ are [Kerala (Engg.) 2004]
(A) 2^{99} (B) 99^2
(C) 100 (D) 9

64. If U is the universal set and $A \cup B \cup C = U$, then $[(A - B) \cup (B - C) \cup (C - A)]'$ is equal to
(A) $A \cup B \cup C$ (B) $A \cup (B \cap C)$
(C) $A \cap B \cap C$ (D) $A \cap (B \cup C)$

65. In a class of 30 pupils, 12 take needle work, 16 take Physics and 18 take History. If all the 30 students take at least one subject and no one takes all three, then the number of pupils taking 2 subjects is **[J AND K 2005]**
(A) 16 (B) 6
(C) 8 (D) 20

66. If A and B are any two sets, then A – B is not equal to
(A) $A \cap B^c$ (B) $B \cap A^c$
(C) $(A^c \cup B)^c$ (D) $A - (A \cap B)$

67. If $N_a = \{an : n \in N\}$ and $N_b \cap N_c = N_d$, where a, b, c, d $\in$ N and b, c are relatively prime, then
(A) $d = b + c$ (B) $d = b - c$
(C) $d = bc$ (D) $d = \frac{b}{c}$

68. If a set A contains 4 elements and a set B contains 8 elements, then maximum number of elements in $A \cup B$ is
(A) 4 (B) 12
(C) 8 (D) 16

69. The set $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$ is equal to
(A) $B \cap C'$ (B) $B' \cap C'$
(C) $A \cap C$ (D) $A \cap C'$

70. If $A = \{(x, y) : y = e^x, x \in R\}$ and $B = \{(x, y) : y = e^{-x}, x \in R\}$, then
(A) $A \cap B = \phi$ (B) $A \cap B \neq \phi$
(C) $A \cup B = R^2$ (D) none of these

71. If A and B are two sets, then $(A \cup B)' \cup (A' \cap B)$ is equal to **[DCE 2008]**
(A) A′ (B) A
(C) B′ (D) none of these

72. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) : n \in N\}$, then $X \cup Y$ is equal to **[Karnataka CET 1997]**
(A) X (B) Y
(C) N (D) none of these

73. In a town of 10,000 families, it was found that 40% family buy newspaper A, 20% buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, then number of families which buy A only is **[Roorkee 1997]**
(A) 3100 (B) 3300
(C) 2900 (D) 1400

74. In a class of 55 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics, 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all three subjects. The number of students who have taken exactly one subject is **[UPSEAT 1990]**
(A) 6 (B) 9
(C) 7 (D) 22

75. Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total, 64 played both basketball and hockey; 80 played cricket and basketball and 40 played cricket and hockey; 24 played all the three games. The number of boys who did not play any game is **[DCE 1995; MP PET 1996]**
(A) 128 (B) 216
(C) 240 (D) 160

76. A survey shows that 63% of the Americans like cheese whereas 76% like apples. If x% of the Americans like both cheese and apples, then
(A) $x = 39$ (B) $x = 63$
(C) $39 \le x \le 63$ (D) none of these

77. Which of the following is an empty set?
(A) The set of prime numbers which are even
(B) The set of reals which satisfy $x^2 + ix + i - 1 = 0$
(C) $(A \times B) \cap (B \times A)$, where A and B are disjoint
(D) The solution set of the equation $\frac{2(2x+3)}{x+1} - \frac{2}{x+1} + 3 = 0, x \in R$

78. If $A = \{x : x^2 - x + 2 > 0\}$ and $B = \{x : x^2 - 4x + 3 \le 0\}$, then $A \cap B$ is
(A) $(1, 3)$ (B) $[1, 3]$
(C) $(-\infty, \infty)$ (D) $(-\infty, 1) \cup (3, \infty)$

79. If $A = \left\{x : |\sin x| \le \frac{1}{2}\right\}$ and $B = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then $A \cap B$ is equal to
(A) $\left[-\frac{\pi}{6}, \frac{5\pi}{6}\right]$ (B) $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$
(C) $\left[0, \frac{\pi}{6}\right]$ (D) $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

80. If $X = \left\{(x, y) : y = \left(\frac{1}{4}\right)^x, x \in R\right\}$ and $Y = \{(x, y) : y = x, x \in R\}$, then
(A) $X = Y$ (B) $X \cap Y = \phi$
(C) $X \cap Y \neq \phi$ (D) none of these

81. Suppose $A_1, A_2, \ldots, A_{30}$ are thirty sets each with five elements and $B_1, B_2, \ldots, B_n$ are n sets each with three elements such that $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$. If each element of S belongs to exactly 10 of the A_i's and exactly 9 of the B_j's, then the value of n is
[DCE 2009]
(A) 15 (B) 30
(C) 40 (D) 45

82. If $A = \{\theta : 2\cos^2\theta + \sin\theta \leq 2\}$ and $B = \left\{\theta : \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}\right\}$, then $A \cap B$ is equal to
(A) $\left\{\theta : \pi \leq \theta \leq \frac{3\pi}{2}\right\}$
(B) $\left\{\theta : \frac{\pi}{2} \leq \theta \leq \frac{5\pi}{6}\right\}$
(C) $\left\{\theta : \frac{\pi}{2} \leq \theta \leq \frac{5\pi}{6}\right\} \cup \left\{\theta : \pi \leq \theta \leq \frac{3\pi}{2}\right\}$
(D) none of these

1.2 Relations

1.2.1 Relation

83. If R is a relation from a non-empty set A to a non-empty set B, then
(A) $R = A \cap B$ (B) $R = A \cup B$
(C) $R = A \times B$ (D) $R \subseteq A \times B$

84. If R is a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is
(A) 2^{mn} (B) $2^{mn} - 1$
(C) 2mn (D) m^n

85. If R is a relation on a finite set A having n elements, then the number of relations on A is
(A) 2^n (B) 2^{n^2}
(C) n^2 (D) n^n

86. The relation R is defined on the set of natural numbers as {(a, b) : a = 2b}. Then, R^{-1} is given by
(A) {(2, 1), (4, 2), (6, 3),….}
(B) {(1, 2), (2, 4), (3, 6),….}
(C) R^{-1} is not defined
(D) none of these

87. If A = {1, 2, 3}, then domain of the relation R = {(1, 1), (2, 3), (2, 1)} defined on A is
(A) {1, 2} (B) {1, 3}
(C) {2, 3} (D) {1, 2, 3}

88. If P = {3, 4, 5}, then range of the relation R = {(3, 3), (3, 4), (5, 4)} defined on P is
(A) {3, 4} (B) {3, 5}
(C) {4, 5} (D) {3, 4, 5}

89. Let A = {1, 2, 3}, B = {1, 3, 5}.
If relation R from A to B is given by
R = {(1, 3), (2, 5), (3, 3)}, then R^{-1} is
(A) {(3, 3), (3, 1),(5, 2)}
(B) {(1, 3), (2, 5), (3, 3)}
(C) {(1, 3), (5, 2)}
(D) {(1, 2), (3, 5)}

90. If A and B are two finite sets such that n(A) = 2, n(B) = 3, then total number of relations from A to B is
(A) 64 (B) 8
(C) 16 (D) 32

91. If A is the set of even natural numbers less than 8 and B is the set of prime numbers less than 7, then the number of relations from A to B is **[NDA 2003]**
(A) 2^9 (B) 9^2
(C) 3^2 (D) $2^9 - 1$

92. If R is a relation from {11, 12, 13} to {8, 10, 12} defined by $y = x - 3$, then R^{-1} is
(A) {(8, 11), (10, 13)}
(B) {(11, 18), (18, 10)}
(C) {(11, 8), (13, 10)}
(D) {(11, 13), (8, 10)}

93. If R = {(x, y) : $x \in$ N, $y \in$ N and $x + y = 5$}, then the range of R is
(A) {1, 2, 3, 5} (B) {1, 2, 3, 4}
(C) {1, 2, 4, 5} (D) {1, 3, 4, 5}

94. Number of relations that can be defined on the set A = {1, 2, 3} is
(A) 2 (B) 2^3
(C) 2^6 (D) 2^9

95. If P = {a, b, c, d} and Q = {1, 2, 3}, then which of the following is a relation from A to B?
(A) R_1 = {(1, a), (2, b), (3, c)}
(B) R_2 = {(a, 1), (2, b), (c, 3)}
(C) R_3 = {(a, 1), (d, 3), (b, 2), (b, 3)}
(D) R_4 = {(a, 1), (b, 2), (c, 3), (3, d)}

96. If P = {1, 2, 3, …., 10} and R = {(x, y) : $x + 2y = 10$, $x, y \in$ A} be a relation on P, then R^{-1}=
(A) {(4, 2), (3, 4), (2, 6)}
(B) {(2, 4), (4, 3), (6, 2), (8, 1)}
(C) {(4, 2), (3, 6), (4, 3)}
(D) {(4, 2), (3, 4), (2, 6), (1, 8)}

97. If R = {(x, y) : $x, y \in$ Z, $x^2 + y^2 \le 4$} is a relation in Z, then domain of R is
(A) {0, 1, 2} (B) {0, –1, –2}
(C) {–2, –1, 0, 1, 2} (D) {–1, 0, 1, 2}

1.2.2 Types of relations

98. If A = {1, 2, 3}, then the relation R = {(1, 1), (1, 2), (2, 1)} on A is
(A) reflexive (B) transitive
(C) symmetric (D) none of these

99. If P = {(x, y) / $x^2 + y^2 = 1$, $(x, y) \in$ R}, then P is
(A) reflexive (B) symmetric
(C) transitive (D) anti-symmetric

100. A relation R on a non-empty set A is an equivalence relation iff it is
(A) reflexive
(B) reflexive and transitive
(C) reflexive, symmetric and transitive
(D) symmetric and transitive

101. If R is a relation from a set A to a set B and S is a relation from B to C, then the relation SoR
(A) is from A to C
(B) is from C to A
(C) does not exist
(D) none of these

102. The void relation on a set A is
(A) reflexive
(B) symmetric and transitive
(C) reflexive and symmetric
(D) reflexive and transitive

103. If R $\subset$ A × B and S $\subset$ B × C be two relations, then $(SoR)^{-1}$ =
(A) S^{-1} o R^{-1} (B) R^{-1} o S^{-1}
(C) SoR (D) RoS

104. $x^2 = xy$ is a relation which is
(A) symmetric (B) reflexive
(C) transitive (D) none of these

105. The relation "less than" in the set of natural numbers is **[UPSEAT 1994,98,99; AMU 1999]**
(A) only symmetric
(B) only transitive
(C) only reflexive
(D) equivalence relation

106. For real numbers x and y, x R $y \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. The relation R is
(A) reflexive (B) symmetric
(C) transitive (D) none of these

107. If A = {1, 2, 3, 4} and R = {(2, 2), (3, 3), (4, 4), (1, 2)}be a relation in A, then R is
(A) reflexive (B) symmetric
(C) transitive (D) none of these

108. If R be a relation < from A = {1, 2, 3, 4} to B = {1, 3, 5} i.e., (a, b) $\in$ R iff a < b, then RoR^{-1} is
(A) {(1, 3), (1, 5), (2, 3), (2, 5), (3, 5)}
(B) {(3, 1), (5, 1), (3, 2), (5, 3), (5, 4)}
(C) {(3, 3), (3, 5), (5, 3), (5, 5)}
(D) {(3, 3), (3, 4), (4, 5)}

109. If R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a relation on the set A = {1, 2, 3, 4}, then R is **[AIEEE 2004]**
(A) reflexive (B) transitive
(C) not symmetric (D) a function

110. Let R be the relation on the set R of all real numbers defined by a R b iff $|a - b| < 1$. Then R is **[Roorkee 1998]**
(A) reflexive and symmetric
(B) symmetric only
(C) transitive only
(D) anti-symmetric only

111. With reference to a universal set, the inclusion of a subset in another, is relation, which is **[Karnataka CET 1995]**
(A) symmetric only
(B) an equivalence relation
(C) reflexive only
(D) not symmetric

112. If R and R′ are symmetric relations on a set A, then the relation R $\cap$ R′ is
(A) reflexive (B) symmetric
(C) transitive (D) none of these

113. The number of reflexive relations of a set with four elements is equal to **[UPSEAT 2004]**
(A) 2^{16} (B) 2^{12}
(C) 2^{8} (D) 2^{4}

114. Consider the following statements on a set A = {1, 2, 3}:
(1) R = {(1, 1), (2, 2)} is a reflexive relation on A.
(2) R = {(3, 3)} is symmetric and transitive but not a reflexive relation on A.
Which of the following given above is/are correct? **[NDA 2005]**
(A) (1) only
(B) (2) only
(C) both (1) and (2)
(D) neither (1) nor (2)

115. Let L be the set of all straight lines in the Euclidean plane and R be the relation defined by the rule l_1 R l_2 iff $l_1 \perp l_2$. Then relation R is
(A) reflexive (B) symmetric
(C) transitive (D) not symmetric

116. Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if ad(b + c) = bc(a + d), then R is **[Roorkee 1995]**
(A) symmetric only
(B) reflexive only
(C) transitive only
(D) an equivalence relation

117. Let S be the set of all real numbers. Then the relation R = {(a, b) : 1 + ab > 0} on S is **[NDA 2003]**
(A) reflexive and symmetric, but not transitive.
(B) reflexive and transitive, but not symmetric.
(C) reflexive, symmetric and transitive.
(D) symmetric and transitive, but not reflexive.

118. On the set N of all natural numbers define the relation R by aRb iff the G. C. D. of a and b is 2. Then R is **[Kerala CET 2007]**
(A) reflexive but not symmetric
(B) symmetric only
(C) reflexive and transitive
(D) reflexive, symmetric and transitive

119. Let W denote the words in English dictionary. Define the relation R by R = {(x, y) ∈ W × W: the words x and y have at least one letter in common}, then R is **[AIEEE 2006]**
(A) reflexive, not symmetric and transitive
(B) not reflexive, symmetric and transitive
(C) reflexive, symmetric and not transitive
(D) reflexive, symmetric and transitive

120. If R_1 and R_2 are two equivalence relations on a non-empty set A, then
(A) $R_1 \cup R_2$ is an equivalence relation on A
(B) $R_1 \cap R_2$ is an equivalence relation on A
(C) $R_1 - R_2$ is an equivalence relation on A
(D) none of these

121. Let R be a relation such that
R = {(1, 4), (3, 7), (4, 5), (4, 6), (7, 6)}
Then $R^{-1}oR^{-1}$ is equal to
(A) {(1, 4), (4, 5), (6, 7)}
(B) {(5, 1), (6, 1), (6, 3)}
(C) {(3, 7), (4, 6), (7, 6)}
(D) {(4, 5), (4, 6), (7, 6)}

122. Let R be a relation over the set of integers such that mRn iff m is a multiple of n, then R is
(A) reflexive and transitive
(B) symmetric
(C) only transitive
(D) an equivalance relation

1.3 Functions

1.3.1 Real valued functions, Algebra of functions and Kinds of functions

123. If f and g are two functions with domains D_1 and D_2 respectively, then the domain of the function (f + g) (x) is
(A) $D_1 \cup D_2$ (B) $D_1 \cap D_2$
(C) $D_1 - D_2$ (D) none of these

124. The range of the function f(x) = $\frac{x-2}{2-x}$ when $x \neq 2$ is
(A) R (B) R – {1}
(C) {–1} (D) R – {–1}

125. If f(x) = x^3 + sin x, then f(x) is
(A) an even function
(B) an odd function
(C) a power function
(D) none of these

126. Which of the following functions is a polynomial function? **[K.U.K.C.E.E.T. 1997]**
(A) $\frac{x^2-1}{x+4}, x \neq -4$
(B) $x^4 + x^3 + 3x^2 - 7x + \sqrt{2x^2}$
(C) $\frac{2x^2+7x+4}{3}$
(D) $2x^2 + x^{2/3} + 4$

127. $f(x) = \frac{|x|}{x}$, $x \neq 0$, then the value of function is

(A) 1 (B) 0
(C) -1 (D) does not exist

128. Which of the following is a rational function? **[DCE 1995]**

(A) $\frac{\sqrt{1+x}}{2+5x}, x \neq -\frac{2}{5}$
(B) $\frac{3x^5+5x^3+2x+7}{x^{3/2}}, x > 0$
(C) $\frac{3x^3-7x+1}{x-2}, x \neq 2$
(D) $\frac{1}{3}\sqrt{4x^3+4x+7}$

129. If $\phi(x) = a^x$, then $[\phi(p)]^3$ is equal to **[MP PET 1999]**

(A) $\phi(3p)$ (B) $3\phi(p)$
(C) $6\phi(p)$ (D) $2\phi(p)$

130. If $f(x) = \frac{x-|x|}{|x|}$, then $f(-1) =$ **[SCRA 1996]**

(A) 1 (B) -2
(C) 0 (D) 2

131. If $f(x) = \frac{x}{x-1}$, then $\frac{f(a)}{f(a+1)} =$ **[MP PET 1996]**

(A) $f(-a)$ (B) $f\left(\frac{1}{a}\right)$
(C) $f(a^2)$ (D) $f\left(\frac{-a}{a-1}\right)$

132. If $f(x) = 4x^3 + 3x^2 + 3x + 4$, then $x^3 f\left(\frac{1}{x}\right)$ is **[SCRA 1996]**

(A) $f(-x)$ (B) $\frac{1}{f(x)}$
(C) $\left(f\left(\frac{1}{x}\right)\right)^2$ (D) $f(x)$

133. The range of the function

$$f(x) = \begin{cases} \frac{|x|}{x}; \text{ for } x \neq 0 \\ 0; \text{ for } x = 0 \end{cases} \text{ is}$$

(A) $\{-1, 1\}$ (B) $[-1, 1]$
(C) $\{-1, 0, 1\}$ (D) $\{0, 1\}$

134. If $f(x) = x$ and $g(x) = |x|$, then $f(x) + g(x)$ is equal to **[AMU 1988]**

(A) 0
(B) $2x$
(C) $2x, x \geq 0; 0, x < 0$
(D) $2x, x \geq 0; 2x, x < 0$

135. Domain of $\sqrt{4-x^2}$ is

(A) $(-2, 2)$ (B) $(-2, 2]$
(C) $[-2, 2]$ (D) $\{-2, 2\}$

136. Let $f : R \to R$ be defined by $f(x) = 2x + |x|$, then $f(2x) + f(-x) - f(x) =$ **[EAMCET 2000]**

(A) $2x$ (B) $-2|x|$
(C) $-2x$ (D) $2|x|$

137. If $f(x) = \frac{2^x + 2^{-x}}{2}$, then $f(x+y).\ f(x-y) =$ **[RPET 1998]**

(A) $\frac{1}{2}$ $[f(2x) + f(2y)]$
(B) $\frac{1}{4}$ $[f(2x) + f(2y)]$
(C) $\frac{1}{2}$ $[f(2x) - f(2y)]$
(D) $\frac{1}{4}$ $[f(x) - f(2y)]$

138. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then $f(x)$ is **[Kerala CEE 2002]**

(A) an even function
(B) an odd function
(C) $f(x_1)\ f(x_2) = f(x_1 + x_2)$
(D) $\frac{f(x_1)}{f(x_2)} = f(x_1 - x_2)$

139. Domain of $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ is **[K.U.K.C.E.E.T. 1999]**

(A) $(1, \infty)$ (B) $(0, \infty)$
(C) $(-\infty, \infty)$ (D) none of these

140. The function $f(x) = \log\left(x + \sqrt{x^2+1}\right)$ is **[AIEEE 2003; MP PET 2003; UPSEAT 2003]**

(A) an even function
(B) an odd function
(C) a periodic function
(D) neither an even nor an odd function

141. The period of $|\cos x|$ is **[RPET 1998]**

(A) 2π (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$ (D) π

142. If $y = 3[x] + 1 = 4[x - 1] - 10$, then $[x + 2y] =$

(A) 61 (B) 67

(C) 88 (D) 107

143. The domain of the function $f(x) = \frac{|x+2|}{x+2}$ is

(A) $R - \{2\}$ (B) R

(C) $R - \{0\}$ (D) $R - \{-2\}$

144. The domain of the function $f(x) = \log(1 - x) + \sqrt{x^2 - 1}$ is

(A) $[-1, 1]$ (B) $(0, 1)$

(C) $(1, \infty)$ (D) $(-\infty, -1]$

145. The value of b and c for which the identity $f(x + 1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d$, are **[Roorkee 1992]**

(A) $b = -1, c = 1$ (B) $b = 4, c = -1$

(C) $b = 2, c = 1$ (D) $b = -1, c = 4$

146. The domain of the function $f(x) = \log\left(\sqrt{x-4} + \sqrt{6-x}\right)$ is **[RPET 2001]**

(A) $[4, \infty)$ (B) $(-\infty, 6]$

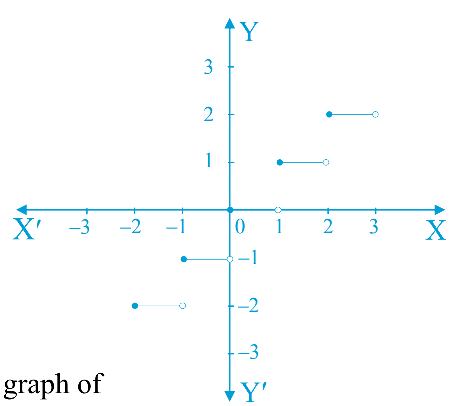
(C) $[4, 6]$ (D) none of these

147. The inverse of the function $y = 2x - 3$ is **[UPSEAT 2002]**

(A) $\frac{x+3}{2}$ (B) $\frac{x-3}{2}$

(C) $\frac{1}{2x-3}$ (D) $\frac{1}{2x+3}$

148.



is the graph of

(A) Modulus function

(B) Signum function

(C) Greatest integer function

(D) Fractional part function

149. If $f(x) = \sqrt{x^2 + 13}$, then the graph of the function $y = f(x)$ is symmetric about

(A) the X-axis (B) the Y-axis

(C) the origin (D) the line $x = y$

150. If $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$, then f is

(A) an absolute value function

(B) a signum function

(C) the greatest integer function

(D) a constant function

151. The period of the function $f(x) = \sin(2x)$ is

(A) 2π (B) π

(C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{2}$

152. If $f(x) = e^{-x}$, then $\frac{f(-a)}{f(b)}$ equals **[AMU 1986]**

(A) $f(a + b)$ (B) $f(a - b)$

(C) $f(-a + b)$ (D) $f(-a - b)$

153. Which of the following functions is an even function? **[Kerala CEE 1987, DCE 1993, RPET 2000]**

(A) $f(x) = \frac{a^x + a^{-x}}{a^x - a^{-x}}$

(B) $f(x) = \frac{a^x + 1}{a^x - 1}$

(C) $f(x) = \frac{x(a^x - 1)}{a^x + 1}$

(D) $f(x) = \log_2(x + \sqrt{x^2 - 1})$

154. If $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$, then $f(\theta)$ is **[IIT Screening 2000]**

(A) ≥ 0 only when $\theta \geq 0$

(B) ≤ 0 for all real θ

(C) ≥ 0 for all real θ

(D) ≤ 0 only when $\theta \leq 0$

155. Domain of $\frac{1}{\sqrt{9 - x^2}}$ is

(A) $(-3, 3)$ (B) $(-3, 3]$

(C) $[-3, 3]$ (D) none of these

156. If f(x) = sin (log x), then the value of f(xy) + f$\left(\frac{x}{y}\right)$ − 2f(x) cos log y is equal to

[Orissa JEE 2004]

(A) −1
(B) 1
(C) 0
(D) sin (log x). cos (log y)

157. The equivalent function of log x^2 is

[MP PET 1997]

(A) 2 log x (B) 2 log $|x|$
(C) $|\log x^2|$ (D) $(\log x)^2$

158. The graph of the function y = f(x) is symmetrical about the line x = 2, then

[AIEEE 2004]

(A) f(x) = − f(− x)
(B) f(2 + x) = f(2 − x)
(C) f(x) = f(− x)
(D) f(x + 2) = f(x − 2)

159. Domain of $\sqrt{x^2 - 16}$ is
(A) [− 4, 4]
(B) (− ∞, 4) ∪ (4, ∞)
(C) (− ∞, − 4] ∪ [4, ∞)
(D) {− 4, 4}

160. The domain of definition of the function f(x) given by equation $2^x + 2^y = 2$ is

[IIT Screening 2000]

(A) $0 < x \leq 1$ (B) $0 \leq x \leq 1$
(C) $-\infty < x \leq 0$ (D) $-\infty < x < 1$

161. If f(x) is an odd periodic function with period 2, then f(4) equals
(A) 0 (B) 2
(C) 4 (D) − 4

162. The domain of definition of f(x) = $\frac{\log_2(x+3)}{x^2+3x+2}$ is

[IIT 2001; UPSEAT 2001]

(A) R − {−1, −2}
(B) (−2, ∞)
(C) R − { −1, −2, −3}
(D) (−3, ∞) − {−1, −2}

163. The fundamental period of the function f(x) = $2\cos\frac{1}{3}(x - \pi)$ is
(A) 6π (B) 4π
(C) 3π (D) 2π

164. The period of the function $\sin\left(\frac{\pi x}{2}\right) + \cos\left(\frac{\pi x}{2}\right)$ is
(A) 4 (B) 6
(C) 12 (D) 18

165. The domain of $\frac{1}{\sqrt{(x-4)(x-5)}}$ is
(A) (− ∞, 4) ∪ (5, ∞)
(B) (− ∞, 4] ∪ [5, ∞)
(C) (− ∞, 4] ∪ (5, ∞)
(D) (− ∞, 4) ∪ [5, ∞)

166. Range of f(x) = 3 cos x + 4 sin x + 5 is
(A) [0, 10] (B) (0, 10)
(C) (0, 10] (D) none of these

167. The range of the function f(x) = ${}^{7-x}P_{x-3}$ is

[AIEEE 2004]

(A) {1, 2, 3, 4} (B) {1, 2, 3, 4, 5}
(C) {1, 2, 3} (D) {1, 2, 3, 4, 5, 6}

168. A real valued functional equation f(x − y) = f(x) f(y) − f(a − x) f(a + y) where 'a' is a constant and f(0) = 1, then f(2a − x) =

[AIEEE 2005]

(A) f(a) + f(a − x) (B) f(−x)
(C) f(x) (D) −f(x)

169. If [x] denotes the greatest integer less than or equal to x, then the range of the function f(x) = [x] − x is [NDA 2005]
(A) (−1, 0) (B) [−1, 0]
(C) [−1, 0) (D) (−1, 0]

170. If f(x) = a cos (bx + c) + d, then range of f(x) is

[UPSEAT 2001]

(A) [d + a, d + 2a] (B) [a − d, a + d]
(C) [d + a, a − d] (D) [d − a, d + a]

171. The range of the function f(x) = $\frac{x+2}{|x+2|}$ is

[RPET 2002]

(A) {0, 1} (B) {−1, 1}
(C) R (D) R − {−2}

172. If f(1) = 1 and f(n + 1) = 2f(n) + 1, n ≥ 1, then f(n) is
(A) 2^{n+1} (B) 2^n
(C) $2^n - 1$ (D) $2^{n-1} - 1$

173. The domain of the function

$f(x) = \sqrt{\dfrac{x+3}{(2-x)(x-5)}}$ is

(A) $(-\infty, -3] \cup (2, 5)$
(B) $(-\infty, -3) \cup (2, 5)$
(C) $(-\infty, -3] \cup [2, 5]$
(D) $(-3, 5)$

174. The domain of $f(x) = \sqrt{\dfrac{1-|x|}{2-|x|}}$ is

(A) $(-\infty, \infty) - [-1, 1]$
(B) $(-\infty, \infty) - [-2, 2]$
(C) $[-1, 1] \cup (-\infty, -2) \cup (2, \infty)$
(D) none of these

175. The domain of the function $f(x) = \dfrac{1}{\sqrt{|\sin x| + \sin x}}$ is

(A) $(-2n\pi, 2n\pi)$
(B) $(2n\pi, (2n+1)\pi)$
(C) $\left((4n-1)\dfrac{\pi}{2}, (4n+1)\dfrac{\pi}{2}\right)$
(D) none of these

176. The range of the function $y = \dfrac{x}{1+x^2}$ is

(A) $\left[0, \dfrac{1}{2}\right]$ (B) $\left[0, \dfrac{1}{2}\right)$
(C) $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$ (D) $\left[-\dfrac{1}{2}, 0\right]$

177. The range of the function $y = \dfrac{x^2}{1+x^2}$ is

(A) $(0, 1]$ (B) $[0, 1)$
(C) $(0, 1)$ (D) $[0, 1]$

178. The period of the function $f(x) = |\sin 4x| + |\cos 4x|$ is

(A) $\dfrac{\pi}{2}$ (B) $\dfrac{\pi}{4}$
(C) $\dfrac{\pi}{8}$ (D) π

179. The period of the function $f(x) = a \sin kx + b \cos kx$ is

(A) $-\dfrac{2\pi}{k}$ (B) $-\dfrac{\pi}{k}$
(C) $\dfrac{2\pi}{|k|}$ (D) $\dfrac{\pi}{|k|}$

180. If the domain of function $f(x) = x^2 - 6x + 7$ is $(-\infty, \infty)$, then the range of the function is **[MP PET 1996]**

(A) $(-\infty, \infty)$ (B) $[-2, \infty)$
(C) $(-2, 3)$ (D) $(-\infty, -2)$

181. The period of the function $f(x) = \sin^4 x + \cos^4 x$ is

(A) π (B) 2π
(C) $\dfrac{\pi}{4}$ (D) $\dfrac{\pi}{2}$

182. The domain of the function $f(x) = \log_2 \log_3 \log_4 x$ is

(A) $[4, \infty)$ (B) $(4, \infty)$
(C) $(-4, \infty)$ (D) $(-\infty, 4)$

183. The range of the function $f(x) = \sqrt{3x^2 - 4x + 5}$ is

(A) $\left(-\sqrt{\dfrac{11}{3}}, \infty\right)$ (B) $\left[\sqrt{\dfrac{11}{3}}, \infty\right)$
(C) $\left(-\infty, \sqrt{\dfrac{11}{3}}\right]$ (D) $\left(-\infty, -\sqrt{\dfrac{11}{3}}\right)$

184. The inverse of the function $f(x) = \dfrac{e^x + e^{-x}}{e^x + e^{-x}} + 2$ is **[Kurukshetra CEE 1996]**

(A) $\log_e \left(\dfrac{x-2}{x-1}\right)^{\frac{1}{2}}$ (B) $\log_e \left(\dfrac{x-1}{3-x}\right)^{\frac{1}{2}}$
(C) $\log_e \left(\dfrac{x}{2-x}\right)^{\frac{1}{2}}$ (D) $\log_e \left(\dfrac{x-1}{x+1}\right)^{-2}$

185. If $f(x) = \cos [\pi^2]x + \cos [-\pi^2]x$, then **[Orissa JEE 2002]**

(A) $f\left(\dfrac{\pi}{4}\right) = 2$ (B) $f(-\pi) = 2$
(C) $f(\pi) = 1$ (D) $f\left(\dfrac{\pi}{2}\right) = -1$

186. If $e^{f(x)} = \dfrac{10+x}{10-x}$, $x \in (-10, 10)$ and $f(x) = kf\left(\dfrac{200x}{100+x^2}\right)$, then k = **[EAMCET 2003]**

(A) 0.5 (B) 0.6
(C) 0.7 (D) 0.8

187. The function $f(x) = \sin(\log(x + \sqrt{x^2+1}))$ is **[Orissa JEE 2002]**
(A) an even function
(B) an odd function
(C) neither even nor odd
(D) a periodic function

188. If the real valued function $f(x) = \dfrac{a^x - 1}{x^n(a^x+1)}$ is even, then n equals **[Roorkee 1991, Karnataka CET 1996]**
(A) 2 (B) $\dfrac{-2}{3}$
(C) $\dfrac{1}{4}$ (D) $-\dfrac{1}{3}$

189. If f : R → R satisfies $f(x + y) = f(x) + f(y)$ for all $x, y \in$ R and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is **[AIEEE 2003]**
(A) $\dfrac{7n(n+1)}{2}$ (B) $\dfrac{7n}{2}$
(C) $\dfrac{7(n+1)}{2}$ (D) $7n(n + 1)$

190. If $[x]$ denotes the greatest integer $\le x$, then $\left[\dfrac{2}{3}\right] + \left[\dfrac{2}{3}+\dfrac{1}{99}\right] + \left[\dfrac{2}{3}+\dfrac{2}{99}\right] + \ldots + \left[\dfrac{2}{3}+\dfrac{98}{99}\right] =$ **[Kerala PET 2006]**
(A) 99 (B) 98
(C) 66 (D) 65

191. If the function
$f(x) = \cos^2 x + \cos^2\left(\dfrac{\pi}{3}+x\right) - \cos x \cos\left(\dfrac{\pi}{3}+x\right)$
is constant (independent of x), then the value of this constant is **[Roorkee 1991, Karnataka CET 1997]**
(A) 0 (B) $\dfrac{3}{4}$
(C) 1 (D) $\dfrac{4}{3}$

192. The domain of the function $y = f(x) = \dfrac{1}{\log_{10}(1-x)} + \sqrt{x+2}$ is **[Haryana CEET 2001]**
(A) [–2, 1), excluding 0
(B) [–3, –2], excluding –2.5
(C) [0, 1], excluding 0
(D) none of these

193. The domain of the function $f(x) = \log_{10}\dfrac{x-5}{x^2-10x+24} - \sqrt[3]{x+5}$ is
(A) (4, 5) (B) (6, ∞)
(C) (4, 5] ∪ (6, ∞) (D) (4, 5) ∪ (6, ∞)

194. The domain of the function
$f(x) = \log_{10}[1 - \log_{10}(x^2 - 5x + 16)]$ is
(A) (2, 3) (B) (2, 3]
(C) [2, 3) (D) [2, 3]

195. The domain of the function $f(x) = \dfrac{1}{\sqrt{[x]^2 - [x] - 6}}$ is
(A) (–∞, –2) ∪ [4, ∞)
(B) (–∞, –2] ∪ [4, ∞)
(C) (–∞, –2) ∪ (4, ∞)
(D) none of these

196. Which of the following function has period π?
(A) $2\cos\left(\dfrac{2\pi x}{3}\right) + 3\sin\left(\dfrac{\pi x}{3}\right)$
(B) $|\tan x| + \cos 2x$
(C) $4\cos\left(2\pi x + \dfrac{\pi}{2}\right) + 2\sin\left(\pi x + \dfrac{\pi}{4}\right)$
(D) none of these

197. The domain of the function
$f(x) = \sqrt{\log_{10}\left\{\dfrac{\log_{10} x}{2(3 - \log_{10} x)}\right\}}$ is
(A) $(10, 10^3)$ (B) $(10^2, 10^3)$
(C) $[10^2, 10^3)$ (D) $[10^2, 10^3]$

198. The period of the function
$f(x) = \dfrac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos 2x \cos x - \sin 3x \sin 4x}$ is
(A) $\dfrac{\pi}{2}$ (B) $\dfrac{\pi}{3}$
(C) π (D) 2π

199. The domain of the function $f(x) = \sqrt{\dfrac{4-x^2}{[x]+2}}$, where $[x]$ denotes the greatest integer less than or equal to x, is
(A) (– ∞, – 2)
(B) [– 1, 2]
(C) (– ∞, 2)
(D) (– ∞, – 2) ∪ [–1, 2]

200. Range of the function $f(x) = \dfrac{x+2}{x^2-8x-4}$ is

(A) $\left(-\infty, \frac{1}{4}\right) \cup \left(-\frac{1}{20}, \infty\right)$

(B) $\left(-\infty, \frac{-1}{4}\right] \cup \left[-\frac{1}{20}, \infty\right)$

(C) $\left(-\infty, -\frac{1}{4}\right] \cup \left(-\frac{1}{20}, \infty\right)$

(D) none of these

201. The domain of the function $f(x) = \exp(\sqrt{5x-3-2x^2})$ is **[MP PET 2004]**

(A) $\left[1, \frac{-3}{2}\right]$ (B) $\left[\frac{3}{2}, \infty\right]$

(C) $(-\infty, 1]$ (D) $\left[1, \frac{3}{2}\right]$

202. The range of the function $f(x) = \log_e(3x^2 - 4x + 5)$ is

(A) $\left(-\infty, \log_e \frac{11}{3}\right]$

(B) $\left[\log_e \frac{11}{3}, \infty\right)$

(C) $\left(\log_e \frac{11}{3}, \infty\right)$

(D) $\left[-\log_e \frac{11}{3}, \log_e \frac{11}{3}\right]$

203. The period of the function $f(x) = \dfrac{|\sin x| - |\cos x|}{|\sin x + \cos x|}$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$

(C) π (D) 2π

1.3.2 One-one, Into and Onto functions, Composition of functions

204. The function $f : N \to N$, where N is the set of natural numbers, defined by $f(x) = 2x + 3$, is

(A) surjective (B) bijective

(C) injective (D) none of these

205. If $x, y \in R$ and $x, y \neq 0$; $f(x, y) \to \dfrac{x}{y}$, then the function is a/an

(A) surjective (B) bijective

(C) one-one (D) none of these

206. If $f(x) = x^2$ and $g(x) = \sqrt{x}$, then **[Punjab CET 2000]**

(A) $(gof)(-2) = 2$ (B) $(fog)(2) = 4$

(C) $(gof)(2) = 4$ (D) $(fog)(3) = 6$

207. The composite map fog of the functions $f : R \to R$, $f(x) = \sin x$ and $g : R \to R$, $g(x) = x^2$ is **[UPSEAT 2000]**

(A) $(\sin x)^2$ (B) $\sin x^2$

(C) x^2 (D) $x^2(\sin x)$

208. If $f : R \to R$, $g : R \to R$ and $h : R \to R$ are such that $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$, then the value of $(ho(gof))(x)$, if $x = \sqrt{\frac{\pi}{4}}$ will be

(A) 0 (B) 1

(C) -1 (D) π

209. Let $f : N \to N$ be defined by $f(x) = x^2 + x + 1$, $x \in N$, then f is **[AMU 2000]**

(A) one-one onto

(B) many-one onto

(C) one-one but not onto

(D) none of these

210. Set A has 3 elements and set B has 4 elements. The number of injection that can be defined from A to B is **[UPSEAT 2001]**

(A) 144 (B) 12

(C) 24 (D) 64

211. Let $f : R \to R$ be a function defined by $f(x) = \dfrac{x-m}{x-n}$, where $m \neq n$. Then **[UPSEAT 2001]**

(A) f is one-one onto

(B) f is one-one into

(C) f is many-one into

(D) f is many-one onto

212. If function $f : R \to R$ be defined by $f(x) = 2x + \sin x$, $x \in R$, then f is **[IIT Screening 2002]**

(A) one-one and onto

(B) one-one but not onto

(C) onto but not one-one

(D) neither one-one nor onto

213. Which of the following is a bijective function on the set of real numbers? **[Kerala (Engg.) 2002]**

(A) $x^2 + 1$ (B) $2x - 5$

(C) x^2 (D) $|x|$

214. If $f(x) = \dfrac{2x+1}{3x-2}$, then(fof)(2) is equal to **[Kerala CEE 2002]**

(A) 1 (B) 3
(C) 2 (D) 4

215. If $f(x) = e^{2x}$ and $g(x) = \log\sqrt{x}$ ($x > 0$), then fog(x) is equal to

(A) e^{2x} (B) x
(C) 0 (D) $\log\sqrt{x}$

216. Let $f : I \to I$ be defined by $f(x) = x + i$, where i is a fixed integer, then f is

(A) one-one but not onto
(B) onto but not one-one
(C) non-invertible
(D) both one-one and onto

217. If A = {a, b, c}, then f = {(a, b), (b, c), (c, a)} is

(A) not a function from A to A
(B) a bijection from A to A
(C) one-one but not onto
(D) none of these

218. The number of surjections from A = {1, 2,…., n}, $n \geq 2$, onto B = {a, b} is

(A) ${}^nP_2$ (B) $2^n - 2$
(C) $2^n - 1$ (D) 2^n

219. The number of onto functions from {1, 2, 3} onto {p, q} is

(A) 7 (B) 5
(C) 6 (D) 4

220. The number of bijections from A = {1, 2, 3} onto B = {a, b} is

(A) 1 (B) 0
(C) 3! (D) 2!

221. The function $f : R \to R$, $f(x) = x^2$ is **[MP CET 1997]**

(A) injective but not surjective
(B) surjective but not injective
(C) injective as well as surjective
(D) neither injective nor surjective

222. Let A and B be two finite sets having m and n elements respectively. If $m \leq n$, then total number of injective functions from A to B is

(A) m^n (B) n^m
(C) $\dfrac{n!}{(n-m)!}$ (D) n!

223. If $f(x) = \dfrac{x}{\sqrt{1+x^2}}$, then (fofof)($x$) = **[RPET 2000]**

(A) $\dfrac{x}{\sqrt{1+3x^2}}$ (B) $\dfrac{x}{\sqrt{1+x^2}}$
(C) $\dfrac{x}{\sqrt{1+2x^2}}$ (D) $\dfrac{x}{\sqrt{1+4x^2}}$

224. If for two functions g and f, gof is both injective and surjective, then which of the following is true?

(A) g and f should be injective and surjective
(B) g should be injective and surjective
(C) f should be injective and surjective
(D) none of them may be surjective and injective

225. The function $f : R \to R$ defined by $f(x) = (x - 1)(x - 2)(x - 3)$ is **[Roorkee 1999]**

(A) one-one but not onto
(B) onto but not one-one
(C) both one-one and onto
(D) neither one-one nor onto

226. If $f(x) = (25 - x^4)^{1/4}$ for $0 < x < \sqrt{5}$, then $f\left(f\left(\dfrac{1}{2}\right)\right) =$ **[EAMCET 2001, Him.CET 2002]**

(A) 2^{-4} (B) 2^{-3}
(C) 2^{-2} (D) 2^{-1}

227. If $g(f(x)) = |\sin x|$ and $f(g(x)) = \left(\sin\sqrt{x}\right)^2$, then

(A) $f(x) = \sin x$, $g(x) = |x|$
(B) $f(x) = x^2$, $g(x) = \sin\sqrt{x}$
(C) $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$
(D) f and g cannot be determined

228. If $f(x) = \dfrac{1-x}{1+x}$; $x \neq 0$, then $f[f(x)] + f\left[f\left(\dfrac{1}{x}\right)\right]$ is

(A) equal to 2
(B) greater than or equal to 2
(C) less than 2
(D) none of these

229. If R denotes the set of all real numbers, then the function $f : R \to R$ defined by $f(x) = [x]$ is **[Karnataka CET 2004]**
(A) one-one only
(B) onto only
(C) both one-one and onto
(D) neither one-one nor onto

230. If $f(x) = \sin^2 x$ and the composite function $g(f(x)) = |\sin x|$, then $g(x)$ is equal to **[Orissa JEE 2003]**
(A) $\sqrt{x-1}$ (B) $\sqrt{x+1}$
(C) $\sqrt{x}$ (D) $-\sqrt{x}$

231. If f is an even function and g is an odd function, then fog is
(A) an even function
(B) an odd function
(C) neither an even nor odd function
(D) a periodic function

232. Two functions $f : R \to R$ and $g : R \to R$ are defined as follows: **[EAMCET 2001]**
$$f(x) = \begin{cases} 0; (x \text{ rational}) \\ 1; (x \text{ irrational}) \end{cases}$$
$$g(x) = \begin{cases} -1; (x \text{ rational}) \\ 0; (x \text{ irrational}), \end{cases}$$
then $(gof)(e) + (fog)(\pi) =$
(A) -1 (B) 0
(C) 1 (D) 2

233. Function $f : R \to R$, $f(x) = x^2 + x$, is **[RPET 1999]**
(A) one-one onto (B) one-one into
(C) many-one onto (D) many-one into

234. If $f(x) = x + \sqrt{x^2}$ is a function from R to R, then $f(x)$ is **[Orissa JEE 2004]**
(A) injective
(B) surjective
(C) bijective
(D) not one-one and onto

235. If the functions f, g, h are defined from the sets of real numbers R to R such that
$$f(x) = x^2 - 1,\ g(x) = \sqrt{x^2+1},\ h(x) = \begin{cases} 0, \text{ if } x \le 0 \\ x, \text{ if } x > 0 \end{cases},$$
then the composite function $(hofog)(x) =$ **[Roorkee 1997]**
(A) $\begin{cases} 0, & x = 0 \\ x^2, & x > 0 \\ -x^2, & x < 0 \end{cases}$ (B) $\begin{cases} 0, & x = 0 \\ x^2, & x \ne 0 \end{cases}$
(C) $\begin{cases} 0, & x \le 0 \\ x^2, & x > 0 \end{cases}$ (D) none of these

236. If $f(x) = \log\left(\dfrac{1+x}{1-x}\right)$ and $g(x) = \dfrac{3x+x^3}{1+3x^2}$, then $(fog)(x)$ equals
(A) $-f(x)$ (B) $3\,f(x)$
(C) $[f(x)]^3$ (D) $2\,f(x)$

237. If $g(x) = x^2 + x - 2$ and $\dfrac{1}{2}(gof)(x) = 2x^2 - 5x + 2$, then $f(x)$ is equal to **[Roorkee 1998; MP PET 2002]**
(A) $2x + 3$ (B) $2x - 3$
(C) $2x^2 + 3x + 1$ (D) $2x^2 - 3x - 1$

238. If $f : [0, \infty) \to [0, 2]$ be defined by $f(x) = \dfrac{2x}{1+x}$, then f is
(A) one-one but not onto
(B) onto but not one-one
(C) both one-one and onto
(D) neither one-one nor onto

239. If $f : R \to R$ and $g : R \to R$ are given by $f(x) = |x|$ and $g(x) = [x]$ for each $x \in R$, then $\{x \in R : g(f(x)) \le f(g(x))\} =$ **[EAMCET 2003]**
(A) $Z \cup (-\infty, 0)$ (B) $(-\infty, 0)$
(C) Z (D) R

240. Let $f(x) = \dfrac{\alpha x}{x+1}$, $x \ne -1$. Then, for what values of α, is $f(f(x)) = x$? **[IIT 2001, UPSEAT 2001]**
(A) $\sqrt{2}$ (B) $-\sqrt{2}$
(C) 1 (D) -1

241. A function f from the set of natural numbers to integers defined by **[AIEEE 2003]**
$$f(n) = \begin{cases} \dfrac{n-1}{2}, \text{ when n is odd} \\ -\dfrac{n}{2}, \text{ when n is even} \end{cases}, \text{ is}$$
(A) one-one but not onto
(B) onto but not one-one
(C) one-one and onto both
(D) neither one-one nor onto

242. Let X and Y be the subsets of R, the set of all real numbers. The function f: X $\to$ Y defined by f(x) = x^2 for $x \in$ X is one-one but not onto if (R^+ is the set of all positive real numbers) **[EAMCET 2000]**

(A) X = Y = R (B) X = Y = R^+
(C) X = R^+, Y = R (D) X = R, Y = R^+

243. Let f: R $\to$ R be a function defined by $f(x) = \dfrac{x^2-8}{x^2+2}$, then f is

(A) one-one but not onto
(B) one-one and onto
(C) onto but not one-one
(D) neither one-one nor onto

244. If $f(x) = \dfrac{1}{1-x}$, g(x) = f[f(x)] and h(x) = f[f{f(x)}], then f(x).g(x).h(x) is

(A) 1 (B) 0
(C) –1 (D) x

245. If $f(x) = \dfrac{x-1}{x+1}$, then f(f(ax)) in terms of f(x) is equal to

(A) $\dfrac{f(x)+1}{a(f(x)-1)}$ (B) $\dfrac{f(x)}{a(f(x)+1)}$
(C) $\dfrac{f(x)+1}{a(f(x)+1)}$ (D) $\dfrac{f(x)-1}{a(f(x)+1)}$

246. If f(x) = ax + b and g(x) = cx + d, then f(g(x)) = g(f(x)) is equivalent to **[UPSEAT 2001]**

(A) f(c) = g(a) (B) f(d) = g(b)
(C) f(a) = g(c) (D) f(b) = g(b)

247. The function f : R $\to$ R defined by f(x) = e^x is **[Karnataka CET 2002; UPSEAT 2002]**

(A) onto (B) one-one and into
(C) many-one (D) many-one and onto

248. If $f(x) = \sin^2 x + \sin^2\left(x+\dfrac{\pi}{3}\right) + \cos\left(x+\dfrac{\pi}{3}\right)\cos x$ and $g\left(\dfrac{5}{4}\right) = 1$, then gof($x$) is **[I.I.T. 1996]**

(A) a polynomial of first degree in sin x and cos x
(B) a constant function
(C) a polynomial of second degree in sin x and cos x
(D) none of these

249. The function g : (–∞, –1] $\to$ (0, e^5) defined by g(x) = e^{x^3-3x+2} is

(A) one-one and into
(B) one-one and onto
(C) many-one and into
(D) many-one and onto

250. If the function f : (– ∞, ∞) $\to$ A defined by f(x) = – x^2 + 6x – 8 is bijective, then A is equal to

(A) (– ∞, 1] (B) [1, ∞)
(C) (– ∞, 1) (D) (– ∞,∞)

251. If f: R $\to$ R given by f(x) = x^3 + (a + 2) x^2 + 3a x + 5 is one-one, then a belongs to the interval

(A) (1, ∞) (B) (– ∞, 1)
(C) (4, ∞) (D) (1, 4)

252. The function f : R $\to$ R defined by f(x) = $2^x + 2^{|x|}$ is

(A) one-one and into
(B) one-one and onto
(C) many-one and onto
(D) many-one and into

253. If f(x) = x^3 + 5x + 1 for real x, then **[AIEEE 2009]**

(A) f is one-one and onto in R
(B) f is one-one but not onto in R
(C) f is onto in R but not one-one
(D) f is neither one-one nor onto in R

254. Let g(x) = 1 + x – [x] and **[IIT 2001; UPSEAT 2001]**

$f(x) = \begin{cases} -1, x<0 \\ 0, x=0 \\ 1, x>0 \end{cases}$, then for all x, f(g(x)) is equal to

(A) x (B) 1
(C) f(x) (D) g(x)

255. A function f : [0, ∞) $\to$ [0, ∞) defined as $f(x) = \dfrac{x}{1+x}$ is **[IIT Screening 2003]**

(A) one-one and onto
(B) one-one but not onto
(C) onto but not one-one
(D) neither one-one nor onto

256. If $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$

and $g(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ x, & \text{if } x \text{ is irrational} \end{cases}$,

then f – g is **[IIT Screening 2005]**

(A) one-one and onto
(B) one-one and into
(C) many one and onto
(D) neither one-one nor onto

Miscellaneous

257. If $A \cap B = B$, then **[JMIEE 2000]**

(A) $A \subset B$ (B) $B \subset A$
(C) $A = \phi$ (D) $B = \phi$

258. If A and B are not disjoint sets, then $n(A \cup B)$ is equal to **[Kerala PET 2001]**

(A) n(A) + n(B)
(B) $n(A) + n(B) - n(A \cap B)$
(C) n(A) – n(B)
(D) $n(A) + n(B) + n(A \cap B)$

259. If n(A) = 6, n(B) = 9 and $A \subseteq B$, then the number of elements in $A \cup B$ is equal to

(A) 3 (B) 9
(C) 6 (D) 12

260. If A = {a, b, c}, then the range of the relation R = {(a, b), (a, c), (b, c)} defined on A is

(A) {a, b} (B) {a, b, c}
(C) {c} (D) {b, c}

261. If f and g be two functions with domains D_f and D_g respectively, then domain of the functions $(fg)(x) = f(x)\,g(x)$ is

(A) $D_f \cup D_g$ (B) $D_f \cap D_g$
(C) D_g (D) D_f

262. The range of the function $f(x) = \dfrac{x}{|x|}$ is

(A) R – {0} (B) R – {–1, 1}
(C) R (D) {–1, 1}

263. Domain of $\sqrt{16 - x^2}$ is

(A) (– 4, 4) (B) (– 4, 4]
(C) [– 4, 4] (D) {– 4, 4}

264. Domain of $\dfrac{1}{\sqrt{25 - x^2}}$ is

(A) (– 5, 5) (B) (– 5, 5]
(C) [– 5, 5] (D) none of these

265. Domain of $\sqrt{x^2 - 36}$ is

(A) (– 6, 6)
(B) [– 6, 6]
(C) $(-\infty, 6) \cup (6, \infty)$
(D) $(-\infty, -6] \cup [6, \infty)$

266. If f is an exponential function and g is a logarithmic function, then fog(1) will be

(A) e (B) $\log_e e$
(C) 0 (D) 2e

267. The expression

$\left(x + \sqrt{x^2 - 1}\right)^5 + \left(x - \sqrt{x^2 - 1}\right)^5$ is a polynomial of degree **[IIT 1992]**

(A) 5 (B) 6
(C) 10 (D) 20

268. If $f(x) = \dfrac{1}{\sqrt{x + 2\sqrt{2x - 4}}} + \dfrac{1}{\sqrt{x - 2\sqrt{2x - 4}}}$ for $x > 2$, then f(11) = **[EAMCET 2003]**

(A) $\dfrac{7}{6}$ (B) $\dfrac{5}{6}$
(C) $\dfrac{6}{7}$ (D) $\dfrac{5}{7}$

269. Let R be a reflexive relation on a finite set A having n elements and let there be m ordered pairs in R. Then

(A) $m \geq n$ (B) m = n
(C) $m \leq n$ (D) none of these

270. For all $x \in (0, 1)$ **[IIT Screening 2000]**

(A) $e^x < 1 + x$ (B) $\log_e(1 + x) < x$
(C) $\sin x > x$ (D) $\log_e x > x$

271. If $f(x) = 2x + 1$ and $g(x) = \dfrac{x - 1}{2}$ for all real x, then $(fog)^{-1}\left(\dfrac{1}{x}\right)$ is equal to

[Kerala PET 2008]

(A) x (B) $\dfrac{1}{x}$
(C) $-x$ (D) $-\dfrac{1}{x}$

272. If $\log_{0.3}(x - 1) < \log_{0.09}(x - 1)$, then x lies in the interval **[DCE 2000]**

(A) $(2, \infty)$ (B) (1, 2)
(C) (–2, – 1) (D) none of these

273. The minimum value of $(x - \alpha)(x - \beta)$ is **[EAMCET 2001]**
(A) 0 (B) $\alpha\beta$
(C) $\frac{1}{4}(\alpha - \beta)^2$ (D) $-\frac{1}{4}(\alpha - \beta)^2$

274. If A and B are any two sets, then $A \cap (A \cup B)$ is equal to
(A) A (B) A^c
(C) B (D) B^c

275. If $aN = \{ax : x \in N\}$, then the set $6N \cap 8N$ is equal to
(A) 8N (B) 24N
(C) 12N (D) 48N

276. If $A = \{(x, y) : x^2 + y^2 \le 1; x, y \in R\}$ and $B = \{(x, y) : x^2 + y^2 \ge 4; x, y \in R\}$, then
(A) $A \cap B = \phi$ (B) $A - B = \phi$
(C) $A \cap B \ne \phi$ (D) $B - A = \phi$

277. Let R be the relation defined on N × N by the rule (a, b) R (c, d) ⇔ a + d = b + c where (a, b), (c, d) ∈ N × N. Then R is
(A) reflexive
(B) symmetric
(C) transitive
(D) an equivalence relation

278. If the function $f(x) = \frac{a^x + a^{-x}}{2}$, (a > 2), then $f(x + y) - f(x - y) =$
(A) $2 f(x).f(y)$ (B) $f(x).f(y)$
(C) $\frac{f(x)}{f(y)}$ (D) $4 f(x). f(y)$

279. The period of the function
$f(x) = \sin\left(\frac{\pi x}{n-1}\right) + \cos\left(\frac{\pi x}{n}\right)$, $n \in Z, n > 2$ is **[Orissa JEE 2002]**
(A) $2\pi n(n - 1)$ (B) $4\pi(n - 1)$
(C) $2n(n - 1)$ (D) $4n\pi(n - 1)$

280. If x is real, then value of the expression $\frac{x^2 + 14x + 9}{x^2 + 2x + 3}$ lies between **[UPSEAT 2002]**
(A) 5 and 4 (B) 5 and – 4
(C) – 5 and 4 (D) none of these

281. Let A and B be finite sets containing respectively m and n elements. The number of functions that can be defined from A to B is
(A) 2^{mn} (B) m^n
(C) n^m (D) mn

282. If $x \ne 1$ and $f(x) = \frac{x+1}{x-1}$ is a real function, then $f(f(f(2)))$ is **[Kerala PET 2001]**
(A) 1 (B) 2
(C) 3 (D) 4

283. If f and g are decreasing and fog is defined, then fog is **[Punjab CET 2008]**
(A) an increasing function
(B) a decreasing function
(C) neither increasing nor decreasing
(D) none of these

284. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$, for every real numbers, then the minimum value of f **[Pb. CET 2001]**
(A) Does not exist because f is bounded
(B) Is not attained even though f is bounded
(C) is 1
(D) is – 1

285. Suppose $f(x) = (x + 1)^2$ for $x \ge -1$. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line $y = x$, then $g(x)$ equals **[IIT Screening 2002]**
(A) $-\sqrt{x} - 1, x \ge 0$ (B) $\frac{1}{(x+1)^2}, x > -1$
(C) $\sqrt{x+1}, x \ge -1$ (D) $\sqrt{x} - 1, x \ge 0$

286. If A = {1, 2, 3} and R = {(1, 2), (2, 3)} be a relation on the set A, then the minimum number of ordered pairs which when added to R make it an equivalence relation is
(A) 5 (B) 8
(C) 6 (D) 7

287. If X = {1, 2, 3, 4, 5} and Y = {1, 3, 5, 7, 9}, then the set defined by $F = \{(x, y) : y = x + 2\}$ is a
(A) Mapping (B) Relation
(C) Both (D) None of these

288. If $f(x) = \sqrt{2 - x}$ and $g(x) = \sqrt{1 - 2x}$, then the domain of $f[g(x)]$ is
(A) $\left[\frac{1}{2}, \infty\right)$ (B) $\left[-\frac{3}{2}, \frac{1}{2}\right]$
(C) $\left[\frac{3}{2}, \frac{1}{2}\right]$ (D) $\left(-\infty, \frac{1}{2}\right]$

289. Let f be a function with domain [–3, 5] and $g(x) = |3x + 4|$. Then the domain of (fog) (x) is

(A) $\left[-3, \frac{1}{3}\right)$ (B) $\left(-3, \frac{1}{3}\right)$

(C) $\left(-3, \frac{1}{3}\right]$ (D) $\left[-3, \frac{1}{3}\right]$

290. If $f : R \to S$ is defined by

$f(x) = \sin x - \sqrt{3}\cos x + 1$ is onto, then the interval of S is

[AIEEE 2004; IIT Screening 2004]

(A) [1, 1] (B) [0, 1]

(C) [0, –1] (D) [–1, 3]

291. The function $f: X \to Y$ defined by $f(x) = \sin x$ is one-one but not onto if X and Y are respectively equal to **[Karnataka CET 2006]**

(A) R and R

(B) [0, π] and [–1, 1]

(C) $\left[0, \frac{\pi}{2}\right]$ and [–1, 1]

(D) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and [–1, 1]

292. The domain of the function $f(x) = \log_{10}\sin(x - 3) + \sqrt{16 - x^2}$ is

(A) (3, 4) (B) (3, 4]

(C) (– 4, 4) (D) (– 4, 4]

Answers to Multiple Choice Questions

1. (D)	2. (A)	3. (B)	4. (D)	5. (C)	6. (D)	7. (A)	8. (B)	9. (D)	10. (D)
11. (C)	12. (C)	13. (B)	14. (B)	15. (B)	16. (D)	17. (B)	18. (C)	19. (A)	20. (D)
21. (A)	22. (B)	23. (C)	24. (A)	25. (B)	26. (B)	27. (C)	28. (A)	29. (A)	30. (D)
31. (D)	32. (A)	33. (A)	34. (D)	35. (B)	36. (B)	37. (B)	38. (A)	39. (C)	40. (A)
41. (C)	42. (C)	43. (A)	44. (B)	45. (D)	46. (B)	47. (D)	48. (C)	49. (C)	50. (D)
51. (B)	52. (B)	53. (A)	54. (C)	55. (C)	56. (C)	57. (C)	58. (A)	59. (D)	60. (A)
61. (B)	62. (D)	63. (B)	64. (C)	65. (A)	66. (B)	67. (C)	68. (B)	69. (A)	70. (B)
71. (A)	72. (B)	73. (B)	74. (D)	75. (D)	76. (C)	77. (C)	78. (B)	79. (D)	80. (C)
81. (D)	82. (C)	83. (D)	84. (A)	85. (B)	86. (B)	87. (A)	88. (A)	89 (A)	90. (A)
91. (A)	92. (A)	93. (B)	94. (D)	95. (C)	96. (D)	97. (C)	98. (C)	99. (B)	100. (C)
101. (A)	102. (B)	103. (B)	104. (B)	105. (B)	106. (A)	107. (C)	108. (C)	109. (C)	110. (A)
111. (D)	112. (B)	113. (D)	114. (B)	115. (B)	116. (D)	117. (A)	118. (B)	119. (C)	120. (B)
121. (B)	122. (A)	123. (B)	124. (C)	125. (B)	126. (C)	127. (D)	128. (C)	129. (A)	130. (B)
131. (C)	132. (D)	133. (C)	134. (C)	135. (C)	136. (D)	137. (A)	138. (B)	139. (C)	140. (B)
141. (D)	142. (D)	143. (D)	144. (D)	145. (B)	146. (C)	147. (A)	148. (C)	149. (B)	150. (B)
151. (B)	152. (D)	153. (C)	154. (C)	155. (A)	156. (C)	157. (B)	158. (B)	159. (C)	160. (D)
161. (A)	162. (D)	163. (A)	164. (A)	165. (A)	166. (A)	167. (C)	168. (D)	169. (D)	170. (D)
171. (B)	172. (C)	173. (A)	174. (C)	175. (B)	176. (C)	177. (B)	178. (C)	179. (C)	180. (B)
181. (D)	182. (B)	183. (B)	184. (B)	185. (D)	186. (A)	187. (B)	188. (D)	189. (A)	190. (C)
191. (B)	192. (A)	193. (D)	194. (A)	195. (A)	196. (B)	197. (C)	198. (A)	199. (D)	200. (B)
201. (D)	202. (B)	203. (C)	204. (C)	205. (A)	206. (A)	207. (B)	208. (A)	209. (A)	210. (C)
211. (B)	212. (A)	213. (B)	214. (C)	215. (B)	216. (D)	217. (B)	218. (B)	219. (C)	220. (B)
221. (D)	222. (C)	223. (A)	224 (A)	225. (B)	226. (D)	227. (C)	228. (B)	229. (D)	230. (C)
231. (A)	232. (A)	233. (D)	234. (D)	235. (B)	236. (B)	237. (B)	238. (A)	239. (D)	240. (D)
241. (C)	242. (C)	243. (D)	244. (C)	245. (D)	246. (B)	247. (B)	248. (B)	249. (A)	250. (A)
251. (D)	252. (A)	253. (A)	254. (B)	255. (B)	256. (A)	257. (B)	258. (B)	259. (B)	260. (D)
261. (B)	262. (D)	263. (C)	264. (A)	265. (D)	266. (B)	267. (A)	268. (C)	269. (B)	270. (B)
271. (B)	272. (A)	273. (D)	274. (A)	275. (B)	276. (A)	277. (D)	278. (A)	279. (C)	280. (C)
281. (C)	282. (C)	283. (A)	284. (D)	285. (D)	286. (D)	287. (B)	288. (B)	289. (D)	290. (D)
291. (C)	292. (B)								

269. Since, R is a reflexive relation on A
$\therefore$ $(a, a) \in R\ \forall\ a \in A$
$\therefore$ The minimum number of ordered pairs in R is n.
Hence, m = n

270. Given, $0 < x < 1$

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \ldots.$$
$$= x - x^2\left[\frac{1}{2} - \frac{x}{3}\right] - x^4\left[\frac{1}{4} - \frac{x}{5}\right] + \ldots.$$
$$= x - \left[x^2\left(\frac{1}{2} - \frac{x}{3}\right) + x^4\left(\frac{1}{4} - \frac{x}{5}\right) + \ldots\right] < x$$

271. $(\text{fog})(x) = f(g(x))$
$$= f\left(\frac{x-1}{2}\right) = 2\left(\frac{x-1}{2}\right) + 1$$
$$= x$$
$\Rightarrow (\text{fog})(x) = x$
$\Rightarrow x = (\text{fog})^{-1}(x)$
Hence, $(\text{fog})^{-1}\left(\frac{1}{x}\right) = \frac{1}{x}$

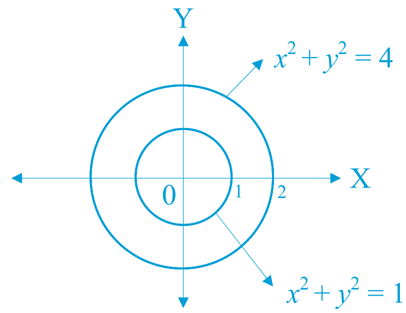
272. $\log_{0.3}(x-1) < \log_{0.09}(x-1)$
$\Rightarrow \log_{0.3}(x-1) < \log_{(0.3)^2}(x-1)$
$\Rightarrow \log_{0.3}(x-1) < \frac{1}{2}\log_{0.3}(x-1)$
$$\left[\because \log_{a^n} x = \frac{1}{n}\log_a x\right]$$
$\Rightarrow 2\log_{0.3}(x-1) - \log_{0.3}(x-1) < 0$
$\Rightarrow \log_{0.3}(x-1) < 0$
$\Rightarrow (x-1) > (0.3)^0$
($\because$ $\log_a x$ is a decreasing function when $0 < a < 1$)
$\Rightarrow x > 2$

273. $(x-\alpha)(x-\beta) = x^2 - (\alpha+\beta)x + \alpha\beta$
$$= x^2 - (\alpha+\beta)x + \left(\frac{\alpha+\beta}{2}\right)^2 + \alpha\beta - \left(\frac{\alpha+\beta}{2}\right)^2$$
$$= \left(x - \frac{\alpha+\beta}{2}\right)^2 - \left\{\left(\frac{\alpha+\beta}{2}\right)^2 - \alpha\beta\right\}$$
$$= \left(x - \frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha-\beta}{2}\right)^2$$
$\geq -\frac{1}{4}(\alpha-\beta)^2$ for all $x \in R$.

274. Since, $A \subseteq A \cup B$
$\therefore$ $A \cap (A \cup B) = A$

275. $6N \cap 8N = 24N$
[$\because$ 24 is the L. C. M. of 6 and 8]

276. A is the set of all points on the inner circle $x^2 + y^2 = 1$ and B is the set of all points on the outer circle $x^2 + y^2 = 4$.
From figure, it is clear that $A \cap B = \phi$



277. Here, (a, b) R (a, b) for all $(a, b) \in N \times N$
($\because$ a + b = b + a)
$\therefore$ R is **reflexive**
Let (a, b) R (c, d)
$\Rightarrow$ a + d = b + c
$\Rightarrow$ d + a = c + b
$\Rightarrow$ c + b = d + a
$\Rightarrow$ (c, d) R (a, b)
$\therefore$ R is **symmetric**
Let (a, b) R (c, d) and (c, d) R (e, f)
$\Rightarrow$ a + d = b + c and c + f = d + e
$\Rightarrow$ (a + d) + (c + f) = (b + c) + (d + e)
$\Rightarrow$ a + f = b + e
$\Rightarrow$ (a, b) R (e, f)
$\therefore$ R is **transitive**
Hence, R is an equivalence relation.

278. $f(x+y) - f(x-y)$
$$= \frac{1}{2}[a^{x+y} + a^{-x-y} + a^{x-y} + a^{-x+y}]$$
$$= \frac{1}{2}[a^x(a^y + a^{-y}) + a^{-x}(a^y + a^{-y})]$$
$$= \frac{1}{2}(a^x + a^{-x})(a^y + a^{-y})$$
$$= 2\, f(x).f(y)$$

279. Since, period of sin x and cos x is 2π.
$\therefore$ Period of $\sin\frac{\pi x}{n-1}$ is $\frac{2\pi}{\frac{\pi}{n-1}} = 2(n-1)$

and period of $\frac{\cos \pi x}{n}$ is $\frac{2\pi}{\frac{\pi}{n}} = n$

Hence, period of f(x) is L.C.M. of 2(n – 1) and n i.e., 2(n – 1)n.

280. Let $y = \frac{x^2+14x+9}{x^2+2x+3}$

$\Rightarrow x^2 + 14x + 9 = x^2y + 2xy + 3y$

$\Rightarrow x^2(y - 1) + 2x(y - 7) + 3y - 9 = 0$

Since, x is real

$\therefore$ $b^2 - 4ac > 0$

$\Rightarrow 4(y-7)^2 - 4(3y - 9)(y - 1) > 0$

$\Rightarrow 8y^2 + 8y - 160 < 0$

$\Rightarrow y^2 + y - 20 < 0$

$\Rightarrow (y + 5)(y - 4) < 0$

$\therefore$ y lies between –5 and 4.

281. Each of m elements of A can be associated to an element of B in n ways.

$\therefore$ All the m elements can be associated to elements in n^m ways.

$\therefore$ Required number of functions = n^m.

282. Here, $f(2) = \frac{2+1}{2-1} = 3$

$\therefore$ $f(f(2)) = f(3) = \frac{3+1}{3-1} = \frac{4}{2} = 2$

$\therefore$ $f(f(f(2))) = f(2) = \frac{2+1}{2-1} = 3$

283. Let $x_1, x_2 \in D_f$, then

$x_1 < x_2 \Rightarrow g(x_1) \geq g(x_2)$

($\because$ g is decreasing function)

$\Rightarrow f(g(x_1)) \leq f(g(x_2))$

($\because$ f is decreasing function)

$\therefore$ $x_1 < x_2 \Rightarrow (fog)(x_1) \leq (fog)(x_2)$

$\Rightarrow$ fog is an increasing function.

284. Let $f(x) = \frac{x^2-1}{x^2+1} = \frac{x^2+1-2}{x^2+1} = 1 - \frac{2}{x^2+1}$

Since, $x^2 + 1 > 1$

$\therefore$ $\frac{2}{x^2+1} \leq 2$

so, $1 - \frac{2}{x^2+1} \geq 1 - 2$

$\therefore$ $-1 \leq f(x) < 1$

Hence, f(x) has the minimum value equal to –1.

285. The reflection of $y = (x + 1)^2$ in $y = x$ is obtained by interchanging x and y.

$\therefore$ The reflection is $x = (y + 1)^2$

$\Rightarrow y + 1 = \sqrt{x}$ $\quad [\because y \geq -1 \therefore y + 1 \geq 0]$

$\Rightarrow y = \sqrt{x} - 1 \ \forall\ x \geq 0$

286. Given, A = {1, 2, 3} and R = {(1, 2), (2, 3)}

Now, R is reflexive if it contains (1, 1), (2, 2), (3, 3), then (1, 1), (2, 2), (3, 3) $\in$ R

R is symmetric, if (2, 1), (3, 2) $\in$ R

R is transitive if (3, 1), (1, 3) $\in$ R $\Rightarrow$ (1,1) $\in$ R

Thus, R becomes an equivalence relation by adding {(1, 1), (2, 2), (3, 3), (2, 1), (3, 2), (1, 3), (3, 1)}

Hence, the total no of ordered pairs is 7.

287. If $1 \in X \Rightarrow y = 1 + 2 = 3 \in Y$

$\therefore$ $(1,3) \in F$

If $2 \in X \Rightarrow y = 4 \notin Y$

$\therefore$ $(2, 4) \notin F$

If $3 \in X \Rightarrow y = 3 + 2 = 5 \in Y$

$\therefore$ $(3,5) \in F$

If $4 \in X \Rightarrow y = 6 \notin Y$

$\therefore$ $(4, 6) \notin F$

If $5 \in X \Rightarrow y = 7 \in Y$

$\therefore$ $(5, 7) \in F$

$\therefore$ F = {(1,3), (3,5), (5,7)}

Hence, F is a **relation** from X to Y

Since, $F \subset X \times Y$

But it is **not a mapping** or **function**.

Since, elements 2 and 4 of the domain X have no images in Y under F.

288. Here, $f[g(x)] = f\left(\sqrt{1-2x}\right)$

$= \sqrt{2-\sqrt{1-2x}}$

Here, f[g(x)] is defined, if $2-\sqrt{1-2x} \geq 0$ and $1 - 2x \geq 0$

$\Rightarrow 2 \geq \sqrt{1-2x}$ and $1 \geq 2x$

$\Rightarrow x \geq \frac{-3}{2}$ and $x \leq \frac{1}{2}$

$\Rightarrow -\frac{3}{2} \leq x \leq \frac{1}{2}$

$\therefore$ domain of $f[g(x)] = \left[-\frac{3}{2}, \frac{1}{2}\right]$

289. Here, (fog) (x) = f[g(x)] = f($|3x + 4|$)
The domain of f is $[-3, 5]$

$\therefore$ $-3 \le |3x + 4| \le 5$

$\Rightarrow -5 \le 3x + 4 \le 5$

$\Rightarrow -9 \le 3x \le 1$

$\Rightarrow -3 \le x \le \dfrac{1}{3}$

$\therefore$ domain of fog is $\left[-3, \dfrac{1}{3}\right]$

290. Since, maximum and minimum values of a cos θ + b sin θ are $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$ respectively.

$\therefore$ $-\sqrt{1+\left(-\sqrt{3}\right)^2} \le (\sin x - \sqrt{3}\cos x) \le \sqrt{1+\left(-\sqrt{3}\right)^2}$

$\Rightarrow -2 \le (\sin x - \sqrt{3}\cos x) \le 2$

$\Rightarrow -2 + 1 \le (\sin x - \sqrt{3}\cos x + 1) \le 2 + 1$

$\Rightarrow -1 \le (\sin x - \sqrt{3}\cos x + 1) \le 3$

Range = $[-1, 3]$

$\therefore$ For f to be onto, S = $[-1, 3]$

291. Given, f(x) = sin x

$\therefore$ f : R $\rightarrow$ R is neither one-one nor onto as $R_f = [-1, 1]$.

f : $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right] \rightarrow [-1, 1]$
is both one-one and onto.

f : $[0, \pi] \rightarrow [-1, 1]$
is neither one-one nor onto as
$R_f = [0, 1]$.

f : $\left[0, \dfrac{\pi}{2}\right] \rightarrow [-1, 1]$ is one-one but not onto as
$R_f = [0, 1]$.

292. Let g(x) = $\log_{10}\sin(x-3)$ and h(x) = $\sqrt{16-x^2}$
Now, g(x) = $\log_{10}\sin(x-3)$ is defined, if
$\sin(x - 3) > 0$

[If $\sin x > 0$, then $2n\pi < x < 2n\pi + \pi$, $n \in I$]

$\Rightarrow 2n\pi < x - 3 < 2n\pi + \pi$

$\Rightarrow 2n\pi + 3 < x < 2n\pi + \pi + 3$ ….(i)

and h(x) = $\sqrt{16-x^2}$ is defined, if

$16 - x^2 \ge 0$

$\Rightarrow (4 - x)(4 + x) \ge 0$

$\Rightarrow (x - 4)(x + 4) \le 0$

$\Rightarrow -4 \le x \le 4$ ….(ii)

From (i) and (ii), we get

$D_f = D_g \cap D_h$
$= (3, 4]$