Module 8

(Lecture 32)

PILE FOUNDATIONS

Topics

1.1 COMPARISON OF THEORY WITH FIELD LOAD TEST RESULTS
1.2 SETTLEMENT OF PILES
1.3 PULLOUT RESISTANCE OF PILES

- **Piles in Clay**
- **Piles in Sand**

1.4 LATERALLY LOADED PILES

- **Elastic Solution**
- **Ultimate Load Analysis-Brom's Method**
- **Ultimate Load Analysis-Meyerhof's Method**
- **Piles in Sand**

COMPARISON OF THEORY WITH FIELD LOAD TEST RESULTS

Details of many field studies related to the estimation of the ultimate load-carrying capacity of various types of piles are available in the literature. In some cases, the results generally agree with the theoretical predictions and, in others, they vary widely. The variations between theory and field test results may be attributed to factors such as improper interpretation of subsoil properties, incorrect theoretical assumptions, erroneous acquisition of field test results, and others.

We saw from example 1 that, for similar soil properties, the ultimate point load (Q_p) can vary over 400% or more depending on which theory and equation is used. Also, from the calculation of part of a example 1, it is easy to see that, in most cases, for long piles embedded in sand the limiting point resistance (q_1) [equations (15 or 16)] controls the unit point resistance (q_P). Meyerhof (1976) provided the results of several field load tests on long piles ($L/D \geq 10$) from which the derived values of q_p have been calculated and plotted in **figure 8.28**. Also plotted in this figure 8.28 is the variation of q_1 calculated from equation (16). It can be seen that, for a given friction angle ϕ, the magnitude of q_p can deviate substantially from the theory.

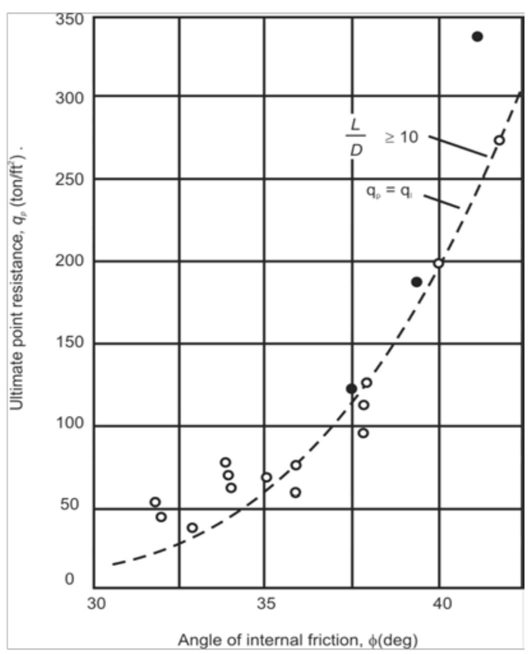

Figure 8. 28.Ultimate point resistances of driven piles in sand (after Meyerhof, 1976)

Briaud et al. (1989) reported the results of 28 axial load tests on impact-driven H-piles and pipe piles in sand performed by the U. S. Army Engineer District (St. Louis) during the construction of the New Lock and Dam No. 26 on the Mississippi River. Typical variations of field standard (uncorrected) penetration numbers with depth are shown in **figure 8.29**.

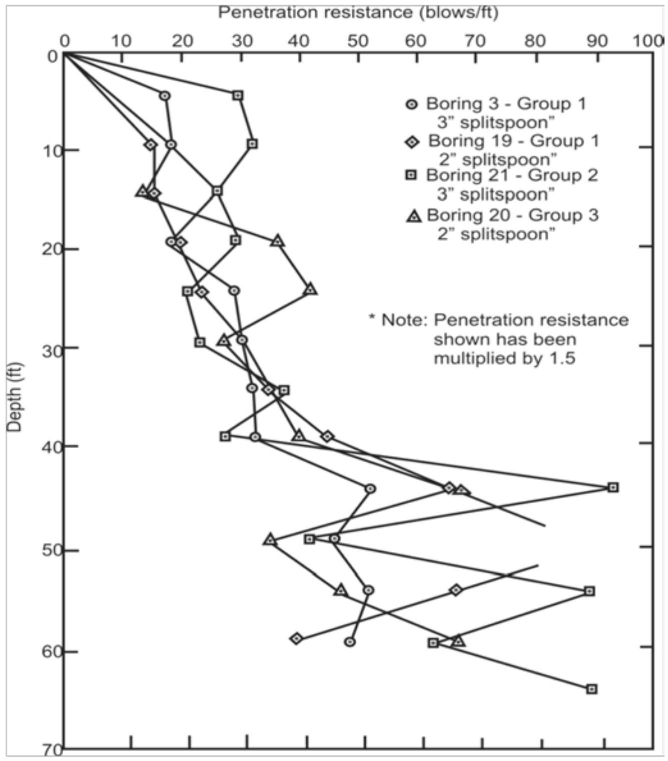

Figure 8.29 Results of the standard penetration test (after Briaud et al., 1989)

The results of the load tests on four H-piles obtained from this program are given in **figure 8.30**. Details of the H-piles and the load test results for these four piles are summarized in table 5. Briaud et al. (1989) made a statistical analysis for the ratio of theoretical ultimate load to the measured ultimate load. The results of this analysis are summarized in table 6 for the plugged case (figure 8. 11c). Note that a perfect prediction would have a mean = 1.0, standard deviation = 0, and a coefficient of variation = 0. Table 6 indicates that no method gave a perfect prediction; in general, Q_p was underestimated. Again, this shows the uncertainty in predicting the load-bearing capacity of piles.

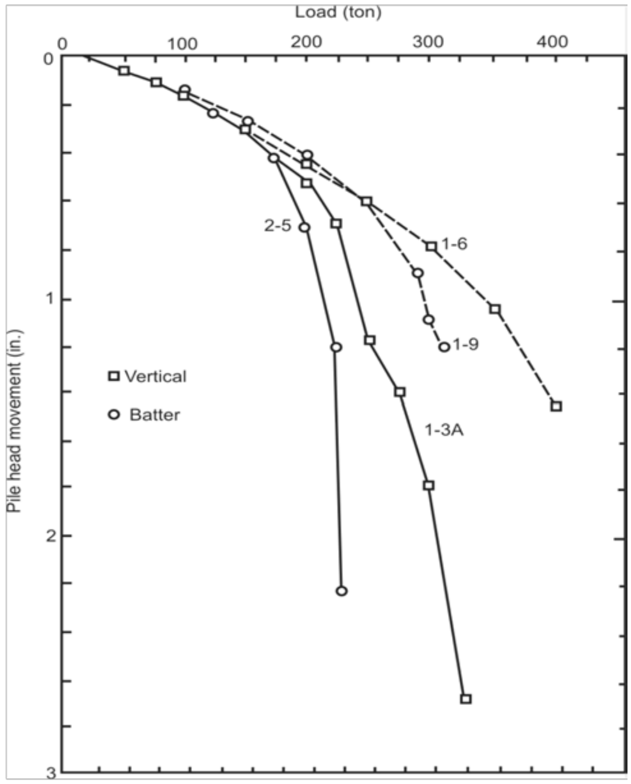

Figure 8.30 Load test results for H-piles in sand (after Briaud et al., 1989)

Table 5 Pile Load Test Results

Pile no.	Pile type	Batter	$Q_p(ton)$	$Q_{sp}(ton)$	$Q_u(ton)$	Pile length (ft)
1-3A	$HP14 \times 73$	Vertical	152	161	313	54
1-6	$HP14 \times 73$	Vertical	75	353	428	53
1-9	$HP14 \times 73$	1:2.5	85	252	337	58
2-5	$HP14 \times 73$	1:2.5	46	179	225	59

Sharma and Hoshi (1988) reported the results of field load tests on two cast-in-place concrete piles in a granular sol deposit in Alberta, Canada. The length of these piles (TP-1 and TP-2) was about 12.3 m. **figure 8.31** shows the general soil conditions, pile dimensions, and load-settlement curves. The load transfer mechanism for the two test piles is shown in **figure 8.32**. The average skin friction, f_{av} is calculated as

$$f_{av} = \frac{Q_{top} - Q_{base}}{\pi D_s L} \quad [8.58]$$

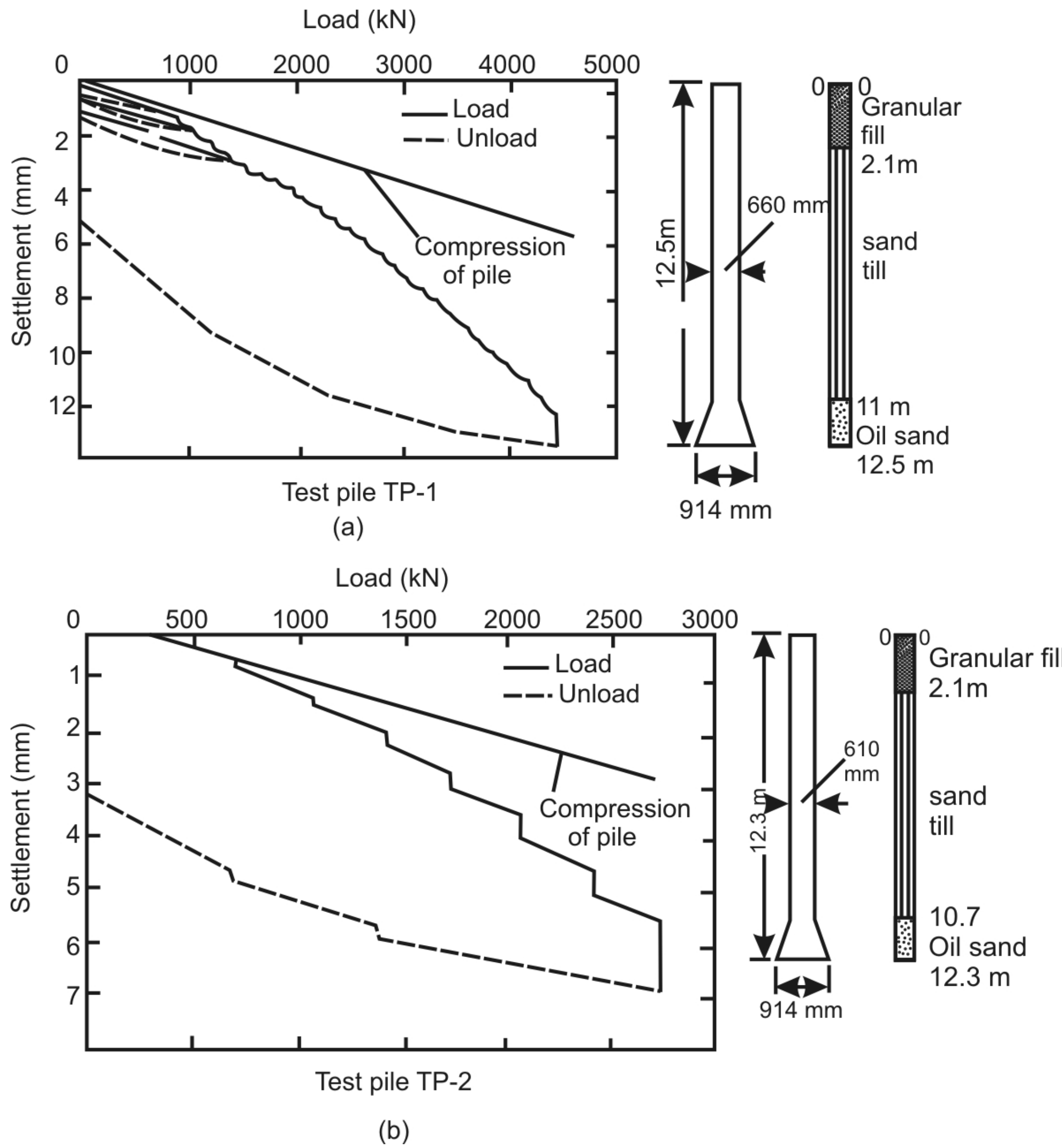

Figure 8.31 General soil condition, pile dimensions, and load-settlement curves (after Sharma and Joshi, 1988)

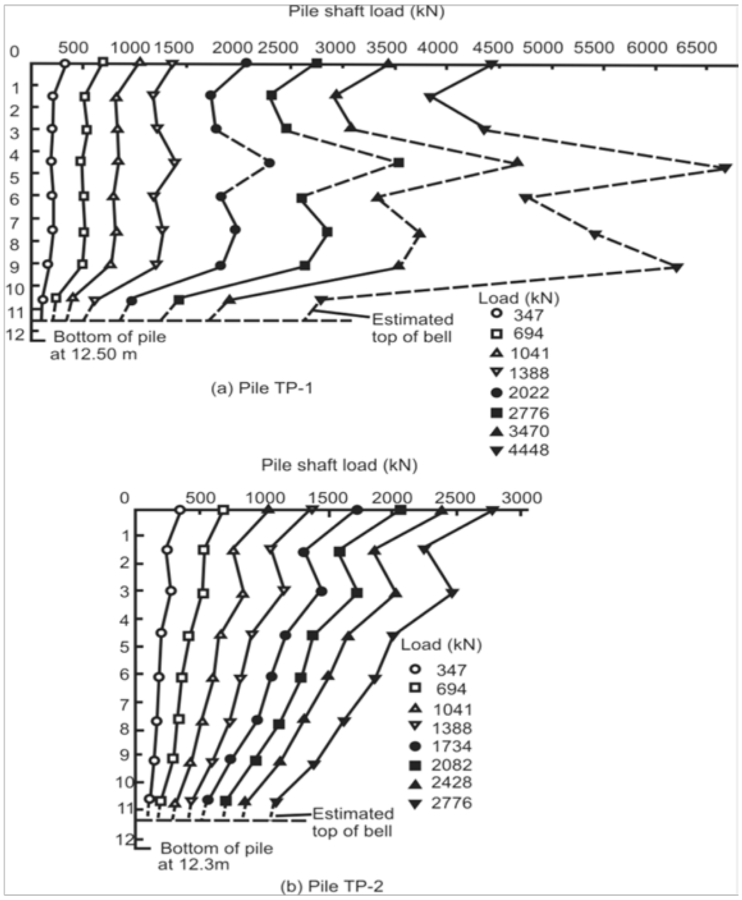

Figure 8. 8.32 Load transfer mechanism for two test piles (after Sharma and Joshi, 1988)

Where

$Q_{top} - Q_{base} = loads\ at\ the\ top\ and\ base\ of\ the\ pile, respectively$

$D_s = diameter\ of\ the\ pile\ shaft$

$L = pile\ length$

The variations of f_{av} with load, Q, for the two piles are plotted in **figure 8.33**. Note that, for the test pile TP-1, the maximum value of f_{av} appears to be about 85 kN/m^2 at a load of about 4000 kN. In figure 8. 31a, it corresponds to a relative displacement of about 7 mm between the soil and the pile. This result confirms that frictional resistance between the pile and the shaft is fully mobilized in about 5-10 mm of pile head movement. Again, referring to equations (38 and 40), we can say that, in general,

$$f_{av}\,(kN/m^2) = m\bar{N} \tag{8.59}$$

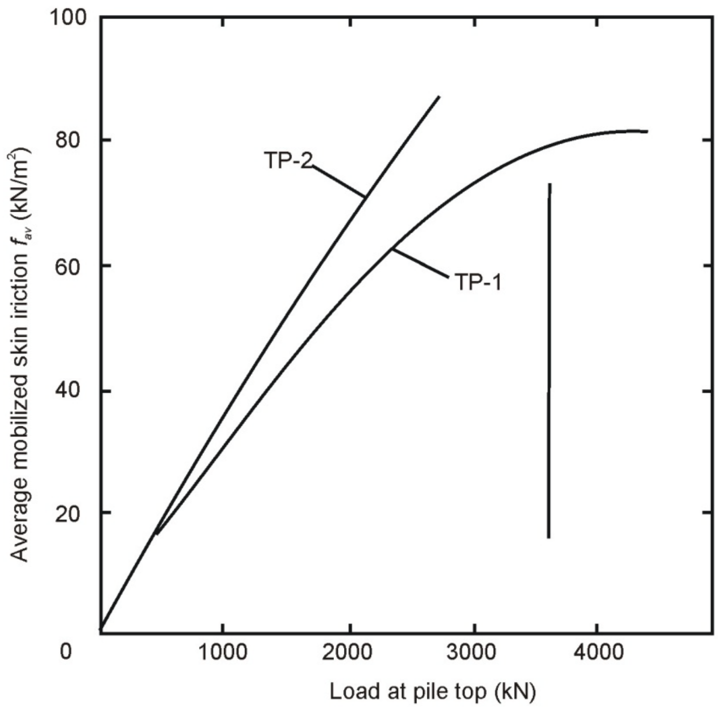

Figure 8.33 Variation of f_{av} with load, Q (after Sharma and Joshi, 1988)

Table 6 Summary of Briaud et al,'s Statistical Analysis for H-Piles-Plugged Case

	Q_p			Q_s			Q_u		
Theoretical method	Mean	Standard deviation	Coefficient of variation	Mean	Standard deviation	Coefficient of variation	Mean	Standard deviation	Coefficient of variation
Coyle and Castello (1981)	2.38	1.31	0.55	0.87	0.36	0.41	1.17	0.44	0.38
Briaud and Tucker (1984)	1.79	1.02	0.59	0.81	0.32	0.40	0.97	0.39	0.40
Meyerhof (1976)	4.37	2.76	0.63	0.92	0.43	0.46	1.68	0.76	0.45
API (1984)	1.62	1.00	0.62	0.59	0.25	0.43	0.79	0.34	0.43

Where

$m = constant\ and\ varies\ between\ 1\ and\ 2$

For test pile TP-1, the shaft length (not including the bell) is about 11 m. hence the following calculations may be determine f_{av}.

Soil	Thickness (m)	$\bar{N}_{cor}$	Average $\bar{N}_{cor}$
Sand and gravel	2.1	15	$\frac{(15)(2.1)+(39)(8.9)}{11} = 34.4$
Sand till	8.9	39	
From Sharma and Joshi (1988)			

The experimental value of f_{av} is about 85 kN/m^2, so from equation (60),

$$m = \frac{f_{av}}{\bar{N}_{cor}} = \frac{85}{34.4} = 2.47$$

This magnitude is somewhat higher than that given by either equation (38) or (40).

Lessons from the case studies above and others available in the literature show that previous experience and good practical judgment are required along with the knowledge of theoretical developments to design safe pile foundations.

SETTLEMENT OF PILES

The settlement of a pile under a vertical working load, Q_w, is caused by three factors:

$$s = s_1 + s_2 + s_3 \qquad [8.60]$$

Where

$s = total\ pile\ settlement$

$s_1 = elastic\ settlement\ of\ pile$

$s_2 = settlement\ of\ pile\ caused\ by\ the\ load\ at\ the\ pile\ tip$

$s_3 = settlement\ of\ pile\ casued\ by\ the\ load\ transmitted\ along\ the\ pile\ shaft$

If the pile material is assumed to be elastic, the deformation of the pile shaft can be evaluated using the fundamental principles of mechanics of materials.

$$s_1 = \frac{(Q_{wp} + \xi Q_{ws})L}{A_p E_p} \qquad [8.61]$$

Where

Q_{wp} $= load\ carried\ at\ the\ pile\ point\ under\ working\ load\ condition$

Q_{ws} $=$
$load\ carried\ by\ frictional\ (skin) resistance\ under\ working\ load\ condition$

$A_p = area\ of\ pile\ cross\ section$

$L = length\ of\ pile$

$E_p = modulus\ of\ elasticity\ of\ the\ pile\ material$

The magnitude of ξ will depend on the nature of unit friction (skin) resistance distribution along the pile shaft. If the distribution of f is uniform or parabolic, as shown in **figure 8.34a and 8.34b**, $\xi = 0.5$. However, for triangular distribution of f (figure 8. 34c), the magnitude of ξ is about 0.67 (Vesic, 1977).

The settlement of a pile caused by the load carried at the pile point may be expressed in a form similar to that given for shallow foundations [equation (33 from chapter 4)]:

$$s_2 = \frac{q_{wp}\,D}{E_s}(1-\mu_s^2)I_{wp} \qquad [8.62]$$

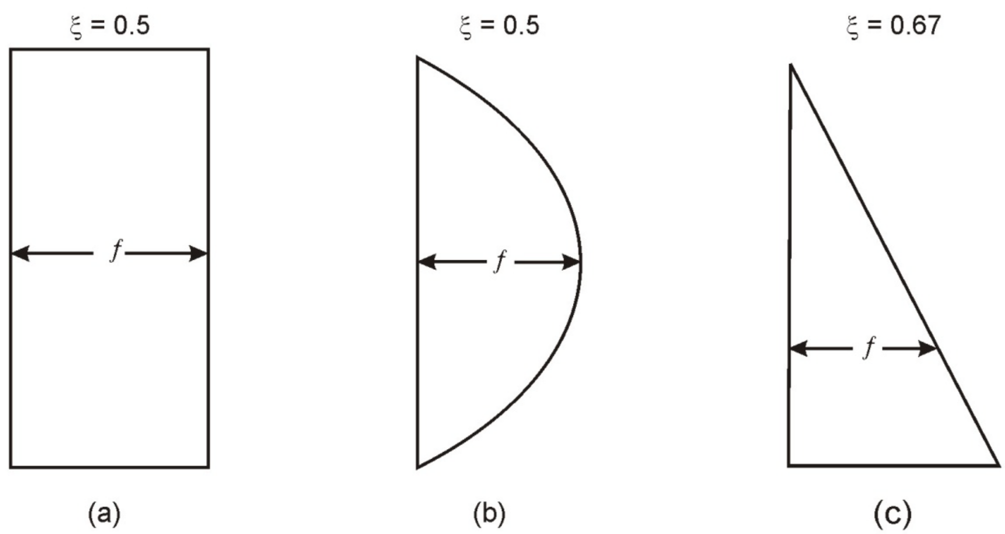

Figure 8.34 Various types of unit friction (skin) resistance distribution along the pile shaft

Where

$D = width\ or\ diameter\ of\ pile$

q_{wp} $= point\ load\ per\ unit\ area\ at\ the\ pile\ point = Q_{wp}/A_p$

$E_s = modulus\ of\ elasticity\ of\ soil\ at\ or\ below\ the\ pile\ point$

$\mu_s = Poisson's\ ratio\ of\ soil$

$I_{wp} = influence\ factor \approx 0.85$

Vesic (1977) also proposed a semi-empirical method to obtain the magnitude of the settlement, s_2:

$$s_2 = \frac{Q_{wp}\, C_p}{D q_p} \quad [8.63]$$

Where

$q_p = ultimate\ point\ resistance\ of\ the\ pile$

$C_p = an\ empirical\ coefficient$

Representative values of C_p for various soils are given in table 7.

The settlement of a pile caused by the load carried by the pile shaft is given by a relation similar to equation (62), or

$$s_3 = \left(\frac{Q_{ws}}{pL}\right)\frac{D}{E_s}(1-\mu_s^2)I_{ws} \quad [8.64]$$

Where

$p = perimeter\ of\ the\ pile$

$L = embedded\ length\ of\ pile$

$I_{ws} = influence\ factor$

Table 7 Typical Values of C_p [equation (64)]

Soil type	Driven pile	Bored pile
Sand (dense to loose)	0.02-0.04	0.09-0.18
Clay (stiff to soft)	0.02-0.03	0.03-0.06
Silt (dense to loose)	0.03-0.05	0.09-0.12
From “Design of Pile Foundations,” by A. S. Vesic in NCHRP *Synthesis of Highway Practice 42*, Transportation Research Board, 1977. Reprinted by permission		

Note that the term Q_{ws}/pL in equation (65) is the average value of along the pile shaft. The influence factor, I_{ws}, has a simple empirical relation (Vesic, 1977):

$$I_{ws} = 2 + 0.35\sqrt{\frac{L}{D}} \quad [8.65]$$

Vesic (1977) also proposed a simple empirical relation similar to equation (63) for obtaining s_3:

$$s_3 = \frac{Q_{ws} C_s}{L q_p} \quad [8.66]$$

Where

$$c_s = an\ empirical\ constant = (0.93 + 0.16\sqrt{L/D})C_p \quad [8.67]$$

The values of C_p for use in equation (66) may be estimated from table 7.

Sharma and Joshi (1988) used equations to estimate the settlement of two concrete piles in sand, as shown previously in **figure 8.31**, and compared them to observed values from the field. For these calculations, they used: $\xi = 0.5\ and\ 0.67, C_p = 0.02\ and\ C_s = 0.02$. Table 8 shows the comparison of s values. Note the fairly good agreement between estimated and observed vales of settlement.

Table 8 Comparison of Observed and Estimated Values of Settlement of Two Concrete Piles (figure 8. 31)

			Calculated s	
Pile	Load on pile (kN)	Measured s (mm)	$\xi = 0.5$	$\xi = 0.67$
TP-1	694	1.08	1.456	1.571
	1388	2.91	3.350	3.55
	2776	6.67	7.195	7.535
	4448	13.41	11.67	13.651
TP-2	694	0.65	1.467	1.610
	1388	2.11	3.118	3.387
	2776	6.72	6.889	7.365

Example 6

The allowable working load on a prestressed concrete pile 21 m long that has been driven into sand is $502\ kN$. The pile is octagonal in shape with $D = 356\ mm$. Skin resistance carries $350\ kN$ of the allowable load, and point bearing carries the rest. Use $E_p = 21 \times 10^6\ kN/m^2$, $E_s = 25 \times 10^3\ kN/m^2$, $\mu_s = 0.35, and\ \xi = 0.62$. Determine the settlement of the pile.

Solution

From equation (61),

$$s_1 = \frac{(Q_{wp} + \xi Q_{ws})L}{A_p E_p}$$

From table D-3 for $d = 356\, mm$, the area of pile cross section $A_p = 1045\, cm^2$. Also, perimeter $p = 1.168\, m$. Given: $Q_{ws} = 350\, kN$, so

$$Q_{wp} = 502 - 350 = 152\, kN$$

$$s_1 = \frac{[152 + 0.62(350)](21)}{(0.1045\, m^2)(21 \times 10^6)} = 0.00353\, m = 3.35\, mm$$

From equation (62),

$$s_2 = \frac{q_{wp}\, D}{E_s}(1 - \mu_s^2)I_{wp} = \left(\frac{152}{0.1045}\right)\left(\frac{0.356}{25 \times 10^3}\right)(1 - 0.35^2)(0.85)$$

$$= 0.0155\, m = 15.5\, mm$$

Again, from equation (64),

$$s_3 = \left(\frac{Q_{ws}}{pL}\right)\left(\frac{D}{E_s}\right)(1 - \mu_s^2)I_{ws}$$

$$I_{ws} = 2 + 0.35\sqrt{\frac{L}{D}} = 2 + 0.35\sqrt{\frac{21}{0.356}} = 4.69$$

$$s_3 = \left[\frac{350}{(1.168)(21)}\right]\left(\frac{0.356}{25 \times 10^3}\right)(1 - 0.35^2)(4.69)$$

$$= 0.00084\, m = 0.84\, mm$$

Hence, total settlement is

$$s = s_1 + s_2 + s_3 = 3.35 + 15.5 + 0.84 = 19.69\, mm$$

PULLOUT RESISTANCE OF PILES

In section 1we noted that, under certain construction conditions piles, are subjected to uplifting forces. The ultimate resistance of piles subjected to such force did not receive much attention among researchers until recently. The gross ultimate resistance of pile subjected to uplifting force (**figure 8.35**) is

$$T_{ug} = T_{un} + W \qquad [8.68]$$

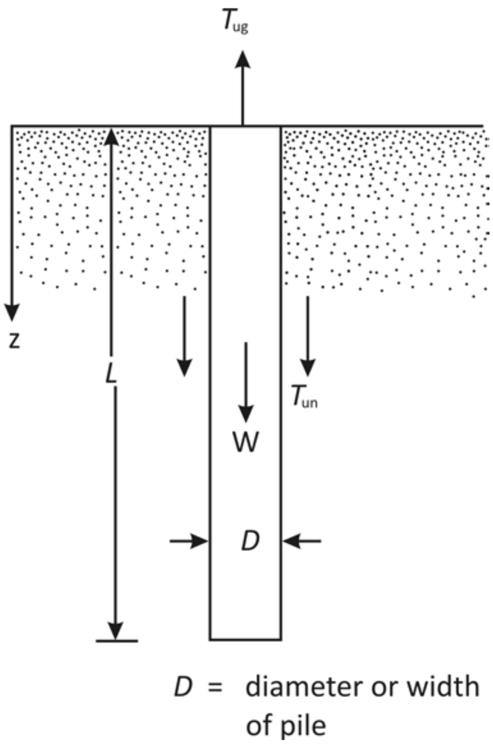

Figure 8.35 Uplift capacity of piles

Where

T_{ug} = gross uplift capacity

T_{un} = net uplift capacity

W = effective weight of the pile

Piles in Clay

The net ultimate uplift capacity of piles embedded in saturated clays was studied by Das and Seeley (1982). According to that study,

$$T_{un} = Lp\alpha' c_u \qquad [8.69]$$

Where

L = length of the pile

p = perimeter of pile section

α' = adhesion coefficient at soil – pile interface

$c_u = undrained\ cohesion\ of\ clay$

For cast-*in-situ* concrete piles,

$$\alpha' = 0.9 - 0.00625c_u \quad (for\ c_u \leq 80\ kN/m^2) \qquad [8.70]$$

And

$$\alpha' = 0.4\ (for\ c_u > 80\ kN/m^2) \qquad [8.71]$$

Similarly, for pile piles,

$$\alpha' = 0.715 - 0.0191c_u\ (for\ c_u \leq 27\ kN/m^2) \qquad [8.72]$$

And

$$\alpha' = 0.2\ (for\ c_u > 27\ kN/m^2) \qquad [8.73]$$

Piles in Sand

When piles are embedded in granular soils ($c = 0$), the net ultimate uplift capacity (Das and Seeley, 1975) is

$$T_{un} = \int_0^L (f_u p) dz \qquad [8.74]$$

Where

$f_u = unit\ skin\ friction\ during\ uplift$

$p = perimeter\ of\ pile\ cross\ section$

The unit skin friction during uplift, f_u, usually varies as shown in **figure 8.36a**. It increases linearly to a depth of $z = L_{cr}$; beyond that is remains constant. For $z \leq L_{cr}$,

$$f_u = K_u \sigma'_v \tan\delta \qquad [8.75]$$

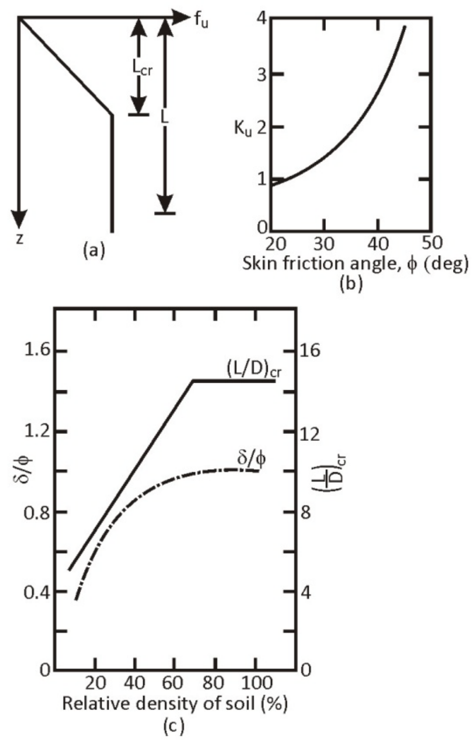

Figure 8.36 (a) Nature of variation of f_u; (b) uplift coefficient K_u; (c) variation of $\delta/\phi\ and\ (L/D)_{cr}$ with relative density of sand

Where

$K_u = uplift\ coefficient$

$\sigma'_v = effective\ vertical\ stress\ at\ a\ depth\ of\ z$

$\delta = soil - pile\ friction\ angle$

The variation of the uplift coefficient with soil friction angle ϕ is given in figure 8. 36b. Based on the author's experience, the values of $L_{cr}\ and\ \delta$ appear to depend on the relative density of soil. Figure 8. 36c shows the approximate nature of these variations with the relative density of soil. For calculating the net ultimate uplift capacity of piles, the following procedure is suggested:

1. Determine the relative density of the soil and, using figure 8.36c, obtain the value of L_{cr}.
2. If the length of the pile, L, is less than or equal to L_{cr},

$$T_{un} = p \int_0^L f_u \, dz = p \int_0^L (\sigma'_v K_u \tan \delta) dz \qquad [8.76]$$

In dry soils, $\sigma'_v = \gamma z$ (where $\gamma = unit\ weight\ of\ soil$), so

$$T_{un} = p \int_0^L (\sigma'_v K_u \tan \delta) dz = p \int_0^L \gamma z \, K_u \tan \delta dz$$
$$= \frac{1}{2} p \gamma L^2 K_u \tan \delta \qquad [8.77]$$

Obtain the values of $K_u\ and\ \delta$ from figure 8.36b and 8.36c.

3. For $L > L_{cr}$,

$$T_{un} = p \int_0^L f_u \, dz = p \left[\int_0^{L_{cr}} f_u \, dz + \int_{L_{cr}}^L f_u \, dz \right]$$
$$= p \left\{ \int_0^{L_{cr}} [\sigma'_v K_u \tan \delta] dz + \int_{L_{cr}}^L \left[\sigma'_{v(at\ z=L_{cr})} K_u \tan \delta \right] dz \right\} \qquad [8.78]$$

For dry soils, equation (79) simplifies to

$$T_{un} = \frac{1}{2} p \gamma L_{cr}^2 K_u \tan \delta + p \gamma L_{cr} K_u \tan \delta (L - L_{cr}) \qquad [8.79]$$

Determine the values of $K_u\ and\ \delta$ from figure 8.36b and 36c.
For estimating the net allowable uplift capacity, a factor of safety of 2-3 is recommended. Thus

$$T_{u(all)} = \frac{T_{ug}}{FS} \qquad [8.80]$$

Where

$T_{u(all)} = allowable\ uplift\ capacity$

Example 7

A concrete pile 50 ft long is embedded in a saturated clay with $c_u = 850\ lb/ft^2$. The pile is $12\ in. \times 12\ in.$ in cross section. Use $FS = 4$ and determine the allowable pullout capacity of the pile.

Solution

Given: $c_u = 850\ lb/ft^2 \approx 40.73\ kN/m^2$. From equation (70),

$$\alpha' = 0.9 - 0.00625 c_u = 0.9 - (0.00625)(40.73) = 0.645$$

From equation (70),

$$T_{un} = Lp\alpha' c_u = \frac{(50)(4\times1)(0.645)(850)}{1000} = 109.7\ kip$$

$$T_{un\,(all)} = \frac{109.7}{FS} = \frac{109.7}{4} = 27.4\ kip$$

Example 8

A precast concrete pile with a cross section of $350\ mm \times 350\ mm$ is embedded in sand. The length of the pile is 15 m. assume that $\gamma_{sand} = 15.8\ kN/m^3$, $\phi_{sand} = 35^\circ$, and the relative density of sand = 70%. Estimate the allowable pullout capacity of the pile $(FS = 4)$.

Solution

From figure 8. 36 for $\phi = 35^\circ$ and relative density = 70%,

$$\left(\frac{L}{D}\right)_{cr} = 14.5;\ L_{cr} = (14.5)(0.35\ m) = 5.08\ m$$

$$\frac{\delta}{\phi} = 1;\ \delta = (1)(35) = 35^\circ$$

$$K_u = 2$$

From equation (80),

$$T_{un} = \frac{1}{2}p\gamma L_{cr}^2 K_u \tan\delta + p\gamma L_{cr} K_u\ (L - L_{cr}) \tan\delta$$

$$= \left(\frac{1}{2}\right)(0.35\times4)(15.8)(5.08)^2(2) \tan 35$$

$$+(0.35\times4)(15.8)(5.08)(2)(15-5.08) \tan 35 = 1961\ kN$$

$$T_{un\,(all)} = \frac{1961}{FS} = \frac{1961}{4} \approx 490\ kN$$

LATERALLY LOADED PILES

A vertical pile resist lateral load by mobilizing passive pressure in the soil surrounding it (figure 8. 1c). The degree of distribution of the soil reaction depends on (a) the stiffness of the pile, (b) the stiffness of the soil, and (c) the fixity of the ends of the pile. In general, laterally loaded piles can be divided into two major categories: (1) short or rigid piles and (2) long or elastic piles. **Figure 8.37a** and 8.**37b** shows the nature of variation of pile deflection and the moment and shear force distribution along the pile length when subjected to lateral loading. Following is summary of the solutions presently available for laterally loaded piles.

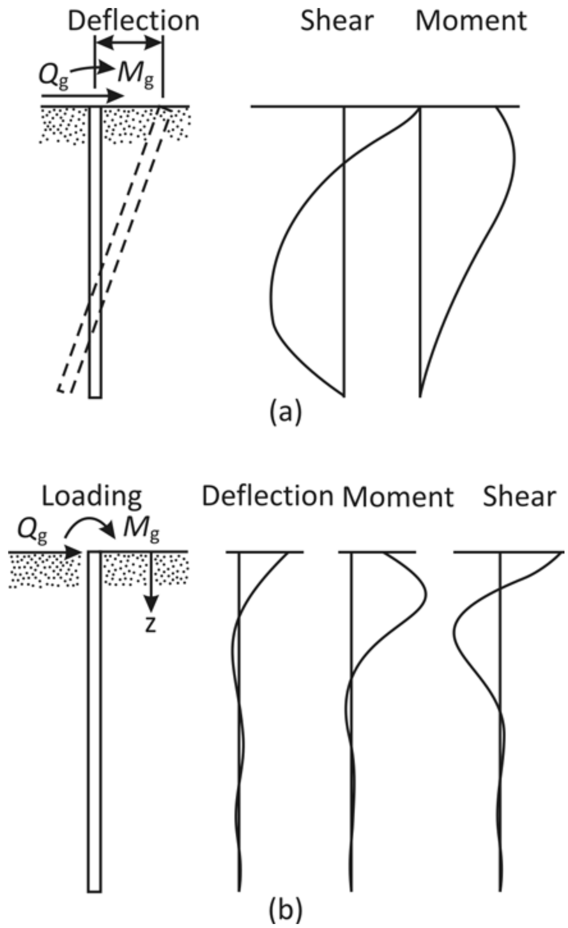

Figure 8.37 Nature of variation of pile deflection, moment, and shear force for (a) rigid pile, (b) elastic pile

Elastic Solution

A general method for determining moments and displacements of a vertical pile embedded in granular soils and subjected to lateral load and moment at the ground surface was given by Matlock and Reese (1960). Consider a pile of length L subjected to a lateral force Q_g and a moment M_g at the ground surface ($z = 0$), as shown in **figure 8. 38a**. figure 8.38b shows the general deflected shape of the pile and the soil resistance caused by the applied load and the moment.

According to a simpler Winkler's model, an elastic medium (soil in this case) can be replaced by a series of infinitely close independent elastic springs. Based on this assumption,

$$k = \frac{p^{'}(kN/m \ or \ lb/ft)}{x(m \ or \ ft)} \qquad [8.81]$$

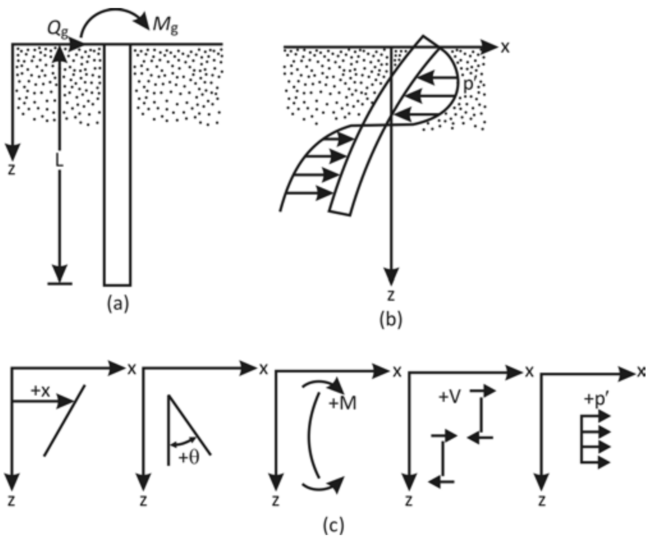

Figure 8.38 (a) Laterally loaded pile; (b) soil resistance on pile caused by lateral load; (c) sign convention for displacement, slope, moment, shear, and soil reaction

Where

$k = modulus\ of\ subgrade\ reaction$

$p^{'} = pressure\ on\ soil$

$x = deflection$

The subgrade modulus for *granular soils* at a depth *z* is defined as

$k_z = n_h z$ [8.82]

Where

$n_h = constant\ of\ modulus\ of\ horizontal\ subgrade\ reaction$

Referring to figure 8. 38b and using the theory of beams on an elastic foundation, we can write

$E_p I_p \frac{d^4x}{dz^4} = p'$ [8.83]

Where

$E_p = modulus\ of\ elasticity\ in\ the\ pile\ material$

$I_p = moment\ of\ inertia\ of\ the\ pile\ section$

Based on Winkler' model

$$p' = -kx \qquad [8.84]$$

The sign in equation (84) is negative because the soil reaction is in the direction opposite to the pile deflection.

Combining equations (83) and (84) gives

$$E_p I_p \frac{d^4x}{dz^4} + kx = 0 \qquad [8.85]$$

The solution of equation (85) results in the following expressions:

Pile Deflection at Any Depth [$x_z(z)$]

$$x_z(z) = A_x \frac{Q_g T^3}{E_p I_p} + B_x \frac{M_g T^2}{E_p I_p} \qquad [8.86]$$

Slope of Pile at Any Depth [$\theta_z(z)$]

$$\theta_z(z) = A_\theta \frac{Q_g T^2}{E_p I_p} + B_\theta \frac{M_g T}{E_p I_p} \qquad [8.87]$$

Moment of Pile at Any Depth [$M_z(z)$]

$$M_z(z) = A_m Q_g T + B_m M_g \qquad [8.88]$$

Shear Force on Pile at Any Depth [$V_z(z)$]

$$V_z(z) = A_v Q_g + B_v \frac{M_g}{T} \qquad [8.89]$$

Soil Reaction at Any Depth [$p'_z(z)$]

$$p'_z(z) = A_{p'} \frac{Q_g}{T} + B_{p'} \frac{M_g}{T^2} \qquad [8.90]$$

Where $A_x, B_x, A_\theta, B_\theta, A_m, B_m, A_v, B_v, A_{p'}, and\ B_{p'}$ are coefficients

$T = characteristics\ length\ of\ the\ soil - pile\ system$

$$= \sqrt[5]{\frac{E_p I_p}{n_h}} \qquad [8.91]$$

n_h has been defined in equation (82)

When $L \geq 5T$, the piles is considered to be a *long pile*. For $L \leq 2T$, the pile is considered to be a *rigid pile*. Table 9 gives the values of the coefficient for long piles ($L/T \geq 5$) in equations (86 to 90). Note that, in the first column of table 9,

Table 9 Coefficients for Long Piles, $k_z = n_h z$

Z	A_x	A_θ	A_m	A_v	A'_p	B_x	B_θ	B_m	B_v	B'_p
0.0	2.435	-1.623	0.000	1.000	0.000	1.623	-1.750	1.000	0.000	0.000
0.1	2.273	-1.618	0.100	0.989	-0.227	1.453	-1.650	1.000	-0.007	-0.145
0.2	2.112	-1.603	0.198	0.956	-0.422	1.293	-1.550	0.999	-0.028	-0.259
0.3	1.952	-1.578	0.291	0.906	-0.586	1.143	-1.450	0.994	-0.058	-0.343
0.4	1.796	-1.545	0.379	0.840	-0.718	1.003	-1.351	0.987	-0.095	-0.401
0.5	1.644	-1.503	0.459	0.764	-0.822	0.873	-1.253	0.976	-0.137	-0.436
0.6	1.496	-1.454	0.532	0.677	-0.897	0.752	-1.156	0.960	-0.181	-0.451
0.7	1.353	-1.397	0.595	0.585	-0.947	0.642	-1.061	0.939	-0.226	-0.449
0.8	1.216	-1.335	0.649	0.489	-0.973	0.540	-0.968	0.914	-0.270	-0.432
0.9	1.086	-1.268	0.693	0.392	-0.977	0.448	-0.878	0.885	-0.312	-0.403
1.0	0.962	-1.197	0.727	0.295	-0.962	0.364	-0.792	0.852	-0.350	-0.364
1.2	0.738	-1.047	0.767	0.109	-0.885	0.223	-0.629	0.775	-0.414	-0.268
1.4	0.544	-0.893	0.772	-0.056	-0.761	0.112	-0.482	0.688	-0.456	-0.157
1.6	0.381	-0.741	0.746	-0.193	-0.609	0.029	-0.354	0.594	-0.477	-0.047
1.8	0.247	-0.596	0.696	-0.298	-0.445	-0.030	-0.245	0.498	-0.476	0.054
2.0	0.142	-0.464	0.628	-0.371	-0.283	-0.070	-0.155	0.404	-0.456	0.140
3.0	-0.075	-0.040	0.225	-0.349	0.226	-0.089	0.057	0.059	-0.213	0.268
4.0	-0.050	0.052	0.000	-0.106	0.201	-0.028	0.049	-0.042	0.017	0.112
5.0	-0.009	0.025	-0.033	0.015	0.046	0.000	-0.011	-0.026	0.029	-0.002

From *Drilled Pier Foundations*, by R. J. Woodwood, W. S. Gardner, and D. M. Greer, Copyright 1972 by McGraw-Hill. Used with the permission of McGraw-Hill Book Company.

Z, is the nondimensional depth, or

$$Z = \frac{z}{T} \quad [8.92]$$

The positive sign conventions for $x_z(z), \theta_z(z), M_z(z), V_z(z), and\ p'_z(z)$ assumed in the derivations in table 9 are shown in figure 8. 38c. Also, **figure 8.39** shows the variation of $A_x, B_x, A_m, and\ B_m$ for various values of $L/T = Z_{max}$. It indicates that, when L/T is greater than about 5, the coefficients do no change, which is true of long piles only.

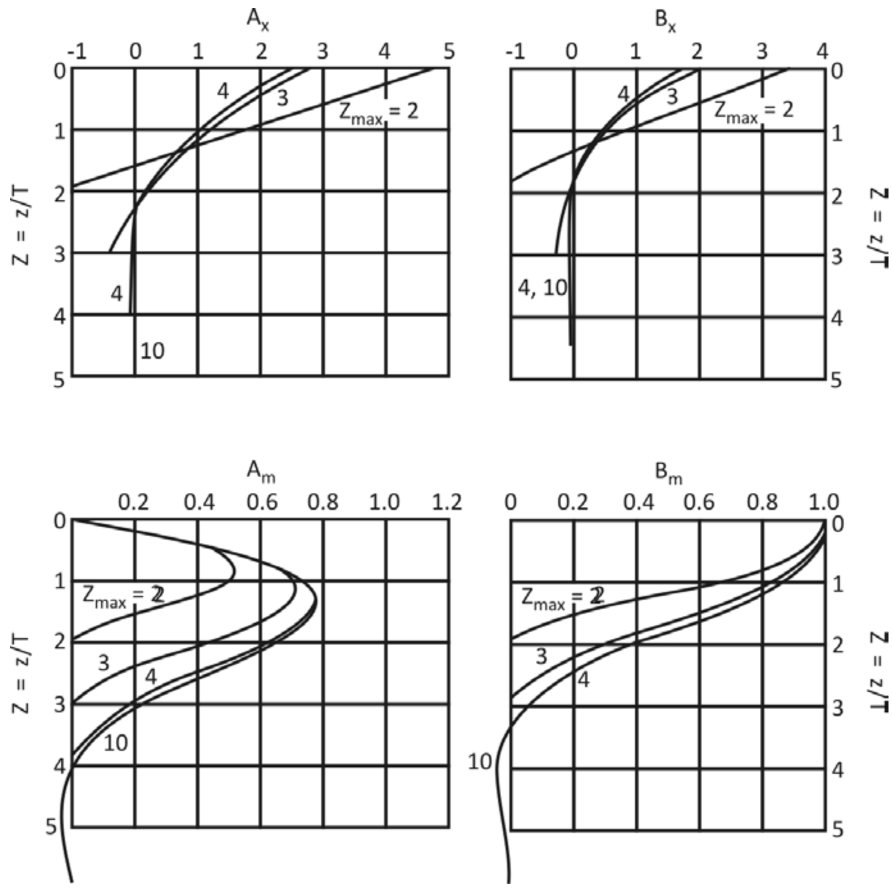

Figure 8.39 Variation of $A_x, B_x, A_m, and\ B_m\ with\ Z$ (after Matlock and Reese, 1960)

Calculating the characteristic length T for the pile requires assuming a proper value of n_h. Table 10 gives some representative values of n_h.

Elastic solutions similar to those given in equations (86)-(90) for piles embedded in *cohesive soil* were developed by Davisson and Gill (1963). These relationships are given in equations 93-97.

Table 10. Representative Values of n_h

	n_h	
Soil	lb/in^3	kN/m^3
Dry or moist and		
Loose	6.5-8.0	1800-2200
Medium	20-25	5500-7000
Dense	55-65	15,000-18,000
Submerged sand		
Loose	3.5-5.0	1000-1400
Medium	12-18	3500-4500
Dense	32-45	9000-12,000

$$\boldsymbol{x_z(z)} = A'_x \frac{Q_g R^3}{E_p I_P} + B'_x \frac{M_g R^2}{E_p I_p} \qquad [8.93]$$

And

$$M_z(z) = A'_m Q_g R + B'_m M_g \qquad [8.94]$$

Where

$A'_x, B'_x, A'_m, and\ B'_m$ are coefficients

$$R = \sqrt[4]{\frac{E_p I_p}{k}} \qquad [8.95]$$

The values of the $A^{'} and\ B'$ coefficients are given in **figure 8.40**. Note that

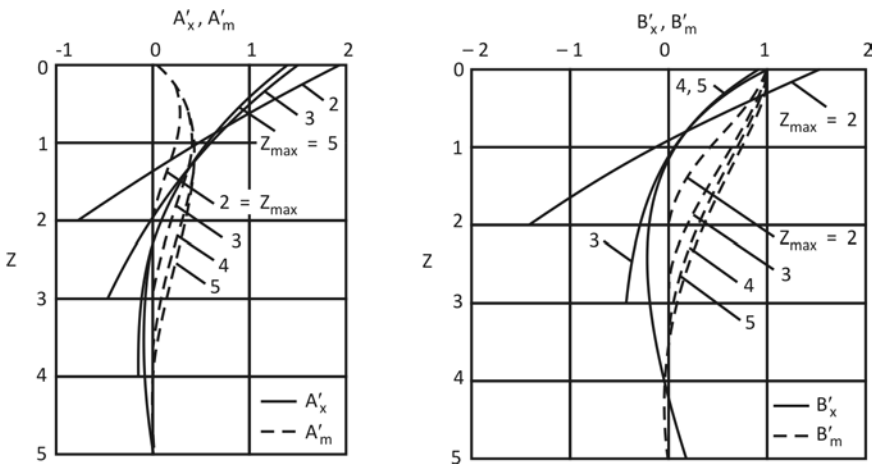

Figure 8.40 Variation of $A'_x, B'_x, A'_m, and\ B'_m\ with\ Z$ (after Davisson and Gill, 1960)

$$Z = \frac{z}{R} \qquad [8.96]$$

And

$$Z_{max} = \frac{L}{R} \qquad [8.97]$$

The use of equations (93 and 94) requires knowing the magnitude of the characteristic length, R. it can be calculated from equation (95), provided the coefficient of the subgrade reaction is known. For sands, the coefficient of subgrade reaction was given by equation (82), which showed a linear variation with depth. However, in cohesive soils, the subgrade reaction may be assumed to be approximately constant with depth. Vesic (1961) proposed the following equation to estimate the value of k:

$$k = 0.65 \sqrt[12]{\frac{E_s D^4}{E_p I_p}} \frac{E_s}{1-\mu_s^2} \qquad [8.98]$$

Where

$E_s = modulus\ of\ elasticity\ of\ soil$

$d = pile\ width\ (or\ diameter)$

$\mu_s = Poisson's\ ratio\ of\ the\ soil$

Ultimate Load Analysis-Brom's Method

Broms (1965) developed a simplified solution for laterally loaded piles based on the assumptions of (a) shear failure in soil, which is the case for short piles, and (b) bending of the pile governed by plastic yield resistance of the pile section, which is applicable for long piles. Brom's solution for calculating the ultimate load resistance, $Q_{u(g)}$, for *short piles* is given in **figure 8.41a**. A similar solution for piles embedded in cohesive soil is shown in figure 8.41b. In using figure 8.41a, note that

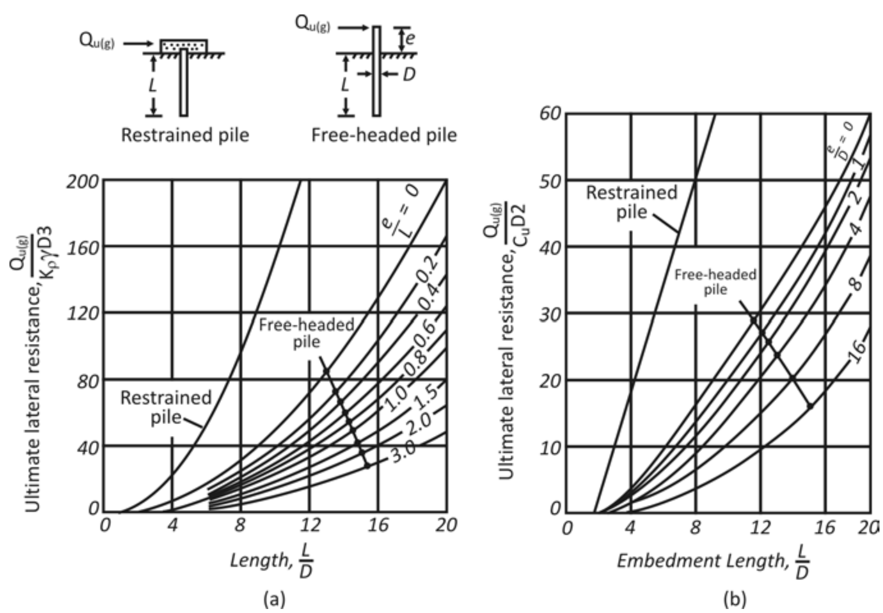

Figure 8.41 Brom's solution for ultimate lateral resistance of short piles (a) in sand, (b) in clay

$$K_p = Rankine\ passive\ earth\ pressure\ coefficient = tan^2\left(45 + \frac{\phi}{2}\right) \qquad [8.99]$$

Similarly, in figure 8.41b,

$$c_u = undrained\ cohesion \approx \frac{0.75q_u}{FS} = \frac{0.75q_u}{2} = 0.375q_u \qquad [8.100]$$

Where

$FS = factor\ of\ safety\ (= 2)$

$q_u = unconfined\ compression\ strength$

Figure 8.42 shows Brom's analysis of long piles. In this figure 8., M_y is the yield moment for the pile, or

$$M_y = SF_y \qquad [8.101]$$

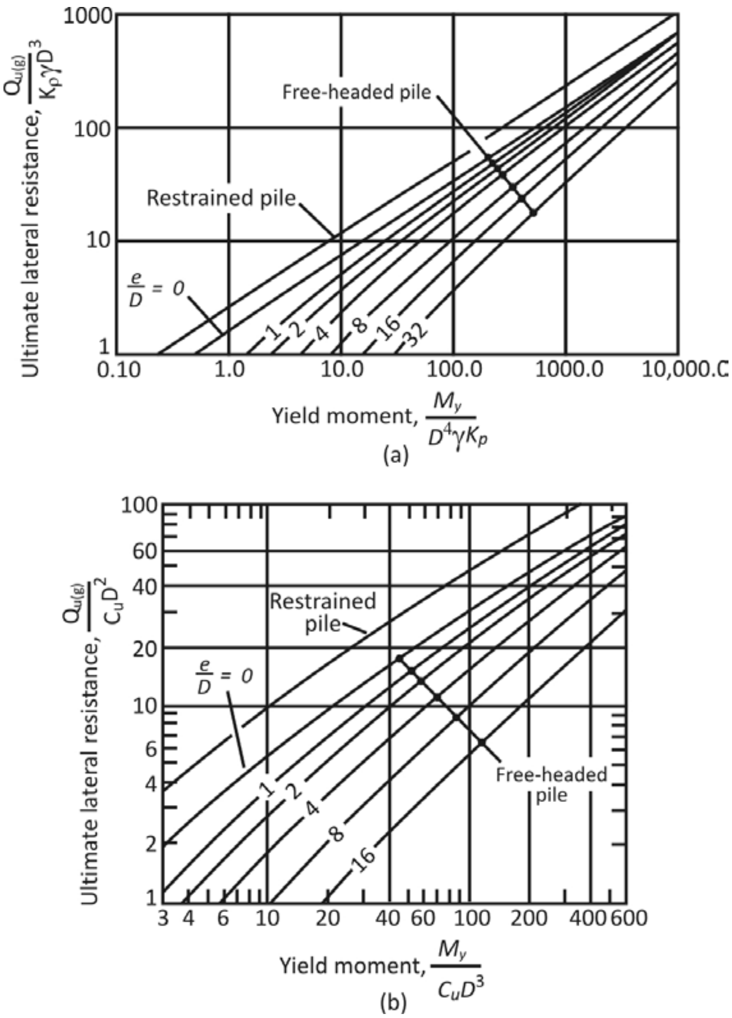

Figure 8.42 Brom's solution for ultimate lateral resistance of long piles (a) in sand, (b) in clay

Where

$S = section\ modulus\ of\ the\ pile\ section$

$F_y = yield\ stress\ of\ the\ pile\ material$

In solving a given problem, both cases (that is, figure 8.41 and figure 8.42) should be checked.

The deflection of the pile head, x_o, under working load conditions can be estimated from **figure 8.43**, the term η can be expressed as

$$\eta = \sqrt[5]{\frac{n_h}{E_p I_p}} \qquad [8.102]$$

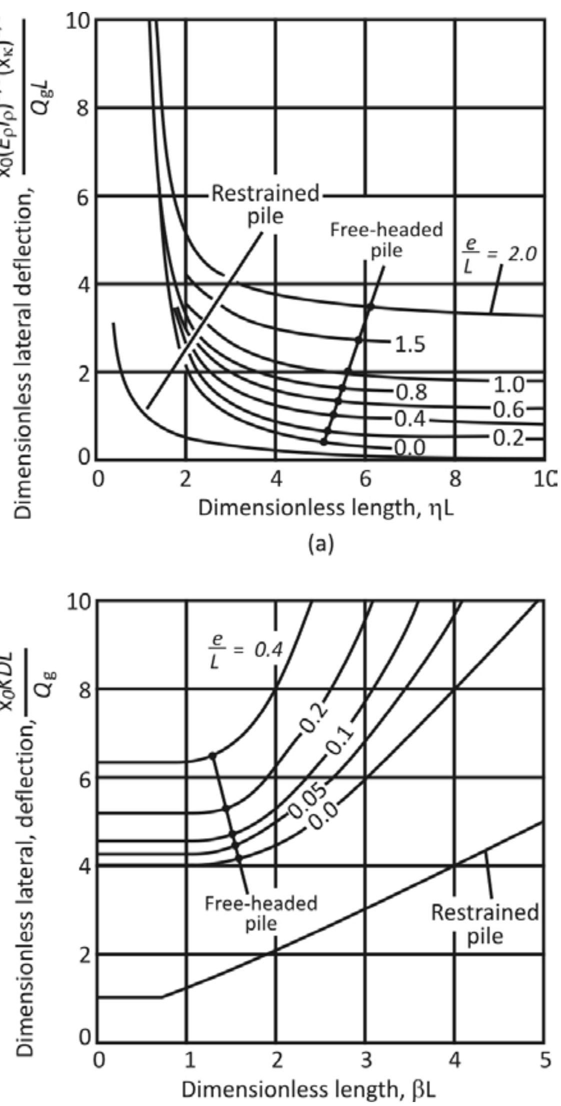

Figure 8.43 Brom's solution for estimating deflection of pile head (a) in sand, and (b) in clay

The range of n_h for granular soil is given in table 10. Similarly in figure 8. 43b, which is for clay, the term *K* is the horizontal soil modulus and can be defined as

$$K = \frac{pressure \ \ (lb/in^2 \ or \ kN/m^2)}{displacement \ \ (in.or \ m)} \qquad [8.103]$$

Also, the term β can be defined as

$$\beta = \sqrt[4]{\frac{KD}{4E_p I_p}} \qquad [8.104]$$

Note that, in figure 8. 43, Q_g is the working load.

Ultimate Load Analysis-Meyerhof's Method

More recently, Meyerhof (1995) provided the solutions for laterally loaded rigid and flexible piles (**figure 8.44**), which are summarized below. According to Meyerhof's method, a pile can be defined as flexible if

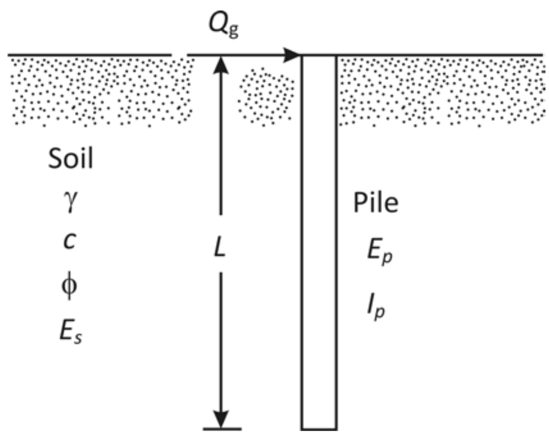

Figure 8.44 Pile with lateral loading at ground level

$$K_r = relative \ stiffness \ of \ pile = \frac{E_p I_p}{E_s L^4} < 0.01 \qquad [8.105]$$

Where

$$E_s = average \ horizontal \ soil \ modulus \ of \ elasticity$$

Piles in Sand

For short (rigid) piles in *sand*, the ultimate load resistance can be given as

$$Q_{u(g)} = 0.12 \, \gamma D L^2 K_{br} \leq 0.4 p_l D L \qquad [8.106]$$

Where

$\gamma = unit\ weight\ of\ soil$

$K_{br} = resultant\ net\ soil\ pressure\ coefficient\ (\textbf{figure 8.45})$

$p_l = limit\ pressure\ obtained\ from\ pressuremeter\ tests\ (chapter\ 2)$

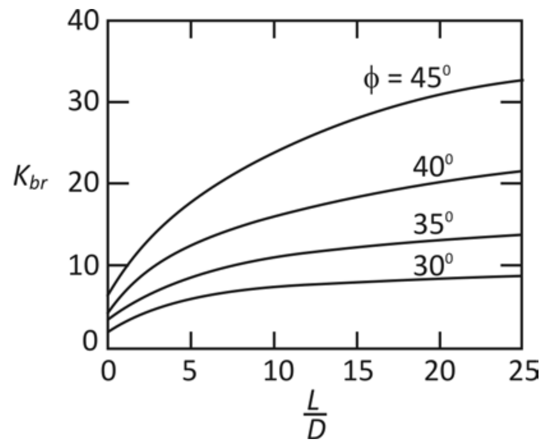

Figure 8.45 Variation of resultant net soil pressure coefficient, K_{br}

The limit pressure, p_l, can be given as

$$p_l = 40N_q \tan\phi\ (kPa) \quad (for\ Menard\ pressuremeter) \qquad [8.107]$$

And

$$p_l = 60N_q \tan\phi\ (kPa) \quad (for\ self-boring\ and\ full\ displacement\ pressuremeters) \qquad [8.108]$$

Where

$N_q = bearing\ capacity\ factor\ (table\ 4\ from\ chapter\ 3)$

The maximum moment, M_{max} , in the pile due to the lateral load, $Q_{u(g)}$, is

$$M_{max} = 0.35Q_{u(g)}L \le M_y \qquad [8.109]$$

For long (flexible) piles in sand, the ultimate lateral load, $Q_{u(g)}$, can be estimated from equation (106) by substituting an effective length (L_e) for L where

$$\frac{L_e}{L} = 1.65K_r^{0.12} \le 1 \qquad [8.110]$$

The maximum moment in a flexible pile due to a working lateral load Q_g applied at the ground surface is

$$M_{max} = 0.3K_r^{0.2}Q_gL \leq 0.3Q_gL \quad [8.111]$$

Piles in Clay The ultimate lateral load, $Q_{u(g)}$, applied at the ground surface for short (rigid) pile embedded in clay can be given as

$$Q_{u(g)} = 0.4c_uK_{cr}DL \leq 0.4p_lDL \quad [8.112]$$

Where

$p_l = limit\ pressure\ from\ pressuremeter\ test$

$K_{cr} = net\ soil\ pressure\ coefficient\ (\boldsymbol{figure\ 46})$

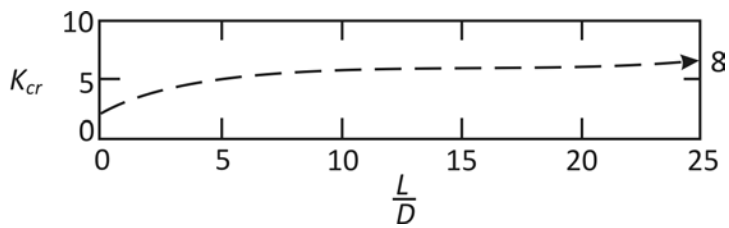

Figure 8.46 Variation of K_{cr}

The limit pressure in clay is

$p_l \approx 6c_u \quad (for\ Menard\ pressuremeter)$

$$p_l \approx 8c_u \quad (for\ self-boring\ and\ full\ displacement\ pressuremeter) \quad [8.114]$$

The maximum bending moment in the pile due to $Q_{u(g)}$ is

$$M_{max} = 0.22Q_{u(g)}L \leq M_y \quad [8.115]$$

For long (flexible) piles, equation (112) can be used to estimate $Q_{u(g)}$ by substituting the effective length (L_e) in place of L.

$$\frac{L_e}{L} = 1.5K_r^{0.12} \leq 1 \quad [8.116]$$

The maximum moment in a flexible pile due to a working lateral load Q_g applied at the ground surface is

$$M_{max} = 0.3K_r^{0.2}Q_gL \leq 0.15Q_gL \quad [8.117]$$

Example 9

Consider a steel H-pile ($HP\ 250 \times 0.834\ 25\ m$ embedded fully in a granular soil. Assume that $n_h = 12{,}000\ kN/m^3$. The allowable displacement at the top of the pile is 8 mm. determine the allowable lateral load, Q_g. Assume that $M_g = 0$. Use the elastic solution.

Solution

From table for an $HP\ 250 \times 0.834$ pile,

$$I_p = 123 \times 10^{-6} m^4 \quad (about\ the\ strong\ axis)$$

$$E_p = 207 \times 10^{6} kN/m^2$$

From equation (91),

$$T = \sqrt[5]{\frac{E_p I_p}{n_h}} = \sqrt[5]{\frac{(207 \times 10^6)(123 \times 10^{-6})}{12{,}000}} = 1.16\ m$$

Here, $L/T = 25/1.16 = 21.55 > 5$, so it is a long pile. Because $M_g = 0$, equation (86) takes the form

$$x_z(z) = A_x \frac{Q_g T^3}{E_p I_p}$$

And

$$Q_g = \frac{x_z(z) E_p I_p}{A_x T^3}$$

At $z = 0, x_z = 8\ mm = 0.008\ m, and\ A_x = 2.435\ (table\ 9),$ so

$$Q_g = \frac{(0.008)(207 \times 10^6)(123 \times 10^{-6})}{(2.435)(1.16^3)} = 53.59\ kN$$

This magnitude of Q_g is based on the *limiting displacement condition only*. However, the magnitude of Q_g based on the *moment capacity* of the pile also needs to be determined. For $M_g = 0$, equation (88) becomes

$$M_z(z) = A_m Q_g T$$

According to table 9, the maximum value of A_m at any depth is 0.772. The maximum allowable moment that the pile can carry is

$$M_{z(max)} = F_Y \frac{I_p}{\frac{d_1}{2}}$$

Let $F_Y = 248{,}000\ kN/m^2$. From table D. $lb, I_p = 123 \times 10^{-6}\ m^4\ and\ d_1 = 0.254\ m$, so

$$\frac{I_p}{\frac{d_1}{2}} = \frac{123 \times 10^{-6}}{\left(\frac{0.254}{2}\right)} = 968.5 \times 10^{-6}\ m^3$$

Now

$$Q_g = \frac{M_{z(max\ )}}{A_m T} = \frac{(968.5 \times 10^{-6})(248{,}000)}{(0.772)(1.16)} = 268.2\ kN$$

Because $Q_g = 268.2\ kN > 53.59\ kN$, the deflection criteria apply. Hence $Q_g = 53 - 59\ kN$.

Example 10

Solve example 9 by Brom's method. Assume that the pile is flexible and is free headed. Given: yield stress of pile material, $F_y = 248\ MN/m^2$; unit weight of soil, $\gamma = 18\ kN/m^3$; and soil friction angle, $\phi = 35^\circ$.

Solution

Check for bending failure. From equation (101),

$$M_y = SF_y$$

From table D. 1b,

$$S = \frac{I_p}{\frac{d_1}{2}} = \frac{123 \times 10^{-6}}{\frac{0.254}{2}}$$

$$M_y = \left[\frac{123 \times 10^{-6}}{\frac{0.254}{2}}\right](248 \times 10^3) = 240.2\ Kn - m$$

$$\frac{M_y}{D^4 \gamma K_p} = \frac{M_y}{D^4 \gamma\ tan^2\left(45 + \frac{\phi}{2}\right)} = \frac{240.2}{(0.250)^4 (18) tan^2\left(45 + \frac{35}{2}\right)} = 925.8$$

From **figure 8.42a**, for $M_y/D^4\gamma K_p = 925.8$, the magnitude of $Q_{u(g)}/K_p D^3 \gamma$ (for free headed pile with $e/D = 0$) is about 140, so

$$Q_{u(g)} = 140 K_p D^3 \gamma = tan^2\left(45 + \frac{35}{2}\right)(0.25)^2(18) = 581.2\ kN$$

Check for pile head deflection. From equation (102),

$$\eta = \sqrt[5]{\frac{n_h}{E_p I_p}} = \sqrt[5]{\frac{12{,}000}{(207 \times 10^6) 123 \times 10^{-6})}} = 0.86\ ,^{-1}$$

$\eta L = (0.86)(25) = 21.5$

From **figure 8.43a,** for $\eta L = 21.5, e/L = 0$ (free-headed pile),

$$\frac{x_o(E_p/I_p)^{3/5}(n_h)^{2/5}}{Q_g L} \approx 0.15 \qquad (by\ interpolation)$$

$$Q_g = \frac{x_o(E_p/I_p)^{3/5}(n_h)^{2/5}}{0.15L}$$

$$= \frac{(0.008)[(207\times10^6)(123\times10^{-6})]^{3/5}(12{,}000)^{2/5}}{(0.15)(25)} = 40.2\ kN$$

Hence, $Q_g = 40.2\ kN\ (< 581.2\ kN)$.