Engineering Mathematics – II

(10 MAT21)

LECTURE NOTES

(FOR II SEMESTER B E OF VTU)

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ENGNEERING MATHEMATICS – II

Content

Unit-IV

PARTIAL DIFFERENTIAL EQUATIONS

Overview:

In this unit we study how to form a P.D.E and various methods of obtaining solutions of P.D.E. This unit consists of 6 sections. In section 1, we learn how to form the P.D.E. by eliminating arbitrary constants and in section 2 we learn the formation of P.D.E by eliminating arbitrary functions. In section 3, the solution of non homogeneous P.D.E by the method of direct integration is discussed. In section 4, the solution of homogeneous equations is discussed. In section 5 we learn the method of separation of variables to solve homogeneous equations. In section 6 we discuss the Lagrange's linear equation and the solution by the method of grouping and multipliers, at end some muliple choice questions prominence the comprehensive unit.

Objective:

At the finish of this unit, we will be able to:

- Form Partial differential equation.
- Solve the first order linear partial differential equation
- Obtain the solution of homogeneous P.D.E by different methods.
- Obtain the solution of non homogeneous P.D.E

Section 1:

Formation of P.D.E by eliminating arbitrary constants

Objective:

At the closing stages of this Section, we will be able to recognize:

- To identify P.D.E order, degree and classification of a P.D.E.
- Formation of P.D.E by elimination of arbitrary constants.

Introduction:

To start with partial differential equations, just like ordinary differential or integral equations, are functional equations. That means the known, or unknowns, we are trying to determine the functions. In the case of partial differential equations(PDE) these functions are to be determined from equations which involve in addition to the usual operations of addition and multiplication, partial derivatives of the functions.

Our objective is to provide an introduction to this important field of mathematics, as well as an entry point, for those who wish it, to the modern, more abstract elements of partial differential equations. A wide variety of partial differential equations occurs in technical computing nowadays

Many real world problems in general involve functions of several independent variables which give rise to partial differential equations more often than ordinary differential equations. Most of the science and engineering problems like vibration of strings, heat conduction, electrostatics etc., flourish with first and second order linear non homogeneous P.D.Es.

A partial differential equation is an equation involving two (or more) independent variables x, y and a dependent variable z and its partial derivatives such as $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}$, etc.,

i.e., F $\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}, \ldots \ldots \ldots\right) = 0$

Order of a P.D.E is the order of the highest ordered derivative appearing in the P.D.E.

Degree of a P.D.E is the degree of the highest order derivative present in the P.D.E after clearing the fractional powers.

Standard notation:

— , — — , — —

Classification of PDEs

The general form of a linear second order PDE, in the two variables x_1, x_2, is given by

$$Au_{11} + \; Bu_{12} + Cu_{22} + Du_1 + Eu_2 + Fu = f$$

where A, B, C, D, E, F and f all depend only on x_1, x_2. There is a classification scheme depending on the values of A, B, C; we say that PDE is:

$$\begin{aligned} \text{Hyperbolic if} \quad B^2 - AC &> 0 \\ \text{Parabolic if} \quad B^2 - AC &= 0 \\ \text{Elliptic if} \quad B^2 - AC &< 0. \end{aligned}$$

If $f = 0$ then the PDE is homogeneous.

If PDE is to represent a nonlinear PDE, then some of the functions A, B, C, D, E, F depend on u, as well as on x_1, x_2. We do not consider nonlinear PDEs in detail in this course.

Formation of partial differential equation by el

Let …….(1) be an equation involving two

Differentiating (1) partially w.r.t x and y,

we get — —— ……………………………(2)

— —— ……………………………..(3)

By eliminating a,b from(1), (2),(3), we get an equation of form

F(x, y, z, p, q)=0…………………….(4)

Which is a partial differential equation of first order.

Note:

1. If the number of arbitrary constants equal to the number of independent variables in (1) ,then the P.D.E obtained is of first order.
2. If the number of arbitrary constants is more than the number of independent variables, then the P.D.E obtained is of 2^{nd} of higher orders.

Examples:

Form the partial differential equation by eliminating the arbitrary constants.

1) $z = (x + a)(y + b)$

Solution: Differentiating z partially w.r.t x and y, we get

$$p = \frac{\partial z}{\partial x} = y + b$$

$$q = \frac{\partial z}{\partial y} = x + a$$

$\Rightarrow z = pq$ is the required p.d.e

2) $z = a\log(x^2 + y^2) + b$

Solution: Differentiating z partially w.r.t x and y, we get

$$p = \frac{\partial z}{\partial x} = \frac{a}{x^2+y^2} \cdot 2x \qquad \Rightarrow p = \frac{2ax}{x^2+y^2}$$

$$q = \frac{\partial z}{\partial y} = \frac{2ay}{x^2+y^2}$$

$$\Rightarrow \quad \frac{p}{q} = \frac{x}{y}$$

$\Rightarrow p\,y - q\,x = 0$ is the required p.d.e

3) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Solution: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ……………………………………….(1)

Differentiating (1) partially w.r.t x and y, we get

$$\frac{2x}{a^2} + \frac{2z}{c^2}p = 0 \qquad \text{and} \qquad \frac{2y}{b^2} + \frac{2z}{c^2}q = 0$$

$\frac{x}{a^2} + \frac{z}{c^2}p = 0$ ………………………………………….(2)

$\frac{y}{b^2} + \frac{z}{c^2}q = 0$ ………………………………………….(3)

Differentiating (2) partially w.r.t x again, we get

$\frac{1}{a^2} + \frac{1}{c^2}(p^2 + zr) = 0$..............................(4)

Now from (2), we have $\frac{1}{a^2} = -\frac{zp}{c^2x}$

Substituting this in (4) we get

$-\frac{zp}{c^2x} + \frac{1}{c^2}(p^2 + zr) = 0$

$zp = (p^2 + zr)x$ is the required p.d.e.

4) Find the P.D.E of the family of all spheres whose centers lie on the plane $z = 0$ and have a constant radius `r'.

Solution: The co-ordinates of the centre of the sphere can be taken as (a,b,0) where a and b are arbitrary , r being the constant radius. The equation of the sphere is

$(x - a)^2 + (y - b)^2 + (z - 0)^2 = r^2$

$(x - a)^2 + (y - b)^2 + z^2 = r^2$....................................(1)

a and b are the arbitrary constants and have to be eliminated .

Differentiating (1) partially w.r.t x and y, we get

$2(x - a) + 2zp = 0$ and

$2(y - b) + 2zq = 0$

$\Rightarrow \quad (x - a) = -zp$

$(y - b) = -zq$

Squaring and adding we get

$(x - a)^2 + (y - b)^2 = z^2p^2 + z^2q^2$

i.e. $r^2 - z^2 = z^2p^2 + z^2q^2$

$\Rightarrow r^2 = (p^2 + q^2 + 1)\ z^2$

Exercise:

1. Form the partial differential equation by eliminating the arbitrary constants.
   a) $z = ax^2 + by^2$
   b) $(x - a)^2 + (y - b)^2 = z^2cot^2\alpha$

Exercise:

1) Form the partial differential equation by eliminating the arbitrary functions.

$a)\ z = (x + y)\varphi(x^2 - y^2)$

$b)\ z = x^n f\left(\frac{y}{x}\right)$

$c) xyz = f(x + y + z)$

Section 3

Solution of a non-homogeneous P.D.E by direct integration method

Objective:

At the end of this section we will be able to:

- Know different types of solution of a P.D.E.
- Solve non homogeneous P.D.E by direct integration

The general form of a first order partial differential equation is $F(x, y, z, p, q) = 0$……………..(1) where x, y are two independent variables , z is the dependent variable and $p = z_x, q = z_y$.

A solution or integral of the p.d.e is the relation between the dependent and independent variable satisfying the equation.

Complete solution:

Any function $f(, y, z, a, b) = 0$…………………………..(2) involving two arbitrary constants a, b and satisfying p.d.e (1) is known as the complete solution or complete integral or primitive. Geometrically the complete solution represents a two parameter family of surfaces .

Eg: $(x + a)(y + b) = z$ is the complete solution of the p.d.e $z = pq$

Particular solution:

A solution obtained by giving particular value to the arbitrary constants in the complete solution is called a particular solution of the p.d.e. It represents a particular surface of a family of surfaces given by the complete solution.

Eg: $(x + 3)(y + 4) = z$ is the particular solution of the p.d.e $z = pq$

General solution:

In the complete solution if we put $b = \varphi(a)$ then we get a solution containing an arbitryary function φ, which is called a general solution. It represents the envelope of the family of surfaces

$f(x, y, z, \ \varphi(a)) = 0.$

Singular solution:

Differentiating the p.d.e (1) w.r.t the arbitrary constants a and b we get

$\frac{\partial F}{\partial a} = 0$ and $\frac{\partial F}{\partial b} = 0$

Suppose it is possible to eliminate a and b from the three equations then the relation so obtained is called the singular solution of the p.d.e.

Singular solution represents the envelope of the two parameter family of surfaces.

Eg: The complete solution of the p.d.e $z = pq$ is $(x + a)(y + b) = z$

Differentiating partially w.r.t a and b we get

$x + a = 0$ and $(y + b) = 0. \Rightarrow z = 0$ is the singular solution.

Solution by Direct integration

Examples:

1) Solve the equation $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$

Solution: The given equation can be written as $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = x^2 y$

Integrating w.r.t. x we get

$$\frac{\partial z}{\partial y} = \frac{x^3 y}{3} + f(y)$$

Where $f(y)$ is an arbitrary function of y

Integrating the above w.r.t. y we get

$$z = \frac{x^3 y^2}{6} + \int f(y) dy + g(x)$$

$$z = \frac{1}{6}(x^3 y^2) + F(y) + g(x)$$

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Where $g(x)$ is an arbitrary function of x

This is the general solution of the given equation.

2) Solve the equation $xy\frac{\partial^2 z}{\partial x\partial y} - x\frac{\partial z}{\partial x} = y^2$

Solution: Diving throughout by x the equation may be written as

$$y\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) - \frac{\partial z}{\partial x} = \frac{y^2}{x}$$

$y\frac{\partial p}{\partial y} - p = \frac{y^2}{x}$ or $\frac{1}{y}\frac{\partial p}{\partial y} - \frac{p}{y^2} = \frac{1}{x}$

$\Rightarrow \quad \frac{\partial}{\partial y}\left(\frac{p}{y}\right) = \frac{1}{x}$

Integrating the above w.r.t. y we get

$\frac{p}{y} = \frac{1}{x}y + f(x)$

$\frac{1}{y}\frac{\partial z}{\partial x} = \frac{y}{x} + f(x)$

Integrating w.r.t. x we get

$\frac{z}{y} = ylogx + \int f(x)dx + g(y)$

$\Rightarrow \quad z = y^2 logx + yF(x) + G(y)$

3)Solve : $\frac{\partial^2 z}{\partial y^2} - x\frac{\partial z}{\partial y} = -siny - xcosy$

Solution: We know that q=$\frac{\partial z}{\partial y}$,the given equation may be written as

$\frac{\partial q}{\partial y} - xq == -siny - xcosy$……………………………….(1)

Since x is treated as constant , this equation is a first order ordinary linear d.e in which q is the dependent variable and y is the independent variable. For this equation

I.F=$e^{\int -xdy} = e^{-xy}$

∴ The solution of (1) is

$$qe^{-xy} = \int (-siny - xcosy)e^{-xy}dy + f(x)$$

$=\int \frac{d}{dy}(cosy)e^{-xy}dy + f(x)$

$qe^{-xy} = e^{-xy}cosy + f(x)$

or

$\frac{\partial z}{\partial y} = cosy + e^{xy}\, f(x)$

Integrating w.r.t y, we get

$z = siny + \frac{e^{xy}}{x} f(x) + g(y)$

Exercise:

1. Solve $\frac{\partial^2 z}{\partial x \partial y} + 9x^2y^2 = \cos(2x - y)$ given that z=0 when y=0 and $\frac{\partial z}{\partial y} = 0$ when x=0.
2. Solve $\frac{\partial^2 z}{\partial x \partial y} = sinxsiny$, given that $\frac{\partial z}{\partial y} = -2siny$ when x=0 and z=0 when y is an odd multiple of $\frac{\pi}{2}$.

Section 4

Solution of Homogeneous equations

Objective:

At the end of this Section we will be able to:

- Solve the homogeneous P.D.E
- Obtain the particular solution of a homogeneous P.D.E using a given set of conditions.

Solution of Homogeneous equations:

In the process of solution we make use of the method of solving ordinary linear differential equations.

Examples:

1) Solve the equation $\frac{\partial^2 z}{\partial x^2} - a^2 z = 0$ under the condition z=0 and $\frac{\partial z}{\partial x} = asiny$ when x=0

Solution: Using the D-Operator, we write the above equation as

$$(D^2 - a^2)z = 0 \text{ where, } D = \frac{\partial}{\partial x}$$

Here we treat y as constant. Then this equation is an ordinary second order linear homogeneous d.e. in which x is the independent variable and z is the dependent variable.

$\therefore$ The A.E is $m^2 - a^2 = 0 \quad \Rightarrow m = \pm a$

$\therefore G.S\ is\ \ z = Ae^{ax} + Be^{-ax}$

Where A and B are arbitrary function of y

Now $\frac{\partial z}{\partial x} = Aae^{ax} - Bae^{-ax}$

Using the given condition i.e., z=0 and $\frac{\partial z}{\partial x} = asiny$ when x = 0

$asiny = Aa - Ba = a(A - B)$

0= A+B

$\Rightarrow A - B = siny$

$A + B = 0$

Solving the above simultaneous equations

$2A = siny$

$\Rightarrow A = \frac{1}{2}siny$

$B = -\frac{1}{2}\,siny$

The solution is

$$z = \frac{1}{2}(e^{ax} - e^{-ax})siny$$

$$z = sinhax\ siny$$

2) Solve the equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + 2z = 0$ Given that z=e^y and $\frac{\partial z}{\partial x} = 0$ when x=0

Solution: The given equation can be put in the form

$(D^2 - 2D + 2)z = 0$ where D=$\frac{\partial}{\partial x}$

If y is treated as constant , then the above equation is a O.D.E with the A.E

$(m^2 - 2m + 2)$=0

$\Rightarrow m = 1 \pm i$

∴The G.S is $z = e^x(Acosx + Bsinx)$

Where A and B are arbitrary functions of y

$$\frac{\partial z}{\partial x} = e^x(Acosx + Bsinx) + e^x(-Asinx + Bcosx)$$

$\frac{\partial z}{\partial x}$=$e^x[(A + B)cosx + (B - A)sinx]$

But $\frac{\partial z}{\partial x}$=0

⇒0=B+A

And z=e^y when x=0

$\Rightarrow e^y = A$

⇒B=-e^y

∴ z=$e^x[e^y cosx - e^y sinx]$

⇒ z=$e^{x+y}[cosx - sinx]$

3. Solve the equation $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial u}{\partial x}$ using the substitution the $\frac{\partial u}{\partial x} = v.$

Sol: Since we have $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

given PDE can be rewritten as

$$\frac{\partial v}{\partial y} = v, \quad \text{where } v = \frac{\partial u}{\partial x}$$

$$\text{or} \quad \frac{\partial v}{\partial y} - v = 0$$

By considering it as a ODE we can write it as

$$\frac{dv}{dy} - v = 0 \Rightarrow (D-1)v = 0$$

A. E is m-1=0 $\Rightarrow m = 1$

$\therefore$ the solution is given by $v = f(x)e^y$

i.e, $\frac{\partial u}{\partial x} = f(x)e^y$

On integrating w. r. to x, we get

u $= F(x)e^y + g(y)$ where F(x)=$\int f(x)dx$

Exercise:

1. Solve $\frac{\partial^3 u}{\partial x^2 \partial y} - \frac{\partial u}{\partial y} = 0$
2. Solve $(u_x^3 - 3u_x^2 u_y + 2u_y^2 u_x)z = 0$

Section 5

Method of separation of variables

Objective:

At the end of this unit reader will know:

- The method of separation of variables
- How to obtain the solution of P.D.E by the method of separation of variables.

Method of separation of variables:

This method consists of the following steps

1. If x and y are indepent variables and z is the dependent variable, we find a solution of the given equation in the form z=XY, where X=X(x) is a function of x alone and Y=Y(y) is a function of y alone.
   Then, we substitute for z and its partial derivative (computed from z=XY) in the equation and rewrite the equation in such a way that the L.H.S involves X and its derivatives and the R.H.S involves Y and its derivatives.
2. We equate each side of the equation obtained in step 1 to a constant and solve the resulting O.D.E for X and Y.
3. Finally we substitute the expression for X and Y obtained in step 2 in z=XY. The resulting expression is the general solution for z.

Examples:

1) By the method of separation of variables solve the equation $\frac{\partial z}{\partial x}y^3+\frac{\partial z}{\partial y}x^2=0$

Solution: Let the solution be in the form z=XY where X=X(x) and Y=Y(y)

$$\Rightarrow \frac{\partial z}{\partial x}=\frac{\partial X}{\partial x}Y \text{ and } \frac{\partial z}{\partial y}=X\frac{\partial Y}{\partial y}$$

Putting these in the given equation we get

$$\left(\frac{\partial X}{\partial x}Y\right)y^3+\left(X\frac{\partial Y}{\partial y}\right)x^2=0 \text{ or}$$

$$\left(\frac{dX}{dx}Y\right)y^3=-\left(X\frac{dY}{dy}\right)x^2$$

$$\frac{1}{X}\left(\frac{dX}{dx}\right)\frac{1}{x^2}=-\frac{1}{Y}\left(\frac{dY}{dy}\right)\frac{1}{y^3}$$

L.S.H is a function of x only and R.S.H is a function of y only

Since x and y are independent variables , this expression can hold only if each side is a constant i.e.,
$\frac{1}{X}\left(\frac{dX}{dx}\right)\frac{1}{x^2} = k$ and $-\frac{1}{Y}\left(\frac{dY}{dy}\right)\frac{1}{y^3} = k$ where k is a constant

These may be written as

$\frac{1}{X}\left(\frac{dX}{dx}\right) = kx^2$ and $\frac{1}{Y}\left(\frac{dY}{dy}\right) = -ky^3$

$\frac{d}{dx}(logX) = kx^2$ and $\frac{d}{dy}(logY) = -ky^3$

Integrating w.r.t x and y we get

$logX = k\frac{x^3}{3} + logC_1$ and $logY = -k\frac{y^4}{4} + logC_2$

$\Rightarrow \quad X = C_1 e^{k\frac{x^3}{3}}$ and $Y = C_2 e^{-k\frac{y^4}{4}}$

Where C_1 *and* C_2 are constants

$\therefore z = XY = Ae^{k\frac{x^3}{3}}e^{-k\frac{y^4}{4}}$ where A=$C_1\ C_2$

2) Using the method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where u(x,0)=$6e^{-3x}$

Solution: Here x and t are independent variables and u id the dependent variable

Let the solution be in the form u=X(x)Y(t)

$$\therefore \frac{\partial u}{\partial x} = \frac{dX}{dx}Y(t)\ and\ \frac{\partial u}{\partial t} = X(x)\frac{dY}{dt}$$

The given equation takes the form

$$\frac{dX}{dx}Y(t) = 2\,X(x)\frac{dY}{dt} + X(x)Y(t)$$

$$\left(\frac{dX}{dx} - X\right)Y = 2X\frac{dY}{dt}$$

$$\frac{1}{X}\frac{dX}{dx} - 1 = \frac{2}{Y}\frac{dY}{dt}$$

$\Rightarrow \frac{1}{X}\frac{dX}{dx} - 1 = k$ and $\frac{2}{Y}\frac{dY}{dt} = k$

$\frac{dX}{X} = (k+1)dx$ and $\frac{dY}{Y} = \frac{k}{2}dt$

On integration we get

$logX = (k+1)x + logC_1$ and $logY = \frac{k}{2}t + logC_2$

$\Rightarrow X = C_1 e^{(k+1)x}$ and $Y = C_2 e^{\frac{k}{2}t}$

∴ The required solution is

u=XY=A $e^{(k+1)x} e^{\frac{k}{2}t}$ where A=C_1C_2

But u(x,0)=6e^{-3x}

$\Rightarrow$ $6e^{-3x} = Ae^{(k+1)x}$

$\Rightarrow$ $A = 6 \; and \; k+1 = -3$

$\Rightarrow$ $A = 6 \; and \; k = -4$

∴ $u = 6e^{-(3x+2t)}$

3. Solve $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + u$

Solution: Let the solution be in the form *u* = XT where X=X(x) and T=T(t)

$$\frac{\partial u}{\partial x} = T\frac{\partial X}{\partial x} \quad \text{and} \quad \frac{\partial u}{\partial t} = X\frac{\partial T}{\partial t}$$

Putting these in the given equation we get

$$T\frac{\partial X}{\partial x} = 2X\frac{\partial T}{\partial t} + XT \quad or \quad \frac{1}{X}\frac{\partial X}{\partial x} = \frac{2}{T}\frac{\partial T}{\partial t} + 1$$

LHS is a function of X only and RHS is of T alone

Since x and t are independent variables, this expression can holds only if each side is a constant i.e.,

$$\frac{1}{X}\frac{dX}{dx} = k \quad \text{and} \quad \frac{2}{T}\frac{dT}{dt} + 1 = k$$

$$\frac{1}{X}dX = kdx \quad \text{and} \quad \frac{1}{T}dT = \frac{k-1}{2}dt$$

$$LogX = kx + Logc_1 \quad \text{and} \quad LogT = \frac{k-1}{2}t + Logc_2$$

$$Log\frac{X}{c_1} = kx \quad \text{and} \quad Log\frac{T}{c_2} = \frac{k-1}{2}t$$

$$X = c_1 e^{kx} \quad \text{and} \quad T = c_2 e^{\frac{k-1}{2}t}$$

Then the complete solution is given by

$$u = XT = c_1 e^{kx} c_2 e^{\frac{k-1}{2}t} = ce^{kx - \frac{k-1}{2}t}$$

4. Solve $xp = yq$ by the method of seperation of variables.

Solution: We have the PDE : $x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$

Let the solution be in the form z = XY where X=X(x) and Y=Y(y)

$$\frac{\partial u}{\partial x} = Y\frac{\partial X}{\partial x} \quad \text{and} \quad \frac{\partial u}{\partial y} = X\frac{\partial Y}{\partial y}$$

Putting these in the given equation we get

$$xY\frac{\partial X}{\partial x} = yX\frac{\partial Y}{\partial y} \Rightarrow \frac{x}{X}\frac{\partial X}{\partial x} = \frac{y}{Y}\frac{\partial Y}{\partial y}$$

$$\frac{x}{X}\frac{dX}{dx} = k \quad and \quad \frac{y}{Y}\frac{dY}{dy} = k$$

$$\frac{dX}{X} = k\frac{dx}{x} \quad and \quad \frac{dY}{Y} = k\frac{dy}{y}$$

$$LogX = kLogx + Logc_1 \ and \ LogY = kLogy + Logc_2$$

$$X = x^k c_1 \ and \ Y = y^k c_2$$

The solution is given by

$$z = XY = cx^k y^k = c(xy)^k$$

4. Solve $\dfrac{\partial^2 z}{\partial x \partial y} = z$

Solution: Let the solution be in the form *z* = XY where X=X(x) and Y=Y(y)

$$\frac{\partial z}{\partial y} = X\frac{\partial Y}{\partial y} \quad \text{and} \quad \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial X}{\partial x}\frac{\partial Y}{\partial y}$$

Putting these in the given equation we get

$$\frac{\partial X}{\partial x}\frac{\partial Y}{\partial y} = XY \qquad \Rightarrow \frac{1}{X}\frac{\partial X}{\partial x} = \frac{Y}{\frac{\partial Y}{\partial y}}$$

LHS is a function of X only and RHS is of Y alone

Since x and y are independent variables, this expression can holds only if each side is a constant i.e.,

$$\frac{1}{X}\frac{dX}{dx} = k \quad \text{and} \quad \frac{Y}{\frac{dY}{dy}} = k$$

$$\frac{1}{X}\,dX = kdx \quad \text{and} \quad \frac{1}{k}dy = \frac{dY}{Y}$$

$$LogX = kx + Logc_1 \quad \text{and} \quad LogY = \frac{y}{k} + Logc_2$$

$$X = c_1 e^{kx} \quad \text{and} \quad Y = c_2 e^{\frac{y}{k}}$$

$\therefore$ The solution is given by

$$Z = XY = c_1 e^{kx} \cdot c_2 e^{\frac{y}{k}} = ce^{kx - \frac{y}{k}}$$

6. Solve $\dfrac{\partial^2 z}{\partial x^2} - 2\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 0$

Solution: Let the solution be in the form *z* = XY where X=X(x) and Y=Y(y)

$$\frac{\partial z}{\partial x} = Y\frac{\partial X}{\partial x},\ \frac{\partial^2 z}{\partial x^2} = Y\frac{\partial^2 X}{\partial x^2} \quad \text{and} \quad \frac{\partial z}{\partial y} = X\frac{\partial Y}{\partial y}$$

Substituting these in the given equation we get

$$Y\frac{\partial^2 X}{\partial x^2} - 2Y\frac{\partial X}{\partial x} + X\frac{\partial Y}{\partial y} = 0$$

$$Y\left(\frac{\partial^2 X}{\partial x^2} - 2\frac{\partial X}{\partial x}\right) = -X\frac{\partial Y}{\partial y}$$

$$\Rightarrow \frac{1}{X}\left(\frac{\partial^2 X}{\partial x^2} - 2\frac{\partial X}{\partial x}\right) = -\frac{1}{Y}\frac{\partial Y}{\partial y}$$

LHS is a function of X only and RHS is of Y alone

Since x and y are independent variables , this expression can holds only if each side is a constant i.e.,

$$\text{Consider} \quad \frac{1}{X}\left(\frac{d^2 X}{dx^2} - 2\frac{dX}{dx}\right) = k$$

$$\frac{d^2 X}{dx^2} - 2\frac{dX}{dx} - Xk - 0$$

$$AE : -m^2 - 2m - k = 0$$

$$Roots\ are\ m = 1 \pm \sqrt{1+k}$$

$$\therefore X = c_1 e^{\left(1+\sqrt{1+k}\right)x} + c_2 e^{\left(1-\sqrt{1+k}\right)x}$$

$$\text{Considering} \quad -\frac{1}{Y}\frac{dY}{dy} = k \quad \Rightarrow \quad \frac{1}{Y}dY = -kdy$$

$$LogY = -ky + Log\ c_2$$

$$Y = c_3 e^{-ky}$$

The complete solution is given by

$$Z = XY = \left(c_1 e^{\left(1+\sqrt{1+k}\right)x} + c_2 e^{\left(1-\sqrt{1+k}\right)x}\right) c_3 e^{-ky}$$

Exercise:

1. Solve $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} = 0$ by the method of separation of variables.
2. Use the separation of variables technique to solve $3u_x + 2u_y = 0$, given $u(x,0) = 4e^{-x}$

Section 6

Lagrange's linear partial differential equation

Objective:

At the end of this section we will understand:

- The form of Lagrange's linear equation.
- Solve the Lagrange's linear P.D.E.

Solution of the first order linear partial differential equation in the form *Pp+Qq=R*

Lagrange's linear partial differential equation:

The general solution of Lagrange's linear partial differential equation Pp+Qq=R……….(1)

where P=P(x,y,z) ,Q=Q(x,y,z) and R=R(x,y,z) is given by

F(u,v)=0……………………….(2)

Since the elimination of the arbitrary function F from (2) results in (1)

Here u=u(x,y,z) and v=v(x,y,z) are known functions.

Method of obtaining the general solution:

1. Rewrite the given equation in the standard form Pp+Qq=R

2. Form the Lagrange's auxiliary equation (A.E)

$\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$…………………………..(3)

3. u(x,y,z)=c_1 and v(x,y,z)=c_2 are said to be the complete solution of the system of the simultaneous equations (3) provided u_1 and u_2 are linearly independent i.e.,$\frac{u_1}{u_2} \neq constant$

Case1: One of the variables is either absent or cancels out from the set of auxiliary equations

Case2: If u=c_1 is known but v=c_2 is not possible by case 1, then use u=c_1 to get v=c_2

Case3: Introducing Lagrange's multipliers P_1, Q_1, R_1 which are functions of x, y, z or constants , each fraction in (3) is equal to $\frac{P_1dx+Q_1dy+R_1dz}{P_1P+Q_1Q+R_1R}$..............................(4)

P_1, Q_1, R_1 are chosen that $P_1P+Q_1Q+R_1R=0$ then $P_1dx+Q_1dy+R_1dz=0$ which can be integrated.

Case4: Multipliers may be chosen (more than once) such that the numerator

$P_1dx+Q_1dy+R_1dz$ is an exact differential of the denominator $P_1P+Q_1Q+R_1R$. Combining (4) with a fraction of (3) to get an integral.

4. General solution of (1) is F(u, v)=0 or v=Φ(u).

Examples:

1) Solve *xp+yq = z*

Solution: The given equation is of the form Pp+Qq=R

The A.E are $\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}$

Consider $\frac{dx}{x}=\frac{dy}{y}$ which on integration give

$$logx = logy + logc_1$$

$$\Rightarrow \frac{x}{y} = c_1$$

Similarly by considering $\frac{dy}{y}=\frac{dz}{z}$ we get

$$\frac{y}{z} = c_2$$

Hence the general solution of the give equation is $\varphi\left(\frac{x}{y},\frac{y}{z}\right)=0$

2) Solve $(mz-ny)\frac{\partial z}{\partial x}+(nx-lz)\frac{\partial z}{\partial y}=ly-mx$

Solution: The given equation is of the form Pp+Qq=R

The auxiliary equations are

$\frac{dx}{(\mathrm{mz-ny})}=\frac{dy}{(nx-lz)}=\frac{dz}{ly-mx}$.....................(1)

Using multipliers l, m, n each ratio is equal to

$$\frac{ldx+mdy+ndz}{l(mz-ny)+m(nx-lz)+n(ly-mx)}=\frac{ldx+mdy+ndz}{0}$$

$\Rightarrow \quad ldx+mdy+ndz=0$ which on integration gives $lx+my+nz=c_1$

Using multipliers x, y, z each ratio in (1) equal to

$$\frac{xdx+ydy+zdz}{x(mz-ny)+y(nx-lz)+z(ly-mx)}=\frac{xdx+ydy+zdz}{0}$$

$\Rightarrow xdx+ydy+zdz=0$ which on integration gives $\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2}=c_2$

or $x^2+y^2+z^2=c_2$

Hence the general solution is $\varphi(lx+my+nz, x^2+y^2+z^2)=0$

3) Solve $x^2(y-z)\frac{\partial z}{\partial x}+y^2(z-x)\frac{\partial z}{\partial y}=z^2(x-y)$

Solution: The given equation is of the form Pp+Qq=R

The auxiliary equations are

$\frac{dx}{x^2(y-z)}=\frac{dy}{y^2(z-x)}=\frac{dz}{z^2(x-y)}$…………………(1)

Using multipliers $\frac{1}{x^2},\frac{1}{y^2},\frac{1}{z^2}$ each ratio is equal to

$$\frac{\frac{1}{x^2}dx+\frac{1}{y^2}dy+\frac{1}{z^2}dz}{(y-z)+(z-x)+(x-y)}=\frac{\frac{1}{x^2}dx+\frac{1}{y^2}dy+\frac{1}{z^2}dz}{0}$$

$\frac{1}{x^2}dx+\frac{1}{y^2}dy+\frac{1}{z^2}dz=0$ which on integration gives

$-\frac{1}{x}-\frac{1}{y}-\frac{1}{z}=c_1$ or $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=c_1$

Now using multipliers $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ each ratio is equal to

$$\frac{\frac{1}{x}dx+\frac{1}{y}dy+\frac{1}{z}dz}{x(y-z)+y(z-x)+z(x-y)}=\frac{\frac{1}{x}dx+\frac{1}{y}dy+\frac{1}{z}dz}{0}$$

$\frac{1}{x}dx+\frac{1}{y}dy+\frac{1}{z}dz=0$ which on integration gives

$logx+logy+logz=logc_2$ or $xyz=c_2$

Hence the general solution is $\varphi\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}, xyz\right)=0$

4. Solve $p\tan x+q\tan y=\tan z$

Solution: The given equation is of the form

$$Pp + Qq = R$$

The Subsidiary equation is given by

$$\frac{dx}{Tanx} = \frac{dy}{Tany} = \frac{dz}{Tanz}$$

Taking the first two relations we get,

$$Log\ (Sinx) = Log(Siny) + \log\ c_1$$

$$\Rightarrow \frac{Sinx}{Siny} = c_1$$

Simillarly taking the last two relations we get,

$$Log\ (Siny) = Log(Sinz) + \log\ c_2$$

$$\Rightarrow \frac{Siny}{Sinz} = c_2$$

Thus a general solution of the PDE is given by

$$\varphi = \left(\frac{Sinx}{Siny}, \frac{Siny}{Sinz}\right) = 0$$

3. Solve $xp - yq = y^2 - x^2$

Solution: The given equation is of the form

$$Pp + Qq = R$$

The Subsidiary equation is given by

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{y^2 - x^2}$$

Taking the first two relations we get,

$$Log\ x = Logy + \log\ c_1 \Rightarrow \frac{x}{y} = c_1$$

Here to get another relation we choose the multipliers $x,\ y,\ 1\ (\because xP + yQ + R = 0)$

ie, we get $xdx + ydy + dz = 0$

$$\Rightarrow x^2 + y^2 + 2z = 2c_2$$

The complete solution is given by

$$\varphi\left(\frac{x}{y}, x^2 + y^2 + 2z\right) = 0$$

4. Solve $(mz-ny)p+(nx-lz)q=ly-mx$

Solution: The given equation is of the form

$$Pp + Qq = R$$

The Subsidiary equation is given by

$$\frac{dx}{mz-ny}=\frac{dy}{nx-lz}=\frac{dz}{ly-mx}$$

Here we choose the multipliers $x,\ y, z$

$$(\because xP+yQ+zR=0)$$

$ie,$ we get $xdx+ydy+zdz=0$

$$\Rightarrow x^2+y^2+z^2=2c_1$$

and again we choose the multipliers $l,\ m, n$

$$(\because lP+mQ+nR=0)$$

$ie,$ we get $ldx+mdy+ndz=0$

$$\Rightarrow lx+my+nz=c_2$$

The complete solution is given by

$$\varphi\left(x^2+y^2+z^2, lx+my+nz\right)=0$$

4. Solve $\frac{y-z}{yz}p+\frac{z-x}{zx}q=\frac{x-y}{xy}$

Solution: The given equation is of the form

$$Pp + Qq = R$$

The Subsidiary equation is given by

$$\frac{dx}{\frac{y-z}{yz}}=\frac{dy}{\frac{z-x}{zx}}=\frac{dz}{\frac{x-y}{xy}}$$

Here we choose the multipliers $x,\ y, z$

$$(\because xP+yQ+zR=0)$$

ie, we get $xdx + ydy + zdz = 0$

$$\Rightarrow x^2 + y^2 + z^2 = 2c_1$$

and again we choose the multipliers $yz,\ zx, xy$

$$(\because yzP + zxQ + xyR = 0)$$

ie, we get $yzdx + zxdy + xydz = 0$

$$\Rightarrow d(xyz) = 0 \Rightarrow xyz = c_2$$

The complete solution is given by

$$\varphi\left(x^2 + y^2 + z^2, xyz\right) = 0$$

5. Solve $(x^2 - y^2 - z^2)p + (2xy)q = 2xz$

Solution: The given equation is of the form

$$Pp + Qq = R$$

The Subsidiary equation is given by

$$\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz} = \frac{xdx + ydy + zdz}{x(x^2 - y^2 - z^2) + y(2x) + z(2xz)}$$

Taking the IIand III relations we get,

$$Log\ y = Logz + \log\ c_1 \Rightarrow \frac{y}{z} = c_1$$

Taking the III and IV relations we get,

$$\frac{dz}{2xz} = \frac{xdx + ydy + zdz}{x(x^2 + y^2 + z^2)}$$

$$\Rightarrow \frac{dz}{z} = \frac{d(x^2 + y^2 + z^2)}{(x^2 + y^2 + z^2)}$$

$$\left(\text{Which is of the form } \frac{f'(x)}{f(x)}\right)$$

$$\Rightarrow \text{Log } z + \text{Log } c_2 = \log(x^2 + y^2 + z^2)$$

$$\Rightarrow c_2 = \frac{x^2 + y^2 + z^2}{z}$$

Hence the required solution is

$$\varphi\left(\frac{y}{z}, \frac{x^2 + y^2 + z^2}{z}\right) = 0$$

6. Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

Solution: The given equation is of the form

Pp + Qq = R

The Subsidiary equation is given by

$$\frac{dx - dy}{(x^2 - yz) - (y^2 - zx)} = \frac{dy - dz}{(y^2 - zx) - (z^2 - xy)} = \frac{dz - dx}{(z^2 - xy) - (x^2 - yz)} \quad ---(1)$$

consider first two expressions of (1), we get

$$\Rightarrow \frac{dx - dy}{(x^2 - yz) - (y^2 - zx)} = \frac{dy - dz}{(y^2 - zx) - (z^2 - xy)}$$

$$\Rightarrow \frac{dx - dy}{(x - y)(x + y + z)} = \frac{dy - dz}{(y - z)(x + y + z)}$$

$$\Rightarrow \frac{dx - dy}{(x - y)} = \frac{dy - dz}{(y - z)}$$

$$\Rightarrow Log(x - y) = Log(y - z) + Logc_1$$

$$\Rightarrow \frac{x - y}{y - z} = c_1$$

similarly considering last two expressions of (1), we get

$$\Rightarrow \frac{dx - dy}{(y - z)} = \frac{dy - dz}{(z - x)}$$

$$\Rightarrow Log(y - z) = Log(z - x) + Logc_2$$

$$\Rightarrow \frac{y - z}{z - x} = c_2$$

hence the required solution is,

$$\varphi\left(\frac{x - y}{y - z}, \frac{y - z}{z - x}\right) = 0$$

Exercise:

Solve

1) $xp + yq = 3z$
2) $yzp - xzq = xy$
3) $p - q = \ln(x + y)$
4) $(x^2 - y^2 - yz)p + (x^2 - y^2 - zx) = z(x - y)$

Pick out the most appropriateanswer:

1. The solution of $\frac{\partial^2 z}{\partial y^2}$ = sin(xy) is (B)

   A) $Z=\frac{1}{x^2}\cos(xy)+y\,f(x)+g(x)$ B) $Z=\frac{-1}{x^2}\sin(xy)+y\,f(x)+g(x)$

   B) $Z=-\sin(xy)+y\,f(x)+g(x)$ D) None of these

2. A solution of $(y-z)\,p + (z-x)\,q = x-y$ is (A)

   A) $\varphi(x^2+y^2+z^2, x+y+z)$ B) $\varphi(x^2+y^2+z^2, x+y-z)$

   C) $\varphi(x^2-y^2-z^2, x+y+z)$ D) $\varphi(x^2+y^2-z^2, x+y+z)$

3. The partial differential equation obtained from $z= ax+by+ab$ is (D)

   A) $px+qy=z$ B) $px+qy+z^2=0$ c) $px-qy=z$ D) $px+qy+pq=z$

4. The partial differential equation obtained from $z = e^y f(x+y)$ is (A)

   A) $p+z=q$ B) $p-z=q$ C) $p-q=z$ D) none of these

5. In a partial differential equation by eliminating a & b from the relation $Z= (x^2+a)(y^2+b)$ is (D)

   A) $z_x z_y = xyz$ B) $z_{xy} = xyz$ C) $z_{xy}=4xyz$ D) $z_x z_y = 4xyz$

6. The solution of $z_{xy}= \sin xy$ is $Z=$……………… (C)

   A) $\sin(xy) + f(x) + g(y)$ B) $-1/x^2\cos(xy)+f(x)+g(y)$

   C) $-1/xy\sin(xy) + f(x) + g(y)$ D) $\sin(xy) +f(x)+xg(y)$

7. For the Lagrange's linear partial differential equation $Pp+Qq= R$ the subsidiary equation is (C)

A) $\frac{dx}{P} = \frac{-dy}{Q} = \frac{dz}{R}$ B) $\frac{-dx}{P} = \frac{-dy}{Q} = \frac{dz}{R}$

C) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ D) $\frac{dx}{P^2} = \frac{-dy}{Q^2} = \frac{dz}{R^2}$

8. In the method of separation of variable, to solve $u_{xx} - 2u_x + u_t = 0$ the trial solution $u = \ldots\ldots$ (A)

A) $X(x)T(t)$ B) $\sqrt{\frac{X(x)}{T(t)}}$ C) $\frac{X(x)}{T(t)}$ D) $X(x)\sqrt{T(t)}$.

9. By eliminating a & b from $(x-a)^2 + (y-b)^2 + z^2 = c^2$ the partial differential equation formed is (A)

A) $z^2(p^2+q^2+1)=c^2$ B) $Z^2(p-q-1)=c^2$

C) $(p+q+1)=c$ D) none of these

10. The equation $\frac{\partial^2 z}{\partial x^2} + 2xy\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial z}{\partial y} = 5$ is order ……….. and degree ….. (A)

A) 2 ,1 B) 1,2 C) 1,1 D) none of these

11. The solution of $x\frac{\partial z}{\partial x} = 2x+y$ is (B)

A) $z = 2x + y\log x + f(x)$ B) $z = 2x + y\log x + f(y)$

C) $z = 2x - y\log x + f(xy)$ D) $z = x^2 + xy + f(y)$

12. By eliminating a & b from $z = a(x+y)+b$ the partial differential formed is (A)

A) $p=q$ B) $pq=0$ C) $p+q=0$ D) $p^2=q$

13. A solution $u_{xy} = 0$ is of the form (D)

A) $U = \int f(y)dy$ B) $U = \int f(y)dy - \emptyset(x)$

C) $U = \int f(x)dy + \emptyset(x)$ D) $U = \int f(x)dx + \emptyset(y)$

14. If $u = x^2 + t^2$ is a solution of $c^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ then $c = \ldots\ldots\ldots\ldots\ldots\ldots$ (A)

A) 1 B) 0 C) 2 D) None of these

15. The general solution of $u_{xx} = xy$ is (B)

A) $U = \frac{1}{6}x^3y - x\,f(y) - \emptyset(y)$ B) $U = \frac{1}{6}x^3y + x\,f(y) + \emptyset(y)$

C) $U = \frac{1}{6}x^3 + f(y) + \emptyset(y)$ D) $U = \frac{1}{6}x^3y + x\,f(x) + \emptyset(x)$

16. The partial differential equation by eliminating *a* & *b* from the relation $z = a\log(x^2+y^2) + b$ is (B)

A) $px\text{-}qy=0$ B) $py\text{-}qx=0$ C) $py+qx=0$ D) $px+qy=0$

17. The partial differential equation by eliminating arbitrary function from the relation $z= f(x^2 + y^2)$ is………………. (B)

A) $px\text{-}qy=0$ B) $py\text{-}qx=0$ C) $py+qx=0$ D) $px+qy=0$

18. The solution of $r+6s+9t=0$ is (A)

A) $z = f_1(y\text{-}3x) + x f_2(y\text{-}3x)$ B) $z = f_1(y\text{-}3x) - x f_2(y\text{-}3x)$

C) $z = f_1(y+3x) + x f_2(y+3x)$ D) none of these

19. If $\frac{\partial^2 u}{\partial x^2} = x+y$ then u =……………….. (B)

A) $\frac{y^3}{6} + \frac{yx^2}{2} + xf(y) + g(y)$ B) $\frac{x^3}{6} + \frac{yx^2}{2} + xf(y) + g(y)$

C) $\frac{y^3}{6} - \frac{yx^2}{2} - xf(y) + g(y)$ D) $\frac{x^3}{6} - \frac{yx^2}{2} + f(y) + g(y)$

20. The nature of the partial differential equation $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 0$ (A)

A) Parabolic B) Elliptic C) Hyperbolic D) None of these

21. $xp + yq = z$ is the general solution of the equation (D)

A) $\emptyset\left(\frac{x}{y},\frac{y}{z}\right)=0$ B) $\emptyset\left(\frac{y}{x},\frac{y}{z}\right)=0$ C) $\emptyset\left(\frac{x}{y},\frac{z}{y}\right)=0$ D) all of these

22. The Lagrange's linear partial differential equation of the form (D)

A) $Pp- Qq= R$ B) $Pp+ Qq= 0$ C) $Pq+ Qp= R$ D) $Pp+ Qq= R$

23. The auxiliary equation of the equation $(y - z) p + (z - x) q = (x - y)$ is (B)

A) $\frac{dx}{(z-x)} = \frac{dy}{(y-x)} = \frac{dz}{(x-y)}$ B) $\frac{dx}{(y-z)} = \frac{dy}{(z-x)} = \frac{dz}{(x-y)}$

C) $\frac{dx}{(z-x)} = \frac{dy}{(x-y)} = \frac{dz}{(y-x)}$ D) none of these

24. The multiplies of the equation $x (y^2+z) p - y(x^2 +z) q= z (x^2-y^2)$ is (A)

A) $(1/x, 1/y, 1/z), (x, y, -1)$ B) $(x, y, z), (x, y, -1)$

C) $(1/x, 1/y, 1/z), (x ,y, 1)$ D) none of these

25. The order of the partial differential equation obtained by eliminating f from $z= f (x^2+y^2)$ is……………… (B)

A) second B) first C) constant D) none of these

26. The solution of $\sqrt{p} + \sqrt{q}$ =1 is (B)

A) $z=ax+((1+\sqrt{a})^2y+c$ B) $z=ax+(1-\sqrt{a})^2y+c$

C) $z=ax-((1-\sqrt{a})^2y$ D) none of these

27. The nature of the partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} - \frac{\partial z}{\partial x} + z = 0$ (B)

A) Parabolic B) Elliptic C) Hyperbolic D) None of these

28. The nature of the partial differential equation $x\frac{\partial^2 z}{\partial x^2} + 3xy\frac{\partial^2 z}{\partial x \partial y} + y\frac{\partial^2 z}{\partial y^2} = 0$ (C)

A) Parabolic B) Elliptic C) Hyperbolic D) None of these

29) The order and the degree of the P.D.E $\frac{\partial^2 z}{\partial x^2} + 2xy\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial z}{\partial y} = 5$ is...... and... (c)

a) 1 and 2 b) 2 and 2 c)2 and 1 d)none

30) By eliminating a and b from $(x-a)^2 + (y-b)^2 + z^2 = c^2$, the P.D.E formed is .... (d)

a) $c^2 = (1+p^2+q^2)z^2$ b) $c = (1+p+q)z$

c) $z^2 = (1+p^2+q^2)c^2$ d) $z = (1+p+q)c$

31) The partial differential equation obtained from $z = ax + by + ab$ by eliminating a and b is ... (b)

a) $z = x + y + pq$ b) $z = px + qy + pq$

c) $z = qx + py + pq$ d) $z = px + qy + xy$

32) The partial differential equation $f_{xx} + 2f_{xy} + 4f_{yy} = 0$ is classified as .... (b)

a)non –homogeneous P.D.E b)homogeneous P.D.E

c)linear P.D.E d)none

33) If z=f(x, y, a, b) then the P.D.E formed by eliminating the arbitrary constants a and b is of ... (c)

a) second order b)third order

c) first order d) forth order

34) The solution of $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$ is .... (a)

a) $\sqrt{x} - \sqrt{y} = f(\sqrt{x} - \sqrt{y})$ b) $x - y = f(x - y)$

c) $\sqrt{x} - \sqrt{y} = f(x-y)$ d) $x - y = f(\sqrt{x} - \sqrt{y})$

35) A general solution of $u_{xy} = 0$ is of the form ……………………… (b)

a) $u = f(y) + \varphi(x)$ b) $u = \int f(y)dy + \varphi(x)$

c) $u = \int f(y)dy$ d) none

36) $u = e^{-t} sinx$ is a solution of ……………………… (c)

a) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ b) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = u$

c) $\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 u}{\partial x^2} = 0$ d) none

37) The auxiliary equations of Lagrange's linear equation $Pp + Qq = R$ is………………… (a)

a) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ b) $\frac{dz}{P} = \frac{dy}{Q} = \frac{dx}{R}$

c) $\frac{dx}{R} = \frac{dy}{P} = \frac{dz}{Q}$ d)none

38) The differential equation $z = \frac{\partial^2 z}{\partial x^2}$ gets reduced to the form : (a)

a) Linear homogeneous differential equation   b)Partial differential equation of first order

c) Both a and b   d) None of these

39. A partial differential equation requires (B)

A) exactly one independent variable

B) two or more independent variables

C) more than one dependent variable

D) equal number of dependent and independent variables

40. A Solution to the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$ is (A)

A)Cos(3x-y)   B)$x^2 + y^2$

C) Sin(3x-y)   D)$e^{-3xy}\sin(\pi y)$

41. The following is true for the following partial differential equation used in nonlinear mechanics known as the Korteweg-de Vries equation $\frac{\partial w}{\partial t} + \frac{\partial^3 w}{\partial x^3} - 6w\frac{\partial w}{\partial x} = 0$ (B)

A) Linear, 3rd order B) Nonlinear, 3rd order

C) Linear 1st order D) Nonlinear 1st order

42. A Singular solution exists to the equation given by (B)

A) f(p, q)=0 B) Z= p x + q y + f(p,q) C)f(x, p)=g(x, q) D) None of these

43. The partial differential equation obtained by eliminating the arbitrary functions from

z = f (x+ky) + g(x-ky) is

A) $\frac{\partial^2 z}{\partial x^2} = k\frac{\partial^2 z}{\partial x \partial y}$ B) $\frac{\partial^2 z}{\partial y^2} = k^2\frac{\partial^2 z}{\partial x^2}$ (B)

C) $\frac{\partial^2 z}{\partial x^2} = k^2\frac{\partial^2 z}{\partial y^2}$ D) $\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial x \partial y}$

44. The initial equation of finding the differential equation of all planes which are at a constant distace 'a' from the origin is

A) $lx+my+nz = a$ B) $x^2+y^2+z^2 = a^2$ (A)

C) $y^2 = 4ax$ D) None of these