1

Sets, Relations and Functions

QUICK REVIEW

Sets

Set is a collection of well-defined objects which are distinct from each other.

Operation on Sets

(*i*) **Union of sets** $A \cup B = \{x : x \in A \text{ or } x \in B\}$

(*ii*) **Intersection of sets** $A \cap B = \{x : x \in A \text{ and } x \in B\}$

(*iii*) **Difference of sets** $A - B = \{x : x \in A, x \notin B\}$

(*iv*) **Symmetric difference** $A\Delta B = (A - B) \cup (B - A)$

(*v*) **Complement of a set** $A' = U - A$

Laws of Algebra of Sets

For three sets *A, B* and *C*

(*i*) **Idempotent law**

(a) $A \cup A = A$ (b) $A \cap A = A$

(*ii*) **Identity law**

(a) $A \cup \phi = A$ (b) $A \cap U = A$

(*iii*) **Commutative law**

(a) $A \cup B = B \cup A$ (b) $A \cap B = B \cap A$

(*iv*) **Associative law**

(a) $(A \cup B) \cup C = A \cup (B \cup C)$

(b) $A \cap (B \cap C) = (A \cap B) \cap C$

(*v*) **Distributive law**

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(*vi*) **De-Morgan's law**

(a) $(A \cup B)' = A' \cap B'$

(b) $(A \cap B)' = A' \cup B'$

(c) $A - (B \cap C) = (A - B) \cap (A - C)$

(d) $A - (B \cup C) = (A - B) \cup (A - C)$

(*vii*) (a) $A - B = A \cap B'$

(b) $B - A = B \cap A'$

(c) $A - B = A \Leftrightarrow A \cap B = \phi$

(d) $(A - B) \cup B = A \cup B$

(e) $(A - B) \cap B = \phi$

(f) $A \cap B \subseteq A$ and $A \cap B \subseteq B$

(g) $A \cup (A \cap B) = A$

(h) $A \cap (A \cup B) = A$

(*viii*) (a) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

(b) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(c) $A \cap (B\Delta C) = (A \cap B)\,\Delta\,(A \cap C)$

(d) $(A \cap B) \cup (A - B) = A$

(e) $A \cup (B - A) = (A \cup B)$

(*ix*) (a) $U' = \phi$ (b) $\phi' = U$

(c) $(A')' = A$ (d) $A \cap A' = \phi$

(e) $A \cup A' = U$ (f) $A \subseteq B \Leftrightarrow B' \subseteq A'$

Results on Number of Elements in Sets

(*i*) $n(A \cup B) = n(A) + (B) - n(A \cap B)$

(*ii*) $n(A \cup B) = n(A) + n(B)$, if A and B are disjoint.

(*iii*) $n(A - B) = n(A) - n(A \cap B)$

(*iv*) $n(A\Delta B) = n(A) + n(B) - 2n(A \cap B)$

(*v*) $n(A \cup B \cup C) = n(A) + n(B) + n(C)$
$- n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(*vi*) n (number of elements in exactly two of the sets A, B, C)
$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(*vii*) n (number of elements in exactly one of the sets A, B, C)
$= n(A) + n(B) + n(C) - 2n(A \cap B)$
$- 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

(*viii*) $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$

(*ix*) $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

(*x*) $n(B - A) = n(B) - n(A \cap B)$

Cartesian Product

Cartesian product of two sets A and B is

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Properties of Cartesian Product

For three sets A, B and C

(*i*) $n(A \times B) = n(A)\, n(B)$

(*ii*) $A \times B = \phi$, if either A or B is an empty set.

(*iii*) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(*iv*) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(*v*) $A \times (B - C) = (A \times B) - (A \times C)$

(*vi*) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

(*vii*) If $A \subseteq B$ and $C \subseteq D$, then $(A \times C) \subset (B \times D)$

(*viii*) If $A \subseteq B$, then $A \times A \subseteq (A \times B) \cap (B \times A)$

(*ix*) $A \times B = B \times A \Leftrightarrow A = B$

(*x*) If A and B are any two non-empty sets having n elements in common, then $A \times B$ and $B \times A$ have n^2 elements in common.

(*xi*) If $A \neq B$, then $A \times B \neq B \times A$

(*xii*) If $A = B$, then $A \times B = B \times A$

(*xiii*) If $A \subseteq B$, then $A \times C = B \times C$ for any set C.

Relation

If A and B are two non-empty sets, then a relation R from A to B is a subset of $A \times B$.

If $R \subseteq A \times B$ and $(a, b) \in R$, then we say that a is related to b by the relation R, written as aRb.

Domain and Range of Relation

Let R be a relation from set A to set B defined as

$$R = \{(a, b) : a \in A, b \in B\}$$

Then, domain of $R = \{a : (a, b) \in R\}$

and range of $R = \{b : (a, b) \in R\}$

Types of Relation

(*i*) **Identity relation** The relation $I_A = \{(a, a) : a \in A\}$ is called the identity relation on A.

(*ii*) **Reflexive relation** A relation R is said to be reflexive relation, if every element of A is related to itself.
Thus, $(a, a) \in R, \forall\ a \in A \Rightarrow R$ is reflexive.

(*iii*) **Symmetric relation** A relation R is said to be symmetric relation, iff

$$(a, b) \in R \Rightarrow (b, a) \in R, \forall\ a, b \in A$$

i.e. $aRb \Rightarrow bRa, \forall\ a, b \in A \Rightarrow R$ is symmetric.

(*iv*) **Anti-symmetric relation** A relation R is said to be anti-symmetric relation, iff
$(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b, \forall\ a, b \in A$
i.e. $a \not R\, b \Rightarrow b \not R\, A, \forall\ a, b \in A$

(*v*) **Transitive relation** A relation R is said to be transitive relation, iff

$$(a, b) \in R \text{ and } (b, c) \in R$$
$$\Rightarrow \quad (a, c) \in R, \forall\ a, b, c \in A$$

(*vi*) **Equivalence relation** A relation R is said to be an equivalence relation, if it is simultaneously reflexive, symmetric and transitive on A.

(*vii*) **Inverse relation** If R is a relation from A to B such that $R = \{(a, b) : a \in A, b \in B\}$, then the inverse of R i.e. $R^{-1} = \{(b, a) : a, b \in R\}$.

Some Important Results on Relation

(*i*) If set A has n elements, then number of reflexive relations from A to A is $2^{n^2 - n}$.

(*ii*) Let A and B be two non-empty finite sets consisting of m and n elements, respectively. Then, $A \times B$ consists of mn ordered pairs. So, total number of relations from A to B is 2^{nm}.

Mapping (Function)

Let A and B be two non-empty sets, then a function f from set A to set B is a rule which associates each element of A to a unique element of B. It is represented as $f : A \to B$ and function is also called mapping.

Range of a real function, f is a set of values $f(x)$ which it attains on the points of its domain.

Types of Mapping (Function)

(*i*) **One-one function** The mapping $f : A \to B$ is called one-one function, if different elements in A have different images in B. Such a mapping is known as injective function or an injection.

(*ii*) **Many-one function** The function $f : A \to B$ is called many-one function, if two or more than two different elements in A have the same image in B.

(*iii*) **Onto** (surjective) **function** If the function $f : A \to B$ is such that each element in B (codomain) is the image of atleast one element of A, then we say that f is a function of A 'onto' B. Thus, $f : A \to B$, such that $f(A) = B$.
i.e. Range = Codomain

Note Every polynomial function $f : R \to R$ of degree odd is onto.

(*iv*) **Into function** If $f : A \to B$ is such that there exists atleast one element in codomain which is not the image of any element in domain, then $f(x)$ is into.
Thus, $f : A \to B$, such that $f(A) \subset B$
i.e. Range $\subset$ Codomain

Even and Odd Functions

(*i*) **Even functions** A real function $f(x)$ is an even function, if $f(-x) = f(x)$.

(*ii*) **Odd functions** A real function $f(x)$ is an odd function, if $f(-x) = -f(x)$.

Periodic Functions

A function $f(x)$ is said to be a periodic function of x, provided there exists a real number $T > 0$, such that

$$F(T + x) = f(x), \forall\ x \in R$$

The smallest positive real number T, satisfying the above condition is known as the period or the fundamental period of $f(x)$.

Inverse of a Function

Let $f : A \to B$ be a bijective function, i.e. it is one-one and onto function.

We define $g : B \to A$, such that $f(x) = y \Rightarrow g(y) = x$, g is called inverse of f and *vice-versa.* Symbolically, we write $g = f^{-1}$.

Thus, $$f(x) = y \Rightarrow f^{-1}(y) = x$$

Composite Functions (Function of Function)

Let $f : A \to B$ and $g : B \to C$ be two functions.

We define $gof : A \to C$, such that

$$gof(x) = g[f(x)], \forall\ x \in A$$

Types of Real Function

(*i*) **Identity function** The function that associates to each real number x for the same number x, is called the identity function.

i.e. $$y = f(x) = x, \forall\ x \in R$$

$$\text{Domain of } f(x) = R$$

$$\text{Range of } f(x) = R$$

(*ii*) **Logarithmic function** A logarithmic function may be given by $y = f(x) = \log_a x$, where $a > 0$, $a \neq 1$ and $x > 0$.

(*iii*) **Modulus function** (absolute value) Modulus function is given by $y = f(x) = |x|$, where $|x|$ denotes the absolute value of x.

i.e. $$|x| = \begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x < 0 \end{cases}$$

Domain of $f(x) = R$

Range of $f(x) = [0, \infty)$

(*iv*) **Signum function** Signum function is defined as

$$y = f(x) = \begin{cases} \dfrac{|x|}{x}, \text{ if } x \neq 0 \\ 0, \quad \text{if } x = 0 \end{cases} \text{ or } \begin{cases} \dfrac{x}{|x|}, \text{ if } x \neq 0 \\ 0, \quad \text{if } x = 0 \end{cases}$$

Domain of sgn $(x) = R$

Range of sgn $(x) = \{-1, 0, 1\}$

(*v*) **Greatest integer function** The greatest integer function is defined as

$$y = f(x) = [x]$$

where, $[x]$ represents the greatest integer less than or equal to x i.e. for any integer n,

$$[x] = n, \text{if } n \leq x < n + 1$$

Domain of $f(x) = R$, Range of $f(x) = I$

(*vi*) **Least integer function** The least integer function which is greater than or equal to x. It is denoted by (x).

In general, if n is an integer and x is any real number between n and $(n + 1)$.

i.e. $n < x \leq n + 1$, then $(x) = n + 1$

$\therefore$ $$f(x) = (x)$$

Domain of $f = R$

Range of $f = I$

(*vii*) **Fractional part function** It is denoted as $f(x) = \{x\}$ and defined as

(a) $\{x\} = f$, if $x = n + f$, where $n \in I$ and $0 \leq f < 1$

(b) $\{x\} = x - [x]$

(c) $\{x\} = x$, if $0 \leq x < 1$

(d) $\{x\} = 0$, if $x \in I$

(e) $\{-x\} = 1 - \{x\}$, if $x \notin I$

Topic 1
Types of Sets, Operations and Cartesian Products

2014

1. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to **[JEE Mains]**

(a) N (b) $Y - X$ (c) X (d) Y

2. The total number of subsets of a finite set A has 56 more elements than the total number of subsets of another finite set B. What is the number of elements in the set A?

(a) 5 (b) 6 **[BITSAT]**
(c) 7 (d) 8

3. In a class of 60 students, if 25 students play cricket, 20 students play tennis and 10 students play both the games, then the number of students who play neither is

(a) 45 (b) 0 **[KCET]**
(c) 25 (d) 35

4. The set $A = \{x : |2x + 3| < 7\}$ is equal to the set **[KCET]**

(a) $D = \{x : 0 < x + 5 < 7\}$ (b) $B = \{x : -3 < x < 7\}$
(c) $E = \{x : -7 < x < 7\}$ (d) $C = \{x : -13 < 2x < 4\}$

5. The number of students who take both the subjects Mathematics and Chemistry is 30. This represents 10% of the enrolment in Mathematics and 12% of the enrolment in Chemistry. How many students take atleast one of these two subjects? **[Kerala CEE]**

(a) 520 (b) 490 (c) 560
(d) 480 (e) 540

6. There is a group of 265 persons who like either singing or dancing or painting. In this group, 200 like singing, 110 like dancing and 55 like painting. If 60 persons like both singing and dancing, 30 like both singing and painting and 10 like all three activities, then the number of persons who like only dancing and painting is **[WB JEE]**

(a) 10 (b) 20 (c) 30 (d) 40

7. If the number of elements of the sets A and B are p and q, respectively. Then, the number of relations from the set A to the set B is **[WB JEE]**

(a) 2^{p+q} (b) 2^{pq} (c) $p + q$ (d) pq

8. If $X_n = \left\{ z = x + iy : |z|^2 \le \frac{1}{n} \right\}$ for all integers $n \ge 1$. Then, $\bigcap_{n=1}^{\infty} X_n$ is **[WB JEE]**

(a) a singleton set
(b) not a finite set
(c) an empty set
(d) a finite set with more than one element

2013

9. If $A = \{(x, y) : y = e^{-x}\}$ and $B = \{(x, y) : y = -x\}$. Then,

(a) $A \cap B = \phi$ (b) $A \subset B$ **[BITSAT]**
(c) $B \subset A$ (d) $A \cap B = \{(0, 1), (0, 0)\}$

2012

10. There are 100 students in a class. In the examination, 50 of them failed in Mathematics, 45 failed in Physics, 40 failed in Biology and 32 failed in exactly two of the three subjects. Only one student passed in all the subjects. Then, the number of students failing in all the three subjects

(a) is 12 **[WB JEE]**
(b) is 4
(c) is 2
(d) Cannot be determined

11. Out of 64 students, the number of students taking Mathematics is 45 and number of students taking both Mathematics and Biology is 10. Then, the number of students taking only Biology is **[OJEE]**

(a) 18 (b) 19
(c) 20 (d) None of these

12. If $A = \{4^n - 3n - 1 : n \in N\}$ and $B = \{9(n-1) : n \in N\}$, then

(a) $B \subset A$ (b) $A \cup B = N$ **[AMU]**
(c) $A \subset B$ (d) None of these

2011

13. If $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ are two sets. Then,

(a) $P \subset Q$ and $Q - P \neq \phi$ **[IIT JEE]**
(b) $Q \not\subset P$
(c) $P \not\subset Q$
(d) $P = Q$

14. The set $A = \{x : x \in R, x^2 = 16 \text{ and } 2x = 6\}$ is equal to

(a) ϕ (b) {14, 3, 4} **[BITSAT]**
(c) {3} (d) {4}

15. If A and B are two sets, then $(A \cup B)' \cap (A' \cap B)$ is equal to **[GGSIPU]**

(a) A' (b) A
(c) B' (d) None of these

16. If $A = \{1, 2\}$, $B = \{\{1\}, \{2\}\}$, $C = \{\{1, 2\}\}$. Then, which of the following relation is correct? **[J&K CET]**

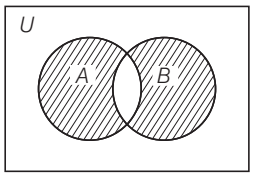
(a) $A = B$ (b) $B \subseteq C$
(c) $A \in C$ (d) $A \subset C$

2010

17. If $S = \{1, 2, 3, 4\}$, then the total number of unordered pairs of disjoint subsets of S is **[IIT JEE]**

(a) 25 (b) 34 (c) 42 (d) 41

18. The shaded region in the figure represents **[Kerala CEE]**



(a) $A \cap B$ (b) $A \cup B$
(c) $B - A$ (d) $A - B$
(e) $(A - B) \cup (B - A)$

2009

19. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then **[AIEEE]**

(a) $A = C$ (b) $B = C$
(c) $A \cap B = \phi$ (d) $A = B$

20. Two finite sets A and B have m and n elements, respectively. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m is

(a) 7 (b) 9 (c) 10 **[Kerala CEE]**
(d) 12 (e) 13

21. For any two sets A and B, if $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, then **[AMU]**

(a) $A - B = A \cap B$
(b) $A = B$
(c) $B - A = A \cap B$
(d) None of the above

22. $\left\{ x \in R : \dfrac{2x-1}{x^3 + 4x^2 + 3x} \in R \right\}$ is equal to **[EAMCET]**

(a) $R - \{0\}$ (b) $R - \{0, 1, 3\}$
(c) $R - \{0, -1, -3\}$ (d) $R - \left\{0, -1, -3, \dfrac{1}{2}\right\}$

23. If S is a set with 10 elements and $A = \{(x, y) : x, y \in S, x \neq y\}$, then the number of elements in A is **[J&K CET]**

(a) 100 (b) 90 (c) 50 (d) 45

24. For any two sets A and B, $A - (A - B)$ is equal to

(a) B (b) $A - B$ **[BCECE]**
(c) $A \cap B$ (d) $A^C \cap B^C$

2008

25. If A and B are two sets, then $(A \cup B)' \cup (A' \cap B)$ is equal to **[BITSAT]**

(a) A' (b) A
(c) B' (d) None of these

26. In a certain town, 25% families own a cell phone, 15% families own a scooter and 65% families own neither a cell phone nor a scooter. If 1500 families own both a cell phone and a scooter, then the total number of families in the town is **[Kerala CEE]**

(a) 10000 (b) 20000 (c) 30000
(d) 40000 (e) 50000

27. Three sets A, B and C are such that $A = B \cap C$ and $B = C \cap A$, then **[WB JEE]**

(a) $A \subset B$ (b) $A \supset B$ (c) $A = B$ (d) $A \subset B'$

28. A survey shows that 63% of the Americans like cheese whereas 76% like apples. If x % of the Americans like both cheese and apples, then **[UPSEE]**

(a) $x = 39$ (b) $x = 63$
(c) $39 \leq x \leq 63$ (d) None of these

29. If $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{a, d, c\}$, then $(A - B) \times (B \cap C)$ is equal to **[Jamia Millia Islamia]**

(a) $\{(a, c), (a, d)\}$ (b) $\{(a, b), (c, d)\}$
(c) $\{(c, a), (d, a)\}$ (d) $\{(a, c), (a, d), (b, d)\}$

30. If $A = \{1, 2, 3\}, B = \{3, 4\}$ and $C = \{4, 5, 6\}$, then $A \cup (B \cap C)$ is equal to **[OJEE]**

(a) $\{1, 2\}$ (b) $\{\phi\}$
(c) $\{4, 5\}$ (d) $\{1, 2, 3, 4\}$

31. If $n(A)$ denotes the number of elements in the set A and if $n(A) = 4, n(B) = 5$ and $n(A \cap B) = 3$, then $n[(A \times B) \cap (B \times A)]$ is equal to **[J&K CET]**

(a) 8 (b) 9 (c) 10 (d) 11

2007

32. The value of $(A \cup B \cup C) \cap (A \cap B^c \cap C^c)^c \cap C^c$ is

(a) $B \cap C^c$ (b) $B^c \cap C^c$ **[BITSAT]**
(c) $B \cap C$ (d) $A \cap B \cap C$

33. If Z denotes the set of all integers and $A = \{(a, b) : a^2 + 3b^2 = 28, a, b \in Z\}$ and $B = \{(a, b) : a > b, a, b \in Z\}$. Then, the number of elements in $A \cap B$ is **[Kerala CEE]**

(a) 2 (b) 3 (c) 4
(d) 5 (e) 6

34. Which of the following is correct? **[OJEE]**

(a) $A \cap \phi = A$ (b) $A \cap \phi = \phi$
(c) $A \cap \phi = U$ (d) $A \cap \phi = A'$

35. If sets A and B are defined as

$$A = \left\{ (x, y) : y = \frac{1}{x}, 0 \neq x \in R \right\}$$

and $B = \{(x, y) : y = -x, x \in R\}$, then **[Guj CET]**

(a) $A \cap B = A$ (b) $A \cap B = B$
(c) $A \cap B = \phi$ (d) None of these

2006

36. Universal set, $U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$, $A = \{x : x^2 - 5x + 6 = 0\}$ and $B = \{x : x^2 - 3x + 2 = 0\}$. Then, $(A \cap B)'$ is equal to **[BITSAT]**
(a) {1, 3} (b) {1, 2, 3}
(c) {0, 1, 3} (d) {0, 1, 2, 3}

37. If $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6\}$. Then, the number of set C such that $A \cap B \subseteq C \subseteq A \cup B$ is **[Kerala CEE]**
(a) 6 (b) 9 (c) 8
(d) 10 (e) 12

38. If X and Y are the sets of all positive divisors of 400 and 1000, respectively (including 1 and the number). Then, $n(X \cap Y)$ is equal to **[Kerala CEE]**
(a) 4 (b) 6 (c) 8
(d) 10 (e) 12

39. If $A = \{x, y\}$, then the power set of A is
(a) $\{x^y, y^x\}$ **[UPSEE, Guj CET]**
(b) $\{\phi, x, y\}$
(c) $\{\phi, \{x\}, \{2y\}\}$
(d) $\{\phi, \{x\}, \{y\}, \{x, y\}\}$

40. If $A = \{x : x$ is a multiple of 3$\}$ and $B = \{x : x$ is a multiple of 5$\}$. Then, $A \cap B$ is given by
(a) {3, 6, 9, …} (b) {5, 10, 15, 20,…} **[AMU]**
(c) {15, 30, 45, …} (d) None of these

2005

41. If $n(A) = 4, n(B) = 3$ and $n(A \times B \times C) = 24$, then $n(C)$ is equal to **[Kerala CEE]**
(a) 288 (b) 1 (c) 12
(d) 17 (e) 2

42. The number of elements in the set $\{(a, b) : 2a^2 + 3b^2 = 35, a, b \in Z\}$, where Z is the set of all integers, is **[Kerala CEE]**
(a) 2 (b) 4 (c) 8
(d) 12 (e) 16

43. $\{n(n+1)(2n+1) : n \in Z\}$ is a subset of **[EAMCET]**
(a) $\{6k : k \in Z\}$
(b) $\{12k : k \in Z\}$
(c) $\{18k : k \in Z\}$
(d) $\{24k : k \in Z\}$

44. In a class of 30 pupils, 12 take Chemistry, 16 take Physics and 18 take History. If all the 30 students take atleast one subject and no one take all three, then the number of pupils taking 2 subjects is
(a) 16 (b) 6 **[J&K CET]**
(c) 8 (d) 20

45. If $A \subseteq B$, then $B \cup A$ is equal to **[BCECE]**
(a) $B \cap A$ (b) A
(c) B (d) None of these

Topic 2
Relation and Equivalence Relation

2014

1. If $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ is a relation on the set $A = \{3, 6, 9, 12\}$. Then, the relation is **[BITSAT]**
(a) an equivalence relation
(b) reflexive and symmetric
(c) reflexive and transitive
(d) only reflexive

2. If S is the set of all real numbers. A relation R has been defined on S by $aRb \Leftrightarrow |a - b| \leq 1$, then R is **[KCET]**
(a) symmetric and transitive but not reflexive
(b) reflexive and transitive but not symmetric
(c) reflexive and symmetric but not transitive
(d) an equivalence relation

3. For any two real numbers θ and ϕ, we define $\theta R \phi$, if and only if $\sec^2\theta - \tan^2\phi = 1$. The relation R is **[WB JEE]**
(a) reflexive but not transitive
(b) symmetric but not reflexive
(c) both reflexive and symmetric but not transitive
(d) an equivalence relation

2013

4. If Z is the set of integers. Then, the relation $R = \{(a, b) : 1 + ab > 0\}$ on Z is **[AMU]**
(a) reflexive and transitive but not symmetric
(b) symmetric and transitive but not reflexive
(c) reflexive and symmetric but not transitive
(d) an equivalence relation

2012

5. If R is a relation on the set N, defined by $\{(x, y): 2x - y = 10\}$, then R is **[AMU]**

(a) reflexive (b) symmetric
(c) transitive (d) None of these

2011

6. If R is the set of real numbers.Then,
Statement I $A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R.
Statement II $B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R. **[AIEEE]**

(a) Statement I is correct, Statement II is correct; Statement II is not a correct explanation for Statement I
(b) Statement I is correct, Statement II is incorrect
(c) Statement I is incorrect, Statement II is correct
(d) Statement I is correct, Statement II is correct; Statement II is a correct explanation for Statement I

7. If A and B are two equivalence relations defined on set C, then **[UPSEE]**

(a) $A \cap B$ is an equivalence relation
(b) $A \cap B$ is not an equivalence relation
(c) $A \cup B$ is an equivalence relation
(d) $A \cup B$ is not an equivalence relation

2010

8. Consider the relations $R = \{(x, y) \mid x, y$ are real numbers and $x = wy$ for some rational number $w\}$,

$S = \left\{ \left(\frac{m}{n}, \frac{p}{q} \right) \middle| \, m, n, p \text{ and } q \text{ are integers such that } n, q \neq 0 \right.$ and $\left. qm = pn \right\}$. Then, **[AIEEE]**

(a) R is an equivalence relation but S is not an equivalence relation
(b) neither R nor S is an equivalence relation
(c) S is an equivalence relation but R is not an equivalence relation
(d) R and S both are equivalence relations

9. If $A = \{x, y, z\}$ and $B = \{a, b, c, d\}$. Then, which one of the following is not a relation from A to B? **[Kerala CEE]**

(a) $\{(x, a), (x, c)\}$ (b) $\{(y, c), (y, d)\}$
(c) $\{(z, a), (z, d)\}$ (d) $\{(z, b), (y, b), (a, d)\}$
(e) $\{(x, c)\}$

2009

10. If R and S are two non-void relations on a set A.Then, which of the following statements is incorrect? **[VITEEE]**

(a) R and S are transitive implies $R \cap S$ is transitive
(b) R and S are transitive implies $R \cup S$ is transitive
(c) R and S are symmetric implies $R \cup S$ is symmetric
(d) R and S are reflexive implies $R \cap S$ is reflexive

11. If r is a relation from R (set of real numbers) to R defined by $r = \{(a, b) \mid a, b \in R$ and $a - b + \sqrt{3}$ is an irrational number$\}$.Then, the relation r is **[AMU]**

(a) an equivalence relation
(b) only reflexive
(c) only symmetric
(d) only transitive

12. If a relation R on the set N of natural numbers is defined as $(x, y) \Leftrightarrow x^2 - 4xy + 3y^2 = 0, \forall\, x, y \in N$. Then, the relation R is **[AMU]**

(a) reflexive (b) symmetric
(c) transitive (d) an equivalence relation

2008

13. If R is the real line. Consider the following subsets of the plane $R \times R$

$$S = \{(x, y): y = x + 1 \text{ and } 0 < x < 2\}$$
$$T = \{(x, y): x - y \text{ is an integer}\}$$

Which one of the following is correct? **[AIEEE]**

(a) T is an equivalence relation on R but S is not
(b) Neither S nor T is an equivalence relation on R
(c) Both S and T are an equivalence relations on R
(d) S is an equivalence relation on R but T is not

14. If R is a relation defined as aRb, iff $|a - b| > 0$, then the relation is **[VITEEE]**

(a) reflexive (b) symmetric
(c) transitive (d) symmetric and transitive

15. R is a relation on N given by $R = \{(x, y): 4x + 3y = 20\}$. Which of the following belongs to R? **[KCET]**

(a) (– 4, 12) (b) (5, 0) (c) (3, 4) (d) (2, 4)

16. If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then which of the following relations is a function from A to B? **[WB JEE]**

(a) {(1, 2), (2, 3), (3, 4), (2, 2)}
(b) {(1, 2), (2, 3), (1, 3)}
(c) {(1, 3), (2, 3), (3, 3)}
(d) {(1, 1), (2, 3), (3, 4)}

17. If R is a relation from {11, 12, 13} to {8, 10, 12} defined by $y = x - 3$. Then, R^{-1} is equal to **[Jamia Millia Islamia]**

(a) {(8, 11), (10, 13)} (b) {(11, 18), (13, 10)}
(c) {(10, 13), (8, 11)} (d) None of these

2007

18. On the set N of all natural numbers define the relation R by aRb, if and only if the GCD of a and b is 2, then R is **[Kerala CEE]**
(a) reflexive but not symmetric
(b) only symmetric
(c) reflexive and transitive
(d) reflexive, symmetric and transitive
(e) not reflexive, not symmetric and not transitive

19. If $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ is a relation on the set $A = \{1, 2, 3, 4\}$. Then, relation R is **[Jamia Millia Islamia]**
(a) a function (b) transitive
(c) not symmetric (d) reflexive

20. If $R = \{(3,3),(6,6),(9,9),(12,12),(6,12),(3,9),(3,12)$ $(3, 6)\}$ is a relation on the set $A = \{3, 6, 9, 12\}$. Then, the relation is **[OJEE]**
(a) reflexive and symmetric (b) an equivalence relation
(c) reflexive only (d) reflexive and transitive

21. If R is an equivalence relation on a set A, then R^{-1} is **[AMU]**
(a) only reflexive
(b) symmetric but not transitive
(c) equivalence
(d) None of the above

22. The relation R defined on the set of natural numbers as $\{(a, b) : a$ differs from b by $3\}$ is given by **[AMU]**
(a) {(1, 4), (2, 5), (3, 6), …}
(b) {(4, 1), (5, 2), (6, 3), …}
(c) {(1, 3), (2, 6), (3, 9), …}
(d) None of the above

2006

23. If W denotes the words in the English dictionary define the relation R by $R = \{(x, y) \in W \times W :$ the words x and y have atleast one letter in common$\}$. Then, R is **[AIEEE]**
(a) reflexive, symmetric and not transitive
(b) reflexive, symmetric and transitive
(c) reflexive, not symmetric and transitive
(d) not reflexive, symmetric and transitive

24. Which of the following statements is not correct for the relation R defined by aRb, if and only if b lives within one kilometre from a? **[BITSAT]**
(a) R is reflexive
(b) R is symmetric
(c) R is anti-symmetric
(d) None of the above

25. If R is a relation on the set of integers given by $aRb \Leftrightarrow a = 2^k .b$ for some integer k. Then, R is
(a) an equivalence relation **[Kerala CEE]**
(b) reflexive but not symmetric
(c) reflexive and transitive but not symmetric
(d) reflexive and symmetric but not transitive
(e) symmetric and transitive but not reflexive

2005

26. $x^2 = xy$ is a relation which is **[BITSAT]**
(a) symmetric
(b) reflexive and transitive
(c) transitive
(d) None of the above

Topic 3
Types of Mapping

2014

1. If $f : N \to N$ is defined by

$$f(n) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ n/2, & \text{if } n \text{ is even} \end{cases}.$$

Then, f is **[KCET]**
(a) onto but not one-one
(b) one-one and onto
(c) neither one-one nor onto
(d) one-one but not onto

2. If $A = \{1, 3, 5, 7\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$, then the number of one-one function from A into B is
(a) 1340 (b) 1860 (c) 1430 **[Kerala CEE]**
(d) 1880 (e) 1680

3. The function $f(x) = x^2 + bx + c$, where b and c are real constants, describes **[WB JEE]**
(a) one-one mapping
(b) onto mapping
(c) not one-one but onto mapping
(d) neither one-one nor onto mapping

2012

4. The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is **[IIT JEE]**
(a) one-one and onto
(b) onto but not one-one
(c) one-one but not onto
(d) neither one-one nor onto

5. The total number of injections (one-one and into mappings) from $\{a_1, a_2, a_3, a_4\}$ to $\{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$ is
(a) 400 (b) 420 **[WB JEE]**
(c) 800 (d) 840

2011

6. The function $f:[0,\infty)\to[0,\infty)$ defined by $f(x)=\dfrac{2x}{1+2x}$ is **[J&K CET]**
(a) one-one and onto
(b) one-one but not onto
(c) not one-one but onto
(d) neither one-one nor onto

2010

7. If $A=\{1, 2, 3, 4\}$ and $B=\{1, 2, 3, 4, 5, 6\}$ are two sets and function $f:A\to B$ is defined by $f(x)=x+2, \forall\, x\in A$, then the function f is **[WB JEE]**
(a) bijective (b) onto (c) one-one (d) many-one

2009

8. For real x, if $f(x)=x^3+5x+1$, then **[AIEEE]**
(a) f is one-one but not onto R
(b) f is onto R but not one-one
(c) f is one-one and onto R
(d) f is neither one-one nor onto R

9. If $f(x)=(x+1)^2-1, x\geq -1$. Then,
Statement I The set $\{x: f(x)=f^{-1}(x)\}=\{0,-1\}$.
Statement II f is a bijection. **[AIEEE]**
(a) Statement I is correct, Statement II is correct; Statement II is a correct explanation for Statement I
(b) Statement I is correct, Statement II is correct; Statement II is not a correct explanation for Statement I
(c) Statement I is correct, Statement II is incorrect
(d) Statement I is incorrect, Statement II is correct

10. On the set of integers Z, define $f:Z\to Z$ as

$$f(n)=\begin{cases}\dfrac{n}{2}, & n \text{ is even.}\\ 0, & n \text{ is odd.}\end{cases}$$

Then, f is **[KCET]**
(a) injective but not surjective
(b) neither injective nor surjective
(c) surjective but not injective
(d) bijective

11. If $n(A)=4$ and $n(B)=6$. Then, the number of one-one function from A to B is **[AMU]**
(a) 24 (b) 60 (c) 120 (d) 360

12. If R and C denote the set of real numbers and complex numbers, respectively.Then, the function $f:C\to R$ defined by $f(z)=|z|$ is **[AMU]**
(a) one-one
(b) onto
(c) bijective
(d) neither one-one nor onto

2008

13. If $f:N\to N$ defined by $f(x)=x^2+x+1, x\in N$, then f is
(a) one-one and onto **[BITSAT]**
(b) many-one and onto
(c) one-one but not onto
(d) None of the above

14. Which one of the following functions is one-one?
(a) $f(x)=\sin x, x\in[-\pi,\pi)$ **[Kerala CEE]**
(b) $f(x)=\sin x, x\in\left[-\dfrac{3\pi}{2},-\dfrac{\pi}{4}\right]$
(c) $f(x)=\cos x, x\in\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$
(d) $f(x)=\cos x, x\in\left[\pi,\dfrac{3\pi}{2}\right)$
(e) $f(x)=\cos x, x\in\left[-\dfrac{\pi}{4},\dfrac{\pi}{4}\right]$

15. A mapping $f:n\to N$, where N is the set of natural numbers is defined as $f(n)=\begin{cases}n^2, & \text{for } n \text{ odd}\\ 2n+1, & \text{for } n \text{ even}\end{cases}$ for $n\in N$. Then, f is **[WB JEE]**
(a) surjective but not injective
(b) injective but not surjective
(c) bijective
(d) neither injective nor surjective

16. The mapping $f:N\to N$ given by $f(n)=1+n^2, n\in N$, where N is the set of natural numbers, is **[WB JEE]**
(a) one-one and onto
(b) onto but not one-one
(c) one-one but not onto
(d) neither one-one nor onto

17. A function $f:A\to B$, where $A=\{x:-1\leq x\leq 1\}$ and $B=\{y:1\leq y\leq 2\}$ is defined by the rule $y=f(x)=1+x^2$. Which of the following statement is correct? **[WB JEE]**
(a) f is injective but not surjective
(b) f is surjective but not injective
(c) f is both injective and surjective
(d) f is neither injective nor surjective

18. If $f: R \to C$ is defined by $f(x) = e^{2ix}$ for $x \in R$, then f is (where, C denotes the set of all complex numbers)
(a) one-one **[AMU, EAMCET]**
(b) onto
(c) one-one and onto
(d) neither one-one nor onto

19. The function $f: R \to R$ given by $f(x) = x^3 - 1$ is
(a) a one-one function **[J&K CET]**
(b) an onto function
(c) a bijection
(d) neither one-one nor onto

2007

20. If $A = [-1, 1]$ and $f: A \to A$ is defined as $f(x) = x|x|$, $\forall\, x \in A$, then $f(x)$ is **[BITSAT]**
(a) many-one and into function
(b) one-one and into function
(c) many-one and onto function
(d) one-one and onto function

21. The function $f: R \to R$ defined by
$$f(x) = (x-1)(x-2)(x-3) \text{ is} \quad \textbf{[AMU]}$$
(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

2006

22. The function $f: X \to Y$ defined by $f(x) = \sin x$ is one-one but not onto, if X and Y are respectively equal to **[KCET]**
(a) R and R (b) $[0, \pi]$ and $[0, 1]$
(c) $\left[0, \frac{\pi}{2}\right]$ and $[-1, 1]$ (d) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and $[-1, 1]$

23. If R denotes the set of all real numbers, then the function $f: R \to R$ defined by $f(x) = |x|$ is **[AMU]**
(a) only one-one (b) only onto
(c) both one-one and onto (d) neither one-one nor onto

2005

24. If $f(x) = \begin{cases} x, \text{ if } x \text{ is rational.} \\ 0, \text{ if } x \text{ is irrational.} \end{cases}$
and $g(x) = \begin{cases} 0, \text{ if } x \text{ is rational.} \\ x, \text{ if } x \text{ is irrational.} \end{cases}$
Then, $f - g$ is **[IIT JEE]**
(a) one-one and into
(b) neither one-one nor onto
(c) many-one and onto
(d) one-one and onto

25. If A is a set containing 10 distinct elements, then the total number of distinct function from A to A is **[DCE]**
(a) 10^{10} (b) 101 (c) 2^{10} (d) $2^{10} - 1$

Topic 4
Domain-Range, Odd-Even and Periodic Functions

2014

1. If $f(x)$ is an odd periodic function with period 2, then $f(4)$ is equal to **[BITSAT]**
(a) -4 (b) 4
(c) 2 (d) 0

2. The period of $\sin^2\theta$ is **[BITSAT]**
(a) π^2 (b) π (c) 2π (d) $\frac{\pi}{2}$

3. Which one of the following is not correct for the feature of exponential function given by
$$f(x) = b^x, \text{where } b > 1? \quad \textbf{[KCET]}$$
(a) For very large negative values of x, the function is very close to 0
(b) The domain of the function is R, the set of real numbers
(c) The point $(1, 0)$ is always on the graph of the function
(d) The range of the function is the set of all positive real numbers

4. The domain of the function $f(x) = \sqrt{\cos x}$ is
(a) $\left[\frac{3\pi}{2}, 2\pi\right]$ (b) $\left[0, \frac{\pi}{2}\right]$ **[Kerala CEE]**
(c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$
(e) All of these

5. If $f(x) = |x - 2|$, where x is a real number. Then, which one of the following is correct? **[Kerala CEE]**
(a) f is periodic
(b) $f(x + y) = f(x) + f(y)$
(c) f is an odd function
(d) f is not one-one function
(e) f is an even function

6. The range of the function $f(x) = x^2 + 2x + 2$ is
(a) $(1, \infty)$ (b) $(2, \infty)$ (c) $(0, \infty)$ **[Kerala CEE]**
(d) $[1, \infty)$ (e) $(-\infty, \infty)$

7. The range of the function $y = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ is

(a) $[0, \sqrt{3/2}]$ (b) $[0, 1]$ **[WB JEE]**
(c) $[0, 3/\sqrt{2}]$ (d) $[0, \infty)$

8. If $A = \{1, 2, 3, 4\}$ and R be the relation on A defined by $\{(a, b) : a, b \in A, a \times b$ is an even number$\}$, then find the range of R. **[J&K CET]**

(a) $\{1, 2, 3, 4\}$ (b) $\{2, 4\}$
(c) $\{2, 3, 4\}$ (d) $\{1, 2, 4\}$

9. Find the domain of the function

$$f(x) = (x^2 + 1)/(x^2 - 3x + 3).$$ **[J&K CET]**

(a) $R - \{1, 2\}$ (b) $R - \{1, 4\}$
(c) R (d) $R - \{1\}$

10. Find the range of the function

$$f : [0, 1] \to R,\ f(x) = x^3 - x^2 + 4x + 2\sin^{-1} x.$$

(a) $[-(\pi + 2), 0]$ (b) $[0, 4 + \pi]$ **[J&K CET]**
(c) $[2, 3]$ (d) $(0, 2 + \pi]$

11. If $A = \{1, 2, 3, 4, 5\}$, then find the domain in the relation from A to A by $R = \{(x, y) : y = 2x - 1\}$. **[J&K CET]**

(a) $\{1, 2, 3\}$ (b) $\{1, 2\}$ (c) $\{1, 3, 5\}$ (d) $\{2, 4\}$

2013

12. If $f(x) = \cos ax + \sin x$ is periodic, then a must be

(a) irrational (b) rational **[OJEE]**
(c) positive real number (d) None of these

2012

13. If $f(x) = \sin\sqrt{x}$, then period of $f(x)$ is **[OJEE]**

(a) π (b) $\frac{\pi}{2}$
(c) 2π (d) None of these

2011

14. The domain of $f(x) = \sin^{-1}\left[\log_2\left(\frac{x}{2}\right)\right]$ is **[KCET]**

(a) $0 \le x \le 1$ (b) $0 \le x \le 4$
(c) $1 \le x \le 4$ (d) $4 \le x \le 6$

15. The range of the function $f(x) = \log_e(3x^2 + 4)$ is equal to **[Kerala CEE]**

(a) $[\log_e 2, \infty)$ (b) $[\log_e 3, \infty)$
(c) $[2\log_e 3, \infty)$ (d) $[0, \infty)$
(e) $[2\log_e 2, \infty)$

16. The period of the function $f(x) = |\sin 2x| + |\cos 8x|$ is

(a) 2π (b) π (c) $\frac{2\pi}{3}$ **[Kerala CEE]**
(d) $\frac{\pi}{2}$ (e) $\frac{\pi}{4}$

17. The even function is **[WB JEE]**

(a) $f(x) = \frac{a^x + a^{-x}}{a^x - a^{-x}}$
(b) $f(x) = \frac{a^x + 1}{a^x - 1}$
(c) $f(x) = x \cdot \frac{a^x - 1}{a^x + 1}$
(d) $f(x) = \log_2(x + \sqrt{x^2 + 1})$

18. The domain of $\cos^{-1}\frac{x-3}{2} - \log_{10}(4 - x)$ is **[GGSIPU]**

(a) $(1, 4)$ (b) $[1, 4)$ (c) $(1, 4]$ (d) $[1, 4]$

19. If $f(x) = 3 - x$, where $-4 \le x \le 4$, then the domain of $\log_e[f(x)]$ is **[J&K CET]**

(a) $[-4, 4]$ (b) $(-\infty, 3]$ (c) $(-\infty, 3)$ (d) $[-4, 3)$

2010

20. The period of the function $f(\theta) = 4 + 4\sin^3\theta - 3\sin\theta$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ **[BITSAT]**
(c) $\frac{\pi}{2}$ (d) π

21. The function $f(x) = \sec[\log(x + \sqrt{1 + x^2})]$ is **[BITSAT]**

(a) odd (b) even
(c) neither odd nor even (d) constant

22. The domain of $\sin^{-1}\left[\log_2\left(\frac{x}{12}\right)\right]$ is **[Kerala CEE]**

(a) $[2, 12]$ (b) $[-1, 1]$ (c) $\left[\frac{1}{3}, 24\right]$
(d) $\left[\frac{2}{3}, 24\right]$ (e) $[6, 24]$

23. The domain of the function $f(x) = \sqrt{\cos^{-1}\left(\frac{1 - |x|}{2}\right)}$ is

(a) $(-3, 3)$ (b) $[-3, 3]$ **[WB JEE]**
(c) $(-\infty, -3) \cup (3, \infty)$ (d) $(-\infty, -3] \cup [3, \infty)$

2009

24. Range of the function $f(x) = \frac{x}{1 + x^2}$ is **[BITSAT]**

(a) $(-\infty, \infty)$ (b) $[-1, 1]$ (c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $[-\sqrt{2}, \sqrt{2}]$

25. If f is a function with domain $[-3, 5]$ and $g(x) = |3x + 4|$. Then, the domain of $(fog)(x)$ is **[BITSAT]**

(a) $\left(-3, \frac{1}{3}\right)$ (b) $\left[-3, \frac{1}{3}\right)$
(c) $\left[-3, \frac{1}{3}\right]$ (d) $\left[-3, -\frac{1}{3}\right]$

26. The domain of the function $f(x)=\log_2[\log_3(\log_4 x)]$ is
(a) $(-\infty, 4)$ (b) $(4, \infty)$ (c) $(0, 4)$ **[Kerala CEE]**
(d) $(1, \infty)$ (e) $(-\infty, 1)$

27. The domain of definition of the function $f(x)=\sqrt{1+\log_e(1-x)}$ is **[WB JEE]**
(a) $-\infty < x \le 0$ (b) $-\infty < x \le \frac{e-1}{e}$
(c) $-\infty < x \le 1$ (d) $x \ge 1-e$

28. If $f(x)=\frac{1}{\sqrt{|x|-x}}$, then domain of $f(x)$ is **[OJEE]**
(a) $(-\infty, 0)$ (b) $(-\infty, 2)$
(c) $(-\infty, \infty)$ (d) None of these

29. If $f:[2, 3] \to R$ is defined by $f(x)=x^3+3x-2$, then the range $f(x)$ is contained in the interval **[EAMCET]**
(a) [1, 12] (b) [12, 34] (c) [35, 50] (d) $[-12, 12]$

30. The domain of the real function $f(x)=\frac{1}{\sqrt{4-x^2}}$ is
(a) the set of all real numbers **[J&K CET]**
(b) the set of all positive real numbers
(c) $(-2, 2)$
(d) $[-2, 2]$

31. The period of the function $f(x)=\text{cosec}^2 3x+\cot 4x$ is
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ **[BCECE]**
(c) $\frac{\pi}{6}$ (d) π

2008

32. The domain of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ is
[BITSAT, Jamia Millia Islamia]
(a) [1, 9] (b) [–1, 9]
(c) [– 9, 1] (d) [– 9, –1]

33. If $f: R \to R$ and $g: R \to R$ are defined by $f(x)=|x|$ and $g(x)=[x-3]$ for $x \in R$, then $\left\{g(f(x)): -\frac{8}{5} < x < \frac{8}{5}\right\}$ is equal to **[BITSAT]**
(a) {0, 1} (b) {1, 2}
(c) {–3, –2} (d) {2, 3}

34. The period of the function
$f(x)=\frac{\sin 8x\cos x-\sin 6x\cos 3x}{\cos 2x\cos x-\sin 3x\sin 4x}$ is **[Manipal]**
(a) π (b) 2π
(c) $\frac{\pi}{2}$ (d) None of these

35. The range of the function $f(x)=x^2+\frac{1}{x^2+1}$ is **[Manipal]**
(a) $[1, \infty)$ (b) $[2, \infty)$
(c) $\left[\frac{3}{2}, \infty\right)$ (d) None of these

36. If $f(x)$ is an even function and $f'(x)$ exists, then $f'(e)+f'(-e)$ is **[KCET]**
(a) >0 (b) $=0$ (c) ≥ 0 (d) <0

37. If n is the natural number. Then, the range of the function $f(n)={}^{8-n}P_{n-4}, 4 \le n \le 6$, is **[Kerala CEE]**
(a) {1, 2, 3, 4} (b) {1, 2, 3, 4, 5, 6}
(c) {1, 2, 3} (d) {1, 2, 3, 4, 5}
(e) ϕ

38. The domain of the function $f(x)=\log_{2x-1}(x-1)$ is
(a) $(1, \infty)$ (b) $\left(\frac{1}{2}, \infty\right)$ **[UPSEE]**
(c) $(0, \infty)$ (d) None of these

39. If $f(x)=\frac{1}{2}-\tan\left(\frac{\pi x}{2}\right), -1 < x < 1$ and $g(x)=\sqrt{3+4x-4x^2}$, then domain $(f+g)$ is given by
(a) $\left[\frac{1}{2}, 1\right]$ (b) $\left[\frac{1}{2}, -1\right)$ **[UPSEE]**
(c) $\left[-\frac{1}{2}, 1\right)$ (d) $\left[-\frac{1}{2}, -1\right]$

40. The range of the function $f(x)=\tan\sqrt{\frac{\pi^2}{9}-x^2}$ is
(a) [0, 3] (b) $[0, \sqrt{3}]$ **[OJEE]**
(c) $(-\infty, \infty)$ (d) None of these

41. The domain of the real valued function $f(x)=\sqrt{1-2x}+2\sin^{-1}\left(\frac{3x-1}{2}\right)$ is **[Guj CET]**
(a) $\left[-\frac{1}{3}, 1\right]$ (b) $\left[\frac{1}{2}, 1\right]$ (c) $\left[-\frac{1}{2}, \frac{1}{3}\right]$ (d) $\left[-\frac{1}{3}, \frac{1}{2}\right]$

42. If $f: R \to R$ is defined by $f(x)=[2x]-2[x], \forall x \in R$, where $[x]$ is the greatest integer not exceeding x, then the range of f is **[BCECE]**
(a) $\{x \in R: 0 \le x \le 1\}$ (b) {0, 1}
(c) $\{x \in R: x > 0\}$ (d) $\{x \in R: x \le 0\}$

43. The function $f(x)=\log(x+\sqrt{x^2+1})$ is **[BCECE]**
(a) an even function
(b) an odd function
(c) a periodic function
(d) neither an even nor an odd function

2007

44. The largest interval lying in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ on which the function $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$ is defined, is **[AIEEE]**

(a) $[0, \pi]$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (c) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (d) $\left[0, \frac{\pi}{2}\right)$

45. If $f : (-1, 1) \to B$ is a function defined by $f(x) = \tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval **[MP PET]**

(a) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (c) $\left[0, \frac{\pi}{2}\right)$ (d) $\left(0, \frac{\pi}{2}\right)$

46. The period of the function $f(x) = a^{\{\tan(\pi x) + x - [x]\}}$, where $a > 0$, [ ] denotes the greatest integer function and x is a real number, is **[Kerala CEE]**

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$
(d) 2π (e) 1

47. The period of the function $f(x) = \sin\left(\sin\frac{x}{5}\right)$ is **[WB JEE]**

(a) 2π (b) $2\pi/5$ (c) 10π (d) 5π

48. The domain of the function $f(x) = \log_{10}(\sqrt{x-4} + \sqrt{6-x})$ is **[WB JEE]**

(a) [4, 6] (b) $(-\infty, 6)$
(c) [2, 3) (d) None of these

49. The domain of the function $\sin^{-1}\left(\log_2\frac{x^2}{2}\right)$ is **[UPSEE]**

(a) $[-1, 2] - \{0\}$ (b) $[-2, 2] - (-1, 1)$
(c) $[-2, 2] - \{0\}$ (d) [1, 2]

50. The range of $f(x) = \cos x - \sin x$ is **[UPSEE]**

(a) [–1, 1] (b) (–1, 2)
(c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $[-\sqrt{2}, \sqrt{2}]$

51. If $f : R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is **[Jamia Millia Islamia]**

(a) [0, 3] (b) [–1, 1]
(c) [0, 1] (d) [–1, 3]

52. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is **[Jamia Millia Islamia]**

(a) [2, 3] (b) [2, 3) (c) [1, 2] (d) [1, 2)

53. The domain of the function $f(x) = \frac{\sqrt{4-x^2}}{\sin^{-1}(2-x)}$ is **[AMU]**

(a) [0, 2] (b) [0, 2) (c) [1, 2) (d) [1, 2]

54. If $f : R \to R$ and is defined by $f(x) = \frac{1}{2 - \cos 3x}$ for each $x \in R$, then the range of f is **[EAMCET]**

(a) (1/3, 1) (b) [1/3, 1] (c) (1, 2) (d) [1, 2]

55. The period of the function $f(x) = |\sin x| + |\cos x|$ is

(a) π (b) $\pi/2$ **[Guj CET]**
(c) 2π (d) None of these

56. If the function $f : R \to R$ defined by $f(x) = [x]$, where $[x]$ is the greatest integer not exceeding x, for $x \in R$, then f is

(a) even **[J&K CET]**
(b) odd
(c) neither even nor odd
(d) strictly increasing

57. The domain of the function $f(x) = \frac{\cos^{-1} x}{[x]}$ is **[BCECE]**

(a) $[-1, 0) \cup \{1\}$ (b) [–1, 1]
(c) [–1, 1) (d) None of these

2006

58. The domain of the real valued function $f(x) = \sqrt{5 - 4x - x^2} + x^2 \log(x + 4)$ is **[Kerala CEE]**

(a) $-5 \le x \le 1$ (b) $-5 \le x$ and $x \ge 1$
(c) $-4 < x \le 1$ (d) ϕ
(e) $0 \le x \le 1$

59. The largest possible set of real numbers which can be the domain of $f(x) = \sqrt{1 - \frac{1}{x}}$, is **[WB JEE]**

(a) $(0, 1) \cup (0, \infty)$ (b) $(-1, 0) \cup (1, \infty)$
(c) $(-\infty, -1) \cup (0, \infty)$ (d) $(-\infty, 0) \cup [1, \infty)$

60. The domain of the function $f(x) = \exp(\sqrt{5x - 3 - 2x^2})$ is

(a) $[3/2, \infty)$ (b) [1, 3/2] **[UPSEE]**
(c) $(-\infty, 1]$ (d) (1, 3/2)

61. The period of the function $f(x) = \sin^4 x + \cos^4 x$ is

(a) π (b) $\frac{\pi}{2}$ **[UPSEE]**
(c) 2π (d) None of these

62. The domain of definition of the function $f(x) = \sqrt{\log_{10}\left(\frac{5x - x^2}{4}\right)}$ is **[OJEE]**

(a) [1, 4] (b) [1, 0] (c) [0, 5] (d) [5, 0]

63. If $f: R \to R$ is defined by $f(x) = x - [x] - \frac{1}{2}$ for $x \in R$, where $[x]$ is the greatest integer not exceeding x, then $\left\{x \in R : f(x) = \frac{1}{2}\right\}$ is equal to **[EAMCET]**

(a) Z, the set of all integers
(b) N, the set of all natural numbers
(c) ϕ, the empty set
(d) R, the set of all rational numbers

2005

64. If $f(x) = 2x^6 + 3x^4 + 4x^2$, then $f'(x)$ is **[BITSAT]**

(a) even function (b) an odd function
(c) neither even nor odd (d) None of these

65. Range of the function $f(x) = \frac{x^2}{x^2 + 1}$ is **[BITSAT, OJEE]**

(a) (–1, 0) (b) (–1, 1) (c) [0, 1) (d) (1, 1)

66. The range of the function $f(x) = \sqrt{(x-1)(3-x)}$ is **[AMU]**

(a) [0, 1] (b) (–1, 1)
(c) (–3, 3) (d) (–3, 1)

67. The domain of the function $f(x) = \log_e (x - [x])$ is

(a) R (b) $R - Z$ **[AMU]**
(c) $(0, +\infty)$ (d) Z

68. The range of the function $f(x) = x^2 - 6x + 7$ is **[BCECE]**

(a) $(-\infty, 0)$ (b) $[-2, \infty)$
(c) $(-\infty, \infty)$ (d) $(-\infty, -2)$

Topic 5
Inverse, Composition and Different Types of Functions

2014

1. If $f_k(x) = 1/k\ (\sin^k x + \cos^k x)$, where $x \in R$ and $k \geq 1$, then $f_4(x) - f_6(x)$ is equal to **[JEE Mains]**

(a) 1/6 (b) 1/3 (c) 1/4 (d) 1/12

2. Let $f(x) = (x+1)^2$ for $x \geq -1$. If $g(x)$ is a function whose graph is the reflection of the graph of $f(x)$ in the line $y = x$, then $g(x)$ is equal to **[KCET]**

(a) $\frac{1}{(x+1)^2}\ x > -1$ (b) $-\sqrt{x} - 1$
(c) $\sqrt{x} + 1$ (d) $\sqrt{x} - 1$

3. If $f(x) = \sqrt{x}$ and $g(x) = 2x - 3$, then domain of $(fog)(x)$ is

(a) $(-\infty, -3)$ (b) $\left(-\infty, -\frac{3}{2}\right)$ **[Kerala CEE]**
(c) $\left[-\frac{3}{2}, 0\right]$ (d) $\left[0, \frac{3}{2}\right]$
(e) $\left[\frac{3}{2}, \infty\right)$

4. If $f(x) = \frac{x+2}{3x-1}$, then $f\{f(x)\}$ is equal to **[Kerala CEE]**

(a) x (b) $-x$ (c) $\frac{1}{x}$
(d) $-\frac{1}{x}$ (e) 0

5. If R is the set of all real numbers and $f: R \to R$ is given by $f(x) = 3x^2 + 1$. Then, the set $f^{-1}([1, 6])$ is **[WB JEE]**

(a) $\left\{-\sqrt{\frac{5}{3}}, 0, \sqrt{\frac{5}{3}}\right\}$ (b) $\left[-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right]$
(c) $\left[-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right]$ (d) $\left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

6. If $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$, then $f(x)$ will be **[RPET]**

(a) $x^2 - 1$ (b) $x^2 - 2$
(c) x^2 (d) x^4

2013

7. If the function $f(x) = \frac{a^x + a^{-x}}{2}$, where $a > 2$. Then, $f(x+y) + f(x-y)$ is equal to **[Manipal]**

(a) $f(x) - f(y)$
(b) $f(y)$
(c) $2f(x)f(y)$
(d) None of these

8. If $f(x) = (x+2)^2 - 2, x \geq -2$. Then $f^{-1}(x)$ is equal to

(a) $-\sqrt{2+x} - 2$ (b) $\sqrt{2+x} + 2$ **[AMU]**
(c) $\sqrt{2+x} - 2$ (d) $-\sqrt{2+x} + 2$

2012

9. If $f:(-1,1)\to R$ be such that $f(\cos 4\theta)=\frac{2}{2-\sec^2\theta}$ for $\theta\in\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{4},\frac{\pi}{2}\right)$. Then, the value(s) of $f\left(\frac{1}{3}\right)$ is/are

(a) $1-\sqrt{\frac{3}{2}}$ (b) $1+\sqrt{\frac{3}{2}}$ **[IIT JEE]**

(c) $1-\sqrt{\frac{2}{3}}$ (d) $1+\sqrt{\frac{2}{3}}$

10. If $f(x)=x^2+2bx+2c^2$ and $g(x)=-x^2-2cx+b^2$ are such that $\min f(x)>\max g(x)$, then relation between b and c is **[Manipal]**

(a) no relation (b) $0<c<b/2$

(c) $|c|<\sqrt{2}\,|b|$ (d) $|c|>\sqrt{2}\,|b|$

11. If $f:R\to R$ is defined by $f(x)=2x+3$, then $f^{-1}(x)$

(a) is given by $\frac{x-3}{2}$ **[KCET]**

(b) is given by $\frac{1}{2x+3}$

(c) does not exist because f is not injective

(d) does not exist because f is not surjective

12. If R is the set of real numbers and the functions $f:R\to R$ and $g:R\to R$ be defined by $f(x)=x^2+2x-3$ and $g(x)=x+1$. Then, the value of x for which $f(g(x))=g(f(x))$ is **[WB JEE]**

(a) -1 (b) 0 (c) 1 (d) 2

2011

13. If $f(x)=x^2$ and $g(x)=\sin x$, $\forall\, x\in R$. Then, the set of all x satisfying $(fogogof)(x)=(gogof)(x)$, where $(fog)(x)=f(g(x))$, is **[IIT JEE]**

(a) $\pm\sqrt{n\pi}$, $n\in\{0,1,2,\ldots\}$

(b) $\pm\sqrt{n\pi}$, $n\in\{1,2,\ldots\}$

(c) $\frac{\pi}{2}+2n\pi$, $n\in\{\ldots,-2,-1,0,1,2,\ldots\}$

(d) $2n\pi$, $n\in\{\ldots,-2,-1,0,1,2,\ldots\}$

14. If $f:(0,1)\to R$ is defined by $f(x)=\frac{b-x}{1-bx}$, where b is a constant such that $0<b<1$. Then, **[IIT JEE]**

(a) f^{-1} is not invertible on (0, 1)

(b) $f\neq f^{-1}$ on (0, 1) and $f'(b)=\frac{1}{f'(0)}$

(c) $f=f^{-1}$ on (0, 1) and $f'(b)=\frac{1}{f'(0)}$

(d) f^{-1} is differentiable on (0, 1)

15. If $f(x)=4x^3+3x^2+3x+4$, then $x^3 f\left(\frac{1}{x}\right)$ is equal to

(a) $f(-x)$ (b) $\frac{1}{f(x)}$ **[MP PET]**

(c) $\left[f\left(\frac{1}{x}\right)\right]^2$ (d) $f(x)$

16. If the function $f:[1,\infty)\to[1,\infty)$ is defined by $f(x)=2^{x(x-1)}$, then $f^{-1}(x)$ is equal to **[Kerala CEE]**

(a) $\left(\frac{1}{2}\right)^{x(x-1)}$ (b) $\frac{1}{2}(1-\sqrt{1+4\log_2 x})$

(c) $\frac{1}{2}\sqrt{1+4\log_2 x}$ (d) $\frac{1}{2}(1+\sqrt{1+4\log_2 x})$

(e) not defined

17. If $f=\{(0,-1),(-1,-3),(2,3),(3,5)\}$ is a function from z to z defined by $f(x)=ax+b$. Then, **[AMU]**

(a) $a=1, b=-2$ (b) $a=2, b=1$

(c) $a=2, b=-1$ (d) $a=1, b=2$

18. If $f(x)$ is a polynomial function of the second degree such that, $f(-3)=6$, $f(0)=6$ and $f(2)=11$, then the graph of the function, $f(x)$ cuts the ordinate $x=1$ at the point

(a) (1, 8) (b) (1, 4) **[GGSIPU]**

(c) (1, −2) (d) None of these

19. If $g(x)$ is the inverse of $f(x)$ and $f'(x)=\cos x$, then $g'(x)$ is equal to **[J&K CET]**

(a) $\sec x$ (b) $\sec[g(x)]$

(c) $\cos[g(x)]$ (d) $-\sin[g(x)]$

2010

20. If $f(x)=\frac{\alpha x^2}{x+1}, x\neq -1$, then the value of α for which $f(a)=a$, where $a\neq 0$, is **[BITSAT]**

(a) $1-\frac{1}{a}$ (b) $\frac{1}{a}$ (c) $1+\frac{1}{a}$ (d) $\frac{1}{a}-1$

21. If $f(x)=x^2-1$ and $g(x)=(x+1)^2$, then $(gof)(x)$ is equal to

(a) $(x+1)^4-1$ (b) x^4-1 **[Kerala CEE]**

(c) x^4 (d) $(x+1)^4$

(e) $(x-1)^4-1$

22. If f is a real valued function such that $f(x+y)=f(x)+f(y)$ and $f(1)=5$, then the value of $f(100)$ is **[Kerala CEE]**

(a) 200 (b) 300 (c) 350

(d) 400 (e) 500

23. If R is the set of real numbers and the mappings $f: R \to R$, $g: R \to R$ are defined by $f(x) = 5 - x^2$ and $g(x) = 3x - 4$, then the value of $(fog)(-1)$ is **[WB JEE]**

(a) -44 (b) -54 (c) -32 (d) -64

2009

24. If $f: [4, \infty[\to [4, \infty[$ is defined by $f(x) = 5^{x(x-4)}$, then $f^{-1}(x)$ is equal to **[BITSAT]**

(a) $2 - \sqrt{4 + \log_5 x}$ (b) $2 + \sqrt{4 + \log_5 x}$

(c) $\left(\frac{1}{5}\right)^{x(x-4)}$ (d) not defined

25. If D_{30} is the set of the divisors of 30, $x, y \in D_{30}$, we define $x + y = \text{LCM}(x, y)$, $x \cdot y = \text{GCD}(x, y)$, $x' = \frac{30}{x}$ and $f(x, y, z) = (x + y) \cdot (y' + z)$, then $f(2, 5, 15)$ is equal to

(a) 2 (b) 5 **[MHT CET]**

(c) 10 (d) 15

26. If the function $f: N \to N$ is defined by $f(x) = \sqrt{x}$, then $\frac{f(25)}{f(16) + f(1)}$ is equal to **[MP PET]**

(a) $\frac{5}{6}$ (b) $\frac{5}{7}$ (c) $\frac{5}{3}$ (d) 1

27. If a function f satisfies $f\{f(x)\} = x + 1$ for all real values of x and $f(0) = \frac{1}{2}$, then $f(1)$ is equal to **[Kerala CEE]**

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$

(d) 2 (e) 0

28. If $f: R \to R$ and $g: R \to R$ are defined by $f(x) = x - 3$ and $g(x) = x^2 + 1$, then the values of x for which $g\{f(x)\} = 10$ are **[Kerala CEE]**

(a) $0, -6$ (b) $2, -2$ (c) $1, -1$

(d) $0, 6$ (e) $0, 2$

29. If $f(x)$ satisfies the relation $2f(x) + f(1-x) = x^2$ for all real x, then $f(x)$ is **[Kerala CEE]**

(a) $\frac{x^2 + 2x - 1}{6}$ (b) $\frac{x^2 + 2x - 1}{3}$

(c) $\frac{x^2 + 4x - 1}{3}$ (d) $\frac{x^2 - 3x + 1}{6}$

(e) $\frac{x^2 + 3x - 1}{3}$

30. If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 1$, then $gof(x)$ is equal to **[UPSEE, Jamia Millia Islamia]**

(a) 1 (b) -1 (c) 2 (d) -2

31. If $f(x) = \frac{4^x}{4^x + 2}$, then $f\left(\frac{1}{97}\right) + f\left(\frac{2}{97}\right) + \ldots + f\left(\frac{96}{97}\right)$ is equal to **[UPSEE]**

(a) 1 (b) 48 (c) -48 (d) -1

32. If $f(x + 2y, x - 2y) = xy$, then $f(x, y)$ is equal to **[Jamia Millia Islamia]**

(a) $\frac{x^2 - y^2}{8}$ (b) $\frac{x^2 - y^2}{4}$ (c) $\frac{x^2 + y^2}{4}$ (d) $\frac{x^2 - y^2}{2}$

33. If $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1, \forall x \in R - \{0\}$, then $f(x^4)$ is equal to **[AMU]**

(a) $\frac{(1 - x^4)(2x^4 + 3)}{5x^4}$ (b) $\frac{(1 + x^4)(2x^4 - 3)}{5x^4}$

(c) $\frac{(1 - x^4)(2x^4 - 3)}{5x^4}$ (d) None of these

34. If a and b are two integers such that $10a + b = 5$ and $P(x) = x + ax + b$. Then, integer n such that $P(10) \cdot P(11) = P(n)$ is **[AMU]**

(a) 15 (b) 65 (c) 115 (d) 165

35. If $f(x) = 2x^4 - 13x^2 + ax + b$ is divisible by $x^2 - 3x + 2$, then (a, b) is equal to **[EAMCET]**

(a) $(-9, -2)$ (b) $(6, 4)$ (c) $(9, 2)$ (d) $(2, 9)$

36. If $f: R \to R$ is defined as $f(x) = (1 - x)^{1/3}$, then $f^{-1}(x)$ is equal to **[RPET]**

(a) $(1 - x)^{-1/3}$ (b) $(1 - x)^3$

(c) $1 - x^3$ (d) $1 - x^{1/3}$

2008

37. If $f: N \to Y$ is a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$. Then, inverse of f is **[AIEEE]**

(a) $g(y) = \frac{y - 3}{4}$ (b) $g(y) = \frac{3y + 4}{3}$

(c) $g(y) = 4 + \frac{y + 3}{4}$ (d) $g(y) = \frac{y + 3}{4}$

38. If $f(x) = e^x$ and $g(x) = \log_e x$, then which of the following is correct? **[MP PET]**

(a) $f\{g(x)\} \neq g\{f(x)\}$

(b) $f\{g(x)\} = g\{f(x)\}$

(c) $f\{g(x)\} + g\{f(x)\} = 0$

(d) $f\{g(x)\} - g\{f(x)\} = 1$

39. If $f: R \to R$ is defined by $f(x) = x^3$, then $f^{-1}(8)$ is equal to **[KCET]**

(a) $\{2\}$ (b) $\{2, 2\omega, 2\omega^2\}$

(c) $\{2, -2\}$ (d) $\{2, 2\}$

40. If $f(x)=\log\left(\frac{1+x}{1-x}\right)$, where $-1<x<1$, then

$f\left(\frac{3x+x^3}{1+3x^2}\right)-f\left(\frac{2x}{1+x^2}\right)$ is equal to **[Kerala CEE]**

(a) $[f(x)]^3$ (b) $[f(x)]^2$ (c) $-f(x)$

(d) $f(x)$ (e) $3f(x)$

41. The function $f(x)=\log\left(\frac{1+x}{1-x}\right)$ satisfies the equation

(a) $f(x+2)-2f(x+1)+f(x)=0$ **[WB JEE]**

(b) $f(x)+f(x+1)=f\{x(x+1)\}$

(c) $f(x)+f(y)=f\left(\frac{x+y}{1+xy}\right)$

(d) $f(x+y)=f(x)f(y)$

42. The function $f(x)$ which satisfies

$f(x)=f(-x)=\frac{f'(x)}{x}$, is given by **[WB JEE]**

(a) $f(x)=\frac{1}{2}e^{x^2}$ (b) $f(x)=\frac{1}{2}e^{-x^2}$

(c) $f(x)=x^2e^{x^2/2}$ (d) $f(x)=e^{x^2/2}$

43. If $f(x)=[x-2]$, where $[x]$ denotes the greatest integer less than or equal to x, then $f(2.5)$ is equal to **[OJEE]**

(a) $\frac{1}{2}$ (b) 0

(c) 1 (d) does not exist

44. If $f:[-6,6]\to R$ is defined by $f(x)=x^2-3$ for $x\in R$, then $(fofof)(-1)+(fofof)(0)+(fofof)(1)$ is equal to

(a) $f(4\sqrt{2})$ (b) $f(3\sqrt{2})$ **[EAMCET]**

(c) $f(2\sqrt{2})$ (d) $f(\sqrt{2})$

2007

45. If $f(x)=\frac{ax+b}{cx+d}$, then $fof(x)=x$ provided that

(a) $d=-a$ (b) $d=a$ **[MHT CET]**

(c) $a=b=c=d=1$ (d) $a=b=1$

46. If the graph of the function of $y=f(x)$ is symmetrical about the line $x=2$, then **[MP PET]**

(a) $f(x+2)=f(x-2)$ (b) $f(2+x)=f(2-x)$

(c) $f(x)=f(-x)$ (d) $f(x)=-f(-x)$

47. If $f:R\to R$ is defined by $f(x)=|x|$, then **[KCET]**

(a) $f^{-1}(x)=-x$

(b) $f^{-1}(x)=\frac{1}{|x|}$

(c) $f^{-1}(x)$ does not exist

(d) $f^{-1}(x)=\frac{1}{x}$

48. Let $[x]$ denotes the greatest integer $\le x$. If $f(x)=[x]$ and $g(x)=|x|$, then the value of $f\left(g\left(\frac{8}{5}\right)\right)-g\left(f\left(-\frac{8}{5}\right)\right)$ is

(a) 2 (b) –2 (c) 1 **[Kerala CEE]**

(d) 0 (e) –1

49. The inverse of the function $y=\frac{10^x-10^{-x}}{10^x+10^{-x}}$ is **[OJEE, WB JEE]**

(a) $\frac{1}{2}\log_{10}\left(\frac{1+x}{1-x}\right)$ (b) $\frac{1}{2}\log_{10}\left(\frac{2+x}{2-x}\right)$

(c) $\frac{1}{2}\log_{10}\left(\frac{1-x}{1+x}\right)$ (d) None of these

50. If the functions f, g and h are defined from the set of real numbers R to R such that $f(x)=x^2-1, g(x)=\sqrt{(x^2+1)}$ and $h(x)=\begin{cases}0, & \text{if } x<0\\ x, & \text{if } x\ge 0\end{cases}$, then $ho(fog)(x)$ is defined by

(a) x (b) x^2 **[UPSEE]**

(c) 0 (d) None of these

51. If $f(x)\cdot f(1/x)=f(x)+f(1/x)$ and $f(4)=65$, then $f(6)$ is equal to **[OJEE]**

(a) 65 (b) 217 (c) 215 (d) 64

52. If $f(x)=ax+b, g(x)=cx+d$, then $f\{g(x)\}=g\{f(x)\}$ is equivalent to **[AMU]**

(a) $f(a)=g(c)$

(b) $f(b)=g(b)$

(c) $f(d)=g(b)$

(d) $f(c)=g(a)$

53. If Q denotes the set of all rational numbers and $f\left(\frac{p}{q}\right)=\sqrt{p^2-q^2}$ for any $\frac{p}{q}\in Q$, then observe the following statements.

I. $f\left(\frac{p}{q}\right)$ is real for each $\frac{p}{q}\in Q$.

II. $f\left(\frac{p}{q}\right)$ is a complex number for each $\frac{p}{q}\in Q$.

Which of the following is correct? **[EAMCET]**

(a) Both I and II are correct

(b) I is correct, II is incorrect

(c) I is incorrect, II is correct

(d) Both I and II are incorrect

54. If $f:R\to R$ and $g:R\to R$ are defined by $f(x)=x-[x]$ and $g(x)=[x]$ for $x\in R$, where $[x]$ is the greatest integer not exceeding x, then for every $x\in R$, $f(g(x))$ is equal to

(a) x (b) 0 **[EAMCET]**

(c) $f(x)$ (d) $g(x)$

55. If $f: R \to R$ is given by

$$f(x) = \begin{cases} -1, & \text{when } x \text{ is rational.} \\ 1, & \text{when } x \text{ is irrational.} \end{cases}$$

Then, $(fof)(1-\sqrt{3})$ is equal to **[J&K CET]**

(a) 1 (b) –1 (c) $\sqrt{3}$ (d) 0

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56. If $g(x) = 1 + x - [x]$ and $f(x) = \begin{cases} -1, x < 0 \\ 0, \ x = 0, \\ 1, \ x > 0 \end{cases}$ then for all x, $f(g(x))$ is equal to **[BITSAT]**

(a) x (b) 1 (c) $f(x)$ (d) $g(x)$

57. If $f(0) = 1$, $f(1) = 5$ and $f(2) = 11$, then the equation of polynomial of degree two is **[MHT CET]**

(a) $x^2 + 1 = 0$ (b) $x^2 + 3x + 1 = 0$

(c) $x^2 - 2x + 1 = 0$ (d) None of these

58. If $f(x) = (a - x^n)^{1/n}$, where $a > 0$ and $n \in N$, then $fof(x)$ is equal to **[MP PET]**

(a) a (b) x (c) x^n (d) a^n

59. If $[x]$ denotes the greatest integer $\le x$, then $\left[\frac{2}{3}\right] + \left[\frac{2}{3} + \frac{1}{99}\right] + \left[\frac{2}{3} + \frac{2}{99}\right] + \cdots + \left[\frac{2}{3} + \frac{98}{99}\right]$ is equal to

(a) 99 (b) 98 (c) 66 **[Kerala CEE]**

(d) 65 (e) 33

60. If $f(x) = \cos(\log x)$, then

$f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$ has the value **[WB JEE, Jamia Millia Islamia]**

(a) –1 (b) 1/2 (c) –2 (d) 0

61. If $f(x) = \frac{2x-1}{x+5}$, $x \neq -5$, then $f^{-1}(x)$ is equal to

(a) $\frac{x+5}{2x-1}, x \neq \frac{1}{2}$ (b) $\frac{5x+1}{2-x}, x \neq 2$ **[UPSEE]**

(c) $\frac{x-5}{2x+1}, x \neq \frac{1}{2}$ (d) $\frac{5x-1}{2-x}, x \neq 2$

62. If $f(x) = \frac{x}{x-1}, x \neq 1$, then $\underbrace{(fofo\ldots of)}_{19 \text{ times}}(x)$ is equal to

(a) $\frac{x}{x-1}$ (b) $\left(\frac{x}{x-1}\right)^{19}$ **[UPSEE]**

(c) $\frac{19x}{x-1}$ (d) x

63. If $y = f(x) = \frac{x+2}{x-1}$, then **[Jamia Millia Islamia]**

(a) $x = f(y)$

(b) $f(1) = 3$

(c) y increases with x for $x < 1$

(d) f is a rational function of x

64. If $f: R \to R$ is defined by

$$f(x) = \begin{cases} x + 4 & \text{for} \quad x < -4 \\ 3x + 2 & \text{for} \ -4 \le x < 4, \\ x - 4 & \text{for} \quad x \ge 4 \end{cases}$$

then the correct matching of List I from List II is

List I	List II
A. $f(-5) + f(-4)$	1. 14
B. $f(\|f(-8)\|)$	2. 4
C. $f[f(-7) + f(3)]$	3. – 11
D. $f[f\{f(f(0))\}] + 1$	4. – 1
	5. 1
	6. 0

[EAMCET]

	A	B	C	D		A	B	C	D
(a)	3	6	2	5	(b)	3	4	2	5
(c)	4	3	2	1	(d)	3	6	5	2

65. Which of the following functions is inverse of itself ?

(a) $f(x) = \frac{1-x}{1+x}$ (b) $f(x) = 3^{\log x}$ **[BCECE]**

(c) $f(x) = 3^{x(x+1)}$ (d) None of these

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66. The second degree polynomial $f(x)$, satisfying $f(0) = 0$, $f(1) = 1$, $f'(x) > 0, \forall\ x \in (0, 1)$ is **[IIT JEE]**

(a) $f(x) = \phi$

(b) $f(x) = ax + (1-a)x^2, \forall\ a \in (0, \infty)$

(c) $f(x) = ax + (1-a)x^2, a \in (0, 2)$

(d) no such polynomials

67. If X and Y are two non-empty sets, where $f: X \to Y$ is function is defined such that

$$f(C) = \{f(x) : x \in C\} \text{ for } C \subseteq X$$

and $f^{-1}(D) = \{x : f(x) \in D\}$ for $D \subseteq Y$,

for any $A \subseteq X$ and $B \subseteq Y$, then **[IIT JEE]**

(a) $f^{-1}\{f(A)\} = A$ only if $A \subset X$

(b) $f^{-1}\{f(A)\} = A$ only if $f(X) = Y$

(c) $f\{f^{-1}(B)\} = B$ only if $B \subseteq f(X)$

(d) $f\{f^{-1}(B)\} = B$

68. The function f satisfies the functional equation $3f(x)+2f\left(\frac{x+59}{x-1}\right)=10x+30$ for all real $x\neq 1$. The value of $f(7)$ is **[Kerala CEE]**

(a) 8 (b) 4 (c) – 8
(d) 11 (e) 44

69. If $f(x)=\frac{\alpha x}{x+1}, x\neq -1$, for what value of α is $f(f(x))=x$? **[Kerala CEE]**

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 1
(d) 2 (e) –1

70. If $f:(2,3)\to(0,1)$ is defined by $f(x)=x-[x]$, then $f^{-1}(x)$ is equal to **[OJEE]**

(a) $x-2$ (b) $x+1$ (c) $x-1$ (d) $x+2$

71. If $f(x+y, x-y)=xy$, then the arithmetic mean of $f(x, y)$ and $f(y, x)$ is **[AMU]**

(a) x (b) y
(c) 0 (d) None of these

72. The function $f:C\to C$ defined by $f(x)=\frac{ax+b}{cx+d}$ for $x\in C$, where $bd\neq 0$ reduces to a constant function, if **[EAMCET]**

(a) $a=c$ (b) $b=d$
(c) $ad=bc$ (d) $ab=cd$

73. The values of b and c for which the identity $f(x+1)-f(x)=8x+3$ is satisfied, where $f(x)=bx^2+cx+d$, are **[BCECE]**

(a) $b=2, c=1$
(b) $b=4, c=-1$
(c) $b=-1, c=4$
(d) $b=-1, c=1$

74. If $f(2x+3)=\sin x+2^x$, then $f(4m-2n+3)$ is equal to **[J&K CET]**

(a) $\sin(m-2n)+2^{2m-n}$
(b) $\sin(2m-n)+2^{(m-n)2}$
(c) $\sin(m-2n)+2^{(m+n)2}$
(d) $\sin(2m-n)+2^{2m-n}$

Types of Sets, Operations and Cartesian Products

1. (d)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (c)
11. (b)	**12.** (c)	**13.** (d)	**14.** (a)	**15.** (d)	**16.** (c)	**17.** (d)	**18.** (e)	**19.** (b)	**20.** (a)
21. (b)	**22.** (c)	**23.** (b)	**24.** (c)	**25.** (a)	**26.** (c)	**27.** (c)	**28.** (c)	**29.** (a)	**30.** (d)
31. (b)	**32.** (a)	**33.** (e)	**34.** (b)	**35.** (c)	**36.** (c)	**37.** (c)	**38.** (e)	**39.** (d)	**40.** (c)
41. (e)	**42.** (c)	**43.** (a)	**44.** (a)	**45.** (c)					

Relation and Equivalence Relation

1. (c)	**2.** (c)	**3.** (d)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (a)	**8.** (c)	**9.** (d)	**10.** (b)
11. (b)	**12.** (a)	**13.** (a)	**14.** (d)	**15.** (d)	**16.** (c)	**17.** (a)	**18.** (b)	**19.** (c)	**20.** (d)
21. (c)	**22.** (b)	**23.** (a)	**24.** (c)	**25.** (a)	**26.** (b)				

Types of Mapping

1. (a)	**2.** (e)	**3.** (d)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (c)
11. (d)	**12.** (d)	**13.** (c)	**14.** (d)	**15.** (d)	**16.** (c)	**17.** (b)	**18.** (d)	**19.** (c)	**20.** (d)
21. (b)	**22.** (c)	**23.** (d)	**24.** (d)	**25.** (a)					

Domain-Range, Odd-Even and Periodic Functions

1. (d)	**2.** (b)	**3.** (c)	**4.** (c,d)	**5.** (d)	**6.** (d)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (b)
11. (a)	**12.** (b)	**13.** (d)	**14.** (c)	**15.** (e)	**16.** (d)	**17.** (c)	**18.** (b)	**19.** (d)	**20.** (a)
21. (b)	**22.** (e)	**23.** (b)	**24.** (c)	**25.** (c)	**26.** (b)	**27.** (b)	**28.** (a)	**29.** (b)	**30.** (c)
31. (d)	**32.** (a)	**33.** (c)	**34.** (c)	**35.** (a)	**36.** (b)	**37.** (c)	**38.** (a)	**39.** (c)	**40.** (b)
41. (d)	**42.** (b)	**43.** (b)	**44.** (d)	**45.** (a)	**46.** (e)	**47.** (c)	**48.** (a)	**49.** (b)	**50.** (d)
51. (d)	**52.** (b)	**53.** (c)	**54.** (b)	**55.** (b)	**56.** (c)	**57.** (a)	**58.** (c)	**59.** (d)	**60.** (b)
61. (b)	**62.** (a)	**63.** (c)	**64.** (b)	**65.** (c)	**66.** (a)	**67.** (b)	**68.** (b)		

Inverse, Composition and Different Types of Functions

1. (d)	**2.** (d)	**3.** (e)	**4.** (a)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (a, b)	**10.** (d)
11. (a)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (d)	**16.** (d)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (c)
21. (c)	**22.** (e)	**23.** (a)	**24.** (d)	**25.** (c)	**26.** (d)	**27.** (c)	**28.** (d)	**29.** (b)	**30.** (a)
31. (b)	**32.** (a)	**33.** (a)	**34.** (a)	**35.** (c)	**36.** (c)	**37.** (a)	**38.** (b)	**39.** (a)	**40.** (d)
41. (c)	**42.** (d)	**43.** (b)	**44.** (a)	**45.** (a)	**46.** (b)	**47.** (c)	**48.** (e)	**49.** (a)	**50.** (b)
51. (b)	**52.** (c)	**53.** (d)	**54.** (b)	**55.** (b)	**56.** (b)	**57.** (b)	**58.** (b)	**59.** (c)	**60.** (d)
61. (b)	**62.** (a)	**63.** (a)	**64.** (a)	**65.** (a)	**66.** (c)	**67.** (a)	**68.** (b)	**69.** (e)	**70.** (d)
71. (c)	**72.** (c)	**73.** (b)	**74.** (d)						