NUMBER SYSTEM

In Hindu Arabic System, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called digits to represent any number. This is the decimal system where we use the numbers 0 to 9. 0 is called insignificant digit whereas 1, 2, 3, 4, 5, 6, 7, 8, 9 are called significant digits.

A group of figures, denoting a number is called a numeral. For a given numeral, we start from extreme right as Unit's place, Ten's place, Hundred's place and so on.

We represent the number 50,78,69,324 as shown below

We read it as Fifty crores, seventy -eight lacs, sixty nine thousands ,three hundred and twenty four.

In this numeral:

The Place value of 4 is 4 × 1 = 4

The place value of 2 is 2 × 10 = 20

The place value of 3 is 3 × 100 = 300

The place value of 9 is 9 × 1000 = 9000 and so on.

The place value of 9 is 6 × 10000 = 60000 and so on

The face value of a digit in a number is the value itself wherever it may be.

Thus, the face value of 7 in the above numeral is 7. The face value of 6 in the above numeral is 6 and so on.

NATURAL NUMBERS

Counting numbers 1, 2, 3, 4, 5, ... are known as natural numbers.

The set of all natural numbers can be represented by N = {1, 2, 3, 4, 5, ...}.

WHOLE NUMBERS

If we include 0 among the natural numbers, then the numbers 0, 1, 2, 3, 4, 5, ... are called whole numbers.

The set of whole numbers can be represented by W = {0,1,2,3,4,5, ...}

Clearly, every natural number is a whole number but 0 is a whole number which is not a natural number.

INTEGERS

All counting numbers and their negatives including zero are known as integers.

The set of integers can be represented by

Z or I = {... ・ 4, –3, –2, –1, 0, 1, 2, 3, 4 ......}

POSITIVE INTEGERS

The set I^+ = {1, 2, 3, 4, ...} is the set of all positive integers. Clearly, positive integers and natural numbers are synonyms.

NEGATIVE INTEGERS

The set { –1, –2, – 3, ...} is the set of all negative integers. 0 is neither positive nor negative.

NON-NEGATIVE INTEGERS

The set {0, 1, 2, 3, ... } is the set of all non-negative integers.

RATIONAL NUMBERS

The numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are known as rational numbers.

The set of all rational numbers is denoted by Q.

i.e. $Q = \left\{ x : x = \frac{p}{q}, p, q \in I, q \neq 0 \right\}$ Since every natural number 'n' can be written as $\frac{n}{1}$, every natural number is a rational number. Since 0 can be written as $\frac{0}{1}$ and every non-zero integer 'n' can be written as $\frac{n}{1}$, ie. every integer is a rational number.

Every rational number has a peculiar characteristic that when expressed in decimal form is expressible either in terminating decimal expansion or in non-terminating repeating decimals.

IRRATIONAL NUMBERS

Those numbers which when expressed in decimal form are neither terminating nor repeating decimals are known as irrational numbers, e.g. Π, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{2}$, $\sqrt{8}$ etc.

Note that the exact value of Π is not22/7, 22/7 is rational but Π is irrational.

22/7 is approximate value of Π . Similarly, 3.14 is not an exact value of it.

REAL NUMBERS

The rational and irrational numbers combined together are called real numbers, e.g13/6,1/2, $\sqrt{5}$,4/5, etc. are real numbers. The set of all real numbers is denoted by R.

EVEN NUMBERS

All those numbers which are exactly divisible by 2 are called even numbers, e.g. 2, 6, 8,10, etc., are even numbers.

ODD NUMBERS

All those numbers which are not exactly divisible by 2 are called odd numbers, e.g. 1, 3, 5, 7 etc., are odd numbers.

PRIME NUMBERS

A natural number other than 1, is a prime number if it is divisible by 1 and itself only.

For example, each of the numbers 2, 3, 5, 7 etc., are prime numbers.

COMPOSITE NUMBERS

Natural numbers greater than 1 which are not prime are known as composite numbers.

For example, each of the numbers 4, 6, 8, 9, 12, etc., are composite numbers.

Note:

1. The number 1 is neither a prime number nor a composite number.

2. 2 is the only even number which is prime.

3. Prime numbers up to 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, i.e. 25 prime numbers between 1 and 100.

4. Two numbers which have only 1 as the common factor are called co-primes or relatively prime to each other, e.g. 3 and 5 are co-primes.

Note that the numbers which are relatively prime need not necessarily be prime numbers, e.g. 16 and 17 are relatively prime although 16 is not a prime number.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, i.e. 25 prime numbers between 1 and 100.

4. Two numbers which have only 1 as the common factor are called co-primes or relatively prime to each other, e.g. 3 and 5 are co-primes.

Note that the numbers which are relatively prime need not necessarily be prime numbers, e.g. 16 and 17 are relatively prime although 16 is not a prime number.

TESTS OF DIVISIBILITY

Divisibility by 2: A number is divisible by 2 if the unit’s digit is zero or divisible by 2.

For example, 24,16,108, etc., are all divisible by 2.

Divisibility by 3: A number is divisible by 3 if the sum of digits in the number is divisible by 3.

For example, the number 6543 is divisible by 3 since 6 + 5 + 4 + 3 = 18, which is divisible by 3.

Divisibility by 4: A number is divisible by 4 if the number formed by the last two digits (ten’s digit and unit’s digit) is divisible by 4 or are both zero.

For example, the number 2936 is divisible by 4 since 36 is divisible by 4.

Divisibility by 5: A number is divisible by 5 if the unit' s digit in the number is 0 or 5.

For example, 14820, 9605, 850, 935, etc., are all divisible by 5.

Divisibility by 6: A number is divisible by 6 if the number is even and sum of its digits is divisible by 3.

For example, the number 6324 is divisible by 6 since it is even and sum of its digits 6 + 3 + 2+ 4 = 15is divisible by 3.

Divisibility by 7: The unit digit of the given number is doubled and then it is subtracted from the number obtained after omitting the unit digit. If the remainder is divisible by 7, then the given number is also divisible by 7.

For example, consider the number 518. On doubling the unit digit 8 of 448 we get 16.

Then, 51 — 16 = 35. Since 35 is divisible by 7, 518 is divisible by 7.

Divisibility by 8: A number is divisible by 8, if the number formed by last 3 digits in divisible by 8.

For example, the number 65784 is divisible by 8 as the number formed by last three digits, i.e. 784 is divisible by 8.

Divisibility by 9: A number is divisible by 9 if the sum of its digits is divisible by 9. For example, the number 25785is divisible by 9 as the sum of its digits 2 + 5 + 7 + 8 + 5 = 27 is divisible by 9.

Divisibility by 10: A number is divisible by 10, if it ends in zero.

For example, the last digit of 630 is zero, therefore, 480 is divisible by 10.

Divisibility by 11: A number is divisible by 11, if the difference of the sum of the digits at odd places and sum of the digits at even places is either zero or divisible by 11.

For example, in the number 51623, the sum of the digits at odd places is 5 + 6 + 3 = 14 and the sum of the digits at even places is 1+2 = 3. The difference is 14— 3 = 11, so the number is divisible by 11.

Divisibility by 12: A number is divisible by 12 if it is divisible by 3 and 4.

Divisibility by 18: An even number satisfying the divisibility test of 9 is divisible by 18.

Divisibility by 25: A number is divisible by 25 if the number formed by the last two digits is divisible by 25 or the last two digits are zero.

For example, the number 83675 is divisible by 25 as the number formed by the last two digits is 75 which is divisible by 25.

Divisibility by 125: A number is divisible by 125 if the number formed by the last three digits is divisible by 125 or the last three digits are zero.

For example, the number 5250 is divisible by 125 as 250 is divisible by 125.

Divisibility by 88: A number is divisible by 88 if it is divisible by 11 and 8.

SOME IMPORTANT PINCIPLES IN NUMBER SYSTEM

1)**Induction Principle:** Let {T(n): n$\in N$} be the set of statements , for each natural number n. If (i) T(a) is true for some a ЄN and(ii) T(k) is true implies T(k+1) is true for all k ≥ a , then T(m) is true for all n ≥ a

2) **The greatest integer function:** [ ] is defined by [x] = the greatest intger not exceeding x, for every real x

3) **Linearity property :** If a/b and a/c then a/ pb+qc

4) **Euclid's Algorithm:** The a and b be two non-zero integers. Then (a,b) [gcd of a and b] existes and is unique. Also , there existes integers m and n such that (a,b)=am+bn

5) **Congruencies:** Let a and b be inegers, m>0. Then we say that a is congruent to b modulo m if, m/(a-b) .We denote this by $a \equiv b(\mathrm{mod}\, m)$.

Let $a \equiv b(\mathrm{mod}\, m)$ and $c \equiv d(\mathrm{mod}\, m)$ then

i) $a + c \equiv (b + d)(\mathrm{mod}\, m)$

ii) $a - c \equiv (b - d)(\mathrm{mod}\, m)$

iii) $ac \equiv bd \pmod{m}$

iv) $pa + qc \equiv (pb + qd) \pmod{m}$ for all integers pand q .

v) $a^n \equiv b^n \pmod{m}$ for all +ve integers m .

vi) $f(a) \equiv f(b) \pmod{m}$ for every polynomial with integer co-efficients.

6) Let N be a +ve integer greaer than 1 , say N=$a^p b^q c^r$…. where a,b,c, .. are distinct primes and p, q, r, .. are +ve integers . The no. of ways in which N can be resolved into two factores is $\frac{1}{2}(p+1)(q+1)(r+1)....$

7)Number of ways in which a composite number can be resolved into factors , which are prime to each other , is 2^{n-1} , where n is the no. of distinct prime factors in the expression for N

8) Let N be +ve integer greater than 1 and let N=$a^p b^q c^r$…. where a,b,c, .. are distinct primes and p, q, r, .. are +ve integers. Then the sum of all the divisors in the product is equal to

$$\frac{a^{p+1}-1}{a-1} \cdot \frac{b^{q+1}-1}{b-1} \cdot \frac{c^{r+1}-1}{c-1} \ldots\ldots\ldots\ldots$$

9) The highest power of prime p which is contained in n! is equal

to $\left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots..$ Where [] is the greatest integer function .

10) **Euler's Totient Function :** Let N be +ve integer greater than 1 . Then the no. of all the +ve intgers less than N and prime to it is denoted by $\phi(N)$. It is obvious $\phi(2)=1$,

$\phi(3)=2$, $\phi(4)=2$, $\phi(5)=4$, $\phi(6)=2$ …. The fuction ϕ is called Euler's Totient Function .

11) If a, b, … are prime to each other , then

$\phi(ab)=\phi(a).\phi(b)$ or $\phi(abcd...)=\phi(a).\phi(b)\phi(c)\phi(d)......$

If N=$a^p b^q c^r$…. where a,b,c, .. are distinct primes and p, q, r, .. are +ve integers then

$$\phi(N)=n(1-\frac{1}{a})(1-\frac{1}{b})(1-\frac{1}{c}).....$$

12) **Euler's Theorem :** If be any +ve integer prime to N . Then $x^{\phi(N)}\equiv 1(\bmod N)$

13) **Fermat's Little Theorem :** If p is prime and n is prime to p then $n^{p-1}\equiv 1(\bmod p)$

14) **WILSON's Theorem :** If p is prime , then $(p-1)!\equiv 0(\bmod p)$

MATHS OLYMPIAD SUMS

EXERCISE 1.1(Number System)

1. **Calculate 5^{2039}(mod 41).**
2. **Find the largest +ve integer n such that n^3+100 is divisible by n+10**
3. **Show that 2^{55}+1 is divisible by 11**
4. **Show that 1^{1997}+ 2^{1997}+…+1996^{1997} is divisible by 1997**

5 (a). Find the difference between the largest and the smallest numbers that can be formed with six digits.

(b) The average of nine consecutive natural numbers is 81. Find the largest of these numbers.

(c) What will be 77% of a number whose 55% is 240?

(d) Flowers are dropped in a basket which become double after every minute. The basket became full in 10 minutes. After how many minutes the basket was half full?

6. A number consists of 3 digits whose sum is 7. The digit at the units place is twice the digit at the ten's place. If 297 is added to the number, the digits of the number are reversed. Find the number.

7 (a)When an integer 'n' is divided by 1995.The remainder is 75. What is the remainder when 'n' is divided by 57?

(b)Find the missing digits in the following multiplication sum:

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 3 5 9 7
   * * *
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*** * * * * ***

*** * *****

*** * * * 5 4 1**

8.)Find the largest prime factor of $3^{14}+3^{13}-12$?

9) a)Find the greatest number of four digits which when divided by 2,3,4,5,6,7 leaves a remainder 1 in each case.

b)How many prime numbers between 10 and 99 remain prime when the order of their digits is reversed?

c) Exactly one of the numbers 234,2345,23456,234567,2345678, 23456789 is a prime. Which one must it be?

10) A two-digit number is such that if a decimal point is placed between its two digits, the resulting number is one- quarter of the sum of two digits. What is the original number?

11) Find the greatest number of five digits which is divisible by 56, 72, 84 and 96 leaves remainders 48, 64, 76 and 88 respectively.

12) Which is greater: ? 31^{11} or 17^{14}

13) Show that is exactly divisible by $1^{99}+2^{99}+3^{99}+4^{99}+5^{99}$ is exactly divisible by 5

14) Find the number of perfect cubes between 1 and 1000001 which are exactly divisible by 7 ?

15) How many numbers from 1 to 50 are divisible by neither 5 nor 7, and have neither 5nor 7 as a digit.

16)The square of a number of two digits is four times the number obtained by reversing its digits . Find the number.

17) Find the sum of the digites in $2^{2000} . 5^{2004}$

18) Arrange the following in ascending order: 2^{5555}, 3^{3333}, 6^{2222}.

19) Find all the positive perfect cubes that divide 9^9

20) Find all the integers closese to $100(12-\sqrt{143})$

21) $(123456)^2$ +123456 +123457 is the square of …..

22) How many four digit numbers can be formed using the digits 1, 2 only so that each of these digits is used at least once ?

23) Find the greatest number of four digits which when increased by 1 is exactly divisible by 2,3 ,4,5,6 and 7 ?

24) Find the last two (ten's and unit's) digit of $(2003)^{2003}$

25) Find the number of perfect cubes between 1 and 1000009 which are exactly divisible by 9.

26) Find the number of positive integers less than or equal to 300 that are multiples of 3 or 5, but are not multiples of 10 or 15.

27) The product of the digits of each of the three – digit numbers 138, 262 and 432 is 24. Write down all three digit numbers having 24 as the product of the digits.

28). (a) Find the number of digits in the number 22005 .52000 when written in full.

(b) Find the remainder when 2^{2005} is divided by 13

29) Find two numbers both lying between 60 and 70, each of which divides 2^{48}-1

30) A number when divided by 7,11 and 13(the prime factors of 1001) successively leave the remainders 6,10 and 12 respectively. Find the remainder if the number is divided by 1001.

31) Find the greatest number of four digits which when divided by 3, 5, 7, 9 leaves remainders 1, 3, 5, 7 respectively.

32) A printer numbers the pages of a book starting with 1. He uses 3189 digits in all. How many pages does the book have?

33) Find the largest prime factor of $3^{12} + 2^{12} – 2.6^{6}$

34) Find the value of S= $1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots\ldots - 98^2 + 99^2$

35) Find the smallest multiple of 15 such that each digit of the multiple is either‘0’or ‘8’.

36) A number ‘X’ leaves the same remainder while dividing 5814, 5430, 5958. What is the largest possible value of ‘X’.

37) Consider the following multiplication in decimal notations (999).(abc) = def132, determine the digits a,b,c,d,e,f.

38) If n is a positive integer such that n/810 = 0.d25d25… where d is a single digit in decimal base. Find 'n'.

39) Let x be the LCM of 3^{2002}-1 and 3^{2002}+1. Find the last digit of x.

40) Let $f_0(X)=1/(1-X)$ and $f_n(x) = f_0(f_{n-1}(x))$ Where n = 1,2,3….Calculate $f_{2009}(2009)$

EXERCISE 1.2(GEOMETRY)

1. If a, b,c are measures which form a triangle for all n= 2,3,4 etc, prove that $\sqrt[n]{a},\sqrt[n]{b},\sqrt[n]{c}$ also will form a triangle

2. Given the vertex A , the orthocentre H and the centroid G ,construct the triangle. Justify your construction.

3. A square sheet of paper ABCD is so folded that the point B falls on the mid point M of CD. Prove that the crease will divide BC in the raion 5:3.

4. In ΔABC , the area is $\frac{1}{2}bc$ sq.units (in the ususal notation). AD is a median to BC . Prove that $\angle ABC = \frac{1}{2}\angle ADC$

5. Prove in any ΔABC , if one angle is 120^0 , the angle formed by the feet of the angle bisectors is right angled.

6. In ΔABC , the incircle touches the sides BC , CA, AB respectively at D, E, F respectively . If the radius of the incircle is 4 units and if BD, CE, AF are consecutive integers , find the sides of ΔABC.

7. Through a point P, within a ΔABC, straight lines are drawn from the angular points A,B and C to cut the opposite sides in D, E and F respectively.

Prove the following :

i) $\frac{PD}{AD}+\frac{PE}{BE}+\frac{PF}{CF}=1$

ii) $\frac{AP}{AD}+\frac{BP}{BE}+\frac{CP}{CF}=2$

8. The parallel sides of a trapezoid are 3 cms and 9 cms . The non- parallel sides are 4 cm and 6 cm. A line parallel to the base divides the trapeziod into two trapezoids of equal perimeteres. Find the ratio into which each of the non-parallel sides is divided.

9. You are given three parallel lines. Construct an equlateral traingle ABC such that A will be on line l1 , B will be on line l2 and C will be on third line l3. Justify your construction. (The three parallel lines are not of equal height)

10. O is the circumcentre of ΔABC and M is the middle point of the median through A . Join OM and produce it to N so that OM = MN . Prove that N lies on the altitude through A.

11. Two circles C1 and C2 intersect at two distinct points P and Q in a plane. Let a line passing through P meet the circles C1 and C2 in A and B respectively. Let Y be the middle point of AB , let QY meet the circles C_1 and C_2 in X and Z respectively . Prove that Y is hte mid point of XZ also.

12. AB is a diameter of a circle and P is apoint in its exterior. Using only an unmarked ruler and a pencil , explain how you construct a perpendicular from P to AB. Justify uour construction.

13. The mid point of the hypotenuse through M in such a way that the position of it lying inside the triangle is 3 cms long and outside the triangle upto the other side is 9 cms. Find the length of the hypotenuse.

14. In ΔABC , $\angle A = 2\angle B$**. Prove in usual notation that** $a^2 = b(b+c)$

15. L is a point on the side QR of ΔPQR . LM **and** LN **are drawn parallel to PR**

and QP meeting QP , PR at M, N respectively . MN produced meets QR in T. Then , prove that , LT is the geometric mean between RT and QT.

16. In ΔABC , M is the midpoint of BC. P is any point on AM and PE , PF are perpendiculars to AB , AC respectively. If EF || BC , prove that $\angle A$ is a right angle or $\angle B = \angle C$.

17. Three equal circles of radius r touch each other. Prove that the line through the centres of any two circles meets any one of the circles at the pont which is at a distance of $\sqrt{7}$ r from the centre of the remaining circle.

18. A circle of radius 2 cms with centre O contains three smaller circles. Two of them touch the outer circle and touch each other at O. The third circle touches each of the other three circles . Find the radius of the third circle.

19. If ' u =cot 22^0 30' and v = $\frac{1}{Sin 22°30}$, prove that u satisfies a quadratic and v a quadratic equation with integral coefficients and with leading coefficients as unity . Use a geometrical construction to prove this proposition.

20. The median AD of ΔABCis perpendicular to AB . Prove that tanA + 2 tanB = 0.

21. In ΔABC AB = 5 , BC = 6 and AC = 7 . Points P and Q are located on AB and AC respectively such that PA +AQ equals half the perimeter of ΔABC. The area of ΔAPQ is half of the area of ΔABC. If PB = x , prove that , P satisfies the quadratic equation $2x^2 - 2x - 5 = 0$.

22. The circumference of a unit circle is dividied into eight equal arcs by points A, B, C, D, E, F, G, H. Chords connecting point A to each of the other points are joined. Find the product of the lengths of these seven chords. Generalize your result.

23. O is the orthocentre of ΔABC and K,L,M are the mirror images on the three sides .Show that the traingle KLM has the same circumcentre of ΔABC.

24. The sides of a triangle are of length a, b and c , where a,b,c are integers and a > b. The angle opposite to the side c is 60^0. Prove that 'a ' cannot be a prime number.

25. Construct a regular hexagon using ruler and compass only . Use this construction to draw two circles which will intersect orthogonally.

26. A triangle has sides of lentghts 6, 8,10 . Calculate the distance between its incentre and circumcentre.

27. Let ABCD be a convex quadrilateral in which $\angle$BAC=50^0; $\angle$CAD=60^0 ; $\angle$CBD=30^0 ; and $\angle$BDC= 25^0 . If E is the point of intersection of AC and BD , find $\angle$AEB.

28. In ΔABC let D be the mid-point of BC . If $\angle$ADB=45^0 ; and $\angle$ACD=30^0 , determine $\angle$BAD.

29. Prove that in ΔABC whose sides AB = 4 cms , BC = 3 cms and AC = $\sqrt{5}$ cms, the medians AK and CL are mutually perpendicular.

30. In a non- degenerate triangle ΔABC , $\angle$C = 3 $\angle$A . BC = 27; AB = 48 ; Find AC.

NOTE:
You are Welcome

i) FOR suggestion And corrections my be sent at maddipatla17@yahoo.co.in

ii) You can contribute Question along with solutions for the Question bankand send them to maddipatla17@yahoo.co.in

iii) Solution for **exercise 1.2** will be Available soon