COURSES OF STUDY 2009

DEPARTMENT OF MATHEMATICS

PROGRAMMES: M.Sc. (Mathematics)

COURSE CURRICULA
M.Sc. (Mathematics)

First Year

I Semester

Course No.	Course Name	L	T	P	C
CS 101	Computer Programming & Utilization	2	0	2	6
MA 401	Linear Algebra	3	1	0	8
MA 403	Real Analysis	3	1	0	8
MA 417	Ordinary Differential Equations	3	1	0	8
MA 419	Basic Algebra	3	1	0	8
	Total	**14**	**4**	**2**	**38**

II Semester

Course No.	Course Name	L	T	P	C
MA 406	General Topology	3	1	0	8
MA 408	Measure Theory	3	1	0	8
MA 410	Multivariable Calculus	2	1	0	6
MA 412	Complex Analysis	3	1	0	8
MA 414	Algebra I	3	1	0	8
	Total	**14**	**5**	**0**	**38**

Second Year

III Semester

Course No.	Course Name	L	T	P	C
MA 503	Functional Analysis	3	1	0	8
MA 515	Partial Differential Equations	3	1	0	8
	Elective I	3	1	0	8
	Elective II	2	1	0	6
	Elective III	2	1	0	6
MA 593	Optional Project Stage I	Pass / No Pass			
	Total	**13**	**5**	**0**	**36**

IV Semester

Course No.	Course Name	L	T	P	C
ES 200	Environmental Science	3	0	0	3
HS 200	Environmental Science	3	0	0	3
MA522	Fourier Analysis and Applications	3	1	0	8
	Elective IV	3	1	0	8
	Elective V	2	1	0	6
	Elective VI	2	1	0	6
MA598	Optional Project Stage II	Pass / No Pass			
	Total	**13**	**4**	**0**	**34**
	Grand Total	**54**	**18**	**2**	**146**

Elective I
MA 505 Algebra - II
SI 507 Numerical Analysis

Elective II and III
SI 419 Combinatorics
MA 521 Theory of Analytic Functions
MA 523 Basic Number Theory
MA 525 Dynamical Systems
MA 533 Advanced Probability Theory
MA 539 Spline Theory and Variational Methods
MA 581 Elements of Differential Topology

Elective IV
MA 508 Mathematical Methods
MA 516 Algebraic Topology

Elective V and VI
MA 504 Operators on Hilbert Spaces
MA 510 Introduction to Algebraic Geometry
MA 518 Spectral Approximation
MA 524 Algebraic Number Theory
MA 526 Commutative Algebra
MA 530 Nonlinear Analysis
MA 532 Analytic Number Theory
MA 534 Modern Theory of Partial Differential Equations
MA 538 Representation Theory of Finite Groups
MA 540 Numerical Methods for Partial Differential Equations
MA 556 Differential Geometry
MA 562 Mathematical Theory of Finite Elements
SI 416 Optimization
SI 527 Introduction to Derivative Pricing

COURSE CONTENTS

CS 101 Computer Programming & Utilization 2 0 2 6

Functional organization of computers, algorithms, basic programming concepts, FORTRAN language programming. Program testing and debugging, Modular programming subroutines: Selected examples from Numerical Analysis, Game playing, sorting/ searching methods, etc.

Texts / References

N.N. Biswas, FORTRAN IV Computer Programming, Radiant Books, 1979.

K.D. Sharma, Programming in Fortran IV, Affiliated East West, New Delhi, 1976.

ES 200 Environmental Studies 3 0 0 3

Multidisciplinary nature of environmental problems; Ecosystems, Biodiversity and its conservation; Indicators of environmental pollution; Environment and human health; Utilization of natural resources and environmental degradation. Sustainable development; Environmental policy and law; Environmental impact assessment; Pollution of lakes, rivers and groundwater. Principles of water and wastewater treatment; Solid and hazardous waste management. Air Pollution: sources and effects, Atmospheric transport of pollutants; Noise pollution; Global issues and climate change: Global warming, Acid rain, Ozone layer depletion.

Texts / References

Cunningham W.P. and Cunningham M.A., Principles of Environmental Science, Tata McGraw-Hill Publishing Company, New Delhi, 2002.

Nathanson, J.A., Basic Environmental Technology: Water Supply Waste Management and Pollution Control, 4th Ed. Prentice Hall of India, New Delhi, 2002.

Masters, G.M., Introduction to Environmental Engineering and Science, Prentice-Hall of India, Second Indian Reprint, 2004.

Davis, M. L. and Cornwell D. A., Introduction to Environmental Engineering, 2nd Ed., McGraw Hill, Singapore, 1998.

Wright, R.T., Environmental Science: Towards a Sustainable Future, 9th Ed, Prentice Hall of India, New Delhi, 2007.

Supplementary Reading Materials (Selected Book Chapters and Papers)

HS 200 Environmental Studies 3 0 0 3

Social Issues and the environment, Public awareness and Human rights, Indicators of sustainability, Governance of Natural Resources - Common pool resources: issues and management.

Environmental ethics, Religion and environment, Wilderness and Developing Trends, Environmental movements and Activism, Social Ecology and Bioregionalism, Environmental justice.

Environmental economics, Trade and environment, Economics of environmental regulation, Natural resource accounting, Green GDP.

Environment and development, Resettlement and rehabilitation of people, Impacts of climate change on economy and society, Vulnerability and adaptation to climate change.

Text / References

Agar, N., 2001. *Life's Intrinsic Value*, New York: Columbia University Press.

Dasgupta, P. and Maler, G. (eds.), (1997), The Environment and Emerging Development Issues, Vol. I, OUP.

Guha, Ramachandra (2006): “Mahatama Gandhi and Environmental Movement,”

Debating on Gandhi, in A. Raghuramaraju (ed.), New Delhi: Oxford University Press.

Guha, Ramachandra and Madhav Gadgil (1995): Ecology and Equity: The Use and Abuse of Nature in Contemporary India, New Delhi: Penguin.

Hanley, Nick, Jason F. Shogren and Ben White (2004): Environmental Economics in Theory and Practice, New Delhi: MacMillan.

Naess, A. and G. Sessions (1984): "Basic Principles of Deep Ecology," Ecophilosophy, Vol.6.

Redclift, M. and Woodgate, G. (eds.), (1997), International Handbook of Environmental Sociology, Edward Edgar.

MA 401 Linear Algebra 3 1 0 8

Vector spaces over fields, subspaces, bases and dimension.

Systems of linear equations, matrices, rank, Gaussian elimination.

Linear transformations, representation of linear transformations by matrices, rank-nullity theorem, duality and transpose.

Determinants, Laplace expansions, cofactors, adjoint, Cramer's Rule.

Eigenvalues and eigenvectors, characteristic polynomials, minimal polynomials, Cayley-Hamilton Theorem, triangulation, diagonal-lization, rational canonical form, Jordan canonical form.

Inner product spaces, Gram-Schmidt orthonormalization, orthogonal projections, linear functionals and adjoints, Hermitian, self-adjoint, unitary and normal operators, Spectral Theorem for normal operators, Rayleigh quotient, Min-Max Principle.

Bilinear forms, symmetric and skew-symmetric bilinear forms, real quadratic forms, Sylvester's law of inertia, positive definiteness.

Texts / References

M. Artin, Algebra, Prentice Hall of India, 1994.

K. Hoffman and R. Kunze, Linear Algebra, Pearson Education (India), 2003. Prentice-Hall of India, 1991.

S. Lang, Linear Algebra, Undergraduate Texts in Mathematics, Springer-Verlag, New York, 1989.

P. Lax, Linear Algebra, John Wiley & Sons, New York,. Indian Ed. 1997

H.E. Rose, Linear Algebra, Birkhauser, 2002.

S. Lang, Algebra, 3rd Ed., Springer (India), 2004.

O. Zariski and P. Samuel, Commutative Algebra, Vol. I, Springer, 1975.

MA 403 Real Analysis 3 1 0 8

Review of basic concepts of real numbers: Archimedean property, Completeness.

Metric spaces, compactness, connectedness, (with emphasis on R^n).

Continuity and uniform continuity.

Monotonic functions, Functions of bounded variation; Absolutely continuous functions. Derivatives of functions and Taylor's theorem.

Riemann integral and its properties, characterization of Riemann integrable functions. Improper integrals, Gamma functions.

Sequences and series of functions, uniform convergence and its relation to continuity, differentiation and integration. Fourier series, pointwise convergence, Fejer's theorem, Weierstrass approximation theorem.

Texts / References

T. Apostol, Mathematical Analysis, 2nd ed., Narosa Publishers, 2002.

K. Ross, Elementary Analysis: The Theory of Calculus, Springer Int. Edition, 2004.

W. Rudin, Principles of Mathematical Analysis, 3rd ed., McGraw-Hill, 1983.

MA 406 General Topology 3 1 0 8

Prerequisite: MA 403 Real Analysis

Topological Spaces: open sets, closed sets, neighbourhoods, bases, subbases, limit points, closures, interiors, continuous functions, homeomorphisms.

Examples of topological spaces: subspace topology, product topology, metric topology, order topology.

Quotient Topology : Construction of cylinder, cone, Moebius band, torus, etc.

Connectedness and Compactness: Connected spaces, Connected subspaces of the real line, Components and local connectedness, Compact spaces, Heine-Borel Theorem, Local -compactness.

Separation Axioms: Hausdorff spaces, Regularity, Complete Regularity, Normality, Urysohn Lemma, Tychonoff embedding and Urysohn Metrization Theorem, Tietze Extension Theorem. Tychnoff Theorem, One-point Compactification.

Complete metric spaces and function spaces, Characterization of compact metric spaces, equicontinuity, Ascoli-Arzela Theorem, Baire Category Theorem. Applications: space filling curve, nowhere differentiable continuous function.

Optional Topics:
1. Topological Groups and orbit spaces.
2. Paracompactness and partition of unity.
3. Stone-Cech Compactification.
4. Nets and filters.

Texts / References

M. A. Armstrong, Basic Topology, Springer (India), 2004.

K.D. Joshi, Introduction to General Topology, New Age International, New Delhi, 2000.

J.L. Kelley, General Topology, Van Nostrand, Princeton, 1955.

J.R. Munkres, Topology, 2nd Ed., Pearson Education (India), 2001.

G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, New York, 1963.

MA 408 Measure Theory 3 1 0 8

Prerequisite: MA 403 Real Analysis

Semi-algebra, Algebra, Monotone class, Sigma-algebra, Monotone class theorem. Measure spaces.

Outline of extension of measures from algebras to the generated sigma-algebras: Measurable sets; Lebesgue Measure and its properties.

Measurable functions and their properties; Integration and Convergence theorems.

Introduction to L^p-spaces, Riesz-Fischer theorem; Riesz Representation theorem for L^2 spaces. Absolute continuity of measures, Radon-Nikodym theorem. Dual of L^p-spaces.

Product measure spaces, Fubini's theorem.

Fundamental Theorem of Calculus for Lebesgue Integrals (an outline).

Texts / References

P.R. Halmos, Measure Theory, Graduate Text in Mathematics, Springer-Verlag, 1979.

Inder K. Rana, An Introduction to Measure and Integration (2nd ed.), Narosa Publishing House, New Delhi, 2004.

H.L. Royden, Real Analysis, 3rd ed., Macmillan, 1988.

MA 410 Multivariable Calculus 2 1 0 6

Prerequisites: MA 403 Real Analysis,
MA 401 Linear Algebra

Functions on Euclidean spaces, continuity, differentiability; partial and directional derivatives, Chain Rule, Inverse Function Theorem, Implicit Function Theorem.

Riemann Integral of real-valued functions on Euclidean spaces, measure zero sets, Fubini's Theorem.

Partition of unity, change of variables.

Integration on chains, tensors, differential forms, Poincare Lemma, singular chains, integration on chains, Stokes' Theorem for integrals of differential forms on chains. (general version). Fundamental theorem of calculus.

Differentiable manifolds (as subspaces of Euclidean spaces), differentiable functions on manifolds, tangent spaces, vector fields, differential forms on manifolds, orientations, integration on manifolds, Stokes' Theorem on manifolds.

Texts / References

V. Guillemin and A. Pollack, Differential Topology, Prentice-Hall Inc., Englewood Cliffe, New Jersey, 1974.

W. Fleming, Functions of Several Variables, 2nd Ed., Springer-Verlag, 1977.

J.R. Munkres, Analysis on Manifolds, Addison-Wesley, 1991.

W. Rudin, Principles of Mathematical Analysis, 3rd ed., McGraw-Hill, 1984.

M. Spivak, Calculus on Manifolds, A Modern Approach to Classical Theorems of Advanced Calculus, W. A. Benjamin, Inc., 1965.

MA 412 Complex Analysis 3 1 0 8

Complex numbers and the point at infinity. Analytic functions.

Cauchy-Riemann conditions. Mappings by elementary functions. Riemann surfaces. Conformal mappings.

Contour integrals, Cauchy-Goursat Theorem.

Uniform convegence of sequences and series. Taylor and Laurent series. Isolated singularities and residues. Evaluation of real integrals.

Zeroes and poles, Maximum Modulus Principle, Argument Principle, Rouche's theorem.

Texts / References

J.B. Conway, Functions of One Complex Variable, 2nd ed., Narosa, New Delhi, 1978.

T.W. Gamelin, Complex Analysis, Springer International Edition, 2001.

R. Remmert, Theory of Complex Functions, Springer Verlag, 1991.

A.R. Shastri, An Introduction to Complex Analysis, Macmilan India, New Delhi, 1999.

MA 414 Algebra - I 3 1 0 8

Prerequiste: MA 401 Linear Algebra,
MA 419 Basic Algebra

Simple groups and solvable groups, nilpotent groups, simplicity of alternating groups, composition series, Jordan-Holder Theorem. Semidirect products. Free groups, free abelian groups.

Rings, Examples (including polynomial rings, formal power series rings, matrix rings and group rings), ideals, prime and maximal ideals, rings of fractions, Chinese Remainder Theorem for pairwise comaximal ideals.
Euclidean Domains, Principal Ideal Domains and Unique Factorizations Domains. Polynomial rings over UFD's.

Fields, Characteristic and prime subfields, Field extensions, Finite, algebraic and finitely generated field extensions, Classical ruler and compass constructions, Splitting

fields and normal extensions, algebraic closures. Finite fields, Cyclotomic fields, Separable and inseparable extensions.
Galois groups, Fundamental Theorem of Galois Theory, Composite extensions, Examples (including cyclotomic extensions and extensions of finite fields).

Norm, trace and discriminant.

Solvability by radicals, Galois' Theorem on solvability.

Cyclic extensions, Abelian extensions, Transcendental extensions.

Texts / References

M. Artin, Algebra, Prentice Hall of India, 1994.

D.S. Dummit and R. M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 2002.

J.A. Gallian, Contemporary Abstract Algebra, 4th Ed., Narosa, 1999.

N. Jacobson, Basic Algebra I, 2nd Ed., Hindustan Publishing Co., 1984, W.H. Freeman, 1985.

MA 417 Ordinary Differential Equations 3 1 0 8

Review of solution methods for first order as well as second order equations, Power Series methods with properties of Bessel functions and Legendre polynomials.

Existence and Uniqueness of Initial Value Problems: Picard's and Peano's Theorems, Gronwall's inequality, continuation of solutions and maximal interval of existence, continuous dependence.

Higher Order Linear Equations and linear Systems: fundamental solutions, Wronskian, variation of constants, matrix exponential solution, behaviour of solutions.

Two Dimensional Autonomous Systems and Phase Space Analysis: critical points, proper and improper nodes, spiral points and saddle points.

Asymptotic Behavior: stability (linearized stability and Lyapunov methods).

Boundary Value Problems for Second Order Equations: Green's function, Sturm comparision theorems and oscillations, eigenvalue problems.

Texts / References

M. Hirsch, S. Smale and R. Deveney, Differential Equations, Dynamical Systems and Introduction to Chaos, Academic Press, 2004

L. Perko, Differential Equations and Dynamical Systems, Texts in Applied Mathematics, Vol. 7, 2nd ed., Springer Verlag, New York, 1998.

M. Rama Mohana Rao, Ordinary Differential Equations: Theory and Applications. Affiliated East-West Press Pvt. Ltd., New Delhi, 1980.

D. A. Sanchez, Ordinary Differential Equations and Stability Theory: An Introduction, Dover Publ. Inc., New York, 1968.

MA 419 Basic Algebra 3 1 0 8

Review of basics: Equivalence relations and partitions, Division algorithm for integers, primes, unique factorization, congruences, Chinese Remainder Theorem, Euler φ-function.

Permutations, sign of a permutation, inversons, cycles and transpositions. Rudiments of rings and fields, elementary properties, polynomials in one and several variables, divisibility, irreducible polynomials, Division algorithm, Remainder Theorem, Factor Theorem, Rational Zeros Theorem, Relation between the roots and coefficients, Newton's Theorem on symmetric functions, Newton's identities, Fundamental Theorem of Algebra, (statement only), Special cases: equations of degree 4, cyclic equations.

Cyclotomic polynomials, Rational functions, partial fraction decomposition, unique factorization of polynomials in several variables, Resultants and discriminants.

Groups, subgroups and factor groups, Lagrange's Theorem, homomorphisms, normal subgroups. Quotients of groups, Basic examples of groups (including symmetric groups, matrix groups, group of rigid motions of the plane and finite groups of motions).

Cyclic groups, generators and relations, Cayley's Theorem, group actions, Sylow Theorems.

Direct products, Structure Theorem for finite abelian groups.

Texts / References

M. Artin, Algebra, Prentice Hall of India, 1994.

D.S. Dummit and R. M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 2002.

J.A. Gallian, Contemporary Abstract Algebra, 4th ed., Narosa, 1999.

K.D. Joshi, Foundations of Discrete Mathematics, Wiley Eastern, 1989.

T.T. Moh, Algebra, World Scientific, 1992.

S. Lang, Undergraduate Algebra, 2nd Ed., Springer, 2001.

S. Lang, Algebra, 3rd ed., Springer (India), 2004.

J. Stillwell, Elements of Algebra, Springer, 1994.

MA 503 Functional Analysis 3 1 0 8

Prerequisites: MA 401 Linear Algebra,
MA 408 Measure Theory

Normed spaces. Continuity of linear maps. Hahn-Banach Extension and Separation Theorems. Banach spaces. Dual spaces and transposes.

Uniform Boundedness Principle and its applications. Closed Graph Theorem, Open Mapping Theorem and their applications. Spectrum of a bounded operator. Examples of compact operators on normed spaces.

Inner product spaces, Hilbert spaces. Orthonormal basis. Projection theorem and Riesz Representation Theorem.

Texts / References

J.B. Conway, A Course in Functional Analysis, 2nd ed., Springer, Berlin, 1990.

C. Goffman and G. Pedrick, A First Course in Functional Analysis, Prentice-Hall, 1974.

E. Kreyzig, Introduction to Functional Analysis with Applications, John Wiley & Sons, New York, 1978.

B.V. Limaye, Functional Analysis, 2nd ed., New Age International, New Delhi, 1996.

A. Taylor and D. Lay, Introduction to Functional Analysis, Wiley, New York, 1980.

MA 504 Operators on Hilbert Spaces 2 1 0 6

Prerequisite: MA 503 Functional Analysis

Adjoints of bounded operators on a Hilbert space, Normal, self-adjoint and unitary operators, their spectra and numerical ranges.

Compact operators on Hilbert spaces. Spectral theorem for compact self-adjoint operators.

Application to Sturm-Liouville Problems.

Texts / References

J.B. Conway, A Course in Functional Analysis, 2nd ed., Springer, Berlin, 1990.

C. Goffman and G. Pedrick, First Course in Functional Analysis, Prentice Hall, 1974.

I. Gohberg and S. Goldberg, Basic Operator Theory, Birkhauser, 1981.

E. Kreyzig, Introduction to Functional Analysis with Applications, John Wiley & Sons, New York, 1978.

B.V. Limaye, Functional Analysis, 2nd ed., New Age International, New Delhi, 1996.

MA 505 Algebra- II 3 1 0 8

Prerequisite: MA 414 Algebra I

Modules, submodules, quotient modules and module homomorphisms.

Generation of modules, direct sums and free modules. Tensor products of modules. Exact sequences, projective modules.

Tensor algebras, symmetric and exterior algebras.

Finitely generated modules over principal ideal domains, invariant factors, elementary divisors, rational canonical forms. Applications to finitely generated abelian groups and linear trans-formations.

Noetherian rings and modules, Hilbert basis theorem, Primary decomposition of ideals in noetherian rings.

Integral extensions, Going-up and Going-down theorems, Extension and contraction of prime ideals, Noether's Normalization Lemma, Hilbert's Nullstellensatz.

Localization of rings and modules. Primary decompositions of modules.

Texts / References

M.F. Atiyah and I. G. Macdonald, Introduction to Commutative Algebra, Addison Wesley, 1969.

D.S. Dummit and R. M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 2002.

N. Jacobson, Basic Algebra I and II, 2nd Ed., W. H. Freeman, 1985 and 1989.

S. Lang, Algebra, 3rd Ed., Springer (India), 2004.

O. Zariski and P. Samuel, Commutative Algebra, Vol. I, Springer, 1975.

MA 508 Mathematical Methods 3 1 0 6

Prerequisite: MA 515 Partial Differential Equations

Asymptotic expansions, Watson's lemma, method of stationary phase and saddle point method. Applications to differential equations. Behaviour of solutions near an irregular singular point, Stoke's phenomenon. Method of strained coordinates and matched asymptotic expansions.

Variational principles, Lax-Milgram theorem and applications to boundary value problems. Calculus of variations and integral equations. Volterra integral equations of first and second kind. Iterative methods and Neumann series.

Texts / References

C.M. Bender and S.A. Orszag, Advanced Mathematical Methods for Scientists and Engineers, McGraw-Hill Book Co., 1978.

R. Courant & D.Hilbert, Methods of Mathematical Physics, Vol. I & II, Wiley Eastern Pvt. Ltd. New Delhi, 1975.

J. Kevorkian and J.D. Cole, Perturbation Methods in Applied Mathematics, Springer Verlag, Berlin, 1985.

S.G. Mikhlin, Variation Methods in Mathematical Physics, Pergaman Press, Oxford 1964.

MA 510 Introduction to Algebraic Geometry 2 1 0 6

Prerequisite : MA 414

Varieties: Affine and projective varieties, coordinate rings, morphisms and rational maps, local ring of a point, function fields, dimension of a variety.

Curves: Singular points and tangent lines, multiplicities and local rings, intersection multiplicities, Bezout's theorem for plane curves, Max Noether's theorem and some of its applications, group law on a nonsingular cubic, rational parametrization, branches and valuations.

Texts / References

S.S. Abhyankar, Algebraic Geometry for Scientists and Engineers, American Mathematical Society, 1990.

W. Fulton, Algebraic Curves, Benjamin, 1969.

J. Harris, Algebraic Geometry: A First Course, Springer-Verlag, 1992.

M. Reid, Undergraduate Algebraic Geometry, Cambridge University Press, Cambridge, 1990.

I.R. Shafarevich, Basic Algebraic Geometry, Springer-Verlag, Berlin, 1974.

R.J. Walker, Algebraic Curves, Springer-Verlag, Berlin, 1950.

MA 515 Partial Differential Equations 3 1 0 8

Prerequisites : MA 417 Ordinary Differential Equations, MA 410 Multivariable Calculus
Cauchy Problems for First Order Hyperbolic Equations: method of characteristics, Monge cone.

Classification of Second Order Partial Differential Equations: normal forms and characteristics.

Initial and Boundary Value Problems: Lagrange-Green's identity and uniqueness by energy methods.

Stability theory, energy conservation and dispersion.

Laplace equation: mean value property, weak and strong maximum principle, Green's function, Poisson's formula, Dirichlet's principle, existence of solution using Perron's method (without proof).

Heat equation: initial value problem, fundamental solution, weak and strong maximum principle and uniqueness results.

Wave equation: uniqueness, D'Alembert's method, method of spherical means and Duhamel's principle.

Methods of separation of variables for heat, Laplace and wave equations.

Texts / References

E. DiBenedetto, Partial Differential Equations, Birkhauser, Boston, 1995.

L.C. Evans, Partial Differrential Equations, Graduate Studies in Mathematics, Vol. 19, AMS, Providence, 1998.

F. John, Partial Differential Equations, 3^{rd} ed., Narosa Publ. Co., New Delhi,1979.

E. Zauderer, Partial Differential Equations of Applied Mathematics, 2^{nd} ed., John Wiley and Sons, New York, 1989.

MA 516 Algebraic Topology 3 1 0 8

Prerequiste: MA 406 General Topology

Paths and homotopy, homotopy equivalence, contractibility, deformation retracts.

Basic constructions: cones, mapping cones, mapping cylinders, suspension.
Cell complexes, subcomplexes, CW pairs. Fundamental groups. Examples (including the fundamental group of the circle) and applications (including Fundamental Theorem of Algebra, Brouwer Fixed Point Theorem and Borsuk-Ulam Theorem, both in dimension two). Van Kampen's Theorem, Covering spaces, lifting properties, deck transformations. universal coverings (existence theorem optional).

Simplicial complexes, barycentric subdivision, stars and links, simplicial approximation. Simplicial Homology. Singular Homology. Mayer-Vietoris Sequences. Long exact sequence of pairs and triples. Homotopy invariance and excision (without proof).

Degree. Cellular Homology.

Applications of homology: Jordan-Brouwer separation theorem, Invariance of dimension, Hopf's Theorem for commutative division algebras with identity, Borsuk-Ulam Theorem, Lefschetz Fixed Point Theorem.

Optional Topics:

Outline of the theory of: cohomology groups, cup products, Kunneth formulas, Poincare duality.

Texts / References

M.J. Greenberg and J. R. Harper, Algebraic Topology, Benjamin, 1981.

W. Fulton, Algebraic topology: A First Course, Springer-Verlag, 1995.

A. Hatcher, Algebraic Topology, Cambridge Univ. Press, Cambridge, 2002.

W. Massey, A Basic Course in Algebraic Topology, Springer-Verlag, Berlin, 1991.

J.R. Munkres, Elements of Algebraic Topology, Addison Wesley, 1984.

J.J. Rotman, An Introduction to Algebraic Topology, Springer (India), 2004.

H. Seifert and W. Threlfall, A Textbook of Topology, translated by M. A. Goldman, Academic Press, 1980.

J.W. Vick, Homology Theory, Springer-Verlag, 1994.

MA 518 Spectral Approximation 2 1 0 6

Prerequisite: MA 503 Functional Analysis
Spectral decomposition. Spectral sets of finite type. Adjoint and product spaces.

Convergence of operators: norm, collectively compact and ν convergence. Error estimates.

Finite rank approximations based on projections and approximations for integral operators.

A posteriori error estimates.

Matrix formulations for finite rank operators.

Iterative refinement of a simple eigenvalue.

Numerical examples.

Texts / References

M. Ahues, A. Largillier and B. V. Limaye, Spectral Computations for Bounded Operators, Chapman and Hall/CRC, 2000.

F. Chatelin, Spectral Approximation of Linear Operators, Academic Press, 1983.

T. Kato, Perturbation Theory of Linear Operators, 2nd ed., Springer-Verlag, Berlin, 1980.

MA 521 Theory of Analytic Functions 2 1 0 6

Prerequisites : MA 403 Real Analysis,
MA 412 Complex Analysis.

Maximum Modulus Theorem. Schwarz Lemma. Phragmen-Lindelof Theorem.

Riemann Mapping Theorem. Weierstrass Factorization Theorem.

Runge's Theorem. Simple connectedness. Mittag-Leffler Theorem.

Schwarz Reflection Principle.

Basic properties of harmonic functions.

Picard Theorems.

Texts / References

L. Ahlfors, Complex Analysis, McGraw-Hill, 3rd ed., New York, 1979.

J.B. Conway, Functions of One Complex Variable, 2nd ed., Narosa, New Delhi 1978.

T.W. Gamelin, Complex Analysis, Springer International, 2001.

R. Narasimhan, Theory of Functions of One Complex Variable, Springer (India), 2001.

W. Rudin, Real and Complex Analysis, 3rd ed., Tata McGraw-Hill, 1987.

MA 522 Fourier Analysis and Applications 3 1 0 8

Prerequisite: MA 403 Real Analysis

Basic Properties of Fourier Series: Uniqueness of Fourier Series, Convolutions, Cesaro and Abel Summability, Fejer's theorem, Poisson Kernel and Dirichlet problem in the unit disc. Mean square Convergence, Example of Continuous functions with divergent Fourier series.

Distributions and Fourier Transforms: Calculus of Distributions, Schwartz class of rapidly decreasing functions, Fourier transforms of rapidly decreasing functions, Riemann Lebesgue lemma, Fourier Inversion Theorem, Fourier transforms of Gaussians. Tempered Distributions: Fourier transforms of tempered distributions, Convolutions, Applications to PDEs (Laplace, Heat and Wave Equations), Schrodinger-Equation and Uncertainty principle.

Paley-Wienner Theorems, Poisson Summation Formula: Radial Fourier transforms and Bessel's functions. Hermite functions.
Optional Topics:

Applications to PDEs, Wavelets and X-ray tomography. Applications to Number Theory.

Texts / References:

R. Strichartz, A Guide to Distributions and Fourier Transforms, CRC Press.

E.M. Stein and R. Shakarchi, Fourier Analysis: An Introduction, Princeton University Press, Princeton 2003.

I. Richards and H. Youn, Theory of Distributions and Non-technical Approach, Cambridge University Press, Cambridge, 1990.

MA 523 Basic Number Theory 2 1 0 6

Prerequisites: MA 419 Basic Algebra
Infinitude of primes, discussion of the Prime Number Theorem, infinitude of primes in specific arithmetic progressions, Dirichlet's theorem (without proof).

Arithmetic functions, Mobius inversion formula. Structure of units modulo n, Euler's phi function

Congruences, theorems of Fermat and Euler, Wilson's theorem, linear congruences, quadratic residues, law of quadratic reciprocity.

Binary quadratics forms, equivalence, reduction, Fermat's two square theorem, Lagrange's four square theorem.

Continued fractions, rational approximations, Liouville's theorem, discussion of Roth's theorem, transcendental numbers, transcendence of "e" and "pi".

Diophantine equations: Brahmagupta's equation (also known as Pell's equation), the The equation, Fermat's method of descent, discussion of the Mordell equation.

Optional Topics:

Discussion of Waring's problem.

Discussion of the Bhargava-Conway "fifteen theorem" for positive definite quadratic forms.

The RSA algorithm and public key encryption.

Primality testing, discussion of the Agrawal-Kayal-Saxena theorem.

Catalan's equation, discussion of the Gelfond-Schneider theorem, discussion of Baker's theorem.

Texts / References

W.W. Adams and L.J. Goldstein, Introduction to the Theory of Numbers, 3rd ed., Wiley Eastern, 1972.

A. Baker, A Concise Introduction to the Theory of Numbers, Cambridge University Press, Cambridge, 1984.

I. Niven and H.S. Zuckerman, An Introduction to the Theory of Numbers, 4th Ed., Wiley, New York, 1980.

MA 524 Algebraic Number Theory 2 1 0 6

Prerequisites:
MA 505 Algebra - II (Exposure)
Algebraic number fields.
Localisation, discrete valuation rings.

Integral ring extensions, Dedekind domains, unique factorisation of ideals. Action of the galois group on prime ideals.

Valuations and completions of number fields, discussion of Ostrowski's theorem, Hensel's lemma, unramified, totally ramified and tamely ramified extensions of p-adic fields.

Discriminants and Ramification.

Cyclotomic fields, Gauss sums, quadratic reciprocity revisited.

The ideal class group, finiteness of the ideal class group, Dirichlet units theorem.

Texts / References

K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, 2nd ed., Springer-Verlag, Berlin, 1990.

S. Lang, Algebraic Number Theory, Addison- Wesley, 1970.

D.A. Marcus, Number Fields, Springer-Verlag, Berlin.

MA 525 Dynamical Systems 2 1 0 6

Prerequisite: MA 417 Ordinary Differential Equations

Review of stability for linear systems. Flow defined by nonlinear systems of ODEs, linearization and stable manifold theorem. Hartman-Grobman theorem. Stability and Lyapunov functions. Planar flows: saddle point, nodes, foci, centers and nonhyperbolic critical points. Gradient and Hamiltonian systems. Limit sets and attractors. Poincare map, Poincare Benedixson theory and Poincare index.

Texts / References

V.I. Arnold, Ordinary Differential Equations, Prentice Hall of India, New Delhi, 1998.

M.W. Hirsch and S. Smale, Differential Equations, Dynamical Systems and linear Algebra, Academic Press, NY, 174.

L. Perko, Differential Equations and Dynamical Systems, Springer Verlag, NY, 1991.

S. Wiggins, Introduction to Applied Nonlinear Dynamical Systems and Chaos, TAM Vol.2, Springer-Verlag, NY, 1990.

MA 526 Commutative Algebra 2 1 0 6

Prerequisites: MA 505 Algebra – II

Dimension theory of affine algebras: Principal ideal theorem, Noether normalization lemma, dimension and transcendence degree, catenary property of affine rings, dimension and degree of the Hilbert polynomial of a graded ring, Nagata's altitude formula, Hilbert's Nullstellensatz, finiteness of integral closure.

Hilbert-Samuel polynomials of modules :
Associated primes of modules, degree of the Hilbert polynomial of a graded module, Hilbert series and dimension, Dimension theorem, Hilbert-Samuel multiplicity, associativity formula for multiplicity,

Complete local rings:
Basics of completions, Artin-Rees lemma, associated graded rings of filtrations, completions of modules, regular local rings

Basic Homological algebra:
Categories and functors, derived functors, Hom and tensor products, long exact sequence of homology modules, free resolutions, Tor and Ext, Koszul complexes.

Cohen-Macaulay rings:
Regular sequences, quasi-regular sequences, Ext and depth, grade of a module, Ischebeck's theorem, Basic properties of Cohen-Macaulay rings, Macaulay's unmixed theorem, Hilbert-Samuel multiplicity and Cohen-Macaulay rings, rings of invariants of finite groups.

Optional Topics:

Face rings of simplicial complexes, shellable simplicial complexes and their face rings.

Dedekind Domains and Valuation Theory.

Texts / References

D. Eisenbud, Commutative Algebra (with a view toward algebraic geometry) Graduate Texts in Mathematics 150, Springer-Verlag, Berlin, 2003.

H. Matsumura, Commutative ring theory, Cambridge Studies in Advanced Mathematics No. 8, Cambridge University Press, Cambridge, 1980.

W. Bruns and J. Herzog, Cohen-Macaulay Rings, (Revised edition) Cambridge Studies in Advanced Mathematics No. 39, Cambridge University Press, Cambridge, 1998.

MA 530 Nonlinear Analysis 2 1 0 6

Prerequisites: MA 503 Functional Analysis.

Fixed Point Theorems with Applications: Banach contraction mapping theorem, Brouwer fixed point theorem, Leray-Schauder fixed point theorem.

Calculus in Banach spaces: Gateaux as well as Frechet derivatives, chain rule, Taylor's expansions, Implicit function theorem with applications, subdifferential.

Monotone Operators: maximal monotone operators with properties, surjectivity theorem with applications.

Degree theory and condensing operators with applications.

Texts / References

M.C. Joshi and R.K. Bose, Some Topics in Nonlinear Functional Analysis, Wiley Eastern Ltd., New Delhi, 1985.

E. Zeilder, Nonlinear Functional Analysis and Its Applications, Vol. I (Fixed Point Theory), Springer Verlag, Berlin, 1985.

MA 532 Analytic Number Theory 2 1 0 6

Prerequisites: MA 414 Algebra - I
MA 412 Complex Analysis

The Wiener-Ikehara Tauberian theorem, the Prime Number Theorem.

Dirichlet's theorem for primes in an Arithmetic Progression.

Zero free regions for the Riemann-zeta function and other L-functions.

Euler products and the functional equations for the Riemann zeta function and Dirichlet L-functions.
Modular forms for the full modular group, Eisenstein series, cusp forms, structure of the ring of modular forms.

Hecke operators and Euler product for modular forms.

The L-function of a modular form, functional equations.

Modular forms and the sums of four squares.

Optional topics:

Discussion of L-functions of number fields and the Chebotarev Density Theorem.

Phragmen-Lindelof Principle, Mellin inversion formula, Hamburger's theorem.

Discussion of Modular forms for congruence subgroups.

Discussion of Artin's holomorphy conjecture and higher reciprocity laws.

Discussion of elliptic curves and the Shimura-Taniyama conjecture (Wiles' Theorem)

Texts / References

S. Lang, Algebraic Number Theory, Addison-Wesley, 1970.

J.P. Serre, A Course in Arithmetic, Springer-Verlag, 1973.

T. Apostol, Introduction to Analytic Number Theory, Springer-Verlag, 1976.

MA 533 Advanced Probability Theory 2 1 0 6

Probability measure, probability space, construction of Lebesgue measure, extension theorems, limit of events, Borel-Cantelli lemma.

Random variables, Random vectors, distributions, multidimensional distributions, independence.

Expectation, change of variable theorem, convergence theorems.

Sequence of random variables, modes of convergence. Moment generating function and characteristics functions, inversion and uniqueness theorems, continuity theorems, Weak and strong laws of large number, central limit theorem.

Radon Nikodym theorem, definition and properties of conditional expectation, conditional distributions and expectations.
Texts / References

P. Billingsley, Probability and Measure, 3rd ed., John Wiley & Sons, New York, 1995.

J. Rosenthal, A First Look at Rigorous Probability, World Scientific, Singapore, 2000.

A.N. Shiryayev, Probability, 2nd ed., Springer, New York, 1995.

K.L. Chung, A Course in Probability Theory, Academic Press, New York, 1974.

MA 534 Modern Theory of Partial Differential Equations 2 1 0 6

Prerequisites: MA 503 Functional Analysis
MA 515 Partial Differential Equations.

Theory of distributions: supports, test functions, regular and singular distributions, generalised derivatives.

Sobolev Spaces: definition and basic properties, approximation by smooth functions, dual spaces, trace and imbedding results (without proof).

Elliptic Boundary Value Problems: abstract variational problems, Lax-Milgram Lemma, weak solutions and wellposedness with examples, regularity result, maximum principles, eigenvalue problems.

Semigroup Theory and Applications: exponential map, C_0-semigroups, Hille-Yosida and Lummer-Phillips theorems, applications to heat and wave equations.

Texts / References

S. Kesavan, Topics in Functional Analysis Wiley Eastern Ltd., New Delhi, 1989.

M. Renardy and R.C. Rogers, An Introduction to Partial Differential Equations,2nd ed., Springer Verlag International Edition, New York, 2004.

L.C. Evans, Partial Differential Equations, AMS, Providence, 1998.

MA 538 Representation Theory of Finite Groups 2 1 0 6

Prerequisite : MA 414 Algebra I

Representations, Subrepresentations, Tensor products, Symmetric and Alternating Squares.

Characters, Schur's lemma, Orthogonality relations, Decomposition of regular representation, Number of irreducible representations, canonical decomposition and explicit decompositions. Subgroups, Product

groups, Abelian groups. Induced representations.
Examples: Cyclic groups, alternating and symmetric groups.
Integrality properties of characters, Burnside's
$p^a q^b$ theorem. The character of induced representation, Frobenius Reciprocity Theorem, Meckey's irreducibility criterion, Examples of induced representations, Representations of supersolvable groups.

Texts / References

M. Burrow, Representation Theory of Finite Groups, Academic Press, 1965.

N. Jacobson, Basic Algebra II, Hindustan Publishing Corporation, 1983.

S. Lang, Algebra, 3rd ed. Springer (India) 2004.

J.P. Serre, Linear Representation of Groups, Springer-Verlag, 1977.

MA 539 Spline Theory and Variational Methods 2 1 0 6

Even Degree and Odd Degree Spline Interpolation, end conditions, error analysis and order of convergence. Hermite interpolation, periodic spline interpolation. B-Splines, recurrence relation for B-splines, curve fitting using splines, optimal quadrature.

Tensor product splines, surface fitting, orthogonal spline collocation methods.

Texts / References

C. De Boor, A Practical Guide to Splines, Springer-Verlag, Berlin, 1978.

H.N. Mhaskar and D.V. Pai, Fundamentals of Approximation Theory, Narosa Publishing House, New Delhi, 2000.

P.M. Prenter, Splines and Variational Methods, Wiley-Interscience, 1989.

MA 540 Numerical Methods for Partial Differential Equations 2 1 0 6

Prerequisite:
MA 515 Partial Differential Equations

Finite differences: grids, derivation of difference equations. Elliptic equations, discrete maximum principle and stability, residual correction methods (Jacobi, Gauss-Seidel and SOR methods), LOD and ADI methods. Finite difference schemes for initial and boundary value problems: Stability (matrix method, von-Neumann and energy methods), Lax-Richtmyer equivalence Theorem. Parabolic equations: explicit and implicit methods (Backward Euler and Crank-Nicolson schemes) with stability and convergence, ADI methods. Linear scalar conservation law: upwind, Lax-Wendroff and Lax-Friedrich schemes and CFL condition.

Lab Component: Exposure to MATLAB and computational experiments based on the algorithms discussed in the course.

Texts / References

R. Mitchell and S. D. F. Griffiths, The Finite Difference Methods in Partial Differential Equations, Wiley and Sons, NY, 1980.

G.D. Smith, Numerical Solutions of Partial Differential Equations, 3rd Edition, Calrendorn Press, Oxford, 1985.

J.C. Strikwerda, Finite difference Schemes and Partial Differential Equations, Wadsworth and Brooks/ Cole Advanced Books and Software, Pacific Grove, California, 1989.

J.W. Thomas, Numerical Partial Differential Equations : Finite Difference Methods, Texts in Applied Mathematics, Vol. 22, Springer Verlag, NY, 1999.

J.W. Thomas, Numerical Partial Differential Equations: Conservation Laws and Elliptic Equations, Texts in Applied Mathematics, Vol. 33, Springer Verlag, NY, 1999.

MA 556 Differential Geometry 2 1 0 6

Prerequiste: MA 410 Multivariable Calculus

Graphs and level sets of functions on Euclidean spaces, vector fields, integral curves of vector fields, tangent spaces.

Surfaces in Euclidean spaces, vector fields on surfaces, orientation, Gauss map.

Geodesics, parallel transport, Weingarten map.

Curvature of plane curves, arc length and line integrals.
Curvature of surfaces.

Parametrized surfaces, local equivalence of surfaces.

Gauss-Bonnet Theorem, Poincare-Hopf Index Theorem.

Texts / References

M. doCarmo, Differential Geometry of Curves and Surfaces, Prentice Hall, 1976.

B. O'Neill, Elementary Differential Geometry, Academic Press, New York, 1966.

J.J. Stoker, Differential Geometry, Wiley-Interscience, 1969.

J.A. Thorpe, Elementary Topics in Differential Geometry, Springer (India), 2004.

MA 562 Mathematical Theory of Finite Elements 2 1 0 6

Prerequisite:
MA 515 Partial Differential Equations
MA 503 Functional Analysis

Sobolev Spaces: basic elements, Poincare inequality. Abstract variational formulation and elliptic boundary value problem. Galerkin formulation and Cea's Lemma. Construction of finite element spaces. Polynomial approximations and interpolation errors.

Convergence analysis: Aubin-Nitsche duality argument; non-conforming elements and numerical integration; computation of finite element solutions.

Parabolic initial and boundary value problems: semidiscrete and completely discrete schemes with convergence analysis.

Lab component: Implementation of algorithms and computational experiments using MATLAB.

Texts / References

K.E. Brenner and R. Scott, The Mathematical Theory of Finite Element Methods, Springer-Verlag, Berlin, 1994.

P.G. Ciarlet, The Finite Element Methods for Elliptic Problems, North Holland, Amsterdam, 1978.

C. Johnson, Numerical solutions of Partial Differential Equations by Finite Element Methods, Cambridge University Press, Cambridge, 1987.

C. Mercier, Lectures on Topics in Finite Element Solution of Elliptic Problems, TIFR Lectures on Mathematics and Physics Vol. 63, Narosa Publ. House, New Delhi, 1979.

MA 581 Elements of Differential Topology 2 1 0 6

Prerequisite:
MA 410 Multivariable Calculus

Differentiable Manifolds in R^n: Review of inverse and implicit function theorems; tangent spaces and tangent maps; immersions; submersions and embeddings.
Regular Values: Regular and critical values; regular inverse image theorem; Sard's theorem; Morse lemma.

Transversality: Orientations of manifolds; oriented and mod 2 intersection numbers; degree of maps. Application to Fundamental theorem of Algebra.

*Lefschetz theory of vector fields and flows: Poincare-Hopf index theorem; Gauss-Bonnet theorem.

*Abstract manifolds: Examples such as real and complex projective spaces and Grassmannian varieties; Whitney embedding theorems.

(*indicates expository treatment intended for these parts of the syllabus.)

Texts / References

A. Dubovin, A.T. Fomenko, S.P. Novikov, Modern Geometry Methods and Applications - II, The Geometry and Topology of Manifolds, GTM 104, Springer-Verlag, Berlin, 1985.

V. Guillemin and A Pollack, Differential Topology Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1974.

J. Milnor, Topology from the Differential View-point, University Press of Virginia, Charlottsville 1990.

SI 416 Optimization 2 0 2 6

Unconstrained optimization using calculus (Taylor's theorem, convex functions, coercive functions).
Unconstrained optimization via iterative methods (Newton's method, Gradient/ conjugate gradient based methods, Quasi-Newton methods).

Constrained optimization (Penalty methods, Lagrange multipliers, Kuhn-Tucker conditions. Linear programming (Simplex method, Dual simplex, Duality theory). Modeling for Optimization.

Text / Reference

M. Bazarra, C. Shetty, Nonlinear Programming, Theory and Algorithms, Wiley, 1979.
Beale, Introduction to Optimization, John Wiley, 1988.

M.C. Joshi and K. Moudgalya, Optimization: Theory and Practice, Narosa, New Delhi, 2004.

SI 419 Combinatorics 2 1 0 6

Prerequisites: None. Non-math department students need the consent of the Instructor to register.

Algorithms and Efficiency. Graphs: Paths, Cycles, Trees, Coloring. Trees, Spanning Trees, Graph Searching (DFS, BFS), Shortest Paths. Bipartite Graphs and Matching problems. Counting on Trees and Graphs. Hamiltonian and Eulerian Paths.

Groups: Cosets and Lagrange Theorem, Cyclic Groups etc.. Permutation Groups, Orbits and Stabilizers. Generating Functions. Symmetry and Counting: Polya Theory. Special Topics (depending upon the instructor!)

Text / References

Normal L. Biggs, Discrete Mathematics, Oxford University Press, Oxford, 2002.

J. Hein, Discrete Structures, Logic and Computability, Jones and Barlett, 2002.

C.L.Liu, Elements of Discrete Mathematics, McGraw Hill, 1986.

SI 507 Numerical Analysis 3 0 2 8

Principles of floating point computations and rounding errors.

Systems of Linear Equations: factorization methods, pivoting and scaling, residual error correction method.

Iterative methods: Jacobi, Gauss-Seidel methods with convergence analysis, conjugate gradient methods.

Eigenvalue problems: only implementation issues.

Nonlinear systems: Newton and Newton like methods and unconstrained optimization.

Interpolation: review of Lagrange interpolation techniques, piecewise linear and cubic splines, error estimates.

Approximation: uniform approximation by polynomials, data fitting and least squares approximation.

Numerical Integration: integration by interpolation, adaptive quadratures and Gauss methods

Initial Value Problems for Ordinary Differential Equations: Runge-Kutta methods, multi-step methods, predictor and corrector scheme, stability and convergence analysis.

Two Point Boundary Value Problems : finite difference methods with convergence results.
Lab. Component: Implementation of algorithms and exposure to public domain packages like LINPACK and ODEPACK.

Texts / References

K.E. Atkinson, An Introduction to Numerical Analysis, Wiley, 1989.

S.D. Conte and C. De Boor, Elementary Numerical Analysis – An Algorithmic Approach, McGraw-Hill, 1981.

K. Eriksson, D. Estep, P. Hansbo and C. Johnson, Computational Differential Equations, Cambridge Univ. Press, Cambridge, 1996.

G.H. Golub and J.M. Ortega, Scientific Computing and Differential Equations: An Introduction to Numerical Methods, Academic Press, 1992.
J. Stoer and R. Bulirsch, Introduction to Numerical Analysis, 2nd ed., Texts in Applied Mathematics, Vol. 12, Springer Verlag, New York, 1993

SI 527 Introduction to Derivatives Pricing 2 1 0 6

Prerequisites:
SI 417 Introduction to Probability Theory

Basic notions – Cash flow, present value of a cash flow, securities, fixed income securities, types of markets.

Forward and futures contracts, options, properties of stock option prices, trading strategies involving options, option pricing using Binomial trees, Black – Scholes model, Black – Scholes formula, Risk-Neutral measure, Delta – hedging, options on stock indices, currency options.

Texts / References

D.G. Luenberger, Investment Science, Oxford University Press, Oxford, 1998.

J.C. Hull, Options, Futures and Other Derivatives, 4th ed., Prentice-Hall, New York, 2000.

J.C. Cox and M. Rubinstein, Options Market, Englewood Cliffs, N.J.: Prentice Hall, 1985.

C.P Jones, Investments, Analysis and Measurement, 5th ed., John Wiley and Sons, New York, 1996.