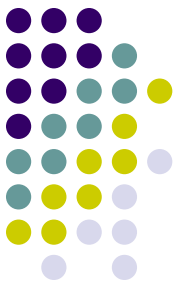


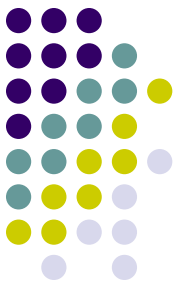
# Arithmetic and Geometric Progression

*Quantitative Aptitude & Business Statistics*

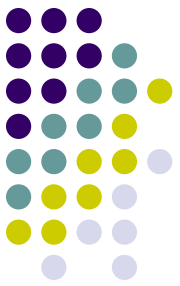


# Sequence

- **An arrangement of numbers in a definite order according to some rule is called a sequence.**
- **The various numbers occurring in a sequence are called its terms we denote the terms of a sequence by  $a_1, a_2, a_3, \dots$  etc.**
- **The  $n$ th term  $a_n$  called general term**

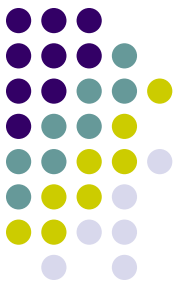


- A sequence has finite or infinite according to its finite or infinite terms.
- Example; 1
- 1, 3, 5..... is an infinite sequence
- Whose nth term is given formula
- $t_n = 2n - 1$



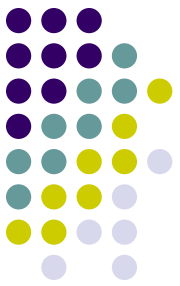
# Series

- A Series is obtained by adding all the terms of a sequence.
- Example
- 1.  $1+3+5+9\dots$  is an infinite series
- 2.  $2+4+6+8+10+12$  is a finite series



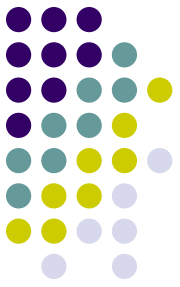
# Progressions

- If the terms of the sequence follow certain pattern ,then the sequence is called a progression.

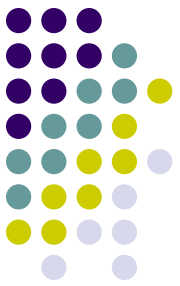


- Example:2
- $1, 1/2, 1/3, \dots$  is an infinite sequence where  $n$ th term is given by formula  $a_n = 1/n$
- Example:3
- $2, 4, 6, 8, 10, 12$  is a finite sequence in which each term is obtained by adding 2 to the previous term

# Arithmetic Progression(A.P)



- A sequence whose each term is obtained adding a fixed number to its term ,the term is called common difference of the A.P



The first in AP is 'a' and  
common difference is 'd'

An arithmetic progression is a  
progression in which *any term  
minus its previous terms is a  
constant.*

$$T(n+1) - T(n) = \text{common difference}$$





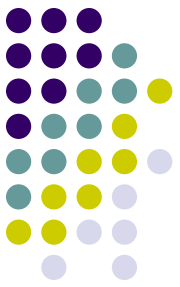
## Examples

- 2, 7, 12, 17, 22, 27, ... is an A.P.
- 2, 4, 8, 12, ... is NOT an A.P.

$$t(n) = a + (n-1)d$$

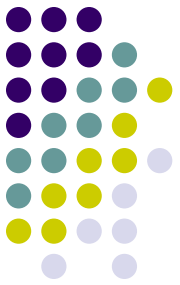
General term

# Arithmetic means

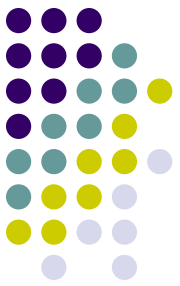


The intermediate terms between two terms of an arithmetic progression are called arithmetic means between the two terms.

# Example



Progression	Between	Arithmetic means
2, 3, 4, 5, 6, ...	2, 6	3, 4, 5
2, 5, 8, 11, 14, ...	2, 11	5, 8



- If  $a$  is first term and 'd' is common difference of an A.P, then  $n$  th term of an AP is denoted by

$$t_n = a + (n-1)d$$

# Sum of terms Arithmetic Series

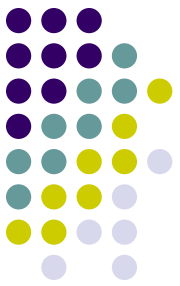


For an arithmetic progression,

$$S(n) = \frac{n}{2} [2a + (n-1)d]$$

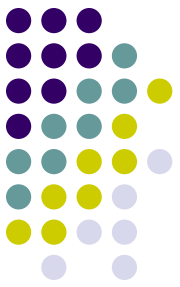
If we use  $l$  to represent the last term,  $T(n)$

$$S(n) = \frac{n}{2} (a + l)$$

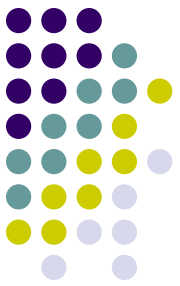


# Properties AP

- 1. If a constant is added or subtracted from term of an AP, then the resulting sequence is also in AP .with same common difference

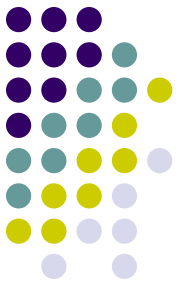


- 2. If each term of an AP is multiplied or divided by non-zero constant  $k$ , then the resulting sequence is also in AP with common difference  $kd$  or  $d/k$

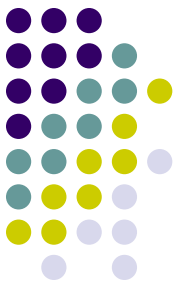


- 3. If  $a_1, a_2, a_3, \dots$  and  $b_1, b_2, b_3, \dots$  are two arithmetic progressions, then the sequence  $a_1 + b_1, a_2 + b_2, \dots$  is also in AP
- 4. In a finite A.P., the sum of terms equidistant from the beginning and end is always same and is equal to the sum of first and last



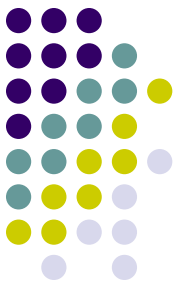


- 5. Three numbers  $a, b, c$  are in A.P if  $2b = a + c$



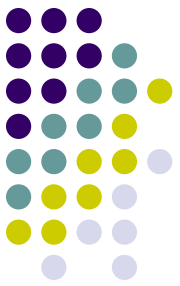
# *1. Sum of the first $n$ natural numbers*

$$\sum n = \frac{n(n+1)}{2}$$



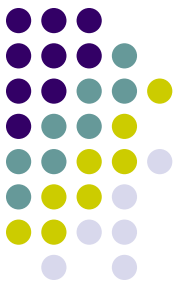
- *2. Sum of the Squares of first  $n$  natural numbers*

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$



- *3. Sum of the Cubes of first  $n$  natural numbers*

$$\sum n^3 = \left( \frac{n(n+1)}{2} \right)^2$$



## Problem ;1

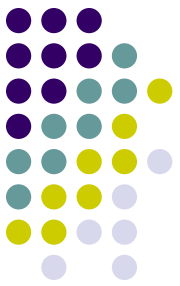
- Find the value of  $k$  for the series  $3k+4, 3k-7, k+12$  an arithmetic sequence

### Solution

If  $a, b, c$  are in A.P then  $2b = a + c$

$$2(3k-7) = 2k+4 = k=12$$

$$6k-14=3k+16; K=10$$



## Problem ;2

- Find the arithmetic mean between 7 and 15

Here  $a=7$  and  $b =15$

The arithmetic mean between a and

b is  $\frac{a + b}{2}$

The required arithmetic mean =

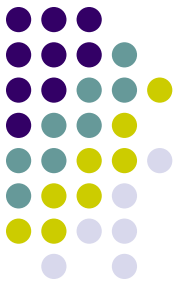
$$\frac{7 + 15}{2} = \frac{22}{2} = 11$$



## Problem ;3

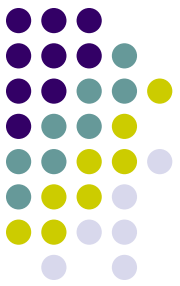
- **Insert 4 arithmetic means between 4 and 29**
- **Solution:**
- **If  $d$  is the common difference ,then**

$$d = \frac{b - a}{n + 1} = \frac{29 - 4}{5} = 5$$



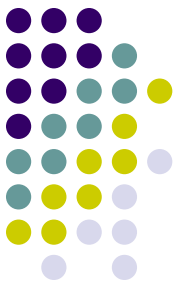
- The arithmetic means are  $4+5, 4+2*5, 4+3*5$  and  $4+4*5$
- i.e 9, 14, 19 and 29





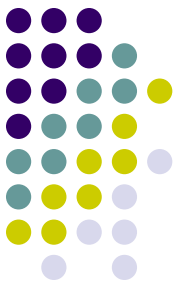
## Problem ;4

- The Tenth term of an arithmetic progression is 25 and fifteenth term is 40. Find the first term and common difference and the find the fifth term

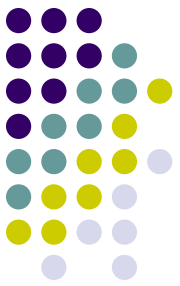


# Solution

- $t_{10}=25$   $t_{15} =40$  ,where  $t_n$  denotes the  $n$ th term .
- By using arithmetic progression.
- $T_n =a+(n-1)d$  ,where  
a= first term and  
d= common difference

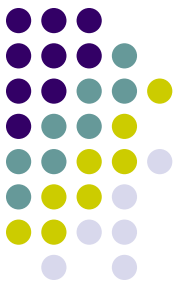


- It is given that
- $25 = a + 9d$  ————— 1
- $40 = a + 4d$  ————— 2
- From 1 and 2 ,we get
- $5d = 15 ; d = 3$
- $a = -2$  ,hence  $t_n = -2 + (n-1) \cdot 3$
- $t_5 = -2 + 4 * 3 = 10$



## Problem ;5

- The Third term of arithmetic progression is 7 and its seventh term is 2 more than twice of its third term. Find the first term ,common difference and the sum of first 20terms of the progression



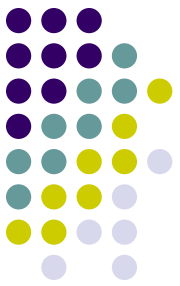
- Let the A.P be  $a, a + d ; a + 2d$   
.....+..... ....  $a + (n-1)d$  ;  $a$  being  
first term and  $d$  the common  
difference.

According to the question  $t_3 = 7$

$$\text{i.e } a + (3-1)d = a + 2d = 7$$

$$t_7 = 2 + 3t_3 \quad \text{—————} \quad \mathbf{1}$$

$$a + 6d = 2 + 3(7) = 2 + 21 = 23$$

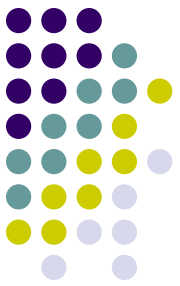


- $a + 6d = 23$  ————— 2
- Solving 1 and 2     $d = 4$  and  $a = -1$
- Also Sum of 20 terms
- $S_{20} = 20/2 \{20 * (-1) + (20-1) * 4\}$
- $= 10(-2 + 76) = 10 * 74 = 740$



## Problem ;6

- Find the increasing arithmetic progression ,the sum of first three terms is 27and sum of their squares is 275.
- Let the first three terms of the progression be  $a-d$  , $a$  and  $a +d$



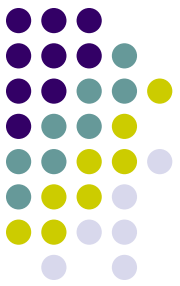
- By the description of the problem

- $(a-d) + a + (a+d) = 27$  .———— 1

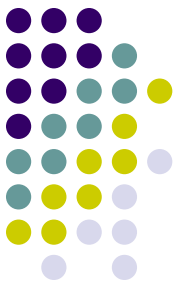
- and

- $(a-d)^2 + a^2 + (a+d)^2 = 275$  ————— 2



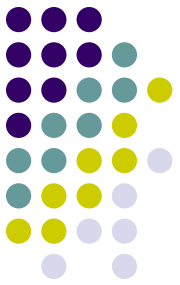


- From 1  $3a=27$  and  $a=9$
- From 2  $3a^2+2d^2=275$
- $2d^2=275-3*81=275-248$
- $d=\pm 4$
- Using  $a=9$  and  $d=4$ , we get required increasing arithmetic progression
- $9-4$ ,  $9$  and  $9+4$  i.e  $5, 9$  and  $13$

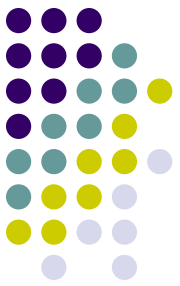


## Problem ;7

- Find the Sum of all numbers between 100 and 1000 which are divisible by 13.
- The numbers divisible by 13 form an arithmetic series. The series starts at 104 and ends at 988
- The term is  $a + (n-1)d$  here  $a=104$   
 $d=13$



- $988 = 104 + (n-1)3 = n = 69$
- Sum of these numbers is given by 37,674

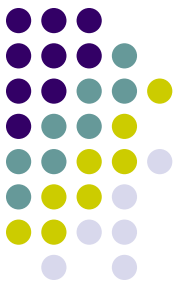


## Problem ;8

- The sum of first  $n$  terms of an AP is

$$3n^2 - 2n + 1$$

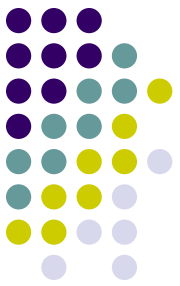
- The common difference is



- The sum of  $n$  terms is

$$3n^2 - 2n + 1$$

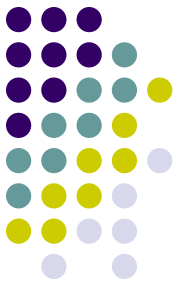
- Putting  $n=1$  then  $S_1 = 2$
- Putting  $n=2$  then  $S_2 = 9$
- Second term is therefore
- $= 9 - 2 = 7$
- And common difference
- $= 7 - 2 = 5$



## Problem ;9

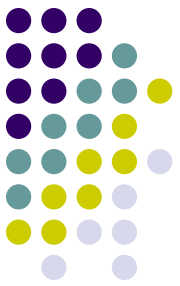
- Show that the sum of an AP, whose first term is 'a' and the second term is b and the last term is 'c' ,is equal to

$$\frac{(a + c)(b + c - 2a)}{2(b - a)}$$



# Solution

- Common difference  $d=b-a$
- Last term  $C = a+(n-1)(b-a)$



$$\frac{c - a}{b - a} = n - 1$$

$$n = 1 + \frac{c - a}{b - a}$$

$$= \frac{b + c - 2a}{b - a}$$

$$S_n = \frac{n}{2}(a + c) = \frac{(b + c - 2a)}{2(b - a)}(a + c)$$



# Geometric progression



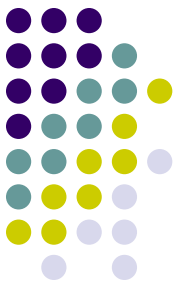
A geometric progression is a progression in which the ratio of each term to the preceding term is a constant.

$$T(n+1):T(n) = \text{Common Ratio}$$

# Geometric mean



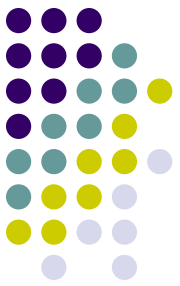
The intermediate terms between two terms of a geometric progression are called geometric means between the two terms.



## Examples

- 2, 4, 8, 16, ... is a G.P.
- 2, 4, 6, 8, ... is NOT a G.P.

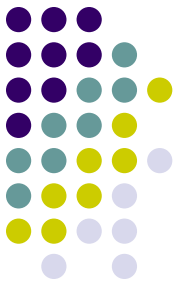
$$t(n) = ar^{n-1} \quad \text{General term}$$



# Example

Progression	Between	geometric means
2, 4, 8, 16, 32, ...	2, 32	4, 8, 16
1, -3, 9, -27, 81, ...	1, -27	-3, 9
4, 16, 64, 256, 1024, ...	4, 1024	16, 64, 256

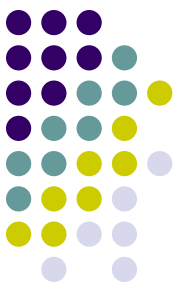
# Geometric Series



## Sum of n terms in Geometric Series

- $$S_n = \frac{a(1 - r^n)}{1 - r} \quad r < 1$$
- $$Sn = \frac{a(r^n - 1)}{r - 1} \quad r > 1$$

# Sum of G.P. find Applications in Mortgage or Installments Payment Calculation



**Formula for Compound Interest Growth**

$$A = P (1 + r\%)^n$$

**Formula for Depreciation**

$$A = P (1 - r\%)^n$$

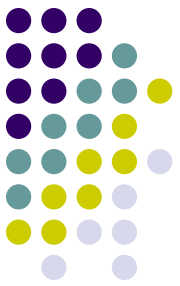
# Sum to infinity of a Geometric Series



$$S(n) = \frac{a(1 - R^n)}{1 - R} \rightarrow \frac{a}{1 - R} \quad (\text{provided } -1 < R < 1)$$

as  $n \rightarrow \infty$ ,  $R^n \rightarrow 0$

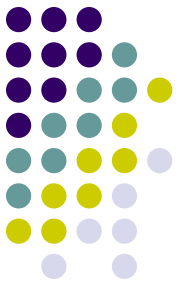
The sum to infinity  $S(\infty) = \frac{a}{1 - R}$



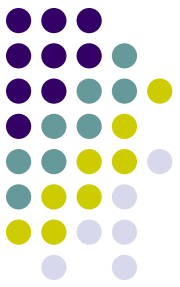
# Problem ;1

- Find the GP .whose 4<sup>th</sup> term is 8 and 8<sup>th</sup> term is  $128/625$ .
- Solution : if  $a$  is the first term and  $r$  is the common ratio of GP ,
- then  $8=t_n=ar^3$  and  $t_8=ar^7=128/625$
- $r=\pm 2/5$





- $r = \frac{2}{5}$  then  $a = 125$
- $r = -\frac{2}{5}$  then  $a = -125$
- Required GP is either  $125, 50, 20, 8, \frac{16}{5}$
- or  $-125, 50$  and  $-20, 8, -\frac{16}{5}$



## Problem ;2

- Find the geometric mean between 3 and 27
- Solution: here  $a = 3$  and  $b = 27$
- The geometric mean between  $a$  and  $b$  is  $= 9$

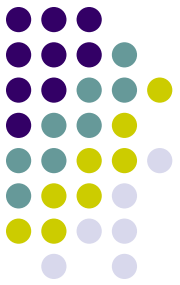


## Problem ;3

- Insert 3 geometric means between  $1/9$  and  $9$ .
- Solution : if  $n$  geometric means are to be inserted between  $a$  and  $b$  ,then the common ratio  $r$  is given by

$$r = \left( \frac{b}{a} \right)^{\frac{1}{n+1}}$$

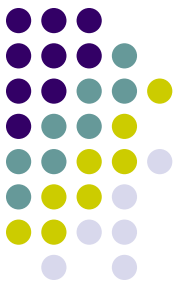
& GP



- Here  $r=1/a$  and  $b=9$   $n=3$

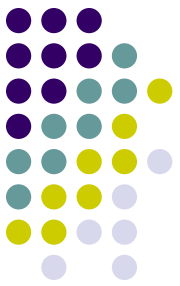
$$r = \left( \frac{9}{\frac{1}{9}} \right)^{\frac{1}{4}} = \pm 3$$

- The required geometric means are
- $1/3, 1, 3$  and  $-1/3, 1, -3$



## Problem ;4

- Find three numbers in GP whose sum is  $57/2$  and whose product is 729.
- Let the three numbers be  $a/r$  ,  $a$  ,  $ar$
- Given  $a/r \cdot a \cdot ar = 729$
- $a^3 = 729 = a = 9$
- It is also given that  $a/r + a + ar = 57/2$



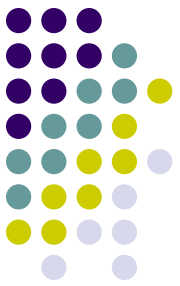
- $r = 2/3, 3/2$
- Therefore, the required numbers are  $27/2, 9, 6$  or  $6, 9, 27/2$



## Problem ;5

- Find the following missing numbers on using suitable formula give sum of the following

$$1 + 3 + 9 + * + 81 + 243 + * + 2187$$



# Solution

- Given

$1 + 3 + 9 + \dots + 81 + 243 + \dots + 2187$ , We may write the sum

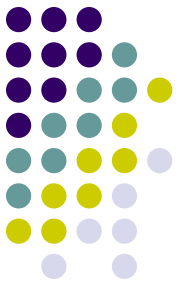
$$S = 1 + 3 + 3^2 + \dots + 3^4 + 3^5 + \dots + 3^7$$

Number of terms = 8 and the series is in GP, with common ratio 3

$$t_4 = 1 \cdot 3^3 = 27$$

$$\text{the seventh term} = 1 \cdot 3^6 = 726$$



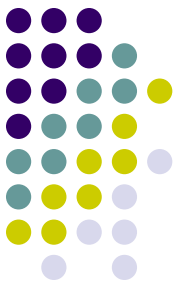


- Required missing numbers are 27 and 729
- And the Sum  $S=3280$



## Problem ;6

- **If  $1/x+y$ ;  $1/2y$ ;  $1/y+z$  are in AP. Then prove that  $x.y.z$  are in GP**
- **Solution: Since  $1/x+y$ ;  $1/2y$ ;  $1/y+z$  are in AP.**



- **Solution**

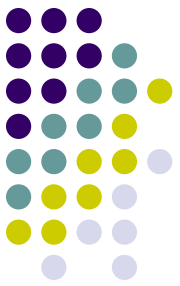
$$\frac{2}{2y} = \frac{1}{x+y} + \frac{1}{y+z}$$

$$\frac{1}{y} = \frac{(y+z) + (x+z)}{(x+y)(y+z)}$$

$$xz = y^2$$

$$\frac{y}{x} = \frac{z}{y}$$

- **Thus x,y and z are in GP**



# Problem ;7

- Find the Sum of the Series

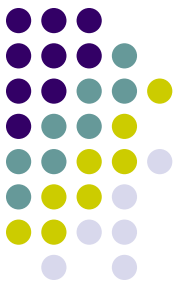
$3+33+333+\dots\dots\dots+$  to n terms

Solution:

$S_n = 3+33+333+\dots\dots\dots+$  to n terms

$=3(1+11+111+\dots\dots\dots+$  to n terms)

$=3/9(9+99+999+\dots\dots\dots+$  to n terms)



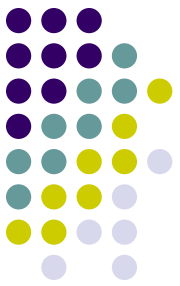
$$= \frac{1}{3} \left\{ 9 + 99 + 999 + \dots \dots n \text{ terms} \right\}$$

$$= \frac{1}{3} \left[ (10 - 1) + (10^2 - 1) + (10^3 - 1) + \dots \dots n \dots \right]$$

$$= \frac{1}{3} \left[ (10 + 10^2 + 10^3 \dots \dots + 10^n) - n \right]$$

$$= \frac{1}{3} \left[ \frac{10(10^n - 1)}{10 - 1} - n \right]$$

$$= \frac{10}{27} \left[ 10^n - 1 \right] - \frac{n}{3}$$



## Problem ;8

- Find the Sum of the Series  
 $0.8+0.88+0.888+\dots+ \text{to } n \text{ terms}$

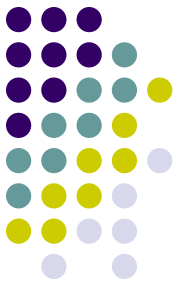
Let  $S_n$  be the Sum of the first  $n$  natural numbers

Solution

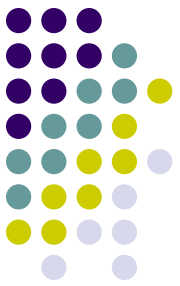
$$S_n = 0.8 + 0.88 + 0.888 + \dots + \text{to } n \text{ terms}$$

$$= 8(0.1 + 0.11 + 0.111 + \dots + \text{to } n \text{ terms})$$

$$= \frac{8}{9}(0.9 + 0.99 + 0.999 + \dots + \text{to } n \text{ terms})$$



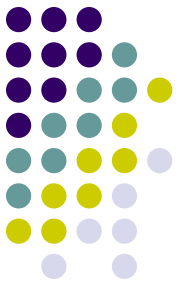
$$\begin{aligned} &= \frac{8}{9} \left\{ .9 + .99 + .999 + \dots n \text{ terms} \right\} \\ &= \frac{8}{9} \left[ \left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + \dots n \text{ terms} \right] \\ &= \frac{8}{9} \left[ n - \left( \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \dots \right) \right] \\ &= \frac{8}{9} \left( n - \frac{1}{10} \times \frac{10}{9} \left( \frac{10^n - 1}{10^n} \right) \right) \\ &= \frac{8}{9} \left( n - \frac{1}{9 \times 10^n} (10^n - 1) \right) \end{aligned}$$



## Example:9

- **By Expressing as an infinite geometric series find the value of 0.2175**
- **Solution**
- **$0.2175 = 0.21757575\dots$**
- **$= 0.21 + 0.0075 + 0.000075$   
 **$+ 0.00000075 + \dots$****

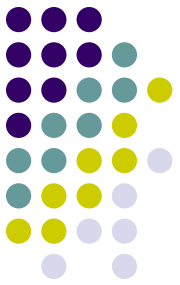




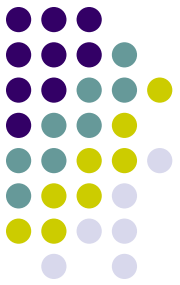
$$\begin{aligned} &= 0.21 + \frac{75}{10^4} + \frac{75}{10^6} + \frac{75}{10^8} + \dots \\ &= 0.21 + \frac{75}{10^4} \left( 1 + \frac{1}{10^2} + \frac{1}{10^4} + \frac{1}{10^8} + \dots \right) \\ &= 0.21 + \frac{75}{10^4} \left( \frac{1}{1 - \frac{1}{10^2}} \right) \\ &= 0.21 + \frac{75}{10^4} \times \frac{100}{99} \\ &= \frac{359}{1650} \end{aligned}$$



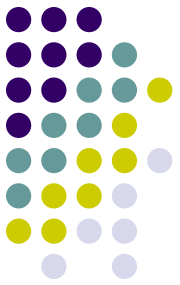
- **1. How many two digit numbers are divisible by 7**
- **A) 14**
- **B) 15**
- **C) 13**
- **D) 12**



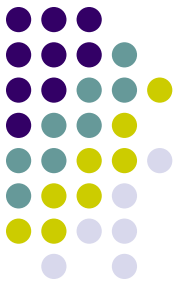
- **1. How many two digit numbers are divisible by 7**
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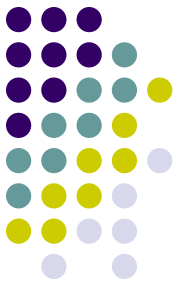
- **2. The two arithmetic means between -6 and 14**
- **A)  $\frac{2}{3}, \frac{1}{3}$**
- **B)  $\frac{2}{3}, \frac{22}{3}$**
- **C)  $-\frac{2}{3}, -\frac{21}{3}$**
- **D) none of these**



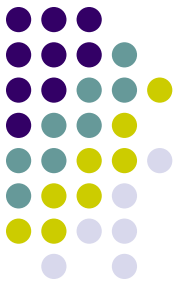
- 2 .The two arithmetic means between -6 and 14
- A) $2/3, 1/3$
- **B) $2/3, 22/3$**
- C) $-2/3, -21/3$
- D) none of these



- **3.The sum of the series 9,5,1 ....to 100 terms**
- **A)-18900**
- **B)18900**
- **C)19900**
- **D) none of these**

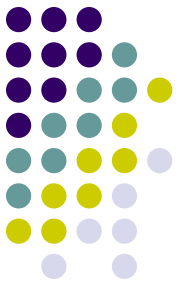


- **3.The sum of the series 9,5,1 ....to 100 terms**
- **A)-18900**
- **B)18900**
- **C)19900**
- **D) none of these**



- **4. The sum of first 64 natural numbers is**
- **A) 2015**
- **B) 2080**
- **C) 1974**
- **D) none of these**

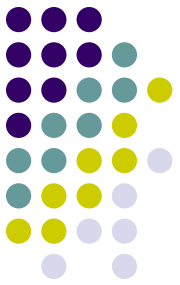




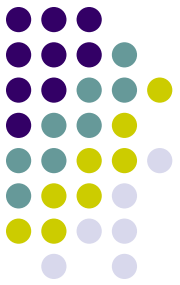
- 4. The sum of first 64 natural numbers is
- A) 2015
- **B) 2080**
- C) 1974
- D) none of these



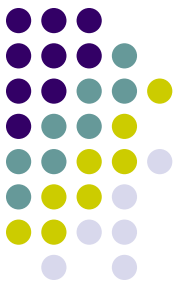
- **5. The sum of first 13 terms of an AP is 21 and the sum of first 21 terms is 13 .The sum of first 34 terms is**
- **A)34**
- **B)-34**
- **C)68**
- **D)-17**



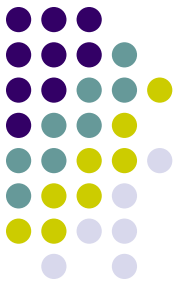
- **5. The sum of first 13 terms of an AP is 21 and the sum of first 21 terms is 13 .The sum of first 34 terms is**
- **A)34**
- **B)-34**
- **C)68**
- **D)-17**



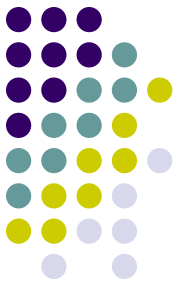
- **6. The sum of the first two terms of a GP is  $\frac{5}{3}$  and the sum of infinity of the series is 3. The common ratio is**
- **A)  $\frac{1}{3}$**
- **B)  $\frac{2}{3}$**
- **C)  $-\frac{1}{3}$**
- **D) none of these**



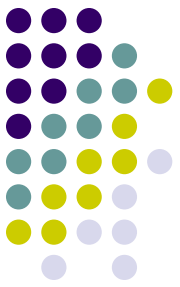
- **6. The sum of the first two terms of a GP is  $\frac{5}{3}$  and the sum of infinity of the series is 3. The common ratio is**
- **A)  $\frac{1}{3}$**
- **B)  $\frac{2}{3}$**
- **C)  $-\frac{1}{3}$**
- **D) none of these**



- **7.The sum of the infinite series**
- **$1+2/3+4/9+\dots$  is**
- **A)  $1/3$**
- **B)  $3$**
- **C)  $2/3$**
- **D) none of these**

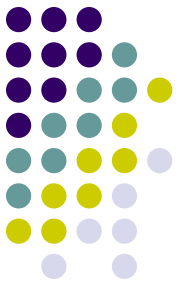


- **7. The sum of the infinite series**
- **$1 + 2/3 + 4/9 + \dots$  is**
- **A)  $1/3$**
- **B)  $3$**
- **C)  $2/3$**
- **D) none of these**

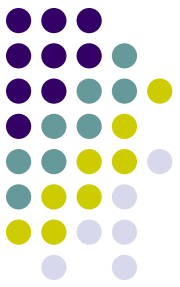


- 8. Sum of the series
- $1+3+9+27+\dots$  is 364. The number of terms is
- A) 5
- B) 6
- C) 11
- D) none of these

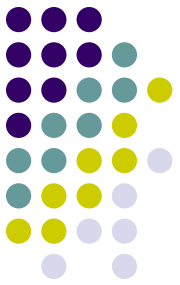




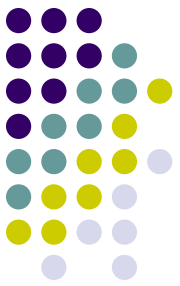
- **8. Sum of the series**
- **$1+3+9+27+\dots$  is 364. The number of terms is**
- **A)5**
- **B)6**
- **C)11**
- **D) none of these**



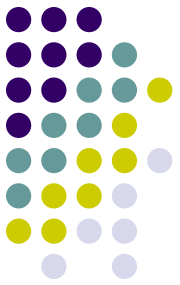
- **9. The  $(m+n)$  th and  $(m-n)$  th terms are  $p$  and  $q$  respectively. The  $m$  th term of GP is**
- **A)  $pq$**
- **B) Square root of  $(pq)$**
- **C)  $p \cdot q^{3/2}$**
- **D) none of these**



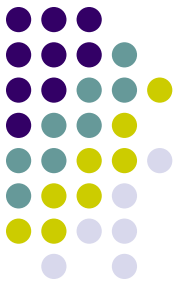
- **9. The  $(m+n)$  th and  $(m-n)$  th terms are  $p$  and  $q$  respectively. The  $m$  th term of GP is**
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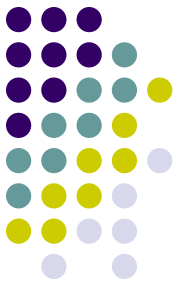
- **10. The  $n$ th terms of two series  $3+10+17+\dots$  and  $63+65+67+\dots$  are equal. Then the value of  $n$  is**
- **A) 9**
- **B) 13**
- **C) 19**
- **D) 21**



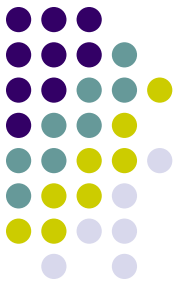
- **10. The  $n$ th terms of two series  $3+10+17+\dots$  and  $63+65+67+\dots$  are equal. Then the value of  $n$  is**
- **A) 9**
- **B) 13**
- **C) 19**
- **D) 21**



- **11. The Sum of three integers in A.P is 15 and their product is 80, The integers are**
- **A) 2, 8, 5**
- **B) 8, 2, 5**
- **C) 2, 5, 8**
- **D) none of these**



- **11. The Sum of three integers in A.P is 15 and their product is 80, The integers are**
- **A) 2, 8, 5**
- **B) 8, 2, 5**
- **C) 2, 5, 8**
- **D) none of these**



- **12. The Sum of all odd numbers between 100 and 200 is**
- **A) 6200**
- **B) 6500**
- **C) 7500**
- **D) 3750**

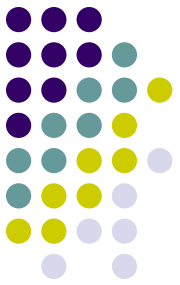




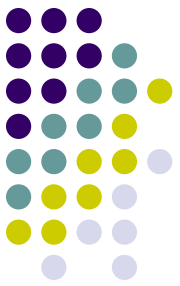
- **12. The Sum of all odd numbers between 100 and 200 is**
- **A) 6200**
- **B) 6500**
- **C) 7500**
- **D) 3750**



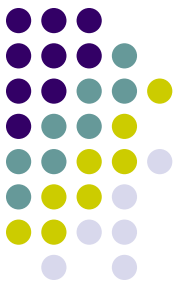
- **13. Which term of the AP 64,60,56,52....is Zero**
- **A)16**
- **B)17**
- **C)15**
- **D)14**



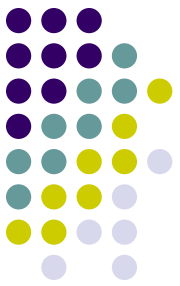
- **13. Which term of the AP 64, 60, 56, 52... is Zero**
- **A) 16**
- **B) 17**
- **C) 15**
- **D) 14**



- **14. The product of 3 numbers in GP is 729 and the sum of squares is 819. The numbers are**
- **A) 9, 3, 27**
- **B) 27, 3, 9**
- **C) 3, 9, 27**
- **D) none of these**



- **14. The product of 3 numbers in GP is 729 and the sum of squares is 819. The numbers are**
- **A) 9, 3, 27**
- **B) 27, 3, 9**
- **C) 3, 9, 27**
- **D) none of these**



- **15.If the first term of a GP exceeds the second term by 2 and the sum of infinity is 50 then the series is**
- **A)10,8,32/5,.....**
- **B)10,8,5/2,.....**
- **C)10,10/3,10/9.....**
- **D) none of these**



- **15.If the first term of a GP exceeds the second term by 2 and the sum of infinity is 50 then the series is**
- **A)10,8,32/5,.....**
- **B)10,8,5/2,.....**
- **C)10,10/3,10/9.....**
- **D) none of these**

**THE END**

*Arithmetic and Geometric  
Progression*