Arithmetic and Geometric Progression

Quantitative Aptitude & Business Statistics

Sequence

- **An arrangement of numbers in a definite order according to some rule is called a sequence.**
- **The various numbers occurring in a sequence are called its terms we denote the terms of a sequence by a_1 ,a_2, a_3….etc.**
- **The nth term a_n called general term**

- A sequence has finite or infinite according to its finite or infinite terms.
- Example;1
- 1,3,5........ is an infinite sequence
- Whose nth term is given formula
- $t_n = 2n-1$

Series

- A Series is obtained by adding all the terms of a sequence.
- Example
- 1. 1+3+5+9.,,,,,,,, is an infinite series
- 2. 2+4+6+8+10+12 is a finite series

Progressions

- If the terms of the sequence follow certain pattern ,then the sequence is called a progression.

- Example:2
- 1,1/2,1/3....is an infinite sequence where nth term is given by formula an =1/n
- Example:3
- 2,4,6,8,10,12 is a finite sequence in which each term is obtained by adding 2 to the previous term

Arithmetic Progression(A.P)

- A sequence whose each term is obtained adding a fixed number to its term ,the term is called common difference of the A.P

The first in AP is ‘a’ and common difference is ‘d’

An arithmetic progression is a progression in which *any term minus its previous terms is a constant*.

T(n+1) - T(n) = common difference

Examples

- 2, 7, 12, 17, 22, 27, ... is an A.P.
- 2, 4, 8, 12,...is NOT an A.P.

$$t(n) = a + (n-1)d$$

General term

Arithmetic means

The intermediate terms between two terms of an arithmetic progression are called arithmetic means between the two terms.

Example

Progression	Between	Arithmetic means
2, 3, 4, 5, 6, …	2, 6	3, 4, 5
2, 5, 8, 11, 14, …	2, 11	5, 8

- **If a is first term and ‘d’ is common difference of an A.P,then n th term of an AP is denoted by**

$$t_n=a+(n-1)d$$

Sum of terms Arithmetic Series

For an arithmetic progression,

$$S(n) = \frac{n}{2}\left[2a + (n-1)d\right]$$

If we use ℓ to represent the last term, $T(n)$

$$S(n) = \frac{n}{2}(a + \ell)$$

Properties AP

- 1.If a constant is added or subtracted from term of an AP ,then the resulting sequence is also in AP .with same common difference

- 2.If each term of an AP is multiplied or divided by non-zero constant k, then the resulting sequence is also in AP with common difference kd or d/k

- 3.If $a_1, a_2 a_3$....and $b_1, b_{2,} b_3$...are two arithmetic progressions, then the sequence a_1+b_1, a_2+b_2,.....is also in AP
- 4.In a finite A.P ,the sum of terms equidistant from the beginning and end is always same and is equal to the sum of first and last

- 5.Three numbers a,b,c are in A.P if 2b=a+c

1.Sum of the first n natural numbers

$$\sum n = \frac{n(n+1)}{2}$$

- *2.Sum of the Squares of first n natural numbers*

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

- *3.Sum of the Cubes of first n natural numbers*

$$\sum n^3 = \left(\frac{n(n+1)}{2} \right)^2$$

Problem ;1

- **Find the value of k for the series 3k+4,3k-7,k+12 an arithmetic sequence**

Solution

If a, b ,c are in A.P then 2b=a +c

2(3k-7) =2k+4 =k=12

6k-14=3k+16; K=10

Problem ;2

- **Find the arithmetic mean between 7 and 15**

Here a=7 and b =15

The arithmetic mean between a and b is $\frac{a+b}{2}$

The required arithmetic mean =

$$\frac{7+15}{2} = \frac{22}{2} = 11$$

Problem ;3

- **Insert 4 arithmetic means between 4 and 29**
- **Solution:**
- **If d is the common difference ,then**

$$d = \frac{b-a}{n+1} = \frac{29-4}{5} = 5$$

- The arithmetic means are 4+5,4+2*5,4+3*5and 4+4*5
- i.e 9,14,19 and 29

Problem ;4

- The Tenth term of an arithmetic progression is 25 and fifteenth term is 40.Find the first term and common difference and the find the fifth term

Solution

- t_{10}=25 t_{15} =40 ,where ,t_n denotes the nth term .
- By using arithmetic progression.
- $T_n = a+(n-1)d$,where

a= first term and

d= common difference

- It is given that
- 25=a+9d —— 1
- 40=a+4d —— 2
- From 1 and 2 ,we get
- 5d =15 ;d=3
- a=-2 ,hence tn =-2+(n-1) .3
- t_5=-2+4*3=10

Problem ;5

- The Third term of arithmetic progression is 7 and its seventh term is 2 more than twice of its third term. Find the first term ,common difference and the sum of first 20terms of the progression

- Let the A.P be a, a+ d ;a+2d ....+..... .... a+(n-1)d ;a being first term and d the common difference.

According to the question $t_3=7$

i.e a+(3-1)d=a+2d=7

$t_7=2+3t_3$ ———— 1

a+6d==2+3(7)=2+21=23

- a+6d=23 ———————— 2
- Solving 1 and 2 d = 4 and a=-1
- Also Sum of 20 terms
- S_{20}= 20/2{20*(-1)+(20-1)X4}
- =10(-2+76)=10*74=740

Problem ;6

- Find the increasing arithmetic progression ,the sum of first three terms is 27and sum of their squares is 275.
- Let the first three terms of the progression be a-d ,a and a +d

- By the description of the problem
- $(a-d) + a + (a+d) = 27$. ______ 1
- and

$(a-d)^2+a^2+(a+d)^2=275$ ______ 2

- From 1 $3a=27$ and $a=9$
- From 2 $3a^2+2d^2=275$
- $2d^2=275-3*81=275-248$
- $d=\pm 4$
- Using $a=9$ and $d=4$,we get required increasing arithmetic progression
- 9-4 ,9 and 9+4 i.e 5,9 and 13

Problem ;7

- Find the Sum of all numbers between 100and 1000which are divisible by 13.
- The numbers divisible by 13 for an arithmetic series .The series starts at 104 and ends at 988
- The term is a+(n-1)d here a=104 d=13

- 988=104+(n-1)3=n=69
- Sum of these numbers is given by 37,674

Problem ;8

- The sum of first n terms of an AP is

$$3n^2 - 2n + 1$$

- The common difference is

- The sum of n terms is

$$3n^2 - 2n + 1$$

- Putting n=1 then $S_1 = 2$
- Putting n=2 then $S_2 = 9$
- Second term is therefore
- =9-2=7
- And common difference
- =7-2=5

Problem ;9

- Show that the sum of an AP, whose first term is ‘a’ and the second tem is b and the last term is ‘c’ ,is equal to

$$\frac{(a+c)(b+c-2a)}{2(b-a)}$$

Solution

- Common difference $d=b-a$
- Last term $C = a+(n-1)(b-a)$

$$\frac{c-a}{b-a} = n-1$$

$$n = 1 + \frac{c-a}{b-a}$$

$$= \frac{b+c-2a}{b-a}$$

$$S_n = \frac{n}{2}(a+c) = \frac{(b+c-2a)}{2(b-a)}(a+c)$$

Geometric progression

A geometric progression is a progression in which the ratio of each term to the preceding term is a constant.

$$T(n+1):T(n) = \text{Common Ratio}$$

Geometric mean

The intermediate terms between two terms of a geometric progression are called geometric means between the two terms.

Examples

- 2, 4, 8, 16, … is a G.P.
- 2, 4, 6, 8,…is NOT a G.P.

$t(n) = ar^{n-1}$ General term

Example

Progression	Between	geometric means
2, 4, 8, 16, 32, …	2, 32	4, 8, 16
1, -3, 9, -27, 81, …	1, -27	-3, 9
4, 16, 64, 256, 1024, …	4, 1024	16, 64, 256

Geometric Series

Sum of n terms in Geometric Series

- $$S_n = \frac{a\left(1-r^n\right)}{1-r}$$ r<1
- $$Sn = \frac{a(r^n - 1)}{r-1}$$ r>1

Sum of G.P. find Applications in Mortgage or Installments Payment Calculation

Formula for Compound Interest Growth

$$A = P\,(1 + r\%)^{n}$$

Formula for Depreciation

$$A = P\,(1 - r\%)^{n}$$

Sum to infinity of a Geometric Series

$$S(n) = \frac{a(1-R^n)}{1-R} \to \frac{a}{1-R} \quad (\text{provided} \quad -1 < R < 1)$$

$$\text{as } n \to \infty,\ R^n \to 0$$

$$\text{The } \textbf{sum to infinity} \quad S(\infty) = \frac{a}{1-R}$$

Problem ;1

- Find the GP .whose 4th term is 8 and 8th term is 128/625.
- Solution : if a is the first term and r is the common ratio of GP ,
- then $8=t_n=ar^3$ and $t_8=ar^7=128/625$
- $r=\pm 2/5$

- r=2/5 then a=125
- r=-2/5then a =-125
- Required GP is either 125,50 20 ,8 16/5
- or -125, 50 and -20, 8 ,-16/5

Problem ;2

- Find the geometric mean between 3 and 27
- Solution: here a= 3 and b=27
- The geometric mean between a and b is =9

Problem ;3

- Insert 3 geometric means between 1/9 and 9.
- Solution : if n geometric means are to be inserted between a and b ,then the common ratio r is given by
- $$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

- Here r=1/a and b=9 n=3

$$r = \left(\frac{9}{\frac{1}{9}}\right)^{\frac{1}{4}} = \pm 3$$

- The required geometric means are
- 1/3,1, 3 and -1/3 ,1 -3

Problem ;4

- Find three numbers in GP whose sum is 57/2 and whose product is 729.
- Let the three numbers be a/r , a, ar
- Given a/r. a. ar=729
- a^3=729= a=9
- It is also given that a/r + a+ ar=57/2

- r=2/3 ,3/2
- Therefore ,the required numbers are 27/2 ,9,6 or 6,9 27/2

Problem ;5

- Find the following missing numbers on using suitable formula give sum of the following

1+ 3 + 9 +* + 81+ 243+ * + 2187

Solution

- Given

1+ 3 + 9 +* + 81+ 243+ * + 2187,We may write the sum

$S=1+ 3 + 3^2 +* + 3^4+ 3^5+ * + 3^7$

Number of terms =8 and the series is in GP, with common ratio3

$t_4=1*3^3=27$

the seventh term $=1*3^6=726$

- Required missing numbers are 27 and 729
- And the Sum S=3280

Problem ;6

- **If 1/x+y; 1/2y;1/y+z are in AP.Then prove that x.y.z are in GP**
- **Solution: Since 1/x+y; 1/2y;1/y+z are in AP.**

- **Solution**

$$\frac{2}{2y} = \frac{1}{x+y} + \frac{1}{y+z}$$

$$\frac{1}{y} = \frac{(y+z)+(x+z)}{(x+y)(y+z)}$$

$$xz = y^2$$

$$\frac{y}{x} = \frac{z}{y}$$

- **Thus x,y and z are in GP**

Problem ;7

- Find the Sum of the Series

3+33+333+.........+ to n terms

Solution:

S n= 3+33+333+.........+ to n terms

=3(1+11+111+........+ to n terms)

=3/9(9+99+999+.......+to n terms)

$$= \frac{1}{3}\left\{ 9 + 99 + 999 + \ldots\ldots.nterms) \right\}$$

$$= \frac{1}{3}\left[(10 - 1) + (10^2 - 1) + (10^3 - 1) + \ldots ton\ldots \right]$$

$$= \frac{1}{3}\left[(10 + 10^2 + 10^3 \ldots\ldots + 10^n) - n \right]$$

$$= \frac{1}{3}\left[\frac{10(10^n - 1)}{10 - 1} - n \right]$$

$$= \frac{10}{27}\left[10^n - 1 \right] - \frac{n}{3}$$

Problem ;8

- Find the Sum of the Series

  0.8+0.88+0.888.........+ to n terms

Let Sn be the Sum of the first n natural numbers

Solution

Sn=0.8+0.88+0.888.........+ to n terms

=8(0.1+0.11+0.111+...+to n terms)

=8/9(0.9+0.99+0.999+....+ to n terms)

$$= \frac{8}{9}\left\{ .9+.99+.999+.......nterms) \right\}$$

$$= \frac{8}{9}\left[(1-\frac{1}{10})+(1-\frac{1}{10^2})+(1-\frac{1}{10^3})+...ton... \right]$$

$$= \frac{8}{9}\left[n-(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+......) \right]$$

$$= \frac{8}{9}\left(n-\frac{1}{10}\times\frac{10}{9}\left(\frac{10^n-1}{10^n}\right)\right)$$

$$= \frac{8}{9}\left(n-\frac{1}{9\times10^n}(10^n-1) \right)$$

Example:9

- **By Expressing as an infinite geometric series find the value of 0.2175**
- **Solution**
- **0.2175=0.21757575………**
- **=0.21+0.0075+0.000075 +0.00000075+…….**

$$= 0.21 + \frac{75}{10^4} + \frac{75}{10^6} + \frac{75}{10^8} + \ldots\ldots\ldots$$

$$= 0.21 + \frac{75}{10^4}\left(1 + \frac{1}{10^2} + \frac{1}{10^4} + \frac{1}{10^8} + \ldots\ldots\right)$$

$$= 0.21 + \frac{75}{10^4}\left(\frac{1}{1 - \frac{1}{10^2}}\right)$$

$$= 0.21 + \frac{75}{10^4} \times \frac{100}{99}$$

$$= \frac{359}{1650}$$

- **1.How many two digit numbers are divisible by 7**
- **A)14**
- **B)15**
- **C)13**
- **D)12**

- **1.How many two digit numbers are divisible by 7**
- **A)14**
- **B)15**
- **C)13**
- **D)12**

- **2.The two arithmetic means between -6 and 14**
- **A)2/3,1/3**
- **B)2/3,22/3**
- **C)-2/3,-21/3**
- **D) none of these**

- **2 .The two arithmetic means between -6 and 14**
- **A)2/3,1/3**
- **B)2/3,22/3**
- **C)-2/3,-21/3**
- **D) none of these**

- **3.The sum of the series 9,5,1 ….to 100 terms**
- **A)-18900**
- **B)18900**
- **C)19900**
- **D) none of these**

- **3.The sum of the series 9,5,1 ….to 100 terms**
- **A)-18900**
- **B)18900**
- **C)19900**
- **D) none of these**

- **4.The sum of first 64 natural numbers is**
- **A)2015**
- **B)2080**
- **C)1974**
- **D) none of these**

- **4.The sum of first 64 natural numbers is**
- **A)2015**
- **B)2080**
- **C)1974**
- **D) none of these**

- **5.The sum of first 13 terms of an AP is 21 and the sum of first 21 terms is 13 .The sum of first 34 terms is**
- **A)34**
- **B)-34**
- **C)68**
- **D)-17**

- **5.The sum of first 13 terms of an AP is 21 and the sum of first 21 terms is 13 .The sum of first 34 terms is**
- **A)34**
- **B)-34**
- **C)68**
- **D)-17**

- **6.The sum of the first two terms of a GP is 5/3 and the sum of infinity of the series is 3 .The common ratio is**
- **A)1/3**
- **B)2/3**
- **C)-1/3**
- **D) none of these**

- **6.The sum of the first two terms of a GP is 5/3 and the sum of infinity of the series is 3 .The common ratio is**
- **A)1/3**
- **B)2/3**
- **C)-1/3**
- **D) none of these**

- **7.The sum of the infinite series**
- **1+2/3+4/9+………is**
- **A)1/3**
- **B)3**
- **C)2/3**
- **D) none of these**

- **7.The sum of the infinite series**
- **1+2/3+4/9+………is**
- **A)1/3**
- **B)3**
- **C)2/3**
- **D) none of these**

- 8.Sum of the series
- 1+3+9+27+…..is 364.The number of terms is
- A)5
- B)6
- C)11
- D) none of these

- **8.Sum of the series**
- **1+3+9+27+…..is 364.The number of terms is**
- **A)5**
- **B)6**
- **C)11**
- **D) none of these**

- **9.The (m+n) th and (m-n) th terms are p and q respectively. The m th term of GP is**
- **A) pq**
- **B)Square root of (pq)**
- **C)p.q3/2**
- **D) none of these**

- **9.The (m+n) th and (m-n) th terms are p and q respectively. The m th term of GP is**
- **A) pq**
- **B)Square root of (pq)**
- **C)p.q3/2**
- **D) none of these**

- **10.The nth terms of two series 3+10+17+……and 63+65+67+……are equal .Then the value of n is**
- **A)9**
- **B)13**
- **C)19**
- **D)21**

- **10.The nth terms of two series 3+10+17+……and 63+65+67+……are equal .Then the value of n is**
- **A)9**
- **B)13**
- **C)19**
- **D)21**

- **11.The Sum of three integers in A.P is 15 and their product is 80,The integers are**
- **A)2,8,5**
- **B)8,2,5**
- **C)2,5,8**
- **D) none of these**

- **11.The Sum of three integers in A.P is 15 and their product is 80,The integers are**
- **A)2,8,5**
- **B)8,2,5**
- **C)2,5,8**
- **D) none of these**

- **12.The Sum of all odd numbers between 100 and 200 is**
- **A)6200**
- **B)6500**
- **C)7500**
- **D)3750**

- **12.The Sum of all odd numbers between 100 and 200 is**
- **A)6200**
- **B)6500**
- **C)7500**
- **D)3750**

- **13.Which term of the AP 64,60,56,52….is Zero**
- **A)16**
- **B)17**
- **C)15**
- **D)14**

- **13.Which term of the AP 64,60,56,52….is Zero**
- **A)16**
- **B)17**
- **C)15**
- **D)14**

- **14.The product of 3 numbers in GP is 729 and the sum of squares is 819.The numbers are**
- **A)9,3,27**
- **B)27,3,9**
- **C)3,9,27**
- **D) none of these**

- **14.The product of 3 numbers in GP is 729 and the sum of squares is 819.The numbers are**
- **A)9,3,27**
- **B)27,3,9**
- **C)3,9,27**
- **D) none of these**

- **15.If the first term of a GP exceeds the second term by 2 and the sum of infinity is 50 then the series is**
- **A)10,8,32/5,….**
- **B)10,8,5/2,……**
- **C)10,10/3,10/9……**
- **D) none of these**

- **15.If the first term of a GP exceeds the second term by 2 and the sum of infinity is 50 then the series is**
- **A)10,8,32/5,….**
- **B)10,8,5/2,……**
- **C)10,10/3,10/9……**
- **D) none of these**

THE END

Arithmetic and Geometric Progression