(184)

3

(182)

-: HAND WRITTEN NOTES:-

OF

(1)

ELECTRONICS & COMMUNICATION ENGINEERING

-: SUBJECT:-

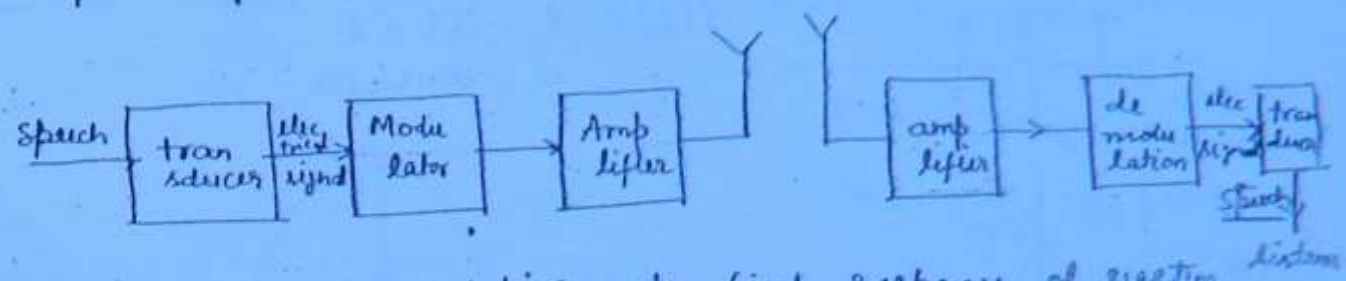
SIGNAL & SYSTEM

3

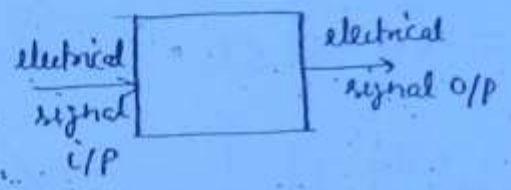
2

Speech signal → 300Hz → 3400Hz

(3)



* SS → we are concentrating to find response of system



to find response of a system, we use following rules/tools-

1. Fourier series
2. Fourier transforms
3. Laplace transforms
4. Z- transforms

Audio signal → 20 Hz - 20 KHz

Video signal → 0 - 5 MHz

data signal → range is based on application

Signal → is a quantity having associated information with it.

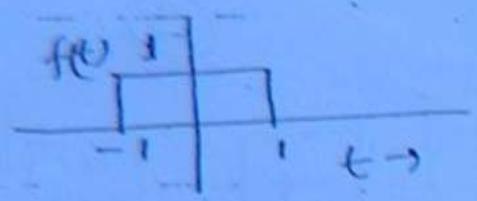
is a function of time.

we deal with electrical signal which are voltages or currents, which are both functions of time. In general signal is a function time f(t). (electrical)

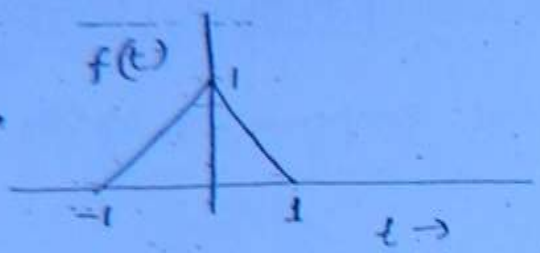
But signal is not always a function of time.

collect still frame and play back to it is video signal (normally 24 frames/ second where play back the still frames).

In Motion picture signal is three dimensional f(x, y, t)

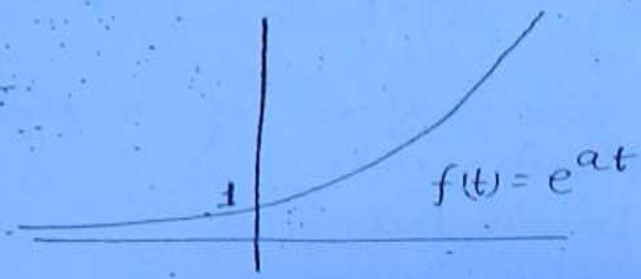


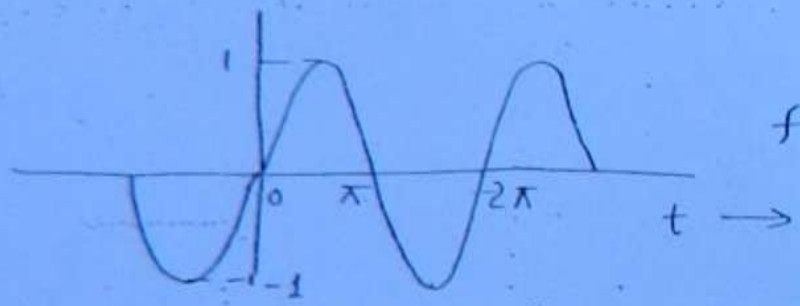
$f(t) = 1 \quad -1 \le t \le 1$
$\quad\;\; 0 \quad$ otherwise



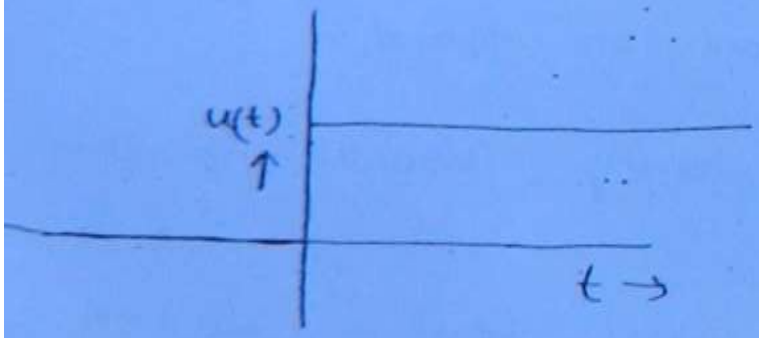
$f(t) = -1 \le t \le 0$
$\quad = t+1$
$\quad = -t+1 \quad 0 \le t \le 1$

$f(t) = e^{-at}$

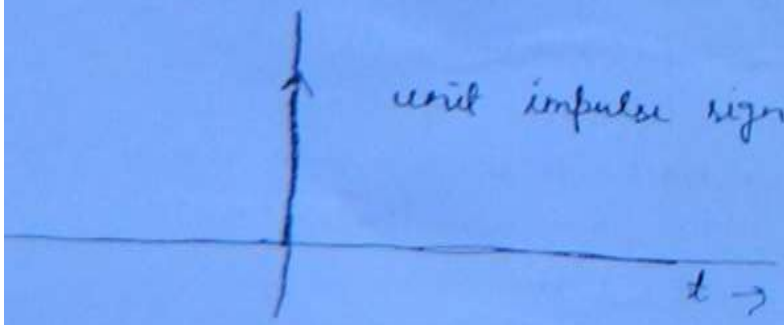




$f(t) = \sin t$



$u(t) = 1 \quad t \ge 0$
$\quad\;\; 0 \quad t < 0$



unit impulse signal

existence is split second
effect is enormous.

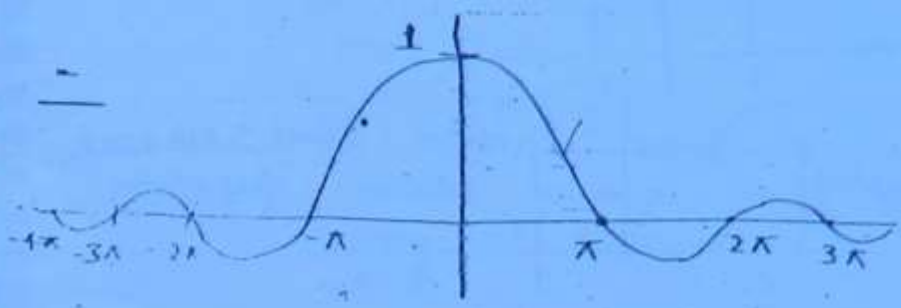
$\delta(t) = 0 \quad t \ne 0$
$\quad = \infty \quad t = 0$ very large
$\quad \ne 0 \quad t = 0 \quad (\infty)$

$\text{rect}\left(\frac{t-1}{2}\right)$ 3/4

$\text{rect}(t+3/4)$

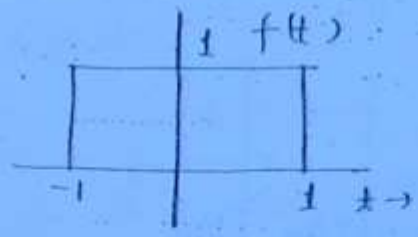
$$\int_{-\infty}^{\infty} f(t)\,dt = 1 \rightarrow \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

5

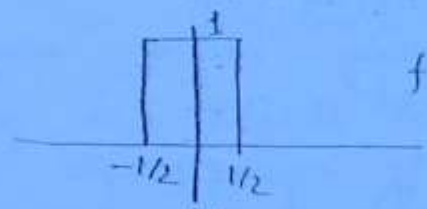


$f(t) = \frac{\sin t}{t} = Sa(t)$

$= \text{Sinc}(t/\pi)$



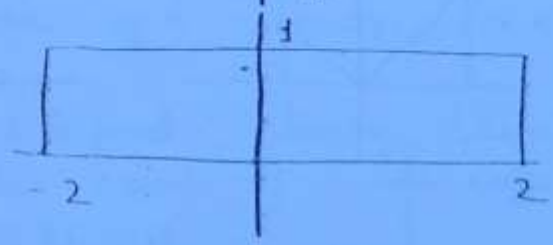
$f(t) = \text{rect}\left(\frac{t}{2}\right)$



$f(2(t))$

$f(t) = \text{rect}(t)$

time scaled version of $f(t)$ (amplified time-axis changed



$f(t/2)$

$f(t) = \text{rect}(t/4)$

$\text{rect}(t-4)$

delay

$f(t-4)$

time shifted version of given signal $f(t)$

$\text{rect}(t+3)$

advanced

$f(t+3)$

[(divide all time (instant) by a) $f(t) \xrightarrow{\text{transformation}} f(at)$ $a>1 \rightarrow$ compression, $0<a<1 \rightarrow$ expansion

(subtract T from all instant) shift

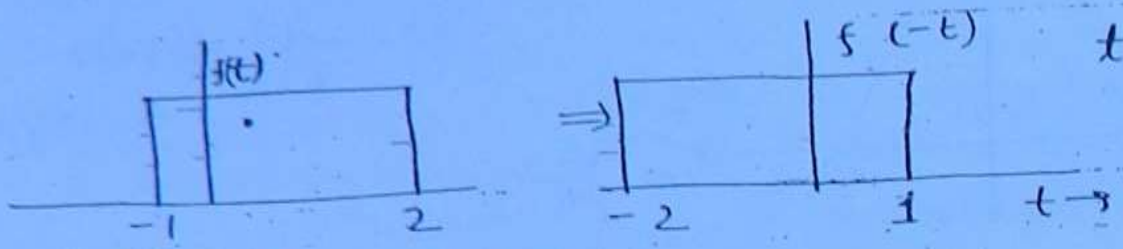
$f(t) \xrightarrow{t \to (t+T)} \rightarrow$ advanced (left)

shift

$t \to (t-T) \rightarrow$ delayed (right)

↳ add (T to all instant)

6



time reversal operation

$f(t) = 1 \quad -1 \leq t \leq 2$

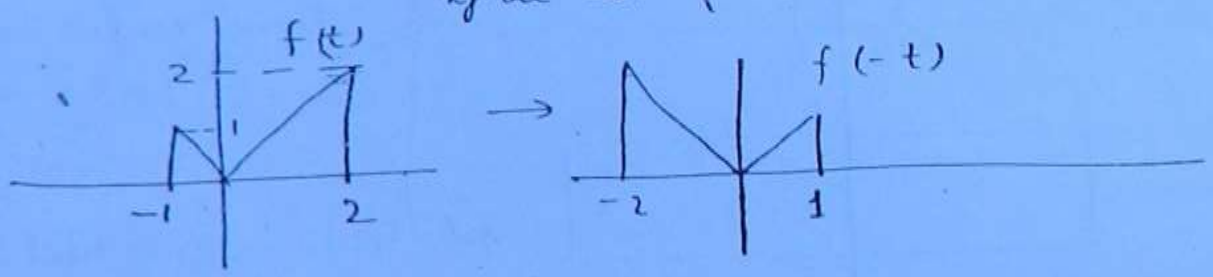
$\quad\quad 0 \quad$ otherwise

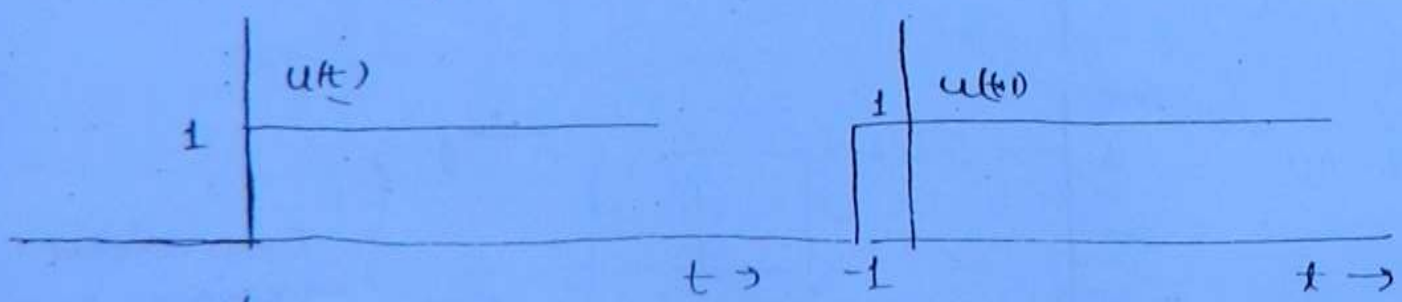
$f(-t) = \quad -1 \leq -t \leq 2$

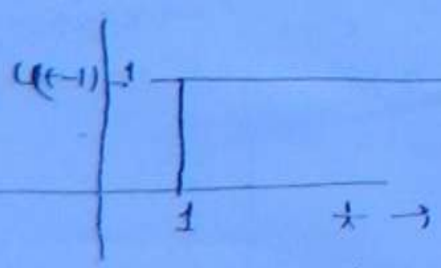
$\quad\quad = \quad 1 \geq t \geq -2$

$\quad\quad 0$ otherwise

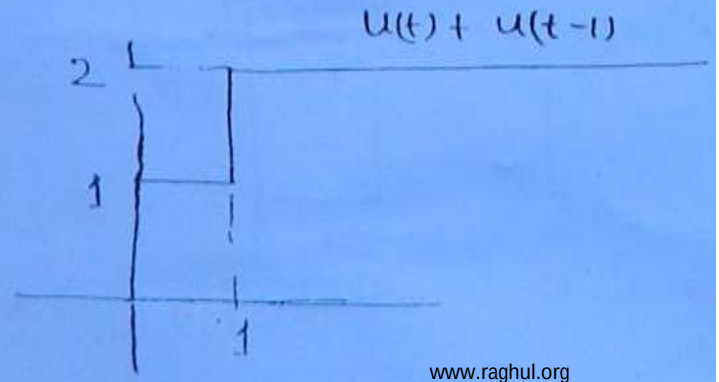
obtained by rotating signal across Y axix/by 180° or by taking mirror image of signal abt Y-axis.



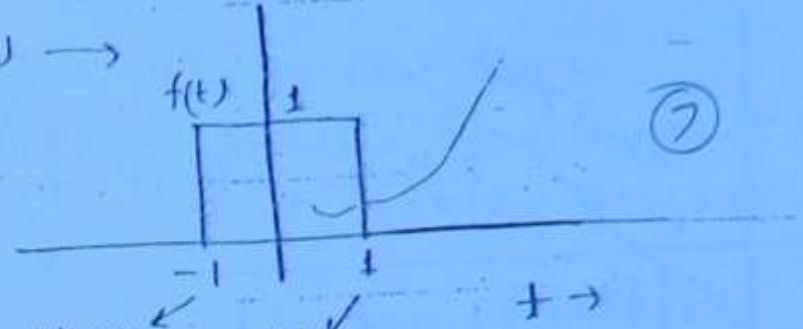




$u(t) + (u(t-1)) \rightarrow$



$f(t) = u(t+1) - u(t-1) \longrightarrow$

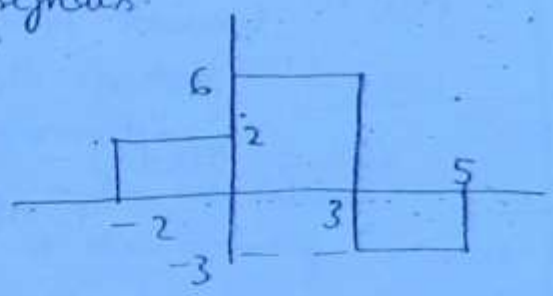


7

$u(t+1) - u(t-1)$

$u(t) \rightarrow$ indicates change in signal value from 0 to 1 exactly at $t=0$

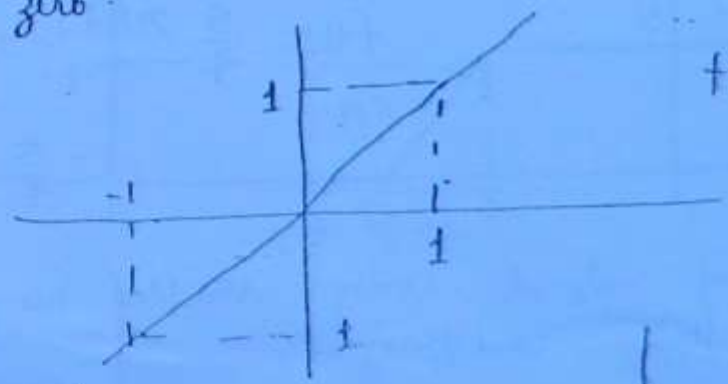
Q. Represent the following signal using shifted unit step signals.



$2u(t+2) + 4u(t) - 9u(t-3) + 3u(t-5)$

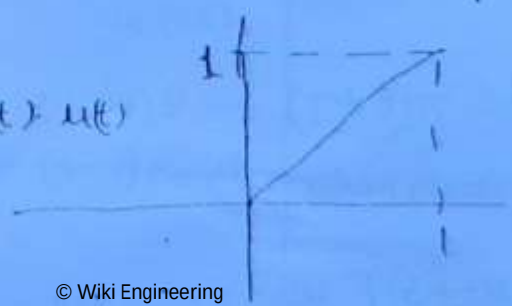
as long as signal is nonzero only for a finite interval of time then some of cofficient will be zero as in above eg → $2+4-9+3=0$

(*) A signal having step changes in finite interval time then some of all cofficient of shifted unit step signal will be zero.

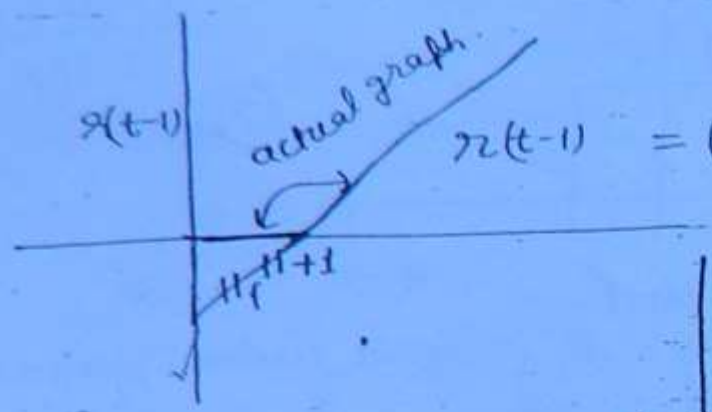


$f(t) = t$ [not ramp signal]

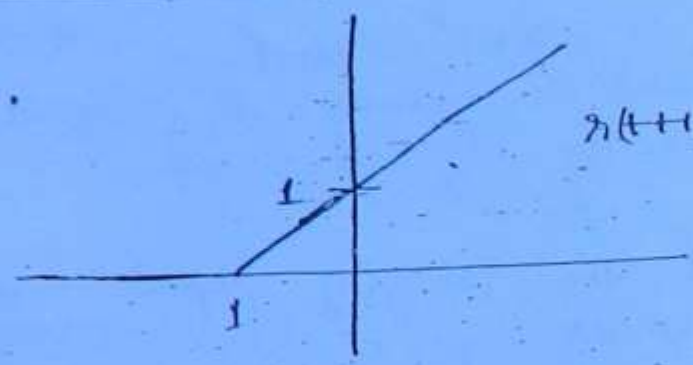
$r(t) \cdot u(t)$



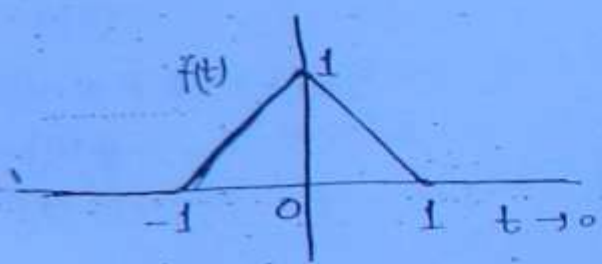
$f(t) = t \quad 0 \le t \le \infty$ or $t \ge 0$ [illegible]



$r(t-1) = (t-1)$ for $t \geq 1$
0 $t < 1$



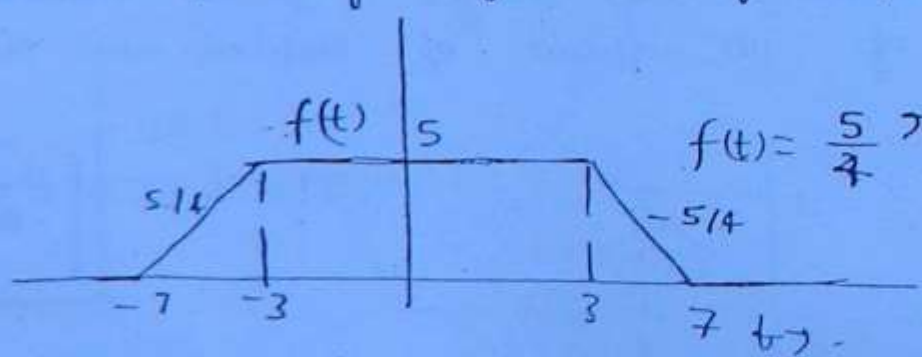
$r(t+1) = (t+1)$ for $t \geq -1$
0 otherwise



$f(t) = r(t+1) - 2\,r(t) + r(t-1)$

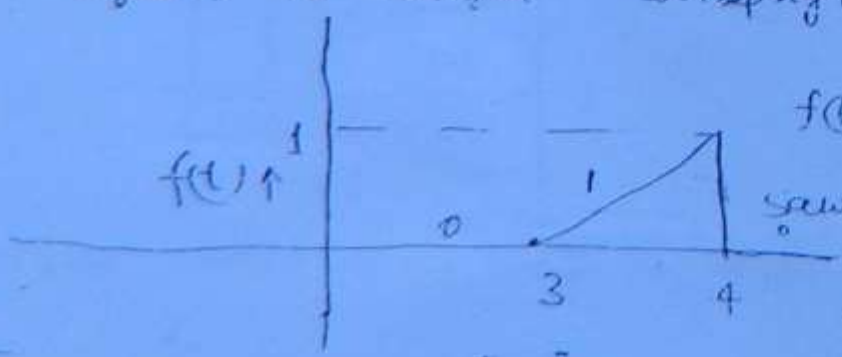
$r(t)$ represent change in slope at $t=0$ from $(0 \to 1)$ slope
if signal is finite interval signal some of coefficients will be zero → eg $1-2+1=0$

Q. Represent following signal using shifted ramp signal.



$f(t) = \frac{5}{4} r(t+7) - \frac{5}{4} r(t+3) - \frac{5}{4} r(t-3) + \frac{5}{4} r(t-7)$

Q. represent following signal using shifted ramp signals and shifted unit step signals.



$f(t) = r(t-3) - u(t-4) + r(t-4)$

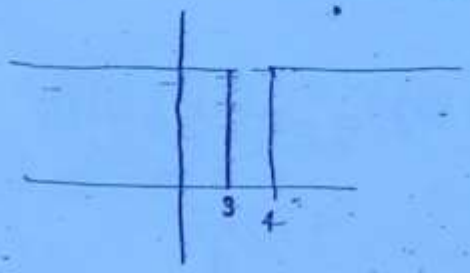
sawtooth pulse

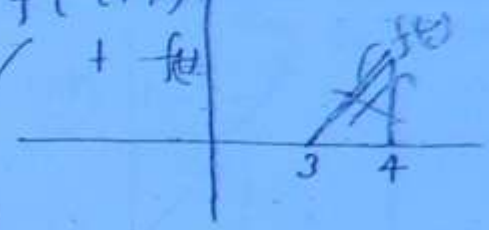
$r(t-3)[u(t-3) - u(t-4)]$

(*) for the above signal $f(t) + f(-t+7)$ will be

$$f(t) + f(-t+7) = r(t-3) - r(t-4) - u(t-4) + r(-t+4) - r(-t+3) - u(-t+3)$$

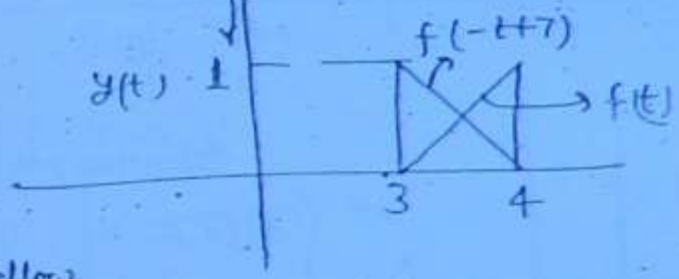
$$= y(t) = f(-t+7) + f(t)$$

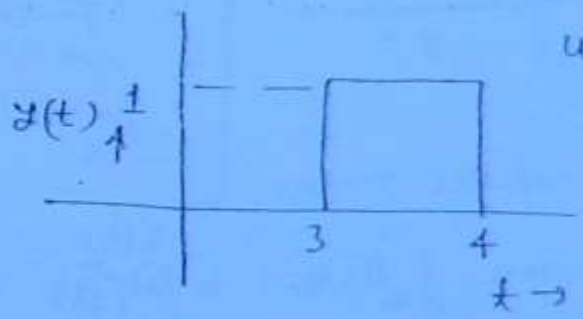




If shifting and reversal operation both involved in given transformation the normal order we follow shifting first and then reversing but we can also do this process in reverse order.

$f(-t+7)$





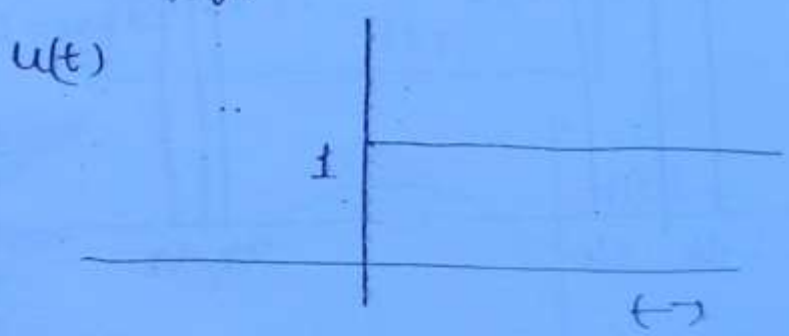
$f(t) \xrightarrow{\text{reversal}} f(-t)$

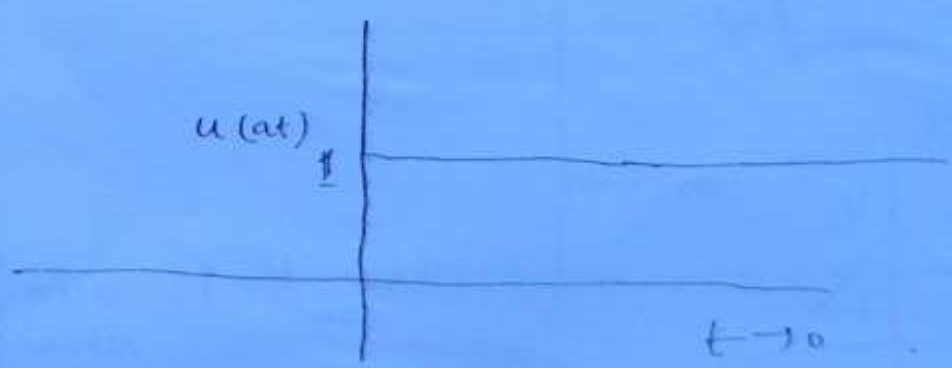
shift $\xrightarrow[\text{delayed}]{\text{by } T}$ $f(-t+7)$

$f(\ominus(t-7))$

$f(at+b)$

$f(a[t+b/a])$

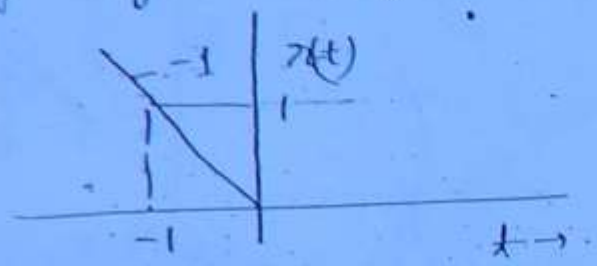




$r_1(at) = \begin{cases} at & at \geq 0 \\ 0 & at < 0 \end{cases} \longrightarrow \{a > 0\}$

$r(at) = a\, r(t)$

* in ramp signal scaling of time results in scaling of magnitude (amplitude).



$r(-t) = \begin{cases} -t & t \leq 0 \\ 0 & t > 0 \end{cases}$

definition of delta function

$$\begin{cases} \delta(t) = 0 & t \neq 0 \\ \quad \neq 0 & t = 0 \\ \int_{-\infty}^{\infty} \delta(t)\, dt = 1 \end{cases}$$

also called dirac delta function.

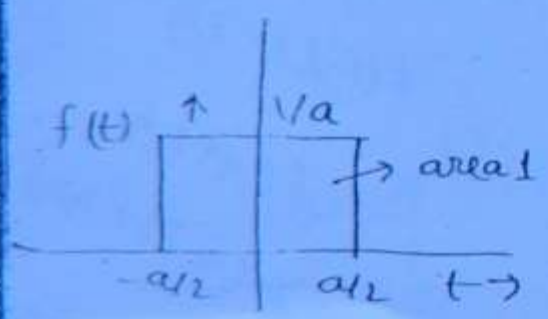
unit impulse means

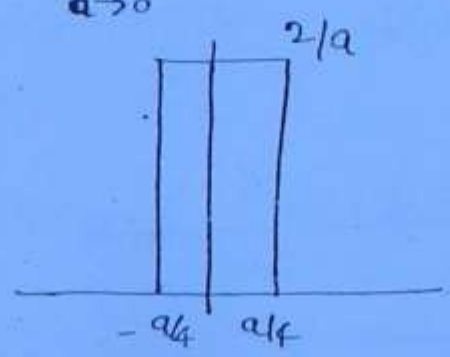
↓

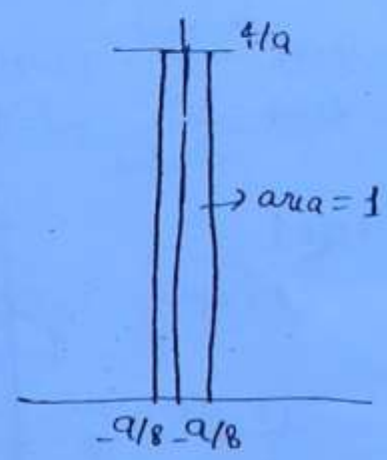
means area $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

$\delta(t) = \lim_{a \to 0} f(t)$

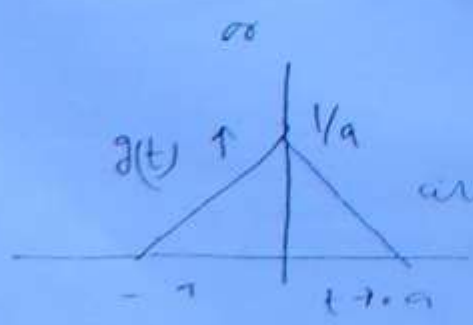
$= \lim_{a \to 0} f''(t)$







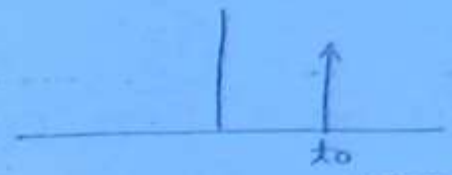
$a \to 0$ height $\to \infty$



area = 1 $\quad a \downarrow \quad 1/a \uparrow$ area remain same

$\delta(t) = \lim_{a \to 0} g(t)$

$\delta(t-t_0) = \lim_{a\to 0} f(t-t_0) =$



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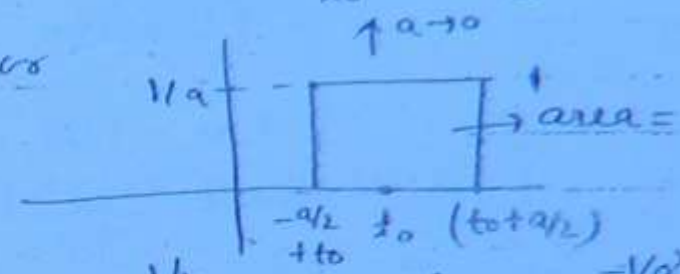
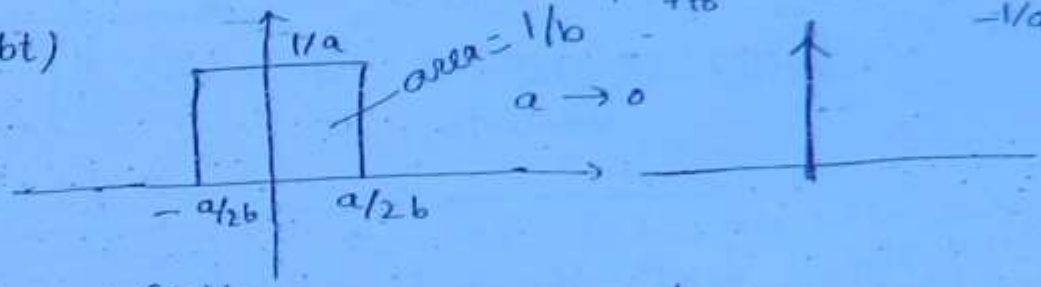
or



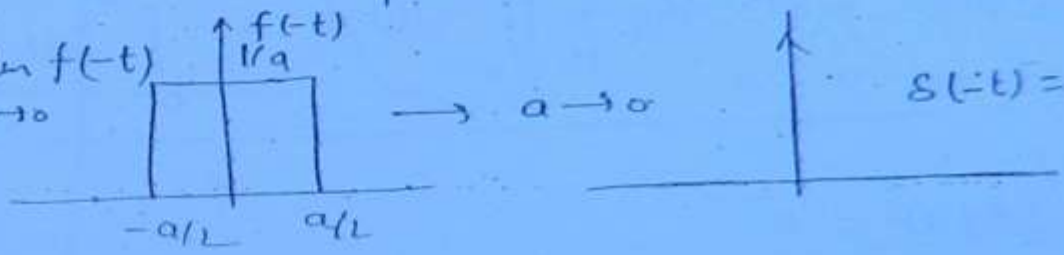
$\delta(bt) = \lim_{a\to 0} f(bt)$

$= \frac{1}{b}\delta(t)$

$= 1/b\, \delta(t)$



$\delta(-t) = \lim_{a\to 0} f(-t)$

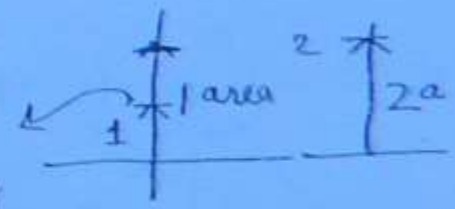


$\delta(-t) = \delta(t)$

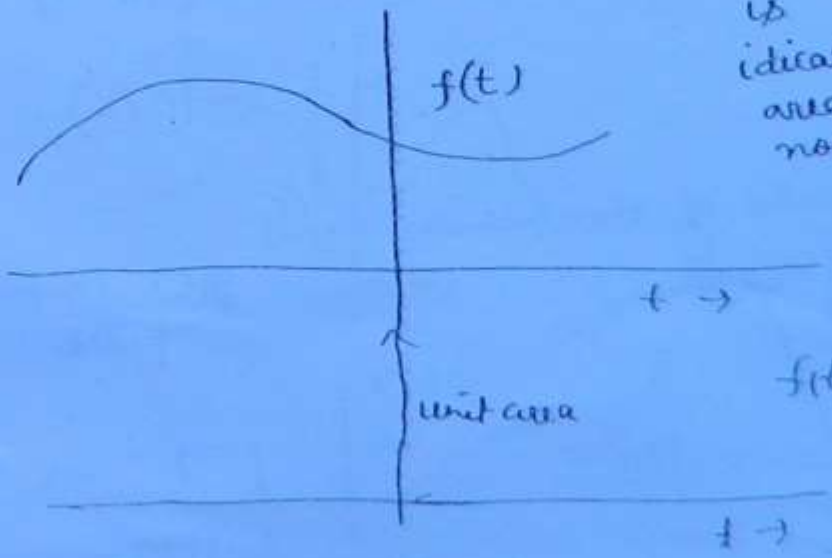
$\delta(-2t) = \frac{1}{2}\delta(t)$

$\delta(-2t) = \delta(2t) = 1/2\, \delta(t)$

$$\boxed{\delta(bt) = \frac{1}{|b|}\delta(t)}$$



height is idicating area not the value



$f(t)\,\delta(t)$

$f(t)\cdot\delta(t) = f(0)\cdot\delta(t) = \delta(t)$ (If $f(0)>0$, area = 1 i.e. line

(12) → amplitude ∞ but effect on area area will become $= f(a)$ b/w ($-\infty$ to ∞) only

$$f(t)\,\delta(t-a) = f(a)\cdot\delta(t-a)$$
$$= f(a)\,\delta(t)$$

sampling property of impulse signal.

$$\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = f(0)\int_{-\infty}^{\infty}\delta(t)\,dt = f(0)$$

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-a)\,dt = f(a) = \int_{-\infty}^{\infty} f(a)\,\delta(t-a)\,dt$$
$$= f(a)\int_{-\infty}^{\infty}\delta(t-a)\,dt = f(a)$$

$$\boxed{\int_{-\infty}^{\infty}\delta(t)\,dt = \int_{-1}^{1}\delta(t)\,dt = \int_{-a}^{a}\delta(t)\,dt = \int_{0^-}^{0^+}\delta(t) = 1}$$

$$\int_{1}^{2}\delta(t)\,dt = 0$$

calculate following → (i) $\int_{-\infty}^{\infty} \cancel{\delta(t-}\ \sin(t-\pi)\,\delta(t-\pi/2)\,dt$

$$= \sin(\pi/2-\pi) = \sin(-\pi/2)$$
$$= -\sin\pi/2 = -1$$

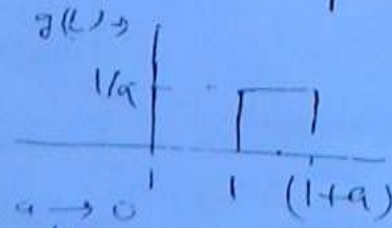
Q $\int_{-\infty}^{\infty}\sin(t-\pi)\,\delta(3t-\pi/2)\,dt = \frac{1}{3}\sin\left(\pi/6-\pi\right)$

$$= +\frac{1}{3}\sin(-\pi/6)$$
$$= -1/6$$

Q-(i) Find the value of the following integral $\int_{-\infty}^{\infty}\frac{\sin(\pi t-\pi/2)}{(t^2+4)}\,g(t)\,dt$ where $g(t)$ is following pulse -

(ii)

$$= \frac{\sin\pi/2}{5} = 1/5$$



as $a \to 0$

as $a \to 0$

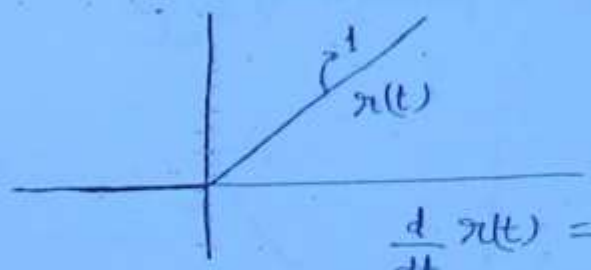
$= \cancel{(1+a/2)}\ \delta(t-1)$

15th oct 10

differentiation →

$f(t)$, $\frac{df(t)}{dt}$

(13)

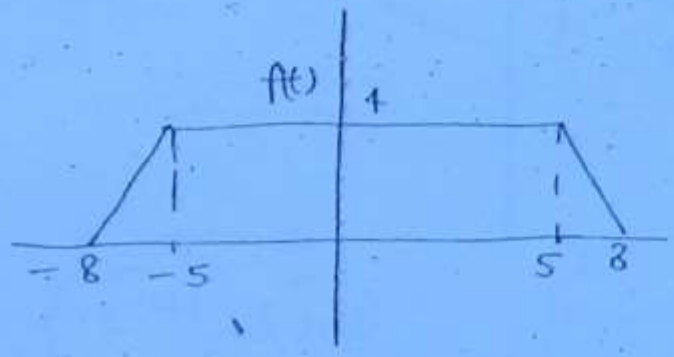


$f(t) = mt + c$

$\frac{df(t)}{dt} = m$

$\frac{d}{dt} r(t) = 1 \quad 0 \le t \le 1 \quad = u(t).$

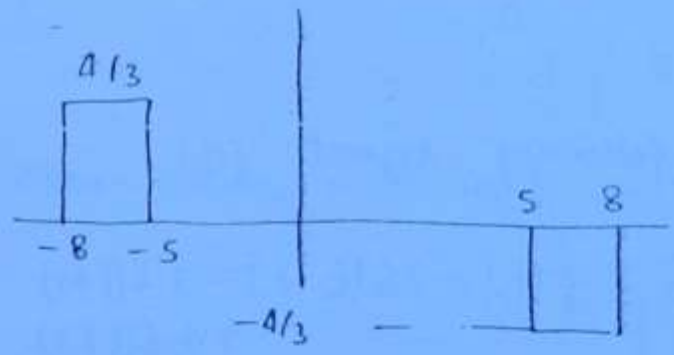
$\quad\quad\quad\quad 0 \quad$ otherwise



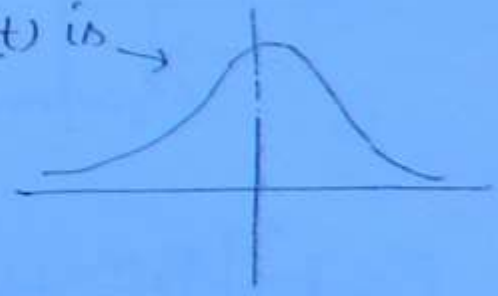
as $f(t) = 4/3\, r(t+8) - 4/3\, r(t+5) - 4/3\, r(t-5) + 4/3$

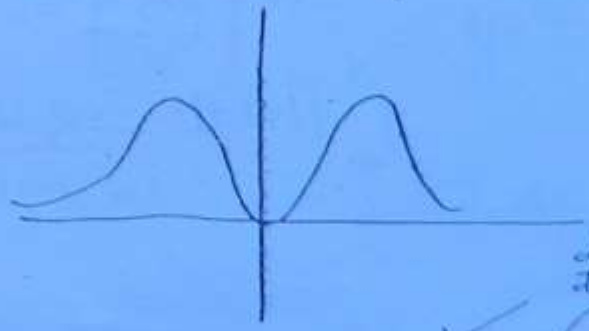
$\frac{d}{dt} f(t) = 4/3 - 4/3 - 4/3 + 4/3$

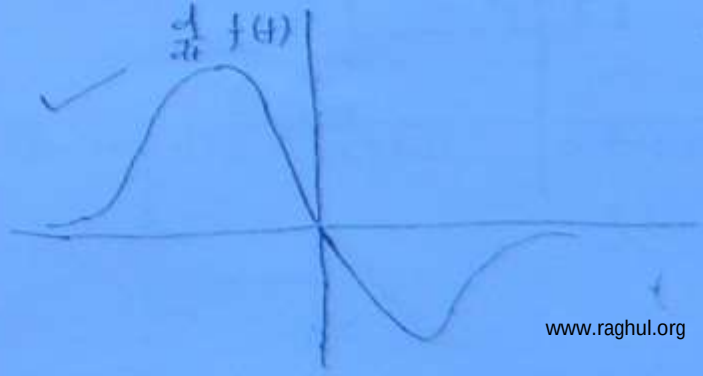
$= 0$ for $t \geq 8$,



Q derivative of the following signal $f(t)$ is →







$$\int_{-\infty}^{\infty} f(t)\,dt = \overset{\text{area}}{K} \text{ (scalar} \rightarrow \text{may be finite or infinite)}$$

$$\int_{-\infty}^{t} f(t)\,dt = g(t) \quad \text{area as a function of time}$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

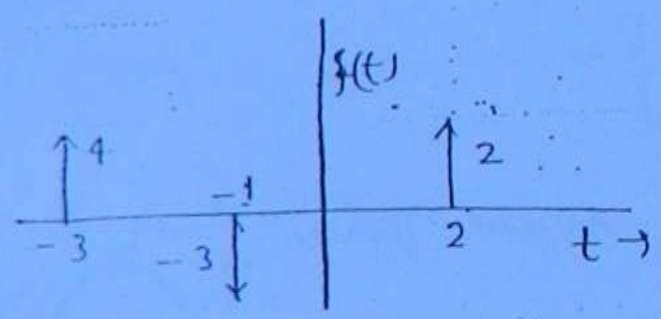
$$\int_{-\infty}^{t} \delta(\tau)\,dt = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$$

$$= u(t)$$

$$\frac{d}{dt} u(t) = \delta(t)$$

$$r(t) = \int_{-\infty}^{t} u(t)\,dt$$

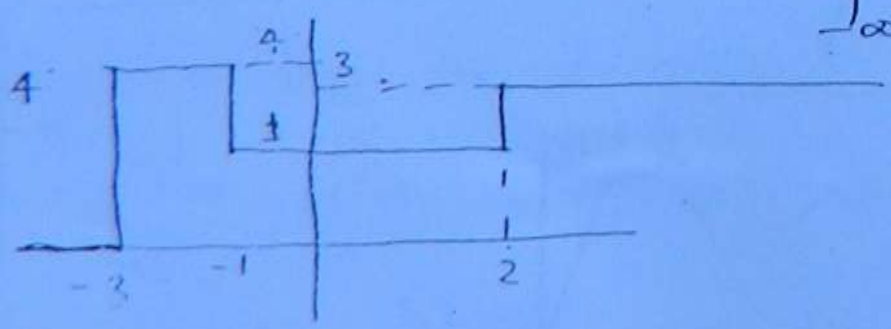
Calculate the integral of following signal $f(t)$.



$$f(t) = 2\delta(t-2) - 3\,\delta(t+1) + 4\,\delta(t+3)$$

$$\int_{-\infty}^{\infty} f(t) = 3$$

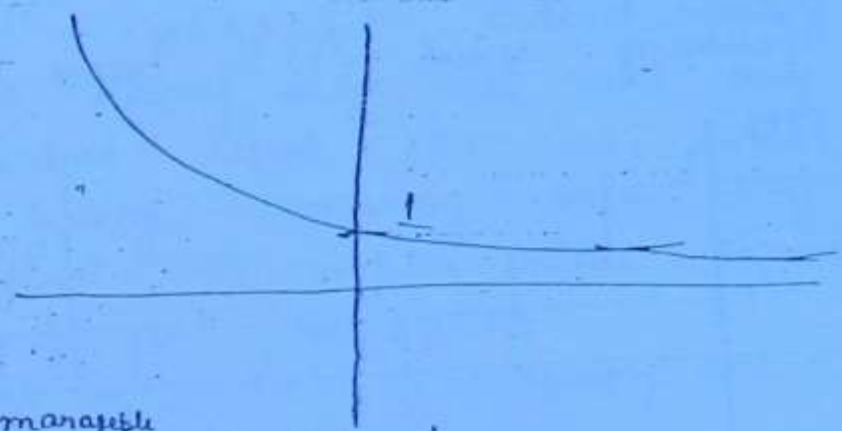
$$\int_{-\infty}^{t} f(\tau)\,dt = 2\,u(t-2) - 3\,u(t+1) + 4\,u(t+3)$$



$$\frac{d}{dt} e^{-at} = -a e^{-at} \quad , \quad \frac{d}{dt} e^{at} = a e^{at}$$

$$\int_{\infty}^{t} e^{-a\tau} d\tau = \left[\frac{e^{-a\tau}}{-a}\right]_{-\infty}^{t} = \frac{1}{a}\left[e^{-at} - \infty\right]$$

= not defined
unmanageable.



$$\int_{-\infty}^{t} \overbrace{e^{-at} u(t)}^{\text{manageble}} dt = \int_{0}^{t} e^{-at} dt$$

$$= -\frac{1}{a}\left[e^{-at} - 1\right]$$

area under $e^{-at} u(t) = \int_{-\infty}^{\infty} e^{-at} u(t) dt$

$$= \int_{0}^{\infty} e^{-at} u(t) dt$$

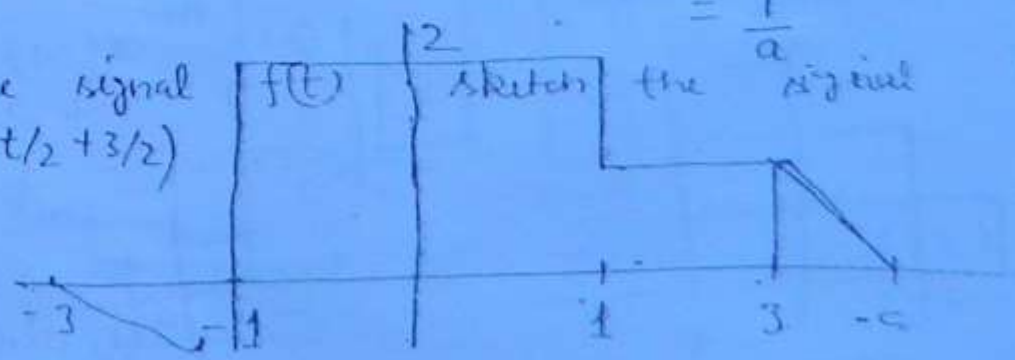
$$= +1/a$$

$$\int_{-\infty}^{t} e^{at} dt = \left[\frac{e^{at}}{a}\right]_{-\infty}^{t} = \frac{1}{a}\left[e^{at}\right] \text{ unmanagble}$$

$$\int_{-\infty}^{t} \overbrace{e^{at} u(-t)}^{\text{manageble}} dt = \int_{-\infty}^{0} e^{at} dt = \frac{1}{a}\left[e^{at}\right]_{-\infty}^{0}$$

$$= \frac{1}{a}$$

for above signal $f(t)$ sketch the signal $f(-t/2 + 3/2)$

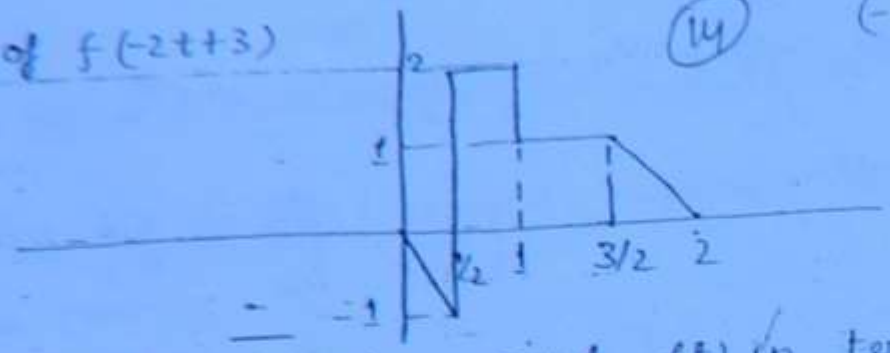


graph of $f(-2t+3)$

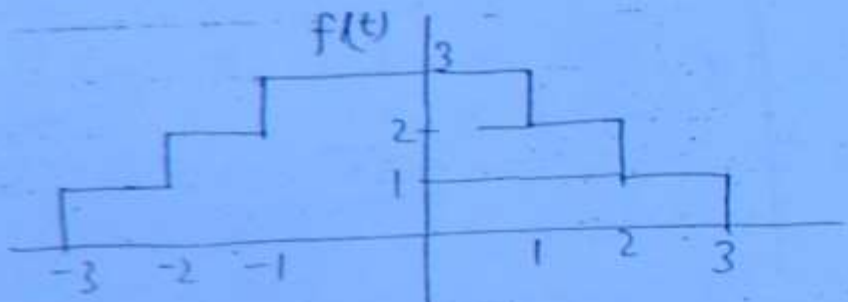
(14)

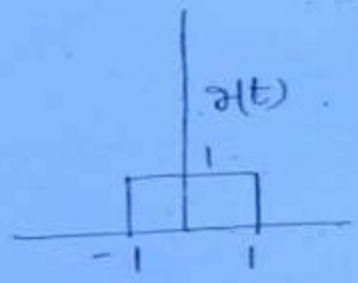
$(-2t+3) = t_1$

$t = -t_1/2 + 3/2$



Q. Represent the following signal $f(t)$ in terms of $g(t)$





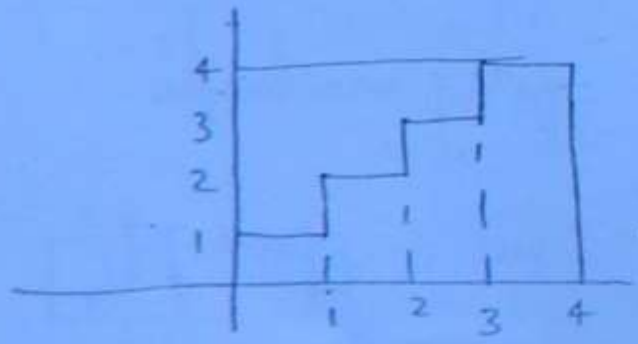
wrong

$$f(t) = \{ g(t+2) + g(t+1) + g(t) + g(t-2) - g(t-3) - g(t-4) \}$$

$$f(t) = g(t/3) + g(t/2) + g(t)$$

Q. Represent the following signal $f(t)$ in terms of $g(t)$

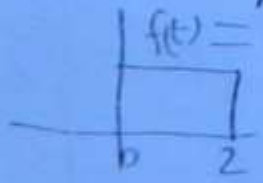
$g(t)$ is same in previous question

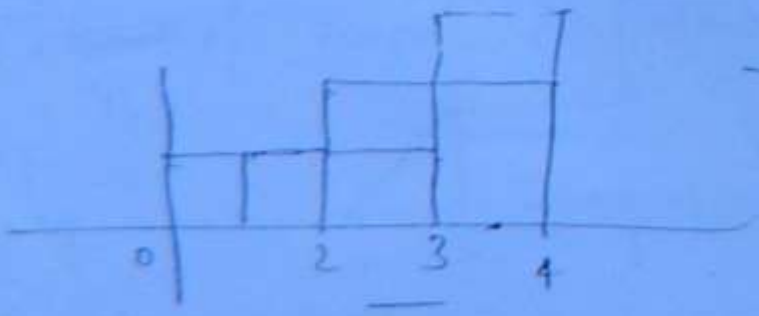


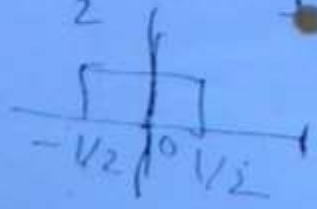
$$g(t-1) + g(t-2) + 2g(t-3) + \cancel{2g(t-4)} + 2g[2(t-7/2)]$$

Ans -

$$f(t) = g(t-1) + g(t-2) + 2g(t-3) + 2g[2t-7]$$







Types of signal →

can be real value signal or complex value signal

$f(t) = at$ real valued

$f(t) = at + ibt$ complex valued

$e^{jt} = \cos t + j\sin t$ Euler's identity

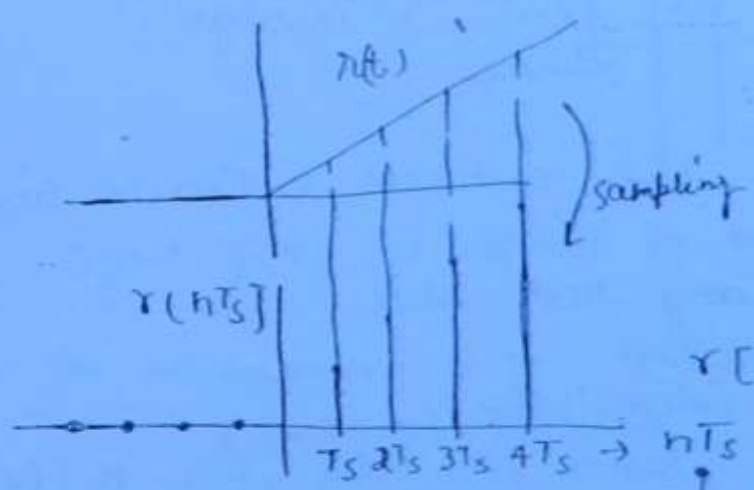
* Signals having only real value they are called as real valued signal $f(t) = \cos t, \sin t$, real valued signal
* Signals having imaginary value along with real value are defined as complex valued signal.

eg. $e^{jt} = \cos t + j\sin t$

real part $= \cos t$, imaginary part $= \sin t$

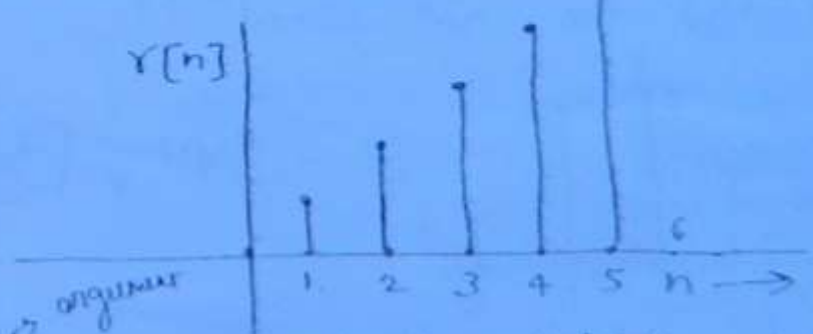
Continuous time & discrete time signal →

(1) Continuous time signals are those signal having def value for every real value of time.



discrete time signals are those signals having defined value at some fixed instant and at other instant undefined.

$r[nTs]$ take $Ts = 1$



$f[n]$ → n must be integer, $f(t)$ → any real value

argument

discrete time signal

$$f[n] = \{-1, \underset{\uparrow}{\overset{n=0}{1}}, 2, -2\}$$

$-1 \Rightarrow n=-1$
$1 \quad n=0$
$2 \quad n=1$
$-2 \quad n=2$
0 otherwise other values of n.

* Continuous time signal is that signal which is defined for all values of time.
* A discrete time signal is a signal which is defined only for specific values of time. It is not defined for other values of time. a discrete time signal is derived from a continuous time signal by a procedure called as uniform sampling and then selecting uniform sampling interval value to be 1.

$f(t) \to f[nT_s] \to T_s = 1 \to f(n)$ n is integer.
continuous discrete

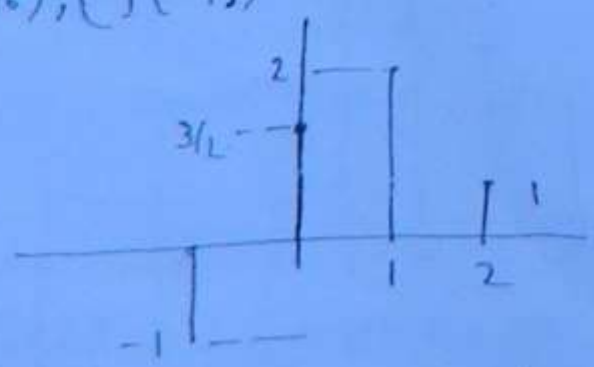
Fundamental difference b/w continuous & discrete time signal is

$f(t)$
↳ t can be any real value

$f(n)$
↳ $n \to$ only integer value.

* For a discrete time signal $f(n)$, value like $f(4/3)$, $f(5/6)$, & $f(-2/3)$ are not defined.

an eg →

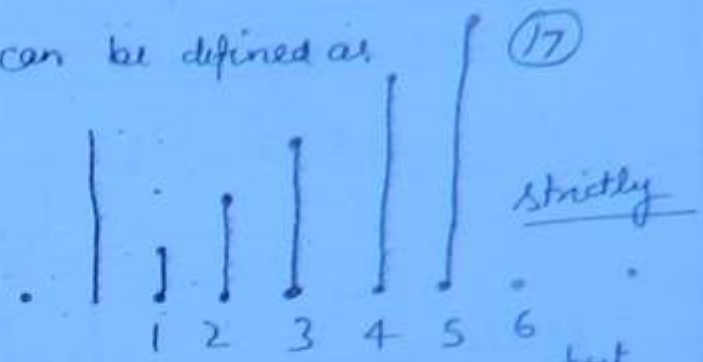


$$f(n) = \{-1, \overset{n=0}{3/2}, 2, 1\}$$

discrete time ramp signal can be defined as

$r(n) = n \quad n \geq 0$

$\quad\quad 0 \quad$ otherwise

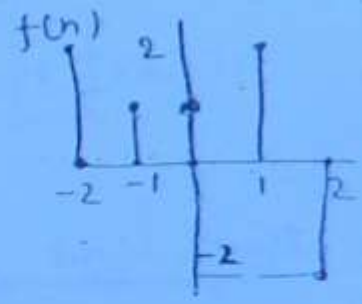


strictly

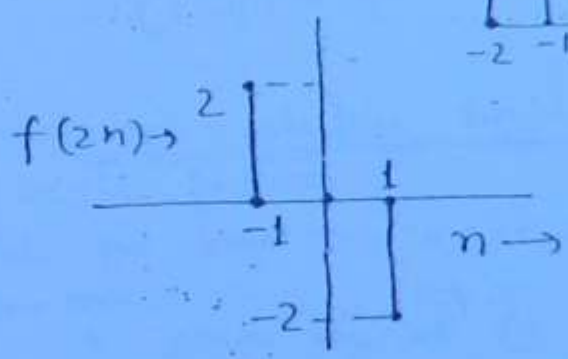
- n-axis discrete amplitude axis is continuous.

but we use it for saren continu

$f(n) \xrightarrow{n \to n-3} f(n-3)$



$f(2n) \longrightarrow$



Q. Signal $f(n)$ is defined to be $f(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

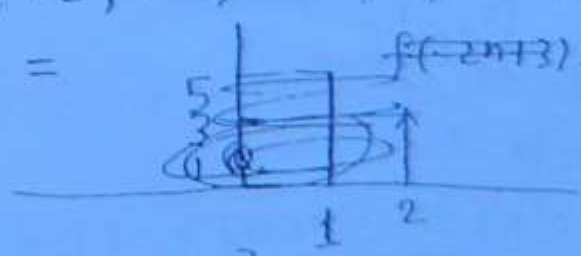
for what values of n will the signal

$f[-2n+3]$ be zero

$f[-2(n-3/2)]$

at $n = 0, -2, -4, \ldots, +2, 4 \ldots$

$f[-2n+3] =$



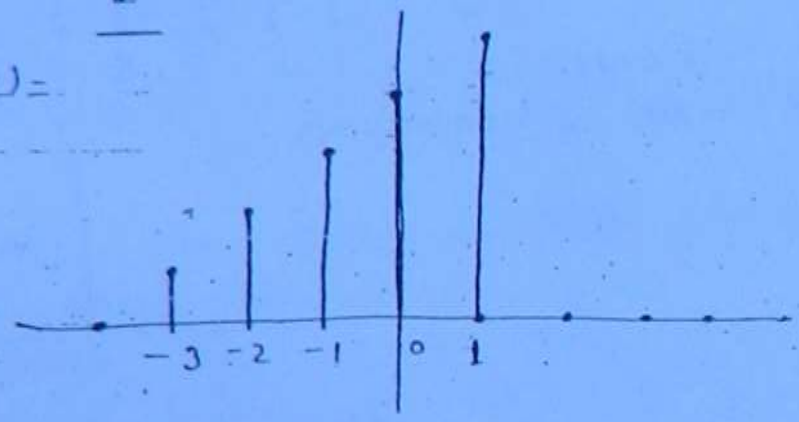
~~$f(-2n+3) = \{-1, 5, 3\}$~~

$f(-2n+3) = \{\underset{\uparrow}{0}, 5, 3, 1\}$ 0 for $n \leq 0$ and 0 $n > 3$

discrete time unit step signal :→

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$f(n) =$



$$f(n) = u[n+3] + u[n+2] + u[n+1] + u[n] + u[n-1] - 5\,u[n-2].$$

discrete time unit impulse function →

area of $\delta(t) = 1$

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

→ ~~real~~ by defining value 1 we are making it manageble.

17th oct 10:

$$\delta[n] = u[n] - u[n-1]$$

$$\delta(t) = \frac{du(t)}{dt}$$

$$u[n] = \sum_{0}^{\infty} \delta[n-K]$$

$$u(t) = \int_{-\infty}^{t} \delta(t)\,dt$$

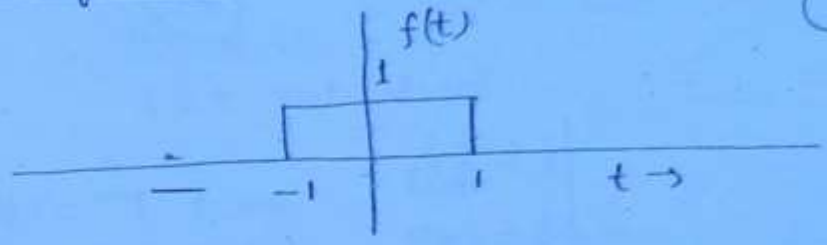
$$u[n] = \sum_{K=-\infty}^{n} \delta[K]$$

~~r[n]~~ =

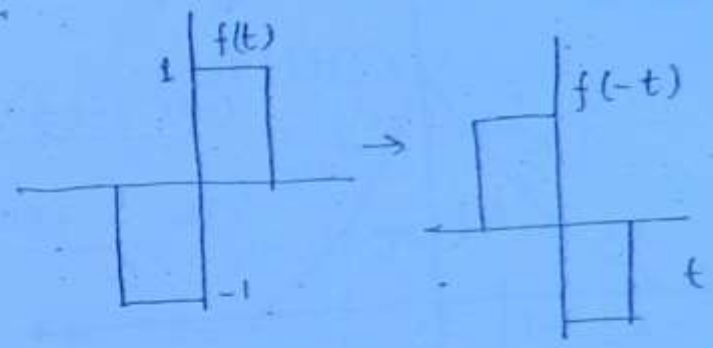
$$\boxed{u[n] = r[n] - r[n-1]}$$

$$r(n) = \sum_{-\infty}^{n} u[K]$$

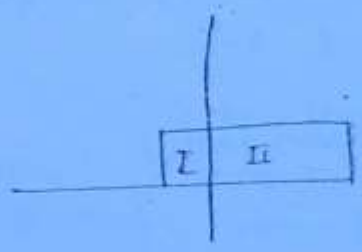
Even signal or odd signals :



$f(-t) = f(t)$ even signal.
or symmetrical signals



$= -f(t)$
odd signal
or antisymmetric signal.

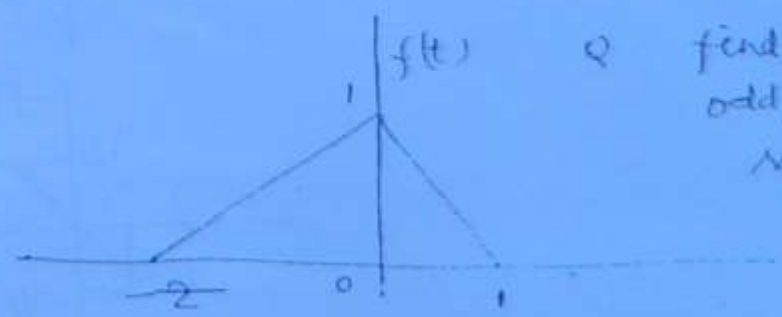


neither even nor odd signal.

$f(t) + f(-t) \rightarrow$ even signal (always)
$f(t) - f(-t) \rightarrow$ odd signal.

$$f(t) = \frac{1}{2}[\underbrace{\text{even signal}}_{\text{even part of } f(t)} + \underbrace{\text{odd signal}}_{\text{odd part of } f(t)}]$$

$\cos(t) = \cos(-t) \rightarrow$ even signal or symmetrical signal
$\sin(t) = -\sin(-t) \rightarrow$ odd signal or antisymmetrical sign
(opposite symmet

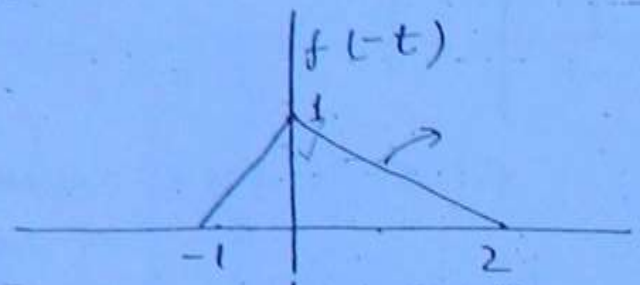


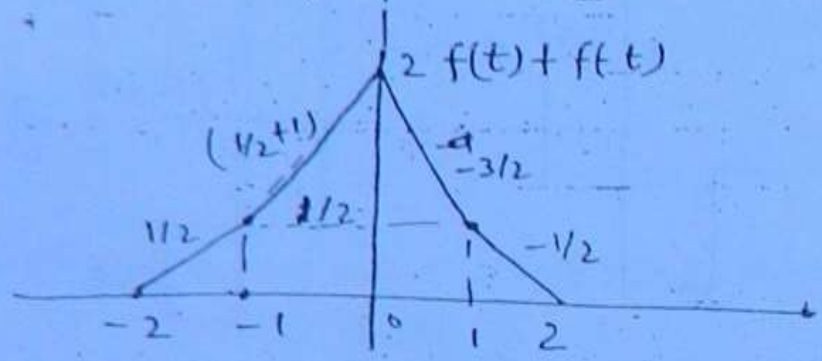
Q find the even and odd part of following signal f(t).

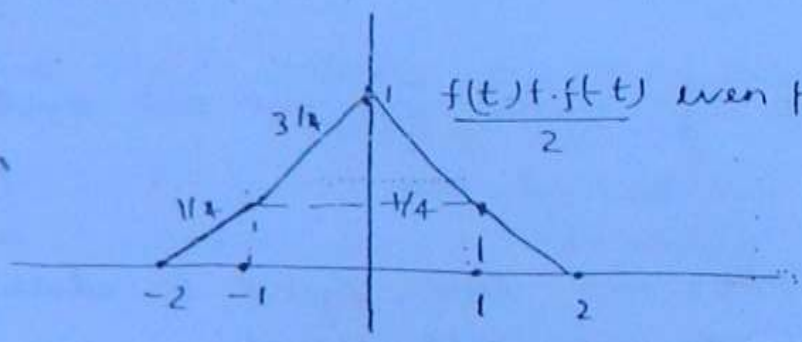
10

even part $= \frac{1}{2}[f(t) + f(-t)]$ (20)

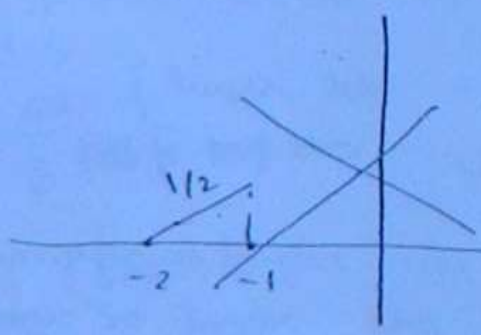
$=$

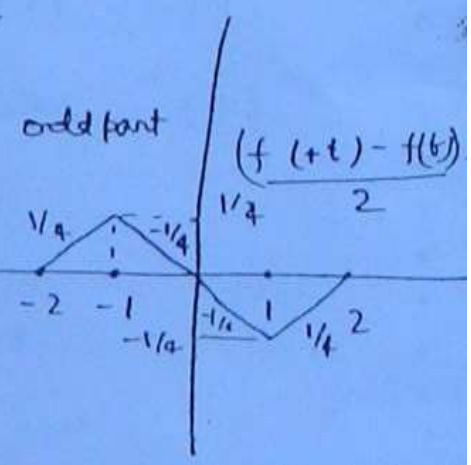


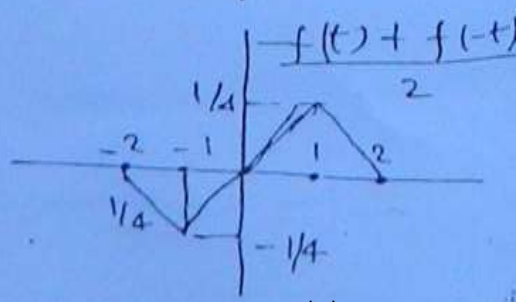




odd part



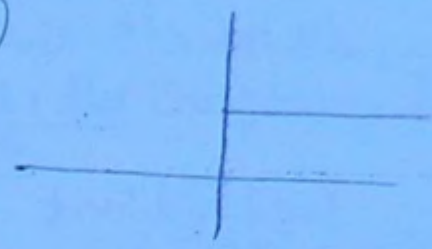




$\frac{1}{2}[u(t) + u(t)]$

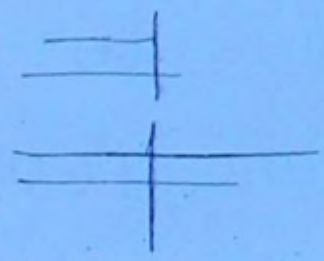
(21)

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \\ \text{not defined} & t = 0 \end{cases}$$



$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \\ 1/2 & t = 0 \end{cases}$$

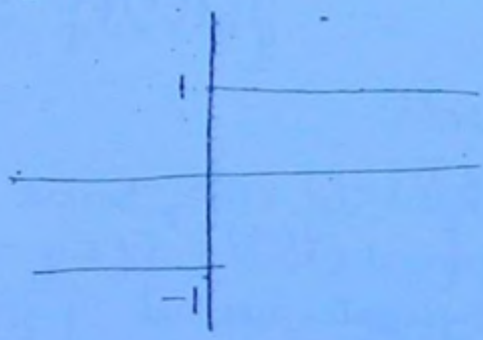
take any definition



$$u_e(t) = \frac{u(t) + u(t)}{2} = 1/2$$

$$u_o(t) = \frac{u(t) - u(t)}{2} = \frac{1}{2}\,\mathrm{sgn}(t)$$

$$\mathrm{sgn}(t) = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \\ \text{undefined} & t = 0 \end{cases}$$



four definition any we can take

or 1

or -1

or = 0

Q. find even and odd part of $f(t) = \sin(t)\,u(t)$

$f(t) = e^{jt} = \cos t + j\sin t$

$f_e(t) = \dfrac{e^{jt} + e^{-jt}}{2} = \cos t$

$f_o(t) = j\sin t$

$f(t), f^*(t)$

* $f(t) = f^*(-t)$ even conjugate signal
  conjugate symmetric signal

$f(t) = -f^*(-t)$ odd conjugate signal
  or conjugate antisymmetric signal

$e^{jt} \rightarrow$ even conjugate signal

$f(t) = f^*(-t)$

$= [e^{j(-t)}]^* = e^{(-j)(t)} = e^{jt}$

je^{jt}

$= -f^*(-t)$

$-[je^{j(-t)}]^* = -[-j\,e^{jt}] = je^{jt}$

odd conjugate signal

$f(t) = e^{jt} = \underbrace{\cos t}_{\text{even}} + j\underbrace{\sin t}_{\text{odd}}$

* for an even conjugate signal

Real part $[f(t)]$ = even part of $f(t)$ → even in nature

odd part $[f(t)]$ = ~~odd~~ imaginary part of $f(t)$

$je^{jt} = \underbrace{-\sin t}_{\text{odd}} + j\underbrace{\cos t}_{\text{even}}$

(1) for a complex valued signal which is even conjugate in nature real part is always even and imaginary part is always odd.

(*) For a complex valued signal which is odd conjugate in nature real part is always odd and imaginary part is always even.

$f(t) = f_{ec}(t) + f_{oc}(t)$.

(23)

$f^*(t) = f^*_{ec}(t) + f^*_{oc}(t)$

$f^*(-t) = f^*_{ec}(t) + f^*_{oc}(-t)$

$= f_{ec}(t) - f_{oc}(t)$

$f_{ec}(t) = \frac{1}{2}[f(t) + f^*(-t)]$

$f_{oc}(t) = \frac{1}{2}[f(t) - f^*(-t)]$

J

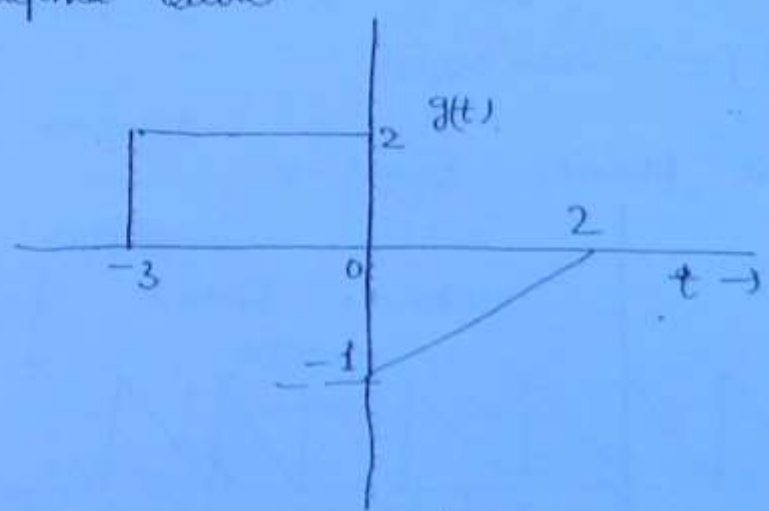
$\cos(t) + j$

$\sin(-t) + j$

$\sin(-t) - j$

$\cos t - j$

Q. A complex valued signal $f(t)$ is defined with a real part $\rightarrow [g(t) + g(-t)]$ and imaginary part which is $[h(t) + \{h(t)\}]$ where $g(t)$ & $h(t)$ are as defined below.

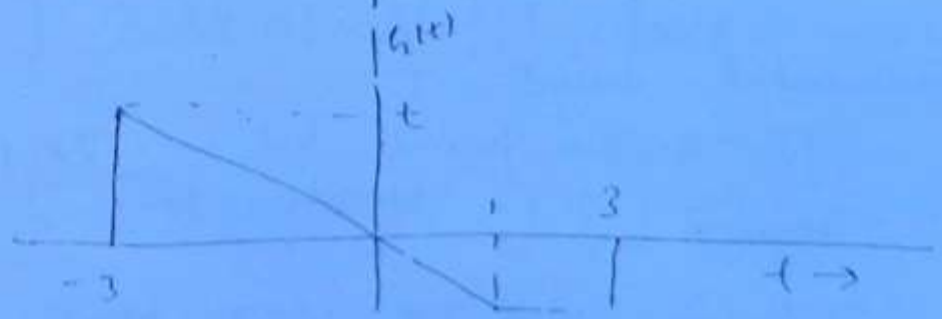


then (*) signal $f(t)$ is even conj

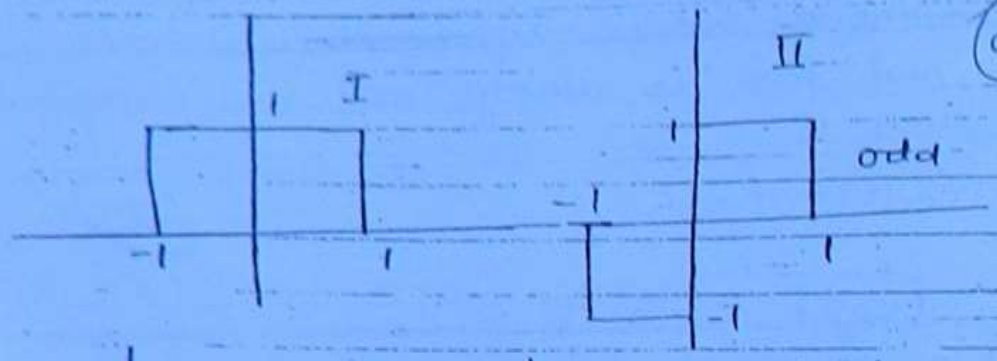
✓ odd conj

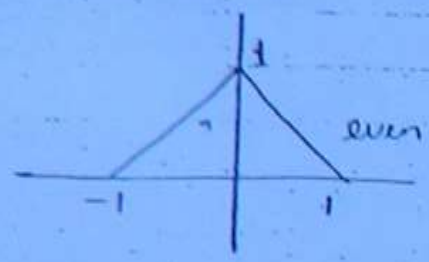
neither even nor odd

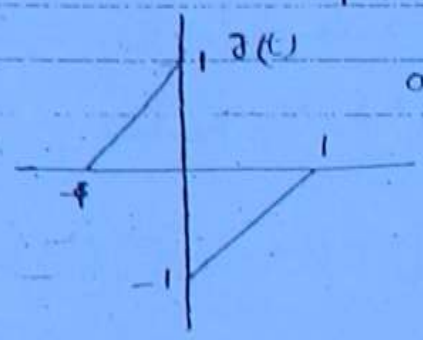
can't comment

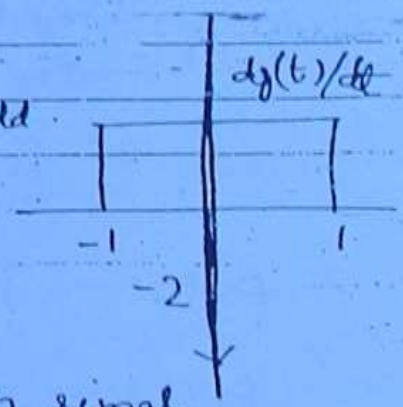


12









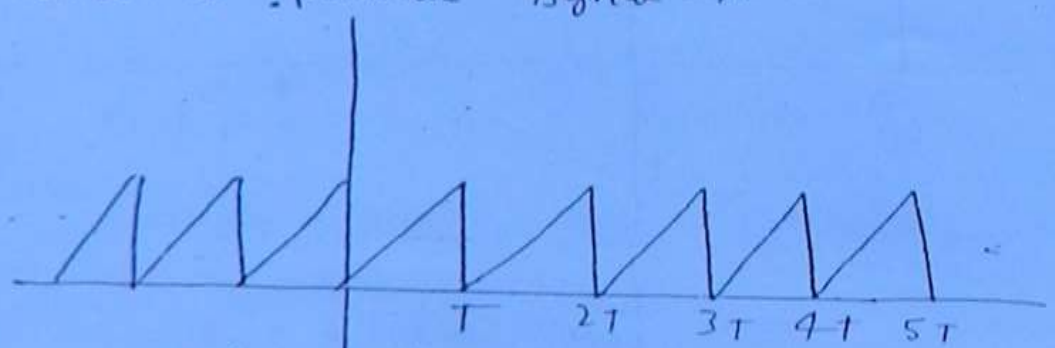
$$\int_{-a}^{a} f(t)\,dt = 2\int_{0}^{a} f(t)\,dt \quad \text{for even signal}$$

$$\int_{a}^{a} f(t)\,dt = 0 \quad \text{for odd signal}$$

$$\frac{d}{dt}[\text{even signal}] = \text{odd signal}$$

$$\frac{d}{dt}[\text{odd signal}] = \text{even signal}$$

periodic and a periodic signal →



fundamental period = T

kT → time period but not the fundamental period.

↓ integer

any shifting will not change period.

$$f(t) = f(t \pm KT)$$

$T \rightarrow$ fundamental period

$K \rightarrow$ integer

(25)

fundamental period of $\sin Kt \rightarrow \frac{2\pi}{|K|}$ (fundamental) $\rightarrow \frac{2\pi}{|K|} \times m$ (also a period)

K integer

m integer

$\downarrow$

Can be 2π.

$\left.\begin{array}{l} \sin t \\ \sin 2t \\ \sin 3t \\ \vdots \\ \sin Kt \end{array}\right\} \rightarrow$ all has period 2π (but this is not fundamental period for all except $\sin t$

$$\int_0^{2\pi} \sin Kt \, dt = 0 \quad \left(K \text{ complete cycle of period } \frac{2\pi}{|K|}\right)$$

so area $= 0$

K is integer.

$\sin \omega_0 t \longrightarrow \frac{2\pi}{\omega_0}$ fundamental period)

$\sin 2\omega_0 t \rightarrow \frac{2\pi}{2\omega_0} \rightarrow \pi/\omega_0$

$\sin 3\omega_0 t \longrightarrow \frac{2\pi}{3\omega_0}$

common period of $\frac{2\pi}{\omega_0}$

$$\int_0^{2\pi/\omega_0} \sin \omega_0 t \cdot \sin 2\omega_0 t \, dt$$

$$= \frac{1}{2}\int_0^{2\pi/\omega_0} (\cos \omega_0 t - \cos 3\omega_0 t) \, dt = 0$$

$$\int_0^{2\pi/\omega_0} \cos \omega_0 t \cos 2\omega_0 t = \frac{1}{2}\int_0^{2\pi/\omega_0} (\cos 3\omega_0 t + \cos \omega_0 t) \, dt$$

$$= 0$$

$$\int_0^{2\pi/\omega_0} \sin \omega_0 t \cos 2\omega_0 t \, dt = \frac{1}{2}\int_0^{2\pi/\omega_0} [\sin 3\omega_0 t - \sin \omega_0 t] \, dt$$

$$= 0$$

$\left.\begin{array}{l}\sin\omega_0 t \\ \cos\omega_0 t\end{array}\right\} 2\pi/\omega_0$

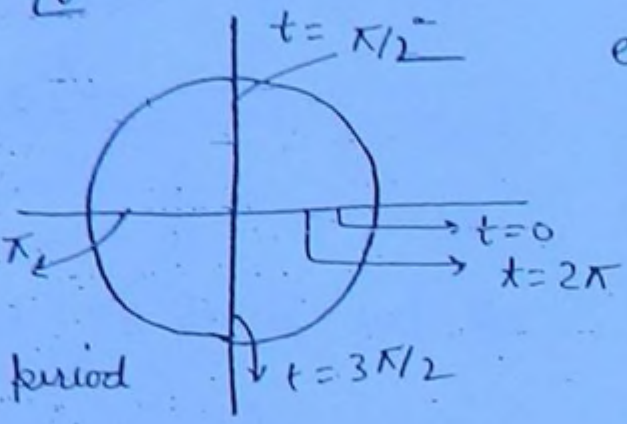
$e^{j\theta} = \cos\theta + j\sin\theta = 1\angle\tan^{-1}(\tan\theta)$

$= \angle\theta$

$e^{jt} = 1\angle t$

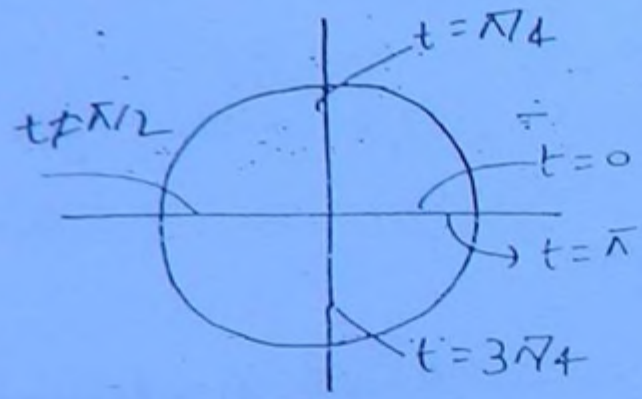
$e^{jt} \xrightarrow{\text{per}} 2\pi$

phasor quantity
covering the circumference of circle of unit radius and periodic with fundamental period 2π.



$e^{j(2t)}$

$= 1\angle 2t$



fundamental period $= \pi$

$e^{jKt} \rightarrow$ fundamental period $\frac{2\pi}{K}$

$e^{j\omega_0 t} \rightarrow \frac{2\pi}{\omega_0}$

$e^{-j\omega_0 t} \rightarrow \frac{2\pi}{\omega_0}$ rotate in opposite direction.

Minimum No. of samples taken to repeat itself is defined as the fundamental time period of a discrete time periodic signal.

- No. of samples is always a integer and hence time period of a discrete time signal is always an integer.

* For a discrete time complex exponential $e^{j\omega_0 n}$ to be period the condition is ratio $\frac{2\pi}{\omega_0}$ must be rational, if it is rational the period

$$N = m \cdot \frac{2\pi}{\omega_0}$$

where m is selected to be a minimum possible integer such that above product is an integer.

$$e^{j\omega_0 n} \rightarrow \omega_0 \rightarrow (\omega_0 + 2\pi K)$$

$\hookrightarrow$ integer.

* A discrete time complex exponential signal there is no change in signal even if ω_0 is replaced by $(\omega_0 + 2\pi K)$ i.e. discrete time complex exponential signals the frequencies $\pi + 2\pi$, $\pi + 4\pi$, $\pi + 6\pi$ - - - so on, $\pi - 2\pi$, $\pi - 4\pi$ - - - so on, all denote the same discrete time complex exponential signal.

same as $\left.\begin{matrix} \cos\omega_0 n \\ \sin\omega_0 n \end{matrix}\right\} \rightarrow \gamma \left(\frac{2\pi}{\omega_0}\right)$ rational

integer

<u>Some of signals</u>

$$\underbrace{\sin t + \sin 2t}_{\text{over all period} = 2\pi}$$

$$\downarrow \qquad \downarrow$$

$$2\pi \qquad \pi$$

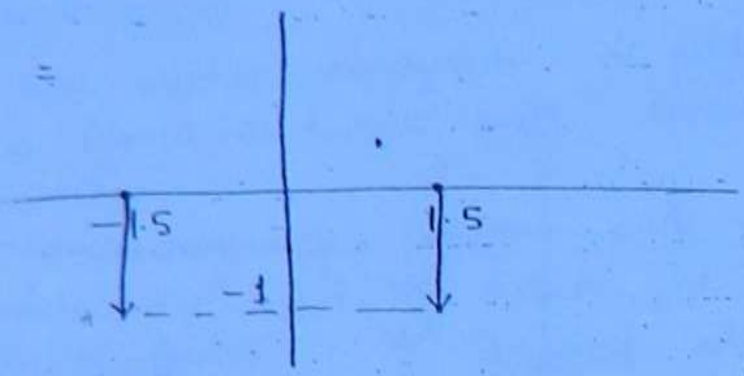
$$\frac{T_1}{T_2} = 2 \rightarrow \text{rational number}$$

$\frac{\sin t + \sin \pi t}{T_1 \quad T_2}$ $\quad \therefore T_1 = \pi$ not rational

Repeat the above problem $g(t)$ is defined as

$$g(t) = f(t)\,[-\delta(t-1.5) + \delta(t+1.5)]$$

then $g(t) =$



Q if the signal

$$f(t) = 3 + 4\sin(\pi/3\, t + \pi/2) + 6\cos(\pi/4\, t + \pi/3)$$

Power, signal, calculate the power.

$$= 9 + \frac{4^2}{2} + \frac{6^2}{2} = 35\,W$$

Q If signal $f(t) = e^{j2t}$ energy signal or power signal

power signal periodic signal

Power = 1

in calculation of power or energy we concentrate on amplitude not on phase.

in case of discrete time signals

$$E = \sum_{n=-\infty}^{\infty} |f[n]|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N} |f[n]|^2$$

Find energy in the even conjugate part of the signal.

$$x[n] = \left\{ -4 - j5 \quad \underset{\uparrow}{1+j2} \quad 4 \right\}$$

$$x_{ec}[n] = \frac{x[n] + x^*[-n]}{2}$$

$$= \left\{ -2.5j \quad \underset{\uparrow}{1} \quad 2.5j \right\}$$

$$\therefore E_{ec} = 1 + (2.5)^2 + (2.5)^2$$
$$= 1 + 6.25 + 6.25$$
$$= 13.5 \text{ J}$$

$$\int_{-\infty}^{\infty} f^2(t)\, dt = \int_{-\infty}^{\infty} [f_e(t) + f_o(t)]^2 dt$$

$$= \int_{-\infty}^{\infty} f_e^2(t)\, dt + \int_{-\infty}^{\infty} f_o^2(t)\, dt + 2\int_{-\infty}^{\infty} f_e(t) f_o(t) \quad \text{odd sign}$$

$$= E_e + E_o + 0$$

$$\boxed{E = E_e + E_o}$$

Causal or Non Causal signal

↳ the signal which not start before zero

↳ If signal start before zero then it is a non causal signal. (all periodic signals are non causal)

<u>Deterministic or Random signal</u>:

* A signal which can be defined by well defined mathematical expression, it is called as deterministic signal.
* A signal for which we can't give a well defined mathematical expression is defined as a random signal.

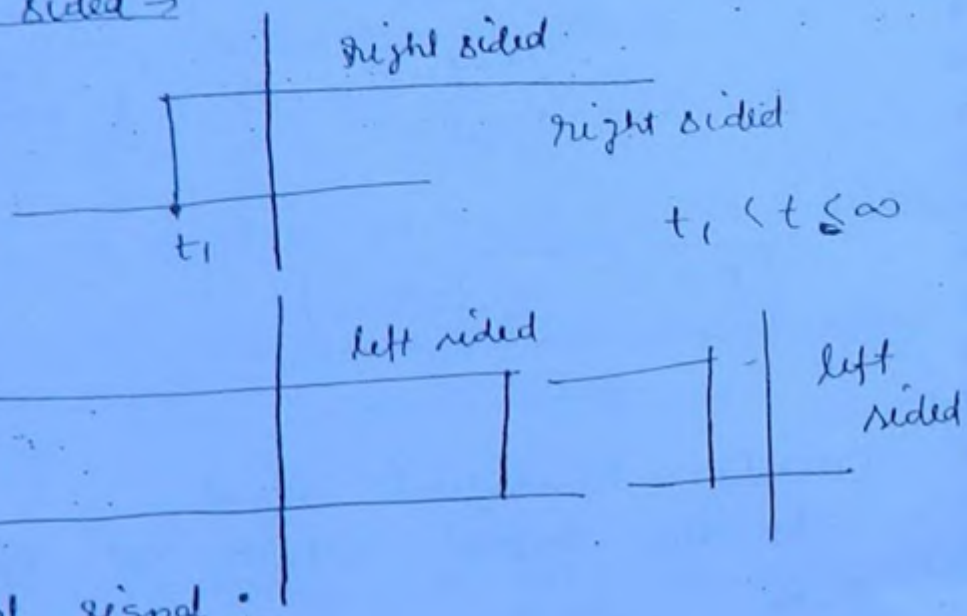
Bounded or unbounded signal →

* If the amplitude of signal have some finite boundaries for all values of time it is called as bounded signal

— $|f(t)| < \infty$
$|f(t)| < M \rightarrow$ finite
} for all t, bounded signal

if signal value become infinite for any value of time, it is called as unbounded signal

Right sided or Left sided →



Analog or digital signal :

* A signal which can assume infinite no. of values for its amplitude is defined as analog signal.
* If a signal is allowed only to assume finite no. of amplitude then the corresponding signal is a digital signal. A digital signal is that signal which ~~disc~~ may discrete in both on time axis & amplitude axis.

$f(t)\,u(-t) = 0$

$f(t)$

Match the following: (31)

List - I expression of $f(t)$	List - II nature of $f(t)$
A. $f(t)[1-u(4+t)] = 0$	(i) Increasing exponential
B. $f(t) + K\dfrac{df(t)}{dt} = 0$ $\{(K = +ve)$	(ii) Causal signal
	(iii) decreasing exponential
C. $f(t) + K\dfrac{d^2f(t)}{dt^2} = 0$	(iv) Sinusoidal
	(v) Impulse
D. $f(t)[g(t) - g(0)] = 0$ arbitrary $g(t)$	

$A \rightarrow f(t) = f(t)\,u(t)$

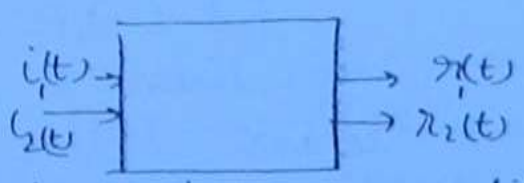
$t < 0$

$f(t) = 0$

$f(t)\,g(t) = f(t)\,g(0)$

$$\left\{\begin{array}{l} C \rightarrow (iv) \\ B \rightarrow (iii), \\ A \rightarrow (ii) \\ D \rightarrow (v) \end{array}\right\}$$

System:



(*) A system is a quantity which maps a set of i/p signals to a set of o/p signals

we can understand a system by

(i) i/p – o/p relationship (or represent)

(ii) Physical composition → (NF, ECD, AE, DE)

(iii) differential equation or difference equations.

(iv) Unit impulse response $h(t)$, $h[n]$

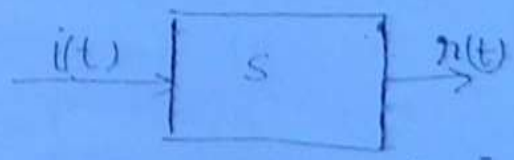
(v). Transfer function $H(\omega)$, $H(S)$, $H(z)$

(vi) State variable

$V = L\dfrac{di}{dt}$

$i = C\dfrac{dv}{dt}$

our case consider



$i(t) \xrightarrow{S} r(t)$

Linear system or Nonlinear system.

$i(t) \xrightarrow{S} r(t)$ where a is a real or imaginary quantity a constant [like 2 or 2j]

$a\,i(t) \xrightarrow{S} a\,r(t)$ homogenity principle

$i_1(t) \xrightarrow{S} r_1(t)$

$i_2(t) \xrightarrow{S} r_2(t)$

$i_1(t) + i_2(t) \xrightarrow{S} r_1(t) + r_2(t)$ superposition principle or additivity principle

$a\,i_1(t) + b\,i_2(t) \xrightarrow{S} a\,r_1(t) + b\,r_2(t)$

Linearty principle.

(*) A system satisfying both homogenity & superposition principle then it is said to linear system.
If it can not satisfy one of these principle or both, it is defined as non linear system.

$r(t) = 2i(t) + 3 \rightarrow$ not linear

$r(t) = \log i(t) \rightarrow$ not linear

$r(t) = i^2(t) \rightarrow$ non linear

$r(t) = t\,i(t) \rightarrow$ linear

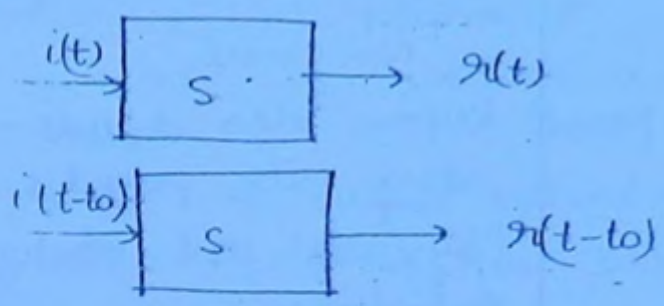
$r(t) = \sin t\; i(t) \rightarrow$ linear

$r(t) = \int_{-\infty}^{t} i(t)\,dz \rightarrow$ linear

$r(t) = \int_{-5}^{5} i(t)\,dz \rightarrow$ linear

$r(t) =$ real part of $\{i(t)\}$ not linear.
it does not hold homogenity for an $a = jb$ (real)
i.e. $a\,i(t)$ will not be $a\,r(t)$ non linear

time varient or time invarient :→



time invarient system.

* The system is defined as time invarient system if the response is delayed by the same amount as delay given to the system.

19th Oct 10

* If response to $i(t-t_0)$ is not equal to $r(t-t_0)$, system called as time varient system

$r(t-t_0) \rightarrow i(-t+t_0)$

$i(-t)$

$r_1[-t-t_0]$

$\downarrow$

$\rightarrow i'(t) \rightarrow r'(t) \rightarrow i'(-t)$

$i(-t-t_0)$

$\downarrow$

$\neq$

$r(t-t_0)$

time varient system

$r(t) = 2\,i(t) + 3 \rightarrow$ time invarient

$r(t) = \log i(t) \rightarrow$ time invarient

$r(t) = i^2(t) \rightarrow$ T.I.

$t\,i(t) \rightarrow$ T.V

$\sin t \cdot i(t) \rightarrow$ T.V

$= \int_{-\infty}^{t} i(\tau)\,d\tau \rightarrow$ T.I

$r(t) = \int_{-5}^{5}$

$r(t) = i$

⊛ Causal system & Non Causal system →

Cause → Causal

Causal	Non Causal
(i) response depends upon present & past i/ps	Response also depends on future along with present & past i/ps
(ii) Physically realizable	Physically not realizable
(iii) Nonanticipatory	Anticipatory

* Non causal system also become physically realizable when the data which is being operated upon or the i/p data is recorded data. But by default we consider data to be real time data and hence only causal systems are physically realizable systems.

	$i(t)$	$i(t-t_0)$	$i(t+t_0)$
$i=0$	$i(0)$	$i(-t_0)$	$i(t_0)$

$$r(t) = f[i(t), i(t-t_0)]$$

$$r(t) = f[i(t-t_0)]$$

$$t_0 \geq 0$$

* $r(t) = i(-t)$

non causal

$r(1) = i(-1)$

$r(-1) = i(1) \rightarrow$ depend future i/p

$r(t) = i(t) + i(t-2) + i(t-4) \rightarrow$ Causal system

$r(t) = i(2t) \rightarrow$ non causal system

$r(1/2) = i(1)$

$r(t) = i(1/2\, t)$ — non causal

$r(-1) = i(-1/2) \rightarrow$ future i/p

(*) $r(t) = i(at) \longrightarrow$ always non causal

$r(t) = i^2(t) \rightarrow$ causal system

$r(t) = i(t^2)$ $i(t^2) \rightarrow$ non causal

$r(t) = i(\sin t) \rightarrow$ non causal

$r(\pi/2) = i(1)$

$r(-\pi/2) = i(-1) \longrightarrow$ future i/p

$r(\pi) = i(\sin \pi) = i(0)$

$r(-\pi) = i(0)$

* Systems can be static or dynamic

If there is no arrangement for memory in an electrical system, it is called as static system

If there is arrangement of memory in electrical system, it is called as dynamic system

$r(t) = i^2(t)$ — static system

or $\log i(t)$, $t\, i(t)$

$\sin t\, i(t)$

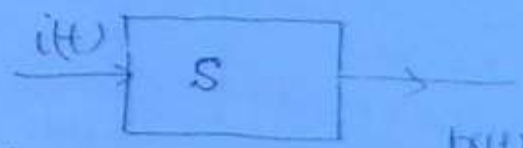
$2\, i(t) + 3$

$A \cdot i(t)$ } - Linear, static, time invariant

* For a static system to be linear & time invariant only way the response can be related to i/p is $r(t) = A\, i(t)$

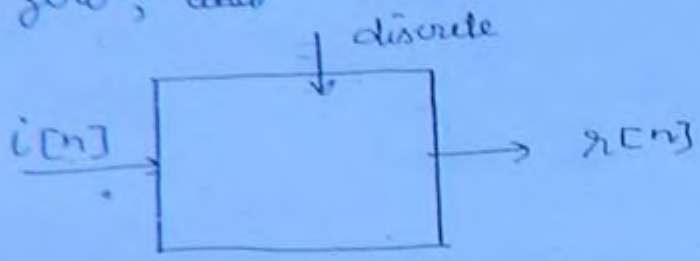
Stable or unstable system :→

BIBO stability



$|i(t)| < M$ $|r(t)| \leq N$

* A system can become noninvertible in following cases
(i) if more than one i/p to system is generating the same response for the system.
(ii) if the response of the system to a nonzero i/p is zero.



36

* Verify whether the following discrete time system is linear, time invarient, causal, static stable, invertible.

$$r[n] = \sum_{k=-\infty}^{n} i[k] \rightarrow \text{accumulator}$$

linear, time invarient, causal, dynamic, unstable, invertible

$i'[n] = i[n-n_0]$

$i'[k]$

$i[k-n_0]$

$i[n] = i[n-n_0]$

$\sum_{k=-\infty}^{n-n_0} i[k] \rightarrow i'[k]$

$\sum_{-\infty}^{n} i[k-n_0]$

$$r'[n] = r[n] - r[n-1]$$
$$= i[n]$$

Q. Find whether system defined as
$$r[n] = i[n] - i[n-1]$$; check for L, T, C, S, I.

this system → linear, time invarient, causal, stable

Invertible system because we can select a system having response $r'[n] = \sum_{k=-\infty}^{n} r[k] = \sum_{k=-\infty}^{n} \{i[k] - i[k-1]\}$

$\{i[n-2] + i[n-1] + i[n]\} = i[n]$

Q. If P → defines linearity
Q → defines Time invarient
R → Causality
S → Stability

(37)

A discrete time system defined by i/p o/p relationship.

$y[n] = x[n] \quad n > 0$

$0 \quad n = 0$

$x[n+1] \quad n < 0$

where $x[n]$ and $y[n]$, i/p and o/p of the system.

system is

(i) P, Q, R, S

(ii) P, Q, S but Not R

✓ (iii) P, S but Not Q, R

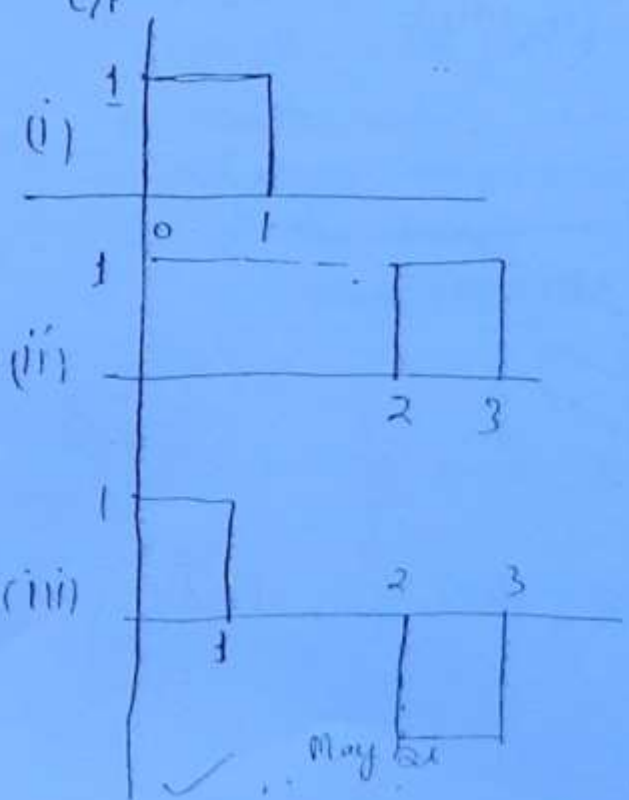
(iv) P but no Q, R, S.

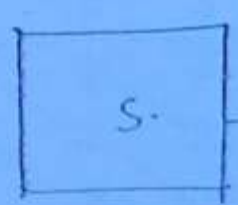
$y[n] = u[n-1]\, x[n] + u[n+1]\, x[n+1]$

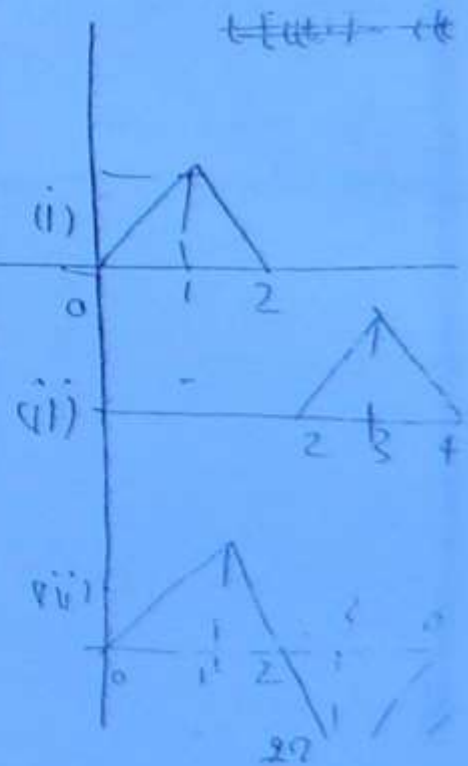
Linear, Not causal,

Stable, time varient

Q. A system S has the following for the considered i/p







May 2e

$\boxed{\text{length of resultant signal } l \text{ always} = L_1 + L_2 - 1}$

lower end of resultant signal $= \underline{(n_1 + n_2)}$

38

$n_1 \rightarrow$ Lower end of $f[n]$

$n_2 \rightarrow$ Lower end of $h[n]$

when we convolve $f[n]$ & $h[n]$

A_1 summation of all sample of $f[n] = \sum_{K=-\infty}^{\infty} f[K]$

A_2 " " " of $h[n] = \sum_{K=-\infty}^{\infty} h[K]$

$$A_1 A_2 = \sum_{K=-\infty}^{\infty} y[K] \qquad y[n] = f[n] * h[n]$$

$=$

$n_2 \rightarrow$ upper end of $f[n]$

$n_3 \rightarrow$ " " " $h[n]$

upper end of $y[n] = f[n] \circledast h[n]$

$= (n_2 + n_3)$

	$h[n]$			
	4	3	2	1
1	4	3	2	1
2	8	6	4	2
3	12	9	6	3
4	16	12	8	4

$$f[n] = \left\{ \overset{n=-1}{4}, \underset{\uparrow}{11}, 20, 30, 20, 11, \overset{n=5}{4} \right\}$$

* whenever two discrete time signals are convolve

(i) ~~Point~~ Resultant of convolution will have a length which is equal to sum of individual lengths of the signal being convolved -1. (minus 1) (39)

$$l = L_1 + L_2 - 1$$

(ii). Resultant of convolution will have extends which is equals to some of individual extends of the signal being convolved

(iii) Some of the sample values is resultant of convolution value is same as the product of the some of the sample values of individuals signals being convolved

if $\boxed{h[n] = u[n]}$ not possible to follow this procedure.

(*) The above method is suitable when the no of samples in the i/p signal and the impulse response are finite in no.

Q. Two discrete time signals $x[n]$ & $h[n]$ each of lengths 3 & 5 are convolved The maximum possible sample value of $x[n]$ is L, max^m possible ~~value~~ sample value is K for $h[n]$

What is maximum value of the some of all the sample value in resultant convolve signal

~~Ans (15K)~~

$$x[n] = \{L, L, L\}$$

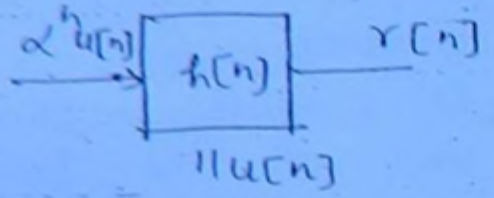
$$h[n] = \{K, K, K, K, K\}$$

$$\sum_{n=-\infty}^{\infty} y[n]_{max} \text{ ~~for comparison~~} = 3L \times 5K = 15LK$$

impulse response i/p

$$\sum_{K=-\infty}^{\infty} r[K] = \sum_{K=-\infty}^{\infty} h[K] \cdot \sum_{K=-\infty}^{\infty} x[K]$$ (48)

$$\boxed{\sum_{K=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x[m]\, h[K-m] = \sum_{K=-\infty}^{\infty} h[K] \cdot \sum_{K=-\infty}^{\infty} x[K]}$$



$$r[n] = \alpha^n [u[n]] \otimes u[n]$$

$$r[n] = \sum_{K=-\infty}^{\infty} \alpha^K u[K]\, u[n-K]$$

$$= \left[\sum_{K=0}^{\infty} \alpha^K u[K]\, u[n-K] \right] \quad \cancel{u[n-K]}$$

$$r[n] = \left[u[n] + \alpha u[n-1] + \alpha^2 u[n-2] + \cdots \alpha^r u[n-r] + \cdots \right]$$

$$= \begin{cases} 0 & n < 0 \\ \left(\dfrac{\alpha^{n+1}-1}{\alpha-1}\right) & n \geq 0 \end{cases}$$

$n=0$ 1

$n=1$ $(1+\alpha)$

$n=2$ $1+\alpha+\alpha^2$

$n=r \rightarrow 1+\alpha+\alpha^2+\cdots\alpha^r$

$$= \frac{1[\alpha^{r+1}-1]}{\alpha-1}$$

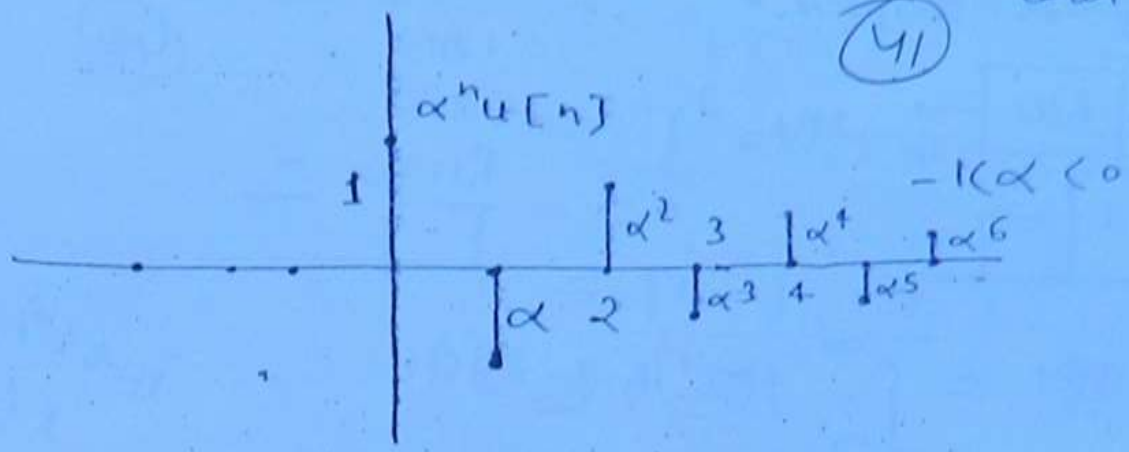
$$r[n] = \left(\frac{\alpha^{n+1}-1}{\alpha-1}\right) u[n]$$

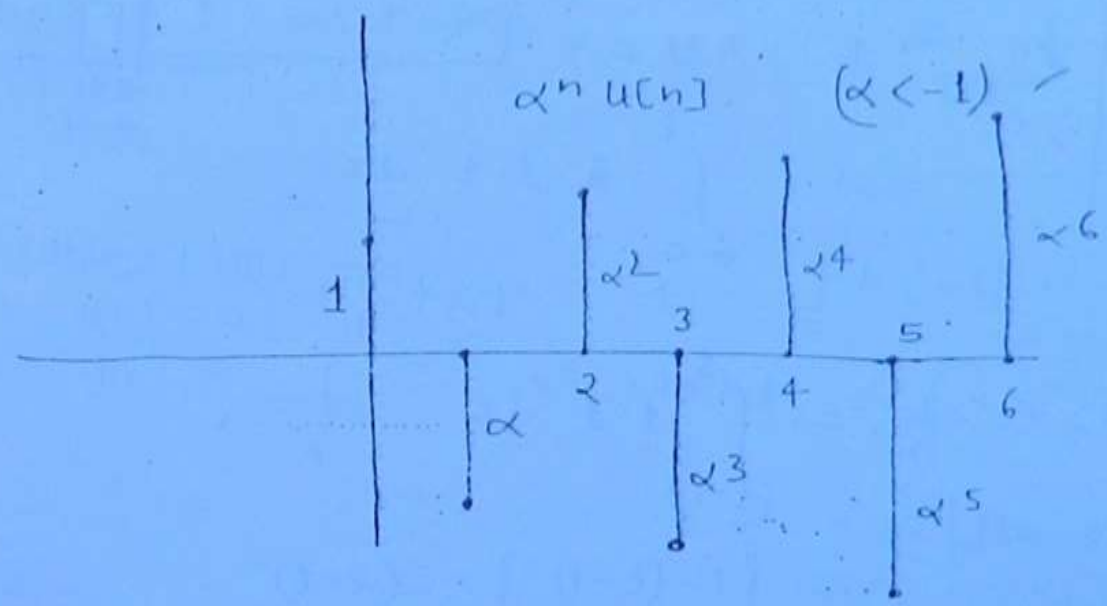
sketch the graphs $\alpha^n u[n]$ when $0<\alpha\leq 1$

when $-1<\alpha<0$

$\alpha<-1$

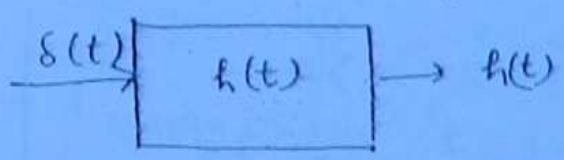
41





20th oct 10

$$f(t) = \int_{-\infty}^{\infty} f(\tau)\,\delta(t-\tau)\,d\tau$$



for any i/p $f(t)$ response $r(t)$

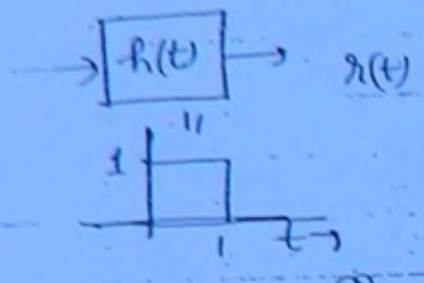
$$r(t) = \int_{-\infty}^{\infty} f(\tau)\,h(t-\tau)\,d\tau$$ → convolution integral

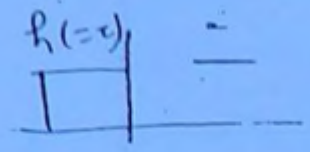
$$\Downarrow$$

$$f(t) \otimes h(t)$$

Q. Find the response of a continuous time LTI system with impulse response.

(42)

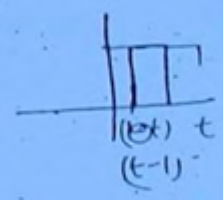




$$r(t) = \int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau$$

$h(-t+\tau)$

$h(-\tau)$



for $t < 0$ $\Rightarrow$ $r(t) = 0$

for $t \geq 0$.

$$= \int_0^t 1 \cdot 1 \, d\tau$$

$$= t \qquad 1 \geq t \geq 0$$

for $2 \geq t > 1$ $= \int_{(t-1)}^{1} 1 \cdot 1 \, d\tau$

$$= [1 - (t-1)] = (2-t)$$

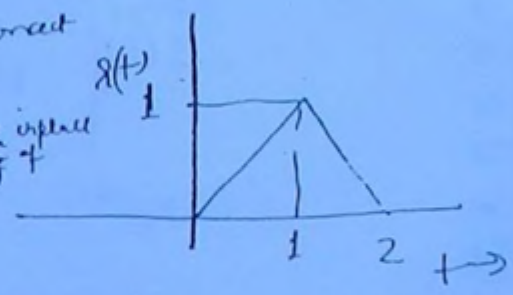
for $t > 2$

$r(t) = 0$

$$r(t) = \begin{cases} t, & 1 \geq t \geq 0 \text{ or } > 0 \\ (2-t), & 2 \geq t > 1 \\ 0, & t > 2 \\ 0, & t < 0 \end{cases}$$

both correct

or $t < 2$ input $(t \leq 2)$



Repeat the above problem if the $f(t) = u(t)$ and the $h(t) = u(t)$

$r(t) = 0 \qquad t < 0$

$r(t) = t \qquad t \geq 0$ Ans

Q. Repeat the problem if $f(t) = u(t) \cdot e^{-t}$, $h(t) = u(t)$

43

$r(t) = 0 \quad t < 0$

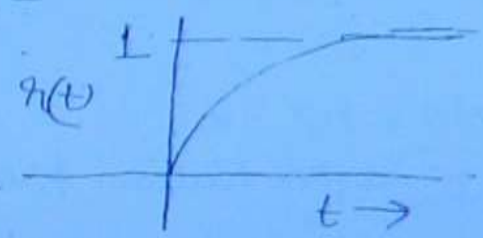
$r(t) = \int_0^t e^{-t} \cdot 1 \, dt \quad t > 0$

$= -\left[e^{-t}\right]_0^t$

$r(t) = (1 - e^{-t}) \quad t > 0$

$0 \quad t = 0$

$\boxed{r(t) \to 1 \text{ as } t \to \infty}$



Q. $e^{-t} u(t) \otimes t\, u(t) \rightarrow$

$t < 0 \quad r(t) = 0$

$t > 0 \quad r(t) = \int_0^t e^{-\tau} (-\tau + t) \, d\tau = -\int_0^t \tau e^{-\tau} d\tau + \int_0^t t e^{-\tau} d\tau$

$= -\left[(1 - e^{-t}) - t e^{-t}\right] + t\left[1 - e^{-t}\right]$

$r(t) = t e^{-t} + (t - 1)(1 - e^{-t}) \quad t \geq 0$

Q. $e^{-t} u(t) \otimes e^{-t} u(t)$

$t < 0 \quad r(t) = 0$

$t > 0 \quad r(t) = \int_0^t e^{-\tau} e^{\tau - t} d\tau$

$r(t) = t e^{-t} \quad t \geq 0$

$e^{\tau} u(-\tau)$

$e^{\tau - t}$

Q. $e^{at} u(t) \otimes e^{-bt} u(t) =$

$t < 0 \quad r(t) = 0$

for $t > 0 \quad = \int_0^t e^{a\tau} e^{b(\tau - t)} d\tau$

$t \geq 0 \quad r(t) = \frac{e^{at} - e^{-bt}}{\ldots}$

$$r(t) = f(t) \otimes h(t) = \int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau$$

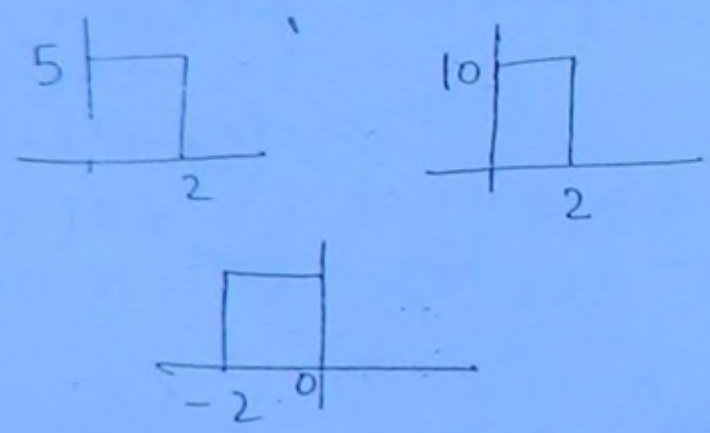
$$\frac{d}{dt} r = \frac{d}{dt}\left[f(t) \otimes h(t) \right] = \int_{-\infty}^{\infty} f(\tau) \frac{d}{dt} h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} f(\tau)\, h'(t-\tau)\, d\tau$$

$$= f(t) \otimes h'(t)$$

differentiation property $\boxed{\frac{dr}{dt} = f(t) \otimes \frac{dh}{dt}}$

Calculate the convolution of following two pulses



$r(t) = 0 \quad t < 0$

$2 \geq t > 0$

$$r(t) = \int_0^t 5 \cdot \times 10\, dt = 50t$$

$t = 2 \quad r(t) = 100$

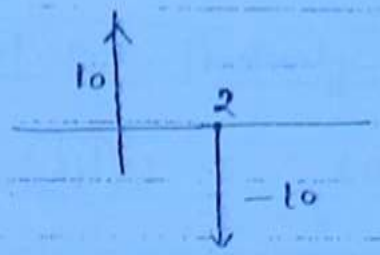
$4 > t > 2$

$$\int_{t-2}^{2} 5 \times 10\, dt$$

$$= 50[4-t]$$

$4 \geq t > 2$

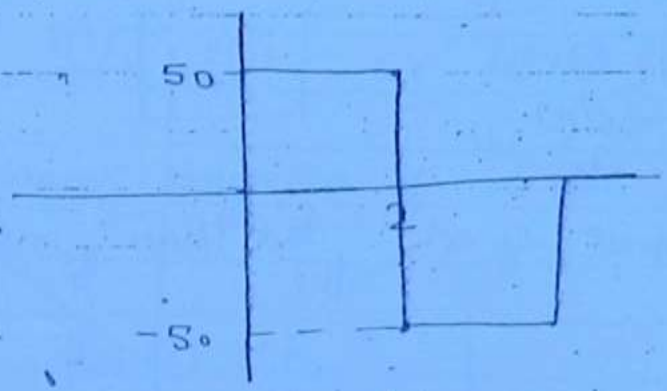
$0 \quad t \geq 4$

$\frac{dh}{dt} =$ 

$\therefore \; = 10[\delta(t) - \delta(t-2)]$

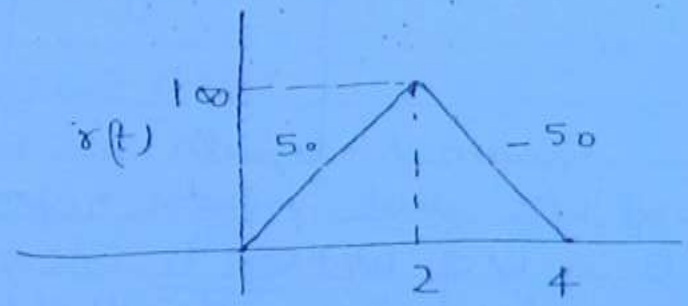
45

$$f(t) \circledast \frac{dh}{dt} = 10\, f(t) - 10\, f(t-2)$$

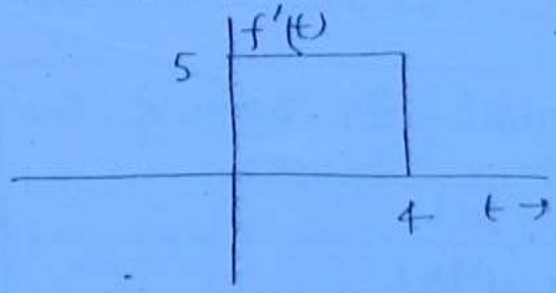
$\frac{dr}{dt}$ 

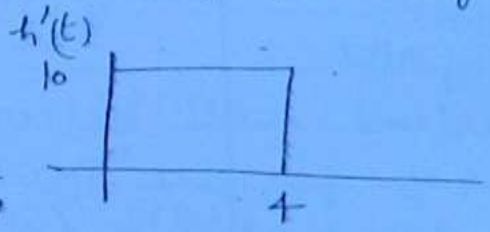
$\Rightarrow \; 50t + (-100)$

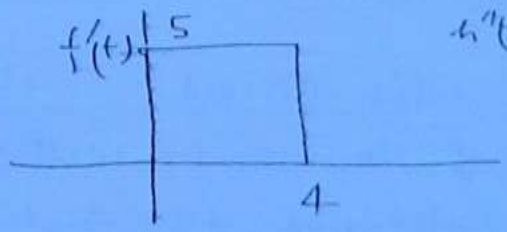
$(-50t)$

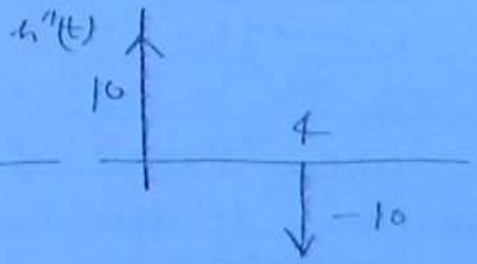
$\int_{-\infty}^{t} \frac{dr}{dt}\, dt =$ 

Q. Calculate the convolution of following signal



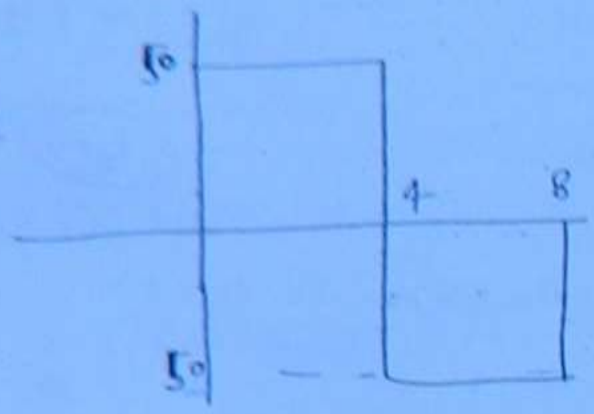




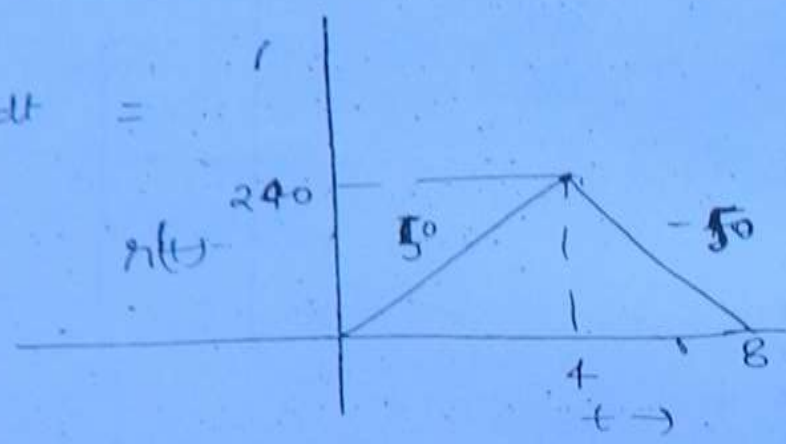


$$f'(t) \circledast h''(t) = 10\, f'(t) - 10\, f'(t-4)$$

$\frac{dr'(t)}{dt}$



$$\therefore r'(t) = \int_{-\infty}^{t} \frac{dr'(t)}{dt}\, dt =$$



* Width of resultant signal will be always some of widths of i/p and impulse respons $h(t)$

i.e. $w = w_1 + w_2$

w → width of $r(t)$; w_1 → width of $f(t)$; w_2 → width of $h(t)$

(*) Lower extend will be equal to some of lower extends of $f(t)$ and $h(t)$

$l = l_1 + l_2$

l → lower end of $r(t)$; l_1 → lower end of $f(t)$; l_2 → lower end of $h(t)$

Similarly upper extend of resultant will be equal to some of upper extends of $f(t)$ & $h_1(t)$

$$\boxed{u = u_1 + u_2}$$

Commutative Property :→

$$f(t) \otimes h(t) = h(t) \otimes f(t)$$

(47)

$f(t) \rightarrow \boxed{h(t)} \rightarrow f(t) \otimes h(t) = r(t)$

$\parallel$

Same

$h(t) \rightarrow \boxed{f(t)} \rightarrow h(t) \otimes f(t) = r(t)$

⊗ Interchanging positions of i/p and impulse response for an LTI system does not change response of the system

$$\boxed{\begin{aligned} \frac{dr}{dt} &= f(t) \otimes \frac{dh}{dt} \\ \frac{dr}{dt} &= h(t) \otimes \frac{df(t)}{dt} \end{aligned}}$$

$$\boxed{\begin{aligned} \frac{d^2r}{dt^2} &= h(t) \otimes \frac{d^2f(t)}{dt} \\ &= f(t) \otimes \frac{d^2h(t)}{dt^2} \\ &= \frac{df(t)}{dt} \otimes \frac{dh(t)}{dt} \\ &= \frac{dh(t)}{dt} \otimes \frac{df(t)}{dt} \end{aligned}}$$

$$\frac{d^m r}{dt^m} = h(t) \otimes \frac{d^m f(t)}{dt^m} = \frac{d^m h(t)}{dt^m} \otimes f(t)$$

$$= \frac{d^{m-1} h(t)}{dt^{m-1}} \otimes \frac{df(t)}{dt}$$

$$= \frac{d^n f(t)}{dt^n} \otimes \frac{d^p h(t)}{dt^p} \qquad = \frac{d^{m-2}}{dt^{m-2}} h(t) \otimes \frac{d^2 f}{dt^2}$$

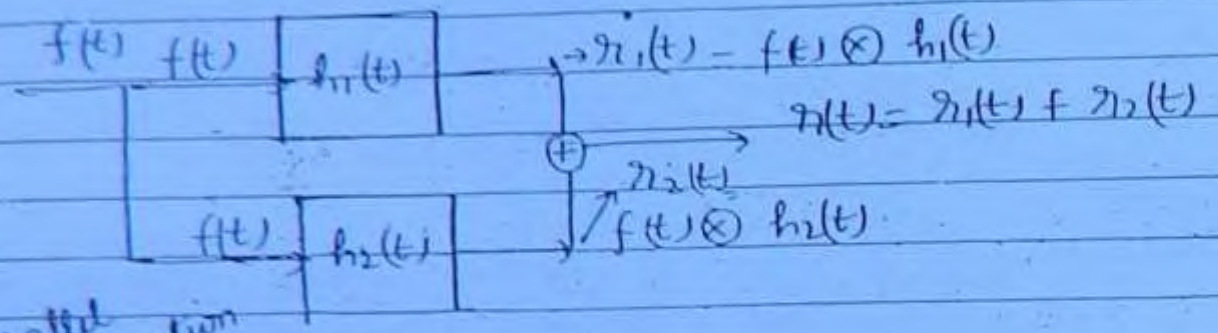
where $\boxed{(n+p) = m}$

$$= d^{m-1} f(t) \otimes \frac{dh}{\ldots}$$

$a \times [b + c] = a \times b + a \times c$

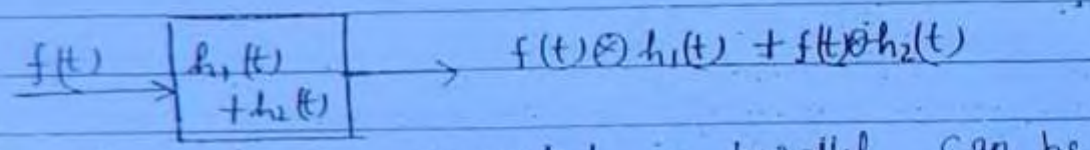
distributive property →

$$h(t) \otimes [f_1(t) + f_2(t)] = h(t) \otimes f_1(t) + h(t) \otimes f_2(t)$$



Parallel combination

⇕

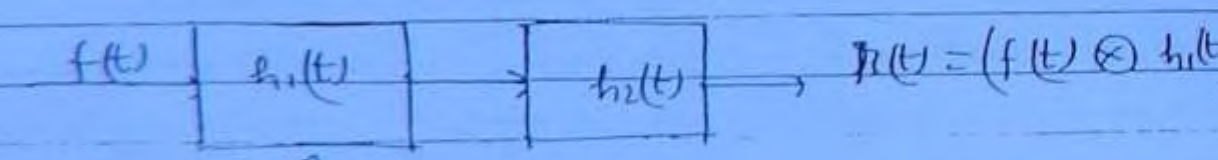


⊗ Any no of systems connected in parallel can be combined into a single system whose impulse response is some of all the individual impulse responses.

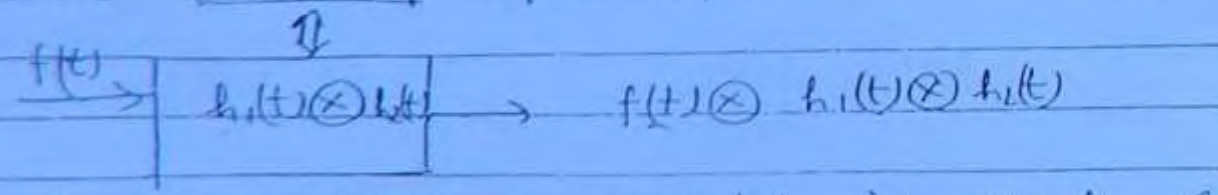
Associative Property :→

$f(t)$, $h_1(t)$, $h_2(t)$

$$f(t) \otimes h_1(t) \otimes h_2(t) = [f(t) \otimes h_1(t)] \otimes h_2(t)$$



⇕



⊗ Any no of systems connected in cascade can be combined into a single system whose impulse response is convolution of all the individual impulse responses.

