SOLUTION & ANSWER FOR KCET-2009
VERSION – A-2

[MATHEMATICS]

1. $\int \operatorname{cosec}(x-a)\operatorname{cosec}x\,dx =$

Ans: $\frac{1}{\sin a}\log[\sin(x-a)\operatorname{cosec}x]+C$

Sol. : sin[x – (x –a) = sinxcos(x-a) – cosx sin(x – a)

$\therefore \int \operatorname{cosec}(x-a)\operatorname{cosec}x\,dx =$

$\int \frac{dx}{\sin a \sin(x-a)} =$

$\frac{1}{\sin a}\int [\cot(x-a)-\cot x]dx$

$= \frac{1}{\sin a}\log\left[\frac{\sin(x-a)}{\sin x}\right]+C$

$= \frac{1}{\sin a}\log[\sin(x-a)\operatorname{cosec}x]+C$

2. If f(x) = $\int_{-1}^{x} |t|dt$, then …..

Ans: $\frac{1}{2}(1+x^2)$

Sol. : f(x) = $\int_{-1}^{x}|t|dt = \int_{-1}^{0}|t|dt + \int_{0}^{x}|t|dt$

$= \int_{-1}^{0} tdt + \int_{0}^{x} tdt$

$= \frac{1}{2}+\frac{x^2}{2} = \frac{1}{2}(1+x^2)$

3. $\int_{1}^{3} \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx =$

Ans: 1.

Sol.: $\int_{a}^{b} \frac{f(x)dx}{f(x)+f(a+b-x)} = \frac{b-a}{2}$

Here a = 1, b = 3

so answer = $\frac{3-1}{2} = 1$

4. The area bounded between the parabola …..

Ans: 9 sq. units.

Sol: Area = $\int_{0}^{1} 2\sqrt{4x}dx + \int_{1}^{4}\left[\sqrt{4x}-(2x-4)\right]dx$

$= \frac{8}{3}+\frac{28}{3}-3$

= 9 sq. units.

5. The differential equation of the family of ….

Ans: $y^2 = x^2 + 2xy\frac{dy}{dx}$

Sol: The family is $x^2 + y^2 - \lambda x = 0$

$\therefore 2x + 2yy' = \lambda$

$\therefore x^2 + y^2 = (2x + 2yy')x$

$y^2 = x^2 + 2xy\frac{dy}{dx}$

6. A population grows at the rate of 10% of the …

Ans: 10log2years

Sol: If p is the population at any time,

$\frac{dp}{dt} = \frac{1}{10}p$

$\frac{dp}{p} = \frac{dt}{10}$

$\log p = c + \frac{t}{10}$

$\log\left(\frac{p}{p_0}\right) = \frac{t}{10}$,

where P_0 = initial population

given $\frac{p}{p_0} = 2$

$\therefore$ t = 10log2years

7. On the set of all natural numbers N, which…

Ans: $a * b = a + 3b$

Sol: obviously, only $a * b = a + 3b$ results in closure property.

8. If $\int_{0}^{1} f(x)dx = 5$, then the value of ….

Ans:

Sol: Question is incomplete

9. If ax + by = 1, where a, b, x and y are ….

Ans: (x, y) = 1

Sol: It is not possible that (x, y) = 1.

10. The digit in the unit place of the number…..

Ans: 9.

Sol: 2009! ends in zero.
Last digit of 3^{7886} is same as last digit of 3^2 since last digit repeats in steps of 4.

11. If $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, then

Ans: in A. P.

Sol: Observe that $R_1 + R_3 = 2R_2$ for the first two columns.
since determinant is zero,
same must be true for column 3.
∴ a + c = 2b.
∴ a, b, c are in A.P.

12. The value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ …

Ans: 0.

Sol: $\log_x y = \frac{\log y}{\log x}$

∴ determinant

$$= \frac{1}{\log x \log y \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix} = 0$$

13. If A = $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$ ….

Ans: 81.

Sol: $|A| = 9$

$\therefore |\text{adj}A| = |A|^2 = 81.$

14. If A and B are square matrices of ….

Ans: B^2

Sol: $(ABA^{-1})^2 = (ABA^{-1})$

15. If $\vec{a}.\vec{b} = -|\vec{a}||\vec{b}|$, then the …..

Ans: 180°.

Sol: $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow \cos\theta = -1$
$\Rightarrow \theta = 180°$.

16. If $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{O}$, then ….

Ans: $3(\vec{c}\times\vec{a})$ and $6(\vec{b}\times\vec{c})$

Sol: $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{O}$

$\vec{a}\times(\vec{a} + 2\vec{b} + 3\vec{c}) = 2\vec{a}\times\vec{b} - 3\vec{c}\times\vec{a} = 0$

$\vec{b}\times(\vec{a} + 2\vec{b} + 3\vec{c}) = -\vec{a}\times\vec{b} + 3\vec{b}\times\vec{c} = 0$

$\vec{c}\times(\vec{a} + 2\vec{b} + 3\vec{c}) = \vec{c}\times\vec{a} - 2\vec{b}\times\vec{c} = 0$

adding,

$\vec{a}\times\vec{b} - 2\vec{c}\times\vec{a} + \vec{b}\times\vec{c} = 0$

$\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} = 3\vec{c}\times\vec{a}$

Also, we can verify that

$6(\vec{b}\times\vec{c})$ is also true.

17. If the volume of the parallelepiped …..

Ans: 80.

Sol: $[\vec{b}+\vec{c}, \vec{c}+\vec{a}, \vec{a}+\vec{b}] = 2[\vec{a}, \vec{b}, \vec{c}]$
= 2 × 40
= 80.

18. In the group G = {0, 1, 2, …….} …..

Ans: 3.

Sol: $3^{-1} = 3$
$(2 \oplus_6 3^{-1} \oplus 4)^{-1} = 3^{-1} = 3$

19. Which one of the following …..

Ans: Fourth roots of unity form an additive abelian group.

Sol: Obviously, fourth roots of unity form an abelian group under multiplication and not under addition.

20. The number of sub groups …..

Ans: 2

Sol: Z_n will have only two sub-groups since it is a group of prime order.

21. The negation of …..

Ans: ~p∨(q∧r).

Sol: question is printed wrongly.
if it is p ∧ (q → ~ r), then the answer is ~p∨(q∧r).

22. If n = 2020, then ----

Ans: 1

Sol: $\log_n(2\times 3\times ....\times 2020) = \log_n n = 1$

23. If `n' is a positive integer, then $n^3 + 2n$ is ----

Ans: 3

Sol: $n(n^2 + 2) = n(n^2 - 1 + 3)$
$= n[(n-1)(n+1) + 3]$
$= n(n-1)(n+1) + 3n$
$= M(3)$.

24. On the set of integers Z, define f : Z → Z as ----

Ans: surjective but not injective

Sol: Obviously, f is surjective but not injective.

25. If α and β are the roots of $x^2 + x + 1 = 0$, ----

Ans: –1

Sol: $\alpha^{16} + \beta^{16} = \omega^{16} + (\omega^2)^{16}$
$\omega + \omega^2 = -1$.

26. The total number of terms in the expansion of $(x + y)^{100} + (x - y)^{100}$ ----

Ans: 51.

Sol: There are 101 terms in each expansion.
But even ordered terms will cancel.
After simplification, 51 terms will remain.

27. $\cot^{-1}(2.1^2) + \cot^{-1}(2.2^2)$ --------

Ans: $\frac{\pi}{4}$

Sol: nth term $= \cot^{-1} 2n^2$
$= \tan^{-1}\frac{1}{2n^2}$
$= \tan^{-1}\frac{2}{4n^2}$
$= \tan^{-1}\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}$
$= \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$
So, sum to n terms
$= \tan^{-1}(2n+1) - \tan^{-1} 1$
sum to $\infty = \frac{\pi}{4}$.

28. If `x' takes negative permissible value, ----

Ans: $-\cos^{-1}\sqrt{1-x^2}$

Sol: We have $-1 \le x \le 0$
$\therefore \sin^{-1}x$ is a negative acute angle
$\therefore \sin^{-1}x = -\cos^{-1}\sqrt{1-x^2}$

29. If $1 + \sin x + \sin^2 x + ...$ up to ∞ ----

Ans: $\frac{\pi}{3}, \frac{2\pi}{3}$

Sol: $\frac{1}{1-\sin x} = 4 + 2\sqrt{3} = \frac{4}{4-2\sqrt{3}}$
$1 - \sin x = 1 - \frac{\sqrt{3}}{2}$
$\Rightarrow \sin x = \frac{\sqrt{3}}{2}$
$\Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}$.

30. The complex number $\frac{1+2i}{1-i}$ -------

Ans: second quadrant

Sol: $\frac{(1+2i)(1+i)}{2} = \frac{-1}{2} + \frac{3}{2}i$

31. If P is the point in the Argand diagram corresponding to the complex number -------

Ans: $-1 + i\sqrt{3}$ or $1 - i\sqrt{3}$

Sol: P is $\sqrt{3} + i = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
Q_1 is $2\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right]$
$= 2\left(-\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)$
$= 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -1 + i\sqrt{3}$
Q_2 is $2\left[\cos\left(\frac{\pi}{6}-\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{6}-\frac{\pi}{2}\right)\right]$
$= 1 - i\sqrt{3}$.

32. The smallest positive integral value of `n' such that -----

Ans: 4

Sol: $\left[\frac{2\cos\frac{3\pi}{16}\left[i\frac{3\pi}{6}\right]}{2\cos\frac{3\pi n}{16}\, e^{-i}\frac{3\pi}{16}}\right]^n = e^{i}\frac{3\pi}{8}n$
$\cos\frac{3\pi n}{8} = 0 \Rightarrow \frac{3n\pi}{8} = 3\frac{\pi}{2}$
$\Rightarrow n = 4$.

33. Which one of the following is possible ---

Ans: $\tan\theta = 45$

Sol: $-\infty < \tan\theta < \infty$

34. If one side of a triangle is double the other and the angles opposite -----

Ans: right angled

Sol: $\frac{a}{\sin\theta} = \frac{2a}{\sin(\theta+60)} \Rightarrow$

$2\sin\theta = \sin(\theta + 60) \Rightarrow$

$\frac{3}{2}\sin\theta = \frac{\sqrt{3}}{2}\cos\theta \Rightarrow$

$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30° \Rightarrow$

$\theta + 60 = 90°$.

35. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2$ -----

Ans: 13.

Sol: $(1 - \sin 2x)^2 + 6(1 + \sin 2x)$

$+ 4\left(1 - \frac{3}{4}\sin^2 2x\right)$

$= 13$.

36. A cow is tied to a post by a rope. The cow moves along the -----

Ans: 35 metres

Sol: $s = r\theta$

$44 = r\frac{72\times\pi}{180} \Rightarrow r = 35$.

37. If $\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 2\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 2\theta \\ \sin^2\theta & \cos^2\theta & 4\sin 2\theta - 1 \end{vmatrix}$ ----

Ans: $\frac{1}{2}$

Sol: $\Delta = \begin{vmatrix} 2 & \cos^2\theta & 4\sin^2\theta \\ 2 & 1+\cos^2\theta & 4\sin 2\theta \\ 1 & \cos^2\theta & 4\sin 2\theta - 1 \end{vmatrix} = 0$

$C_1 \to C_1 + C_2$

$= \begin{vmatrix} 2 & \cos^2\theta & 4\sin 2\theta \\ 0 & 1 & 0 \\ 0 & \frac{1}{2}\cos^2\theta & 2\sin 2\theta - 1 \end{vmatrix} = 0$

$2(2\sin 2\theta - 1) = 0 \Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow$

$\cos 4\theta = 1 - 2\times\frac{1}{4} = \frac{1}{2}$.

38. The locus of the mid points of the chords of the circle -----

Ans: $x^2 + y^2 = 2$.

Sol: Mid point of the chord joining (2, 0), (0, 2) subtending 90° at the origin.
Equation of the locus is $x^2 + y^2 = 1^2 + 1^2 = 2$.

39. The length of the chord joining the points ($4\cos\theta$, $4\sin\theta$) -------

Ans: 4.

Sol: $4\sqrt{2 - 2[(\cos(\theta+60)\cos\theta + \sin(\theta+60)\sin\theta)]}$

$= 4\sqrt{2}\sqrt{1-\cos 60} = 4$.

40. The number of common tangents to the circles ----------

Ans: 3.

Sol: The circles touches externally. Hence there will be 3 common tangents.

41. The co-ordinates of the centre of the smallest circle -----

Ans: $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Sol: Back substitution

42. The length of the diameter of the circle which cuts -----

Ans: 4

Sol: $-g - f = c - 14$

$3g - 5f = c - 10$

$-2g + 3f = c - 27 \Rightarrow$

$g = -3, f = -4, c = 21$

Diameter $= 2\sqrt{9+16-21} = 4$.

43. For the parabola $y^2 = 4x$, the point P whose focal ------

Ans: (16, 8) or (16, –8)

Sol: Focus = (1, 0)
The only points distant 17 from (1, 0) are (16, 8) and (16, –8).

44. The angle between the tangents drawn to the parabola $y^3 = 12x$ from the ------

Ans: 90°

Sol: $y^2 = 12x$; $a = 3$
$x + 3 = 0$ is the direction.

(–3, 2) lies on the direction ⇒ the tangents are ⊥

∴ the angle between the tangents = 90°

45. The number of values of `c' such that the line ----

Ans: 2

Sol: $y = mx + c$

$c^2 = a^2m^2 + b^2$

$= 4(16) + 1$

$= 65$

$c = \pm\sqrt{65}$

There are two values for c.

46. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ ------

Ans: $x_1 + x_2 + x_3 + x_4 = 0$

Sol: $x^2 + y^2 = a^2$

$xy = c^2$

$x^{2++}\left(\frac{c^2}{x}\right)^2 = a^2$

$\Rightarrow x^4 - a^2x^2 + c^4 = 0$

⇒ sum of roots = 0

$\Rightarrow x_1 + x_2 + x_3 + x_4 = 0$

47. The foot of the perpendicular from the point (2, 4) -----

Ans: (1, 3)

Sol: The foot of the ⊥ from (2, 4) upon x + y = 4 is (h, k) and it is given by

$\frac{h-2}{1} = \frac{k-4}{1} = -\frac{2+4-4}{1+1}$

$h - 2 = -1$; $k - 4 = -1$

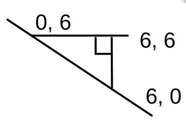
$h = 1, k = 3$

(1, 3) is the point required.

48. The vertices of a triangle are (6, 0), (0, 6) and 96, 6) …..

Ans: $\sqrt{2}$

Sol:



The given Δ is at right angled one.

Circumcentre is the mid point of the hypotenuse

∴ s= (3, 3)

a= (4, 4)

sa = $\sqrt{1+1} = \sqrt{2}$

49. The angle between the pair of lines ----

Ans: $\frac{\pi}{2}$

Sol: x^2 – coeff + y^2 – coeff = 0

⇒ Lines are ⊥

50. $\lim_{n\to\infty}$ -------

Ans: $\frac{-20}{7}$

Sol: $\lim_{n\to\infty} \frac{3.2^n.2 - 4.5^n.5}{5.2^n + 7.5^n}$

$\lim_{n\to\infty} \frac{6\left(\frac{2}{5}\right)^n - 20}{5\left(\frac{2}{5}\right)^n + 7}$

$= \frac{-20}{7} \left(\because \left(\frac{2}{5}\right)^n \to 0 \text{ as } n \to \infty\right)$

51. The function

$f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$

Ans: a + b

Sol: f(x) is continuous at x = 0

$\Rightarrow \lim_{x\to\infty} f(x) = f(0)$

$\therefore f(0) - \lim_{x\to\infty} \frac{\log(1+ax) - \log(1-bx)}{x}$

$= \lim_{x\to\infty}\left[\frac{\log(1+ax)}{ax}a - \frac{\log(1-bx)}{-bx}(-b)\right]$

$= a - (-b) = a + b$

52. If $f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}$ ----

Ans: $n(n-1)2^{n-2}$

Sol: $f(x) = (1 + x)^n$

$f'(x) = n(1 + x)^{n-1}$

$f''(x) = n(n-1)(1 + x)^{n-2}$

$f''(1) = n(n-1)2^{n-2}$

53. if $f(x) = \log_{x^2}(\log_e x)$, then ---

Ans: $\frac{1}{2e}$

Sol: $f(x) = \log_{x^2}(\log x)$

$= \frac{\log(\log x)}{2\log x}$

$f'(x) = \frac{1}{2}\frac{\log x\frac{1}{x\log x} - \frac{\log(\log x)}{x}}{(\log x)^2}$

$f'(e) = \frac{1}{2}\frac{\left[1.\frac{1}{e} - 0\right]}{1} = \frac{1}{2e}$.

54. If y = $\sin^n x \cos nx$, then -----

Ans: $n\sin^{n-1}x\{\cos(n+1)x\}$

Sol: y = $\sin^n x \cos nx$

$\frac{dy}{dx} = n\sin^{n-1}x\cos x\cos nx$

$+ \sin^n x.(-n\sin nx)$

$= n.\sin^{n-1}x\{\cos nx\cos x - \sin x\sin nx$

$= n\sin^{n-1}x\{\cos(n+1)x\}$

55. If f(x) = $\frac{g(x)+g(-x)}{2} + \frac{2}{[h(x)+h(-x)]-1}$ ----

Ans: 0

Sol: f(x) = $\frac{1}{2}[g(x)+g(-x)]+2[h(x)+h(-x)]$

f'(x) = $\frac{1}{2}[g'(x)-g'(-x)]+2[h'(x)-h'(-x)]$

f'(0) = 0

56. The tangent to a given curve y = f(x) is ----

Ans: $\frac{dx}{dy} = 0$

Sol: Conceptual

Tangent is parallel to x – axis

$\Rightarrow \frac{dy}{dx} = 0$

$\therefore$ Tangent || to y – axis

$\Rightarrow \frac{dx}{dy} = 0$

57. The minimum value of $27^{\cos 2x}\ 81^{\sin 2x}$ ---

Ans: $\frac{1}{243}$

Sol: Let y = $27^{\cos 2x}.\ 81^{\sin 2x}$

log y' = cos 2x log 27 + sin 2x log 81

Minimum of log y = $-\sqrt{(\log 27)^2 + (\log 81)^2}$

$= -\sqrt{3^2(\log 3^2 + 4^2(\log 3)^2)} = -5\log 3$

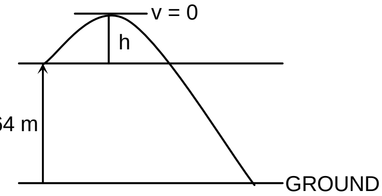
$= \log\left(\frac{1}{243}\right)$

$\therefore$ Minimum y = $\frac{1}{243}$

58. A stone is thrown vertically upwards from the top of a tower 64 metres ----

Ans: 100 m

Sol:



v = 0

$\Rightarrow \frac{dS}{dt} = 0$

$\Rightarrow$ 48 – 32 t = 0

$\Rightarrow t = \frac{3}{2}$

$\therefore (S)_{t=\frac{3}{2}}$ = 36 m (height attained from the tower)

$\therefore$ Height attained from the ground

= 36 + 64 = 100 m

59. The length of the subtangent at `t' on the curve --

Ans: a sint

Sol: y = a (1 – cos t)

x = a (t + sint)

$\frac{dy}{dt} = \frac{a\sin t}{a(1+\cos t)}$

$= \tan\frac{t}{2}$

sub tangent = $\frac{y}{y'} = \frac{a(1-\cos t)}{\tan\frac{t}{2}}$

60. $\int e^{\tan^{-1}}\left(1+\frac{x}{1+x^2}\right)dx$ ---

Ans: $x\,e^{\tan^{-1}x}.x + c$

Sol: Put x = tanθ

$I = \int e^{\theta}\left(1+\frac{\tan\theta}{\sec^2\theta}\right)\sec^2\theta\, d\theta$

$= \int e^{\theta}(\sec^2\theta + \tan\theta)d\theta$

$= e^{\theta}\tan\theta + c$

$= e^{\tan^{-1}x}.x + c$